

THE
PHILOSOPHICAL
TRANSACTIONS
AND
COLLECTIONS,

To the End of the Year 1700.

ABRIDG'D

AND

Dispos'd under GENERAL HEADS

In Three Volumes.

By JOHN LOWTHORP, M. A.
and F. R. S.

The Third Edition.

LONDON:

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Robinson, and J. Osborn.

M.D.CC.XXII

TO HIS
Royal Highness
THE
PRINCE,
LORD High ADMIRAL
OF
ENGLAND, &c.

SIR,
YOUR Royal Highness's Great Con-
descension to Subscribe the Statutes of
the Royal Society, as one of their Fel-
lows, Commands some Tribute from every
Mem-

The Epistle Dedicatory.

Member. This, SIR, I humbly hope, will in some measure excuse the Presumption of laying these Papers at Your Royal Highness's Feet. They have unavoidably, in this Abridgment, lost much of their Original Beauty: But the Order wherein the Remaining Substance is Disposed, will give Your Royal Highness a Nearer Prospect of the Course of those Studies you have the Goodness to Protect.

It was a Noble Design, Worthy of their Royal Founder, by Incorporating this Society, to Perpetuate a Succession of Useful Inventions: But the Discouraging Neglect of the Great, the Impetuous Contradictions of the Ignorant, and the Reproaches of the Unreasonable, have unhappily Retarded them in their Pursuit of those Great Ends. To Restore them therefore to their first Vigour, is a Glory reserv'd for Your Royal Highness; And already

The Epistle Dedicatory.

ready, SIR, we feel the Chearful Influence of a Returning Spring.

The Commands Your Royal Highness has given, for Publisbing, at Your own Expence, a most Magnificent Uranigtaphy, (far exceeding that of all the Arabian Princes, the Noble Tycho Brahe, and the Industrious Hevelius) cannot fail of surprizing Effects. All Art and Nature will exert their Powers upon this Occasion, to keep pace with Astronomy; particularly Navigation (being under Your Royal Highness's Immediate Care) will Industriously apply those Accurate Observations to all Nautical Purposes, and by some Familiar Method, Deliver the Anxious Seamen, from the Fatal Accidents that frequently attend their Mistaken Longitude.

Thus, SIR, the Munificence of the Prince, and the Vigilence of the Lord High Admiral,

The Epistle Dedicatory.

*ral, will be equally a Blessing to the Present, and
to all Future Ages,*

*May Your Royal Highness ever meet with
with Returns of Gratitude, Suitable to the Uni-
versal Beneficence. So Wisbes with great
Sincerity,*

May it please Your ROYAL HIGHNESS,

Your ROYAL HIGHNESS'S

Most Faithful, and

Most Obedient Servant,

John Lowthorp.

T H E

P R E F A C E.

THE Philosophical Transactions having met with General Applause and Encouragement for many Years, it would be a Needless Trouble to give any History of them : 'Tis enough to say, That many of the Discourses were Compos'd, and All of them Collected and Published, by Particular Members of the Royal Society. I shall therefore employ these very few Pages, only to acquaint the Reader with my own Conduct in this Abridgment of them.

When I first resolv'd upon this Undertaking, I had Two Sorts of Readers in View, whom I was desirous to serve ; Those who make use of Books for their private Instruction or Entertainment, and those who consult them in order to publish something of their own. To a Reader of the Former Class, I thought it sufficient to give him the Substance of so many curious Papers, in such Order as would best suit with the Course of those Studies that might Denominate him a General Scholar. But for the Sake of the Latter, I have, in the Margin, given the Title and Author of each Paper, and Directed to the Number and Page of the Transactions or Collections, where he may meet with the Original it self. To the former I design'd this Abridgment to be as Useful as the Volumes at large, and to serve the Latter, instead of a not inconvenient Repertorium : And in the Prosecution of this Design, I have generally confined my self to these Rules.

†

I. I

The PREFACE.

I. *I have not only Retain'd the Essential Parts of the Discourses, but I have kept in many Places to the very Words of their own Authors, (except where I was Forc'd to Vary them a little, to preserve the Connection :) For, I thought it very unwarrantable to Obtrude any thing of mine under the Name of another Person*

II. *But to Shorten the whole Work, wherever I found any Personal Addresses, Long and Unnecessary Excursions, or pompous Citations of Books, I have taken the Liberty to Suppress them, yet, I hope, without Injuring the Force of the Author's Reasoning.*

III. *I have Omitted all Accounts and Extracts of Books, which now, after so many Years Publication, seem almost Useless: Yet, to put the Readers in Mind of them, especially such as are about to Furnish or Enlarge their Libraries, I have added a Catalogue at the End of each Chapter, to which they chiefly belong; and I have also directed them to such Additions, Emendations, or Refutations, as ought to be consulted, when those Books fall under their Examination.*

IV. *I have also Omitted all Heads of Enquiries and Experiments simply propos'd, without further Prosecution; Believing that the Answers already given to many of them, and other Discourses upon the same, or the like Subjects, will sufficiently direct the Notice of an Inquisitive Reader.*

V. *The previous Calculations of Eclipses, Lunar Appulses, and Satellite Eclipses and Occultations; also Tide-Tables, and many other curious Papers of that kind; have long ago Out-liv'd the Reason of their Publication.*

VI. *All*

The PREFACE.

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VI. *All Simple Catalogues of Natural Curiosities (as of Shells, Minerals, Plants, Animals, &c.) without particular Descriptions of them, are little Instructive : And chiefly serve to enlarge the History of the Musæum, where they are Deposited ; which is no Part of the Design of these Volumes.*

VII. *I have commonly omitted such Papers as have been collected into Just Volumes by their own Authors. For this Reason, I have omitted some of those surprizing Microscopial Discoveries by the Famous M. Lecuwenhoeck : But I further confess, I was also less inclin'd to insert them here, because most of them Treat of Subjects not at all convenient (in my Opinion) for Common Readers.*

VIII. *But, to do all the Right I could to the Ingenious Authors of those Papers, which the Limits of this Abridgment obliged me to Omit, I have, at the End of each Chapter, Annexed their Titles, and sometimes a short Account of them.*

These are the Rules I have carefully observed through the whole Conduct of this Tedious Work : Wherein I have faithfully aim'd at the General Good of all sorts of Readers ; if I have fail'd in the Performance, 'tis for Want of Judgment to do it Better : But I am bold to say, That if a Kind Reception of this shall encourage a like Abridgment of the Foreign Philosophical Journals, in the Same or a Better Order, it will much Facilitate the many Discoveries still ready to Reward the Labours and Expences of all Industrious Promoters of Natural Knowledge.

A D-

ADVERTISEMENT
 Of the **BOOKSELLERS**
 To this Third **EDITION.**

THE *Philosophical Transactions*, since the Year 1700. having been abridg'd and dispos'd under General Heads, (after the same Method with this Work) by Mr. MOTTE, and by him published in Two Volumes: We thought it would tend to make the whole complete, if the several *Indices* of the Five Volumes were joined together; by which means, the Reader will have but one Trouble in seeking any Particular he has Occasion for. To this Purpose we have taken Care, that in the *Contents*, the several Tracts should be inserted and referr'd to under their proper Chapters; and that in the *Index* of Authors Names, and the General one of Matters, the whole should be plac'd in Alphabetical Order. At the Head of each *Index*, Directions are given for distinguishing the Matters occurring in these Three Volumes, from those in the Continuation.

N. B. In the ensuing *Contents*, those Tracts whose Titles are printed in Roman, and the Authors in the Italic Character, are to be found in the Continuation.

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THE
PHILOSOPHICAL
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ABRIDG'D

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M D C C X L I I

To the HONOURABLE
Sir *ISAAC NEWTON*, Kt.
PRESIDENT,

And to the
COUNCIL and FELLOWS
OF THE

Royal Society

OF
LONDON,

FOR THE
Advancement of *Natural Knowledge*,

THESE
Mathematical Papers,

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AND
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Are Most Humbly Dedicated

By *John Lowthorp*.

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And to the
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CHAP. I.
Geometry, Algebra, Arithmetick, Logarithmotechny.

I. 1.  UÆ antehac de Augmentis Artium Mathematicarum executus sum, eorum Summa huc fere redit; Quamdiu homines Voluntate, Ingenio, Adminiculis, & Otio ad hæc Studia requisitis destituantur, mirum non esse, si non majores in iis progressus faciant: Mihi itaque videri sequentium mediorum ope, Remedium huic Malo satis commode afferi posse. Nimirum si,
 Vol. I. B §. 1. Con-

An Idea of Mathematics; by Dr. J. Pell, An. 1638. Pb. Col. N. 5. p. 127.

§. 1. Conscribatur Consiliarius Mathematicus (ita appellare libet) qui ad tres hæc Quæstiones respondeat. 1. Qui, qualesq; Fructus ex studiis Mathematicis sperandi? 2. Quænam assequendæ tam fructuosæ scientiæ Auxilia jam extent? 3. Qui Ordo in utendis istis Adminiculis observandus? Itaque continebit.

1. Facilem & perspicuum discursum, de limitibus seu ambitu Artium Mathematicarum, deque ingentibus utilitatibus, primo ad ipsarum studiosos, tum ad gentem complurium in iis peritorum feracem inde redundaturis.

2. Catalogum Mathematicorum & ab iis editorum Operum, qui exhibeat, 1. Synopsis omnis generis Librorum Mathematicorum vel jam Publicatorum, vel Manuscriptorum in publicis Bibliothecis delitescantium, peculiaribus, cuius Generi, numeris appositis. 2. Catalogum Chronologicum omnium Mathematicorum celebrium secundum ordinem Annorum quibus singuli vixere, additis ubique annis quibus Opera ipsorum primo excusa fuere. 3. Catalogum ipsorummet Operum secundum ordinem Annorum, quibus ulla Lingua impressa fuere. In quo concinnando ita procederem, ut notato Anno Domini subjungerem, more usitatorum Catalogorum, Nomina omnia Mathematicorum Librorum in ulla Gente aut Lingua isto Anno editorum, 1. Indicando tamen in singulis, rationem Voluminis, hoc est, non tantum magnitudinem paginarum (*e. g. 4to. 8vo. &c.*) sed earundem summam, ut moles totius Operis sic facile cuius pateat. 2. Ante ipsummet Titulum exprimendo Annum, ad quem retrospicere oporteat cognitum quando opus conscriptum, quando hac illave Lingua ultimo editum fuerit. 3. Notando in Margine post Titulum, 1. Annum quo opus aliquod proxime Typis exscriptum sit. 2. Numerum, qui remittat Lectorem ad Synopsin prima statim Catalogi pagina delineatam. Quorum Numerorum beneficio quilibet è Vestigio omnes unius argumenti libros Mathematicos percurrere queat.

3. Consilium, instruens studiosum, de optimis in quolibet genere libris: Quo ordine & quomodo legendi, quid observandum, quid evitandum in lectione quorundam Mathematicarum, quomodo procedendum, omniaque memoria tenenda.

4. Parænesin, Primo ad omnes, opibus, otio, & ingenio horum Studiosorum capaci instructos, ut, 1. Cum ingentium commodorum ad ipsummet eorum studiosum & ad universum humanum genus inde redeuntium. 2. Tum etiam defæcatæ illius voluptatis, quæ ex scrutinio abstrusarum Veritatum, lucta cum difficilibus Problematibus, & eorundem victoria emergit, respectu, serio ad ea excolenda incumbant, tanto quidem magis, quo, 3. Expeditiora adminicula reperta sunt, quæ laborem, tempus, sumptus, majoribus nostris impendenda, nobis compendifaciant. Deinde ad omnes tum iudicio ad æstimandum horum studiorum pretium, tum & opibus (æternæ nostri memoriæ, si in prudentes sui dispensatores inciderint, promiccondis) pollentes ut majori curæ sibi esse patiantur hoc studiosorum genus, amplisque propositis præmiis lectissimos eorum seponant perficiendis inventis, ad quæ ipso genio suo ducuntur. Denique ad omnes Principes & Respublicas, qui majus ditionibus suis

ornamentum vix consiliaverint, quam operam dando, 1. Ut harum Artium peritis abundant. 2. Via ad eas minus laboriosa sumptuosaq; reddatur. 3. Ipsaq; adeo Ingenia Mathematica magis innotescant, seq; dignis auxiliis gaudeant.

In quem finem utilissimum fuerit, ut,

§. 2. Constituatur Publica Bibliotheca, quæ & omnes supradictos Libros, & unum Instrumentorum unquam inventorum contineat, & præterea redivitibus sufficientibus instructa sit. 1. Ad coemenda singulorum librorum Mathematicorum, quæ quotannis alibi locorum foras dantur, exemplaria. 2. Ad sustentandum Bibliothecarium, cui incumbat.

1. Perlegere omnes hujus Classis libros in Regione isthac publico commit- tendos, 1. Suppressis quæ ab artis regulis exorbitant, ne Scriptorum errores Lectoribus fraudi sint. 2. Monereq; Scriptores, si vetera & jam dicta tan- tum reponant.

2. Sub periculo existimationis suæ approbare inventa insignia, & sincere commendare Inventores Mecænatibus.

3. Recipere, in Catalogum referre, & per Repositoria disponere unum ita perfectorum librorum exemplar, donandum Bibliothecæ bene compactum, Authoris aut Bibliopolæ sumptibus.

4. Promptum Responsum præbere cuivis studiofo, circa Problema aliquod inquirenti, an solutum fuerit nec ne, ni illi necesse sit actum agere, aut contra inventa sua suppressere, metu, ne antiqua jam sint, & forsan in aliquo Li- brorum Bibliothecæ jam elaborata.

5. Recipere, &c. omnia Manuscripta, vel donatione, aut legato Biblio- thecæ obvenientia.

6. Diligens cum sui generis hominibus, extra patriam suam agentibus, Li- terarum commercium exercere, ne ipsum fugiat, quidnam ibi locorum prelo paratum sit.

7. Observare in Conterraneis suis, qui docendis hisce artibus apti sint.

8. In numerato semper habere Artifices omnes conficiendorum Instrumen- torum Mathematicorum & Repræsentationum, adeoque Operationum in Ligno, Magnete, Metallis, Vitro, &c. peritos.

9. Post legitimum Examen, omnis generis practicis, e. g. Gubernatoribus Navium, Gæodetis, Logistis, &c. Speculativæ Peritiæ & Practicæ Dexter- itatis testimonium præbere, ne posthac, quibus hoc hominum genere opus, imperiti magno ipsorum damno imponant.

Catalogus docuerit, in tanta multitudine librorum, mundum tantum non obruentium, quinam ad hanc tantum studiorum partem pertineant. Biblio- theca, & ipsorummet Librorum copiam faciet, & simul cognoscendi ubi loco- rum exemplaria plura venalia sint: Prætereaq; promptuarii instar erit exteris & indigenis, unde extemplo discant, quænam ad hæc studia adminicula Re- gio ista sit suppeditatura.

Atque hæc meo judicio expeditissima Methodus est, utendi illis auxiliis, quibus jam potimur. Si ampliora desiderentur, conveniens fuerit, ut peri- torum Artificum opera.

§. 3. Conficiantur & publicentur tres hi novi Tractatus.

1. Pandectæ Mathematicæ, complectentes, quam fieri potest, perspicue, methodice, compendiose, & ingenue quæcunque ex Mathematicis libris & inventis, quæ ante nos fuere, colligi aut veluti consuetaria inde deduci potuere, citatis sub finem cuiusque periodi aut propositionis, authoribus, in quibus ea reperitur antiquissimis, sequentibusque adeo omnibus nota inusta, ubi vel in furto deprehenfi fuerint, vel mutuati esse suppresso unde sumpserint, aut (quod pessimum omnium) aliorum inventa audacter sibi arrogasse. Hoc pacto magna illa Bibliotheca in multo augustius spatium contraheretur, majori omnibus postfuturis laboris, temporis & sumptuum compendio, quam inpræsentiarum quis imaginetur. Sed cum & hoc opus portatu minus amplum futurum sit: Elaboretur præterea.

2. Comes Mathematicus, libro manuali (adeoque quam licet brevissime) continens utilissimas Tabulas una cum præceptis de usu ipsarum in Problematibus solvendis, seu pure Mathematicis, seu ad diversas materias, prout res requisierit, applicatas.

Et denique, ne in hoc quoque Eruditionis genere libris amplius astricti simus: Excogitetur,

3. Mathematicus *αὐτάγκης* seu Instructio quomodo quilibet Mathematicus, laborem non exhorrens, eam peritiam adipisci possit, ut sane ulla Librorum aut Instrumentorum ope, cujusvis tamen Problematis Mathematici resolutio æque facilis ipsi sit, ac alteri sola librorum Volutione nixo.

Atque hæc est Idea illa Matheſeos, quam, more meo, diu abhinc mihi effinxi, firmiter semper persuasus, tum demum feliciter magnis rebus vires intendi, ubi exactam prius earundem Ideam, ejusque assequendæ media quam maxime opposita animus conceperit: Quam si exprimere re ipsa non detur, multum tamen esse ab ea quam proxime abfuisse. Hancce meam equidem adeo non supra vires humanas arbitror, ut vel viri unius, rei domesticæ aliorumve negotiorum curis non distracti, diligentiam parem ei putem. Bibliothecam enim & Catalogum nummis præsentibus facile parari posse, quis non videt? Et si mihi Pandectarum contexendarum (quales supra descripsi) Provincia demandaretur, longe duriores Leges, quam quas ibidem enumeravi, mihi impositurus forem. Delinearem, primo omnium, infallibilem Rationis humanæ in indagando, quodcunque objicitur, processum, ostenso quomodo à primis principiis seu elementis nunquam interrupta Serie, ad sublimissimas juxta & abjectissimas eorum applicationes conscendendum sit. Qua arte forsan haud diu carerent homines, si in posterum sollicitè vestigarent, quam ratione illis quos mirantur tales obortæ essent cogitationes, quomodo tali fini, tam appositæ media reperta fuissent. Pandectas in Enchiridion, quotidianis usibus opportunum, constringendi ratio forsan haud plures fugerit: At ita mentibus suis eas inscribendi, ut ne Libris quidem amplius opus habeant (quod Mathematicus noster *αὐτάγκης* flagitat) vires ingenii humani transcendere plerisque haud dubie videbitur, cum nemò hucusque, quod sciam, tale quid vel animo concipere ausus fuerit. Nihilominus tamen remittent aliquid, credo, de incredulitate sua homines, postquam quæ roborandæ imaginationi, juvandæ memoriæ,

memoriae, dirigenda rationi artes excogitari, quam miri effectus conjunctione & jugi exercitio earum produci queant, paulo attentius secum consideraverint.

2. Consultius omnino mihi videtur ac præstantius, si loco tanti apparatus quem Author Ideæ proponit de colligendis variis Mathematicorum scriptis, optimi solummodo & digniores seligerentur; Adductis primum Authoribus illis Antiquis quorum libri adhuc supersunt, veluti Euclide, Apollonio, Archimede, Theodosio, Pappo, Ptolemæo, cum reliquis ipsorum Fragmentis & Manuscriptis quæ nondum viderunt lucem, ac quorum aliqua penes Golium Lugduni Batavorum, quædam vero Romæ asservantur. Iphis deinde recentiores possent adjungi, veluti Vieta, Clavius, & noster Herigonius. Similiter inter Opticos, quis seligeret Vitellionem, Keplerum, Aquilonium, & Dem. de Viles. Inter Arithmeticos post Diophantum, eminent Cardanus, Tartalea & Vester Nepperus. Pro Triangulis Sphæricis & Supputatione per Logarithmos extant Briggsius, Gordanus, Pitiscus, Snellius, & noster Morinus. Pro Astronomicis, post Ptolemæum & aliquot Arabes, conducerent omnes illi qui Tabulas composuerunt, sicut Alphonsus, Johannes Regiomontanus, Keplerus, & noster Duretus. Et ut paucis dicam, pro Fortificatoriis & Musicis octo vel decem Authores excerptendi essent, illi scilicet qui meliori inclaruere Praxi. Similiter pro Rebus Mechanicis, Viribus Motuum, Machinis, & Ductibus Aquarum, ita ut enucleatis decem vel duodecim Authoribus, desiderium suum explere posset harum rerum cupidus. Et si duodecim Viri Intelligentes & Amici cordate in se susciperent hoc negotium, ut quilibet unico Volumine convenienti & perspicuo compingeret aliquam Scientiam, additis illis quæ defunt in reliquis Authoribus, & sublatis non necessariis, absque dubio haberemus breviter & nervose, in duodecim Voluminibus totum id quod hac in re à quocumque posset expeti. Et mea quidem opinione, quicquid spectat ad Mathefin, tam Puram quam Mixtam, poterit comprehendi duodecim illis Voluminibus. Sic omnem elegantiorum Philosophiam tribus, & omnes Artes Liberales & Mechanicas etiam Tribus Libris exponere liceret: Ideoque levi pretio ad Eruitionem quis posset pertingere. Quod autem Mathematica illa Instrumenta attinet, parum certe prodesset apparatus omnium illorum quæ hætenus inventa fuere. Præstaret habere quatuor vel quinque, quæ nimirum optima sunt in eo genere, & magis commoda præ reliquis judicantur.

*Consider'd, Oct.
1639. by Mer-
tennus, ibid.
p. 135.*

3. Tibi (Vir Clarissime) si mentem tuam recte assequor, Omnia probatur, nisi quod majorem Apparatum, quam opus sit, à me desideratum censet: Selectiora tantum, non ut nos, Omnia Volumina atque Instrumenta Mathematica, colligi suades. Quod consilium tuum neque ego improbarem, si & omnium calculo definitum esset quinam in tanta Scriptorum Turba reliquis omnibus Palmam præriperent, & id tantum à me spectaretur ut aliquid Laboris, Sumptuumque Compendium harum Artium studiosis suppeditaretur. Nunc cum totius Matheseos Perfectissimam Instaurationem optem, non aliud Consilium à me proficisci debet, quam quod tali scopo per omnia quadret. Hujus Consilii fundamentum in universali ejusmodi Bibliotheca haud immerito collo-

*Answer'd by Dr.
Pell, ib. p. 137.*

care mihi videor. Non audeo quenquam spernere, symbolum suum quocunq; modo huc conferentem, multo minus condemnare inauditum. Et si meo iudicio locus sit, vel Ineptissimum sive Scriptum sive Instrumentum Mathematicum dignum esse puto, cujus unum Exemplar, vel propter Errores, in amplæ cujuscunque regionis loco certo & patente, servetur. Videmus enim non pauca ingeniose inventa in superiorum seculorum rudioribus Instrumentis, etiam nunc non observatu tantum digna, sed & Imitatione, quemadmodum etiam Imperiti Scriptores non paucas optimas egregiorum inventorum ἀπορῶδες sagacioribus ingeniis præbuere; digitum enim intendere possumus, quo pervenire non possumus. Videmus quamplurima Lemmata rectissime demonstrata ab hujusmodi Scriptoribus, & tamen propter unum Pseudographema inter fundamenta collocatum, quicquid superstruxerant corruisse. Quod si non solum falsitate, sed etiam verbositate, sribligine, &c. offensus multos rejiciendos putas, cogita quam varium & multiplex sit hominum Palatum, neque omnes ex tui ingenii perspicacitate judica. Sunt enim nonnulli qui nihil intelligunt nisi idem (& fere iisdem verbis) dictum sit centies; his credamus πειρολογίας istas accommodatas esse. Et quoniam à Notioribus omnino incipiendum est, eadem vero non sunt omnibus notiora, à diversissimis initiis progrediuntur, ut vix reparias Discipulis Ingenium, cujus Tyrocinio non sit jamdudum accommodatum Rudius aliquod Scriptum vel Instrumentum quod nescire nullo modo debet, qui se Consiliarium Mathematicum profitetur. Itaque Librorum ista Perfectior Collectio sanè necessaria mihi videtur.

Quo minus vero nobis placent isti Mathematicastri, eo magis fuerit, cur Bibliothecam hujusmodi optemus, quippe quæ sola infinitam istam hoc in genere scriptitandi pruriginem facile coercuerit. Balbutientes enim isti Magistrelli qui Tyrocinia & Juventutis captum crepantes, nusquam non pueriliter ineptiunt, videbunt sat superque esse hujus farinæ Elementarium jam pridem Compilatorum. Qui vero infinitis inventis Mathesin augere sed frustra cupiunt incautiores, tot καὶ ὁδὸν παρὰ ὁδὸν publice damnata, atque ludibrio omnium exposita videntes, malo cavebunt si sapiunt. Plagiarii vero, publico odio dignissimi, non audebunt posthac veteres aliquos Libellos semel forsan typis impressos, aut aliquas tantum earum particulas, pro suis extrudere. Quin etiam illi, quibus alioqui nec Candor nec Ingenium deesset, ad cogitata sua satis commode proferenda, quia tamen tam multos in singulis materiis se præcessisse videbunt, satis monebuntur, ne alia in publicum edant, quam prioribus indicta. Qualia quæ sint, ex inspecta ampla ejusmodi Bibliotheca, vel si hunc laborem gravantur, ex ipsomet, quem ei assigno Bibliothecario, facile cognoscent. Atque hæc fere rationes sunt, quæ consilii istius de tam universali Bibliotheca pœnitere me adhuc non sinunt.

To the Satisfaction of Mertenius, Dec. 10. 1639. *ibid.* pag. 143.

4. Statim atque tuas Literas perlegi (Vir Eruditissime) non solum totus fui tuus, tuæque subscripsi sententiæ, quam mihi egregie probatam testor; sed & insolitus Ardor Animi me propemodum abstulit, nempe ut tuum illud opus quantumvis arduum Orbis Magnatibus proponendum curarem, si liber ad eos foret Aditus. Sed quis Regum incipiet? Hoc enim Opus Regum ausim appellare.

5. Ideam

5. Ideam Mathematicam non nisi obiter inspexi, jamq; tantum memini nihil me in illa reperisse à quo multum dissentirem, & valde probasse quod primo loco omnis supellex Mathematica ibi enumeretur, & postea ipse Mathematicus tanquam *artifex* ex seipso contentus describatur. In eundem enim fere sensum duo soleo in Mathesi distinguere, Historiam scilicet & Scientiam: Per Historiam intelligo illud omne quod jam inventum est, atque in libris continetur: Per Scientiam vero, Peritiam Quæstiones omnes resolvendi, atque ideo inveniendi propria Industria illud omne quod ab humano ingenio in ea Scientia potest inveniri, quam qui habet, non sane multum aliena desiderat, atque adeo valde proprie *artifex* appellatur. Valde autem optandum foret, ut illa Historia Mathematica, quæ in multis Voluminibus sparsa, nondum integra & perfecta est, in unum Librum tota colligeretur: Neque ad hoc ulli sumptus in perquirendis aut coemendis libris essent faciendi: Cum enim Authores alii ex aliis multa exscripserint, nihil ullibi extat, quod non in quavis mediocriter instructa Bibliotheca alicubi reperiatur; nec tam diligentia opus esset ad omnia colligenda, quam judicio ab superflua rejicienda, & Scientia ad ea quæ nondum inventa sunt supplenda. Atqui si talis Liber extaret, facile ex eo unusquisque omnem Historiam Mathematicam, atque etiam aliquam partem Scientiæ addisceret. Si quis autem omnia quæ ad ejus Praxin pertinent, habere vellet, ut Instrumenta, Machinas, Automata, &c. nã ille si Rex esset, orbis Terrarum impensis omnibus ad hoc necessariis sufficere nunquam posset. Neque vere etiam illis opus habet, sed satis est si omnium norit descriptionem, adeo ut ea, cum usus exiget, vel ipse facere, vel per Artifices fieri curare possit.

*The Judgment
and Approbation
of Des Cartes,
Feb. 1640.
lb. p. 144.*

II. **T**HE Propositions which I shall endeavour to demonstrate independently from all others, shall be these; the 32d and 47th of the *First Book*; most of the *Second* and *Fifth Books*; the 1st and 16th of the *Sixth*; with their Corollaries. In order to demonstrate the 32d; I suppose it known what is meant by an Angle, Triangle, Circle, External Angle, Parallels, and that the Measure of an Angle is the Arch of a Circle intercepted between its Sides; that a Right Angle is measur'd by a Quadrant, and two Right Angles by a Semi-circle. I say then, that in the Triangle ABC, the External Angle BCE is equal to the two opposite Internal ones ABC, BAC; for let a Circle be drawn, C being the Center, and BC the Radius; and let CD be drawn parallel to AB, those two Lines being always equidistant, will both have the same Inclination to any third Line falling upon them; that is, (by the Definition of Angle) they will make Equal Angles with it: For if any Part of CD (for Instance) did incline more to BC than to AB, upon that very Account they would not be parallel; it follows therefore that the Angles ABC, BCD, are equal: Also BAC = DCE, because AE falls upon two Parallels; but the External Angle BCE = BCD + DCE, which were before prov'd to be equal to ABC, BAC. *Q.E.D.* Hence may be infer'd as a Corollary, That the three Angles of every Triangle are equal to two Right ones; for the Angles ACB + BCE, are measur'd by a

*Some of Euclid's
Propositions, de-
monstrated inde-
pendently from
the rest, by
Mr. Añ.
N. 162. p. 672.
32. 1. E.*

Fig. 1.

Semi-

Semicircle, and therefore equal to two Right Angles; Corollaries also from hence are the 20th, 22d, and 31st of the *Third Book*, which contain the Properties of Circles, whose Deduction from hence is most natural and obvious.

47. 1. E. In order to demonstrate the 47th, I suppose the Meaning of the Terms made use of, to be known; and that two Angles or Superficies are equal, when one being put on the other, it neither exceeds, nor is exceeded. This being allow'd, I say, the Sides about the Right Angle are either equal or unequal; if equal, let all the Squares be describ'd; the whole Figure exceeds the Square of the Hypotenuse BC , by the two Triangles M, V , and exceeds also the Squares of the other two Sides, AB, AC , by the two Triangles, ABC , and S ; which Excesses are equal, for M is equal to ABC , the two Sides about the Right Angle, being two Sides of a Square upon AB , by Supposition equal to AC , and the third Side equal to BC ; therefore the whole Triangles are equal. After the same manner, S and V are proved to be equal; therefore the Square of CB is equal to the Squares of the two other Sides. *Q. E. D.*

Fig. 2.

Fig. 3.

But if the Sides be unequal, let the Squares be described, and the Parallelogram LQ compleated, the whole Figure exceeds the Square upon BC , by three Triangles, X, R, Z , and exceeds also the Square LA, AD , by the Triangle ABC , and the Parallelogram PQ : Which Excesses, I say, are equal; for Z is equal to ABC , the Side $OC = BC$, $CD = AC$, the Angle $D = A$, and $OCD = BCA$; which is manifest, by taking the common Angle ACO , out of the two Right Angles BCO, ACD ; therefore by Superimposition the whole Triangles are equal. In like manner X is proved equal to ABC , also R ; and the Parallelogram PQ to be double of the Triangle ABC : Thus the Excesses being proved equal, the Remainders also will be equal; viz. the Square of BC to the Square of AB, AC .

35, 36. 3. E.

12, 13. 2. E.

1, 2, &c. 2. E.

Q. E. D. Manifest Corollaries from hence are, the 35th and 36th of the *Third Book*; also the 12th and 13th of the *Second*.

Fig. 4.

4. 2. E.

Fig. 5.

10. 2. E.

The first Ten Propositions of the *Second Book* are evidently demonstrated, only by substituting Species or Letters instead of Lines, and multiplying them according to the Tenor of the Proposition: Thus, to instance in one or two, Call the whole Line A , and its Parts B and C , therefore $A = B + C$, and consequently $AA = BB + CC + 2BC$, which is the very Sense of the *Fourth* of the *Second Book*. Thus also, Let a Line be cut into equal Parts F, F , and let another Line, S , be added thereto; 'tis manifest, that $4FF + 4SF + 2SS = 2FF + 2FF + 2SS + 4SF$, which is the *Tenth* Proposition of the same Book.

Fig. 6.

Almost the whole Doctrine of Proportionals, viz. Permutation, Inversion, Conversion, Composition, Division of Ratio's, and Proportion *ex aequo*; and consequently the most useful Propositions of the *Fifth Book* are clearly demonstrated by one Definition, and that is of Similar or Like Parts, which are said to be such as are after the same manner, or equally contain'd in their Wholes: Thus the Antecedents A and C , are either equal to their Consequents, or greater, or less; if equal, the thing is manifest; if less, then

(by

(By the Definition of Proportionals) A and C are like Parts of B and E; therefore what Proportions the whole B and E have to one another, the same will A and C have, which is Permutation; likewise $E : C :: B : A$, which is Inversion; so also if from Proportionals you take like Parts, the Remainders will be Proportional; whence Conversion and Division are Demonstrated: And if to Proportionals you add Like Parts, the Wholes will still be Proportional, which is Composition, &c. If the Antecedents be greater than the Consequents, the Consequents will be like Parts of them, and the Demonstration exactly the same with the former.

The first of the *Sixth* Book is prov'd, by considering the Generations of the Parallelograms, which are produced by drawing or multiplying the Perpendicular upon the Basis; that is, taking it so often as there be Parts and Divisions in the Base: Therefore the same Proportion that $R X$ single, hath to $N X$ single, the same hath $R X$ multiplied by $X Z$, that is, repeated a certain number of times, to $N X$ multiplied by $X Z$, that is, repeated the same Number of times; which is as much as to say, $R X : N X :: \text{Paral. } R Z : \text{Paral. } N Z$. Now that this Proposition also is true in Oblique angled Parallelograms, is proved, because they are equal to the Rectangled ones upon the same Basis, and between the same Parallels, as does thus independently appear; the Triangles $R Q X$, and $M P Z$, are equal, for $R X = M Z$, $Q X = P Z$, $R M = Q P$; therefore adding to both $M Q$, $R Q = M P$; if therefore from these equal Triangles you take what is Common, *viz.* $M L Q$, the Remainders will be equal, $R X L M = Q L Z P$; to both which add $X L Z$, and the whole Parallelograms will be Equal, $R Z = Q Z$. *Q. E. D.* That Triangles also having a Common Basis, are in the Proportion of their Altitudes, does hence follow, because they are the halves of Parallelograms upon the same Basis. This also is true, and the Demonstration exactly the same in Prisms, Pyramids, Cylinders, and Cones, having the same Basis.

To prove the 16th of the *Sixth*, I suppose the 4 Lines, A, B, C, E, to be Proportional, that is, granting A and C to be the lesser Terms; the same way that A is contained in B, so is C in E, and that D is the Denominator of the Ratio; 'twill follow then, that B is made up of A, multiplied by D, and E of C, multiplied by D; so that $A D = B$, and $C D = E$; draw therefore the Extremes upon one another, that is, A upon C D, and the Means, that is, C upon A D, the Factors being the same; I say, the Products A C D, and C A D, are the same, and consequently Equal. *Q. E. D.*

The Problem compos'd by M. *Comier*, (with Ostentation enough) as if it contain'd something New, tho' in reality it be nothing but the Old Business of doubling the Cube a little disguis'd, is easily solv'd Algebraically, as follows,

1. 6. E.

Fig. 7.

Fig. 7.

16. 6. E.
Fig. 6.

Fig. 8.

| | | |
|------------|---|---|
| 8. 6. E. | 1 | $a:2x::x:\frac{2x^2}{a}=p$ |
| 47. 1. E. | 2 | $aa+2ax-\frac{2x^2}{a}=\frac{4x^2}{a}$ |
| 2 x by aa | 3 | $a^4+2a^3x=4x^4+2x^3a$ |
| 3 : a + 2x | 4 | $a^3=2x^3$; that is, The Cube upon x , is half the Cube upon a . |

The Squaring of
the Hyperbola by
the L. Viscount
Brouncker.
N. 34. p. 645.
Fig. 9.

III. Let AB be one Asymptote of the Hyperbola E d C; and let AE, and BC, be Parallel to the other: Let also AE, be to BC, as 2 to 1; and let the Parallelogram ABDE, be equal to 1.

Supposing the Reader knows, that EA, a , ζ , KH, β , n , d , θ , γ , x , δ , ν , μ , CB, &c. are in an Harmonick Series, or a Series *Reciproca-Primariorum*, *sem Arithmetice*, *Proportionalium*, (otherwise he is referr'd for Satisfaction to *Arithm. Infinitor. Wallisi*, Prop. 87, 88, 89, &c.) I say,

$$ABCEdEA = \frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + \frac{1}{7 \times 8} + \frac{1}{9 \times 10}, \text{ \&c.}$$

$$EdCDE = \frac{1}{2 \times 3} + \frac{1}{4 \times 5} + \frac{1}{6 \times 7} + \frac{1}{8 \times 9} + \frac{1}{10 \times 11}, \text{ \&c.}$$

$$E d C \gamma B = \frac{1}{2 \times 3 \times 4} + \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} + \frac{1}{8 \times 9 \times 10} + \frac{1}{10 \times 11 \times 12}, \text{ \&c.}$$

The Problem compos'd by M. Cassini (with Observation enough) as it contains something New, tho' in reality it be nothing but the Old Problem of doubling the Cube a little differently, is easily sol'd Algebraically, as follows.

(11)

For (in Fig. 10. and 11.) the Parallelogram,

and (in Fig. 12.) the Triangle, Fig. 10, 11, 12

| | |
|-------------------------------|--|
| $CA = \frac{1}{1 \times 2}$ | $EdC = \frac{1}{2 \times 3 \times 4} = \frac{1}{2} dD - \frac{1}{2} Ed$ |
| $dD = \frac{1}{2 \times 3}$ | $Ebd = \frac{1}{4 \times 5 \times 6} = \frac{1}{2} br - \frac{1}{2} bn$ |
| $br = \frac{1}{4 \times 5}$ | $dfC = \frac{1}{6 \times 7 \times 8} = \frac{1}{2} fG - \frac{1}{2} fk$ |
| $fG = \frac{1}{6 \times 7}$ | $Eab = \frac{1}{8 \times 9 \times 10} = \frac{1}{2} ag - \frac{1}{2} ap$ |
| $ag = \frac{1}{8 \times 9}$ | $bcd = \frac{1}{10 \times 11 \times 12} = \frac{1}{2} cs - \frac{1}{2} cm$ |
| $cs = \frac{1}{10 \times 11}$ | $def = \frac{1}{12 \times 13 \times 14} = \frac{1}{2} et - \frac{1}{2} el$ |
| $et = \frac{1}{12 \times 13}$ | $fgC = \frac{1}{14 \times 15 \times 16} = \frac{1}{2} gu - \frac{1}{2} gb$ |
| $gu = \frac{1}{14 \times 15}$ | $gh = \frac{1}{15 \times 16}$ |
| &c. | &c. |

Now, $CA = dD + dF$
 $dD = br + bn$
 $dF = fG + fk$
 $br = ag + ap$
 $bn = cs + cm$
 $fG = et + el$
 $fk = gu + gb$ &c.

And that therefore in the first Series half the first Term is greater than the Summ of the two next, and half this Summ of the Second and Third greater than the Summ of the four next; and half the Summ of those Four, greater than the Summ of the next Eight, &c. in infinitum. For $\frac{1}{2} dD = br + bn$; but $bn > fG$, therefore $\frac{1}{2} dD > br + fG$, &c. And in the second Series, half the second Term is less than the Summ of the two next, and half this Summ less than the Summ of the four next, &c. in infinitum.

That the first Series are the even Terms, viz. the 2d, 4th, 6th, 8th, 10th, &c. and the second the odd, viz. the 1st, 3d, 5th, 7th, 9th, &c. of the following Series, &c.

$$\frac{1}{1 \times 2}, \frac{1}{2 \times 3}, \frac{1}{3 \times 4}, \frac{1}{4 \times 5}, \frac{1}{5 \times 6}, \frac{1}{6 \times 7}, \text{ \&c. infinitum} = 0.$$

Whereof a , being put for the Number of Terms taken at Pleasure, $\frac{1}{a_2 + a}$

is the last, $\frac{a}{a+1}$ is the Summ of all those Terms from the Beginning, and

$\frac{1}{a+1}$ the Summ of the rest to the End.

That $\frac{1}{a}$ of the first Term in the third Series, is less than the Summ of the two next, and $\frac{1}{a}$ of this Summ less than the Summ of the four next, and $\frac{1}{a}$ of this last Summ less than the next eight, I thus Demonstrate.

Let a , be equal to the Third or Last Number of any Term of the first Column, viz. of Divisors;

$$\frac{1}{a \times a - 1 \times a - 2} = \frac{1}{a^3 - 3a^2 - 2a} = \frac{16a^3 - 48a^2}{16a^6 - 96a^5 + 232a^4 + 56a^3 - 24} = A.$$

$$\left. \begin{aligned} \frac{1}{2a \times 2a - 1 \times 2a - 2} &= \frac{1}{8a^3 - 12a^2 + 4a} \\ \frac{1}{2a - 2 \times 2a - 3 \times 2a - 4} &= \frac{1}{8a^3 - 36a^2 + 52a - 24} \end{aligned} \right\} =$$

$$\frac{16a^3 - 48a^2 + 56a - 24}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} = B.$$

$$\frac{64a^6 - 384a^5 + 928a^4 - 1152a^3 + 736a^2 - 192a}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} \times \frac{1}{4} A = B.$$

And $48a^4 - 192a^3 + 240a^2 - 96a =$ Excess of the Numerator above the Denominator.

But the Affirmat. > the Negat.

That is, $48a^4 + 240a^3 > 192a^3 + 96a^2$ if $a > 2$.

Because $a^4 + 5a^3 > 4a^3 + 2a^2$

$a^3 + 5a > 4a^2 + 2$

Therefore $B > \frac{1}{4} A.$

Therefore one Fourth of any Number of A , or Terms, is less than their so many respective B ; that is, than twice so many of the next Terms.

Q. E. D.

By

By any one of which three Series it is not hard to calculate, as near as you please, these and the other like Hyperbolic Spaces, whatever be the rational Proportion of A E to B C. As for Example; When A E is to B C, as 5 is to 4. (Whereof the Calculation follows, after that where the Proportion is, as to 2 to 1; and both by the *Third* Series.)

First then, when (in *Fig. 9.*) A E : B C :: 2 : 1.

Fig. 9.

$$2 \times 3 \times 4) 1. (0. 0416666666] 0. 0416666666$$

$$4 \times 5 \times 6) 1. (0. 0083333333 \} 0. 0113095237$$

$$6 \times 7 \times 8) 1. (0. 0029761904 \}$$

$$8 \times 9 \times 10) 1. (0. 0013888888 \}$$

$$10 \times 11 \times 12) 1. (0. 0007575757 \} 0. 0029019589$$

$$12 \times 13 \times 14) 1. (0. 0004578754 \}$$

$$14 \times 15 \times 16) 1. (0. 0002976190 \}$$

$$16 \times 17 \times 18) 1. (0. 0002042484 \}$$

$$18 \times 19 \times 20) 1. (0. 0001461988 \}$$

$$20 \times 21 \times 22) 1. (0. 0001082251 \}$$

$$22 \times 23 \times 24) 1. (0. 0000823452 \}$$

$$24 \times 25 \times 26) 1. (0. 0000641026 \}$$

$$26 \times 27 \times 28) 1. (0. 0000508751 \}$$

$$28 \times 29 \times 30) 1. (0. 0000410509 \}$$

$$30 \times 31 \times 32) 1. (0. 0000336021 \}$$

$$32 \times 33 \times 34) 1. (0. 0000278520 \}$$

$$34 \times 35 \times 36) 1. (0. 000023426 \}$$

$$36 \times 37 \times 38) 1. (0. 0000197566 \}$$

$$38 \times 39 \times 40) 1. (0. 0000168691 \}$$

$$40 \times 41 \times 42) 1. (0. 0000145180 \}$$

$$42 \times 43 \times 44) 1. (0. 0000125843 \}$$

$$44 \times 45 \times 46) 1. (0. 0000109793 \}$$

$$46 \times 47 \times 48) 1. (0. 0000096361 \}$$

$$48 \times 49 \times 50) 1. (0. 0000085034 \}$$

$$50 \times 51 \times 52) 1. (0. 0000075415 \}$$

$$52 \times 53 \times 54) 1. (0. 0000067193 \}$$

$$54 \times 55 \times 56) 1. (0. 0000060125 \}$$

$$56 \times 57 \times 58) 1. (0. 0000054014 \}$$

$$58 \times 59 \times 60) 1. (0. 0000048704 \}$$

$$60 \times 61 \times 62) 1. (0. 0000044068 \}$$

$$62 \times 63 \times 64) 1. (0. 0000040002 \}$$

(14)

By any one of which three it is not hard to calculate as near as you please, that the total of the spaces, whatever be the normal proportion of A to B, is to the total of the spaces as 1 is to 4. (Whereof the following series follows, after that where the proportion is as 1 to 1; and nothing but the series.)

$$\begin{aligned} &0.0416666666 \\ &0.0113095237 \\ &0.0029019589 \\ &0.0007396482 \\ &0.0001829939 \end{aligned}$$

$$\begin{aligned} &0.05679179 \\ &+ 0.00000100 \end{aligned}$$

$$0.05685279 < EdCj$$

$$\begin{aligned} \text{But } 0.0007396482 & \\ 0.0001829939 & \\ 0.0000458315 & \end{aligned}$$

$$\begin{aligned} \text{Therefore } 0.05679179 & \\ + 0.00004583 & \\ + 0.00001528 & \end{aligned}$$

$$0.05685290 > EdCj$$

For it has been demonstrated, That if of any Term in the last Column is less than the Term next after it; and therefore that if of the last Term, at which you stop, is less than the remaining Terms; and that the Total of these is less than $\frac{1}{4}$ of a Third proportional to the two last.

$$\begin{aligned} \text{And therefore } ABCjE \text{ being } &= 0.75 \text{ --- } 0.57 \\ \text{and } EdCj &\geq 0.05685279 \text{ --- and } < 0.05685290 \end{aligned}$$

$$\text{And } ABCdE \text{ is } < 0.69314720 \text{ --- and } > 0.69314709$$

Fig. 9.

But when $AE : BC :: 5 : 4$, or as EA to KH; then will the Space ABCE, or now, the Space AHKE ($AH = \frac{1}{2}AB$) be found as follows.

$$\begin{aligned} 8 \times 9 \times 10 & 1. (0.0013888888) \quad 0.0013888888 \\ 16 \times 17 \times 18 & 1. (0.0001042484) \quad 0.0001042484 \\ 18 \times 19 \times 20 & 1. (0.0001461988) \quad 0.0001461988 \\ 32 \times 33 \times 34 & 1. (0.000078520) \quad 0.000078520 \\ 34 \times 35 \times 36 & 1. (0.000039426) \quad 0.000039426 \\ 36 \times 37 \times 38 & 1. (0.000019756) \quad 0.000019756 \\ 38 \times 39 \times 40 & 1. (0.0000168691) \end{aligned}$$

(15)

$$\begin{array}{r}
 0.0013888888 \\
 0.0003504472 \\
 3) 0.0000878204 \quad (0.0000292735) \\
 \hline
 0.0018271564 \\
 + 0.0000292735 \\
 \hline
 0.0018564299 < Eab \\
 \hline
 \text{But } 0.0003504472 \quad \left. \begin{array}{l} 0.0000878204 \\ 0.00002200737 \end{array} \right\} \div \\
 \hline
 \text{Therefore } 0.0018271564 \\
 + 0.0000220074 \\
 + 0.0000073358 \\
 \hline
 0.0018564996 > Eab.
 \end{array}$$

Therefore E M b (Fig. 12.) being = 0.025 ——— 0.025
 $Eab > 0.0018564292 \& < 0.0018564996$

E M b a (Fig. 12.) or E K M (Fig. 9.) > 0.02685643 < 0.02685650
 $A H K E < 0.22314356 > 0.22314349$

Therefore 3 A B C d E = 2.07944154 Therefore,
 and A H K E = 0.2231435 The Logar. of 10, is to the Log. of 2,

A B C d E (when A E : B C :: 10 : 1) = 2.18025850 As 2.3010305, is to 0.6989695,

IV. The Quadrature of the Circle, or the turning it into an equal Square, or any other Right-lin'd Figure, (which depends upon the Ratio of the Circle to the Square of its Diameter, or of the Circumference to its Diameter) may be understood to be Fourfold; to wit: either by Calculation, or by Linear Construction, and each of them again may be either perfectly exact, or else almost, or pretty near. The Quadrature by accurate Calculation, I call the Analytical; That which is done by accurate Calculation, I call the Geometrical: That which is done by Calculation pretty near, I call the Approximate; That which is by Construction pretty near, I call Mechanical.

The Approaches have been further carry'd on by *Adolphus von Cotes*, *Philos. Hygenius*, and others, have given several Mechanical. The Accurate Geometrical Construction may be had, by which not only an entire Circle may be measur'd, but any Section or Arch of it also, which is by an exact and ordinate Motion, but such notwithstanding as runs with Transcendental Curves, which may sufficiently enough be conceived by the Mind. And it is by this way that the

The Quadrature of a Circle by M. Leibnitz, Phil. Coll. N. 7. p. 204.

which erroneously are accounted Mechanical, tho' in truth they are as Geometrical as those which are commonly so esteem'd, tho' they are not Algebraical, nor can be reduced to Equations Algebraical or of certain Degrees, they having Degrees proper to themselves, which tho' they be not Algebraical, are yet nevertheless Analytical.

The Analytical Quadrature, or that which is made by Accurate Calculation, may be again subdivided into three Kinds; namely, into the Analytical Transcendent, the Algebraical, and the Arithmetical. The Analytical Transcendent is to be obtain'd, amongst others, by Equations of Degrees indefinite, hitherto consider'd by none. As if $X^2 + X$, be equal to 30, and X be sought, it will be found to be 3; because $3^2 + 3$, is $27 + 3$, or 30.

The Algebraical is done by Vulgar Numbers, tho' irrationally, Vulgar, or by the Roots of common Equations, which for the General Quadrature of the Circle, or its Sectors, is indeed impossible. Now there remains the Arithmetical Quadrature, which is performed by certain Series exhibiting the Quantity of the Circle exact by a Progression of Terms (first) Rational, such as I shall here propound.

I have found therefore, that if the Square of the Diameter be put 1, the

Area of the Circle will be $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13}$

$- \frac{1}{15} + \frac{1}{17}$, &c. To wit, the entire Square of the Diameter being

diminished (that it may not be too big) by a third part; and again (because hereby too much is taken away) being augmented by one fifth; and again (because by this last too much is added) diminished by one seventh; and so onward continually.

And the first Quantity will be too great, viz. 1. but the Error } $\frac{1}{3}$
will be less than _____

The next too little, viz. $1 - \frac{1}{3}$, but the Error will be less than $\frac{1}{5}$

The 3d too much, viz. $1 - \frac{1}{3} + \frac{1}{5}$, but, &c. $\frac{1}{7}$

The 4th too little, viz. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$, but, &c. $\frac{1}{9}$

The whole Series contains all the Approaches together, or the Values both greater than they ought to be, and less than they ought to be.

So that by continuing the Series, the Errors may be made less than the Fraction given, and consequently less than any Assignable Quantity. Whence it follows, that the whole Series must give the true Value. And tho' the Summ of the whole Series cannot be express'd by one Number, and that the Series be infinitely continued: yet because it consisteth of one regular Method of Progression, the whole may sufficiently enough be conceived by the Mind. And if *Van Ceulen* could

1. Rejectis ab *Æquatione* partibus in quibus *y* vel *v* non invenitur; stantur ab uno latere omnes in quibus est *y*, & ab altero illæ in quibus habetur *v*, cum suis signis + vel -. Hoc dextrum, illud sinistrum latus facilitatis causa, vocabimus.

2. In latere dextro, præfigatur singulis partibus Exponens potestatis quam in illis obtinet *v*; seu, quod idem est, in illum ducantur partes.

3. Fiar idem in latere sinistro, præponendo scilicet unicuique illius parti Exponentem potestatis quam in illa habet *y*. Sed & hoc amplius: unum *y* in singulis partibus vertatur in *a*.

Aio, *Æquationem* sic reformatam modum ostendere ducendæ Tangentis ad punctum D datum. Cum enim eo dato, pariter datæ sunt *y* & *v*, & ceteræ quantitates, quæ Consonantibus exprimuntur, non poterit ignorari.

Si quid forte sit obscuritatis in Regula, aliquot Exemplis illustrabitur: Data sit hæc *Æquatio*, $by - yy = vv$; in qua E B sit *b*; B A, *y*; D A, *v*; & queratur *a*, sive A C, talis ut iuncta D C, tangat Curvam D Q in D. Ex Regula, nihil rejiciendum est ab hac *Æquatione*, cum in singulis ejus partibus reperiatur *y* vel *v*. Ita quoque disposita est, ut ab uno latere sint omnes illius partes in quibus *y*; ab altero, omnes in quibus *v*. Singulis itaque tantum præfigendus est Exponens potestatis, quam in illis habet *y* vel *v*; in latere sinistro unum *y* vertendum in *a*, ut fiat $ba + 2ya = 2vv$. Aio nunc, hanc *Æquationem* ostendere modum ducendæ Tangentis ad punctum D, sive

$$\begin{aligned}
 & 2vv \\
 & a = b - 2y = AC. \\
 & \text{Sic si data fuisset } \mathcal{Æquatio}, qq + by - yy = vv; \text{ eadem plane fieret} \\
 & \text{cum priori } \mathcal{Æquatio} \text{ pro Tangente, abstracto scilicet } qq, \text{ ut Regula præscribit.} \\
 & \text{Sic ex } 2byy = y^3 = v \text{ fit } 4bya - 3yya = 3v^2, \text{ sive } a = \\
 & \frac{3v^2}{4by - 3yy}. \text{ Ex } bby + 2yy + y^3 = qvv, \text{ fit } bba + 2ya + \\
 & 3yya = 2qvv, \text{ \& } a = \frac{2qvv}{bb + 2zy + 3yy}. \text{ Ex } b^4 + by^3 - y^4 \\
 & = 9qvv + 2v^3, \text{ fit } 3byja - 4y^4a = 29qvv + 3zv^3, \text{ \&} \\
 & a = \frac{29qvv + 3zv^3}{3byj - 4y^4}.
 \end{aligned}$$

Verum in similibus *Æquationibus* nullam arbitror accidere posse difficultatem. Aliqua fortasse in illis occurrit, quarum partes quædam constant ex productis *y* in *v*. Ut in yyv, y^2v, v^2y , &c. Sed hæc quoque levis est, ut Exemplis patebit. Detur enim $y^2 = bvv - yvv$. Nihil ab illa rejiciendum erit, cum in singulis ejus partibus reperiatur *y* vel *v*.

Sed ut ex Regula præscripto disponatur, his sumendum erit yvv , & statucendum tam in latere dextro, ut quo sunt partes quæ habent *v*, quam in sinistro, cujus partes habent *y*, si quandoquidem yvv , tam *y* quam *v* contineat. Faciendum igitur erit $y^2 + vv = bvv - yvv$. Tum mutata, ut prius, hac *Æquatione* in aliam $3yya + vva = 2bvv - 2yvv$, dabitur

$$a = \frac{2bvv - 2yvv}{3yy + vv}.$$

Ita enim intelligenda est Regula, ut nempe in latere sinistro non consideretur potestas ipsius y , ideoque ipsius yy , Exponens v , præfigi non debeat, sed tantum ipsius y . Sicut contra ab alio latere, in yv considerari non debet potestas ipsius y , sed v tantum; exque suis Exponens præponi. Sic si foret $y^4 + by^4 = 299v^3 - yv^3$, faciendum esset $y^4 + by^4 + vyy = 299v^3 - yv^3$, & haberetur Aequatio pro Tangente $3ya + 4by^3a + 3vga = 699v^2 - 3yyv$ & $a = \frac{699v^2 - 3yyv}{3y^4 + 4by^3 + 2vy}$.

Fig. 15.

Atque his Exemplis arbitror, me omnem quæ dari posset, Castum varietatem complexum esse. Cæterum non erit fortasse inutile, si ea quæ generatim exposui, ad lineam aliquam singularem applicem. Data sit igitur Curva BD, cujus ea sit proprietas, ut sumpto in illa quolibet puncto D, si jungatur BD, & erigatur ad illam normalis DE, occurrens rectæ BB in E, recta DE sit semper æqualis datæ rectæ BF. Ut habeatur Aequatio in terminis Analyticis, sit $DA = v$, $BA = y$, BF vel DE = q . Erit itaq; EA = $v - y$.

Et cum quadratum DE æquale sit duobus DA, AE; erit Aequatio $qq = v^2 + vv$; sive $qqy = v^2 + yyvv$; quæ pro Tangente ex Regula præscripto, sic reformanda erit, $qqy - vvyy = v^2 + yyvv$, & $2qqya - 2vvya = 4v^2 + 2yyvv$, & $a = \frac{4v^2 + 2yyvv}{2qqy - 2vv}$.

Quomodo autem Aequationes huiusmodi ad faciliores terminos pro constructione reduci debeant, id sane solertem Geometram minime latebit. Ut ecce in hoc exemplo, quoniam Rectangulum BAE supponitur æquale Quadrato AD, si EA dicatur e , erit $vv = ye$, & $v = yye$, & $qq = ye + ee$. Itaq; pro illis, posito in Aequatione eorum valore, sit $a = \frac{4yyee + 2ye}{2ee + 2ey - 2ey}$, sive

$a = \frac{2ey + yy}{2ee + 2ey - 2ey}$, hoc est, $ae = 2ey + yy$, & addito ee utrinque $ae + ee = ee + 2ey + yy$. Erant itaque tres e , $e + y$, $e + a$, sive EA, EB, EC, in continua analogia, & facillima evadet constructio.

Cæterum quoniam hactenus supposuisse videmur, Tangentem versus partes B ducendum esse, cum tamen ex datis accidere possit, ut vel parallela sit ipsi AB, vel eam ducenda ad partes contrarias, definiendum nunc superest, quomodo hæc casuum diversitas in Aequationibus distinguarum facta agitur. Præfatione prænotat in Exemplis supra adductis, consideranda sunt partes tam Numeratoris quam Denominatoris, & earum Signa.

1. Nam si in utroque partes vel habeant omnes Signum +, vel ætiam Affirmatæ prævalent Negatis, ducenda erit Tangens versus B. Si autem Affirmatæ prævalent Negatis in Numeratore, sed æquales sint in Denominatore, ducenda erit Tangens versus B, tanget Curvam in D. hoc autem in casu, est infinite longitudinis.

3. Si

3. Si tam in Denominatore, quam Numeratore, partes Affirmatæ minores sunt Negatis; mutatis omnibus Signis, ducenda erit rursus Tangens versus B: hic enim Casus cum primo in idem recidit.

4. Si in Denominatore prævaleant, in Numeratore minores sint, vel contra; mutatis Signis illius in quo sunt minores, ducenda erit Tangens versus partes contrarias, *h. e.* AC, sumenda erit versus E.

5. Ac tandem si in Numeratore partes Affirmatæ sint æquales Negatis, quomodocunque se habeant in Denominatore, a abibit in nihilum Itaque vel ipsa AD erit Tangens, vel ipsa EA, aut ei Parallela; quod ex datis facile dignoscitur. Horum autem Casuum Varietas explicari potest per Æquationes ad Circulum.

Sit enim Semicirculus, cujus Diameter EB, & in eo punctum D datum, ex quo cadat Normalis DA = *v*. Sit BA = *y*, BE = *b*; erit Æquatio *by*

Fig. 16.

— *yy* = *vv*, & ducta Tangente DC, erit AC, five $a = \frac{2vv}{b-2y}$. Nunc

si *b* major sit *2y*, ducenda est Tangens versus B; si æqualis, sit Parallela EB; sin autem minor, ducenda est versus E; ut *n.* 1, 2, & 4. diximus.

Detur rursus alius Semicirculus inversus, cujus puncta referri intelligentur ad rectam Diametro parallelam, & eidem æqualem, ut in Schemate. Denominatis, ut prius, partibus, & NB = *d*, fit Æquatio *by - yy = dd*

Fig. 17.

+ *vv - 2dv*. Igitur AC, five $a = \frac{2vv - 2dv}{b - 2y}$. Cum vero Ex-

emplo supposuerimus *v* semper esse minorem *d*; si *b* sit major *2y*, ducenda erit Tangens versus E; si æqualis, erit parallela; sin minor, mutatis omnibus Signis, ducenda erit versus B; ut *n.* 4, 5, & 3. Nulla autem ducenda esset Tangens, seu Tangens foret ipsa EB, si supposuissimus NB æqualem Semidiametro, five *2d = b*; ut *n.* 5.

Sit tandem alius Semicirculus, cujus Diameter NB normalis sit ad Rectam BE, ad quam ejus puncta referri intelligentur. NB dicatur *b*, & aliae partes

Fig. 18.

denominentur ut supra; fiet Æquatio *yy = bv - vv*; & $a = \frac{bv - vv}{2y}$.

Jam si *b* sit major *2v*, Tangens ducenda erit versus B; si minor, versus E; si autem æqualis, ipsa DA erit Tangens; ut *n.* 1, 4, & 5.

Et hæc est, ni fallor, Casuum omnium Varietas, quæ ex Æquationum consideratione deprehendi potest.

2. (1.) Differentia duarum dignitatum ejusdem gradus applicata ad differentiam laterum, dat partes singulares gradus inferioris ex binomio laterum; ut

The Lemmata, whereby the preceding Method is demonstrated, by M. Slufius. n. 95. p. 6059. n. 97. p. 6126. June, An. 1673.

$y^2 - x^2 = yy + yx + xx$. Quod facile ostenditur.

(2.) Tot sunt partes singulares ex binomio in gradu quolibet, quot unitates habet Exponens dignitatis immediate superioris; tres nimirum in Quadrato, quatuor in Cubo, &c. Et hoc vulgo notum.

(3.) Si

(3) Si quantitas eadem applicetur ad duas alias, quarum ratio data sit. Quotientes erunt reciproce in eadem ratione data.

His Lemmatibus Methodus mea facile demonstratur, cum eo ordine disposita sint, qui ad illam quasi manu ducit.

*The Testudo
Veliformis
Quadrabilis
Ænigmatically
propos'd by V. V.
n. 196. p. 484.
Jan. An. 1693.*

VI. 1. *ÆNIGMA Geometricum, De miro Opificio Testudinis Quadrabilis Hemisphærica, à D. Pio Lisci Pusillo Geometra propositum.*

Inter Venerabilia Eruditæ olim Græciæ Monumenta extat adhuc, perpetuo equidem duraturum, Templum Augustissimum Ichnographia Circulari, *Alme Geometria* dicatum, quod, à Testudine intus perfecte Hemisphærica, operitur: sed in hac, Fenestrarum quatuor æquales Aræ (circum, & supra Basin Hemisphære ipsius dispositarum) tali Configuratione, Amplitudine, tantaque Industria, ac Ingenii acumine sunt extructæ, ut, his detractis, superstes Curva Testudinis Superficies, pretioso Opere Musivo ornata, Tetragonismi vere Geometrici est capax.

Quæritur modo, Quæ sit, Qua Methodo, Quave Arte, Pars illa Hemisphæricæ Superficie Curvæ Quadrabilis, Tensi ad instar Carbasi, vel Turgidi Veli Nautici, ab Architecto illo Geometra fuerit affecuta? & cui demum Plano Geometrici Quadrabili sit æqualis?

*Solv'd by Dr.
Wallis, ibid.
p. 487.*

2. Accepi, V. C. nudius tertius (noctu decubiturus) Literas tuas quibus heri non vacabat, alias occupato, respondere, eisque inclusam Chartulam, Typis impressam, quam ais Florentia te accepisse mihi mittendam.

Continet ea Chartula *Ænigma Geometricum*, quod (verborum involucri exemptum) hoc innuere judico Problema; Ab Hemisphærii curva Superficie, Segmenta quatuor inter se æqualia sic amputare, ut reliquum sit Tetragonismi capax.

Similique videtur innuere, in veteris Græciæ Monumentis etiamnum extare quiddam, quò illud fiat.

Hoc esse estimo Hippocratis Chii Quadraturam Lunulæ.

Quippe cum Archimedes demonstravit, Curvam Hemisphærii Superficiem æqualem duobus Circulis ejusdem Sphære maximis, (id est quatuor Semicirculis;) docuitque Hippocrates Chius Lunulam quadrare quandam: Si singulis Hemisphærici hujusce Fornicis Quadrantibus tantundem eximatur, quanto deficit à Semicirculo ea Lunula, Reliquum æquabitur Quadrato, quod Circulo Sphære maximo (cui hic insistit Fornix Hemisphæricus) Inscribatur.

Si tamen præter *Ænigmaticam* Problematis Involutionem, subsit aliquid (de Templo) Historicum; putaverim ego S. Sophiæ (quod est Constantinopoli) Templum hic insinuatam.

SCHOLIUM] Per Hippocratis Chii Quadraturam Lunulæ (primo Physicorum Aristotelis & Simplicii in eum locum Commentariis, indicatam,) Si Semicirculo ABD , in duos Quadrantes ACD , BCD diviso, aptetur AD Subtensa Quadrantalibus Arcus, Radio CE bisecta in H ; & Centro H scribatur Semicirculus ADF : Erit (propter Quadratum Rectæ AD Subduplum Quadrati Rectæ AB) Semicirculus ADF Subduplus Semicirculi ABD ; adeoque

adeoque Quadranti ACD æqualis. Et (dempto utrinque communi Segmento ADE) residua Lunula AEDF residuo Triangulo ADC æqualis. Talesque quatuor Lunulæ, talibus quatuor Triangulis; hoc est, Quadrato toti Circulo inscripto ADBG.

Porro, per Archimedis demonstrata, æquatur Sphæræ Superficies, quatuor Circulis in ea Sphæra Maximis; Adeoque Hemisphæræ Superficies Curva, talibus quatuor Semicirculis; talisque Superficie Hemisphæræ Quadrans, uni Semicirculo.

Circulus ADBG esto jam Basis Hemisphæræ Superficie Curvæ; cujus Polus P, Axis CP, plano Basis perpendicularis, ejusq; Quadrans unus DPA; qui plano EPC per Axem transeunte, bifecetur.

Fig. 20.

Ponantur item (ob commodiorem Calculum) Circuli Radius R, Diameter $D = 2R$, Peripheria P, Expositus Arcus a .

Positoque Quadrantali Arcu $DEA = a = \frac{1}{4}P$; est Semicirculus ABD $= \frac{1}{2}R = \frac{1}{4}RP$; Triangulum ADC $= \frac{1}{2}R^2 = \frac{1}{4}RD$; reliquumq; Semicirculo (dempto hoc Triangulo) $\frac{1}{2}RP - \frac{1}{4}RD$; cui æquale auferendum est ex DPA (Quadrante Superficie Hemisphæræ Curvæ, æquali Semicirculo ABD) quo Residuum æquetur Exposito Triangulo ADC.

Quod cum variis modis fieri possit; per ea quæ nos dudum docuimus Anno 1659. (ad Calcem Tractatus de Cycloide, tum editi, p. 122. inferenda ad §. 68.) iterumq; Anno 1670. (in Tractatus de Motu, Cap. V. Prop. 24.) de Figura Plana, æquali cuivis in Superficie Sphæræ Figuræ, Circulis quibuscvis (sive Maximis, sive Minoribus) terminatæ. Sic fiat simplicissime;

Fig. 21.

Cum Superficie Sphæræ Segmenta, Parallelis Planis Abscissa, sint Axis Segmentis proportionalia (quod de exposita quadrantalis Cunei Superficie DPA pariter valet:) Si sumatur, in Axe CP, ut Semicirculus, $\frac{1}{2}RP$, ad Semicirculum dempto Triangulo $\frac{1}{2}RP - \frac{1}{4}RD$; hoc est, ut P ad P - D; sic CP ad CY: (sive, quod tantundem est, ut P ad D; sic CP ad PY:) Planum per YZ Basi parallelum, abscindet hujus Superficie Curvæ portionem Polo adjacentem, æqualem Triangulo ADC. Quod cum in reliquis Superficie Curvæ Quadrantibus, pariter fiat; æquabitur totum Abscissum (Polo, adjacens) toti Quadrato Basi inscripto: Et sic Tensum ut oportuit: Q. E. F.

Vel sic brevius. Est Hemisphæræ Superficies Curva (utpote duobus Circulis Maximis æqualis) $= RP$; Quadratum Circulo Maximo inscriptum $= 2RR = RD$; Illudque ad hoc, ut Pad D. Adeoque (propter Segmenta Superficie Parallelis Planis abscessa, Segmentis Axis proportionalia) sumptis CP ad PY, ut Pad D, erit tum tota Superficies $= RP$; tum Portio ad Polum, plano ZY abscessa, $= RD$, Quadrato Basi inscripto. Q. E. F.

Sic dicatur; Processum hic esse ex præsumpti Circuli Quadratura, aut ratione quam habet Circuli Perimeter ad Diametrum: Id omnino verum est. Sed non est objiciendum. Quia non postulat Ænigma propositum ut Hemisphæræ Superficie Portiones abscessæ, (quas Fenestras vocat) sed ut Portio Superstes, si Tetragonismi capax. Et quidem si utrumq; postularet, postularet Circuli Quadraturam absolute Geometricam: quod haberi non posse satis constat.

Opificium quod spectat; super Basem Planam, extra Basem Hemisphæræ positam, sed ipsi contiguam, cujus duo Latera in Angulum coeant ad A, intra protractæ;

Fig. 21.

Fig. 21.

protractas DA , GA Rectas, (quo Fenestrarum quas vocat utrinque adjacentium liber prospectus pateat, non impeditus) extruatur Moles satis firma; ita quidem ut, assurgente Structura, promineat ejus acies, angulo suffulta, Circuli Arcum efficiens qualis est DZ , ad altitudinem Y assurgens. Et similiter ad reliquos Angulos D , B , G . Atque his demum Structuris (quasi totidem Columnis) ad eam Altitudinem proVectis, imponatur Testudo, sic intus excavata, ut possit Hemisphæricæ Superficies; Adeoque totum opus imperatum absolvitur.

Aliter. Idem fiet si, pro Quadrato Basi inscripto, exponatur Quadratum quodvis QQ , (quod minus sit quam Hemisphærica Superficies curva). Quippe si sumatur, ut RP (Hemisphærica Superficies curva) ad QQ (expositum Quadratum, sic) CP (Axis Hemisphærii) ad PY (Axis Portionem Polo adjacentem: Planum ZY (Basi parallelum) abscindet portionem Superficies Sphæricæ Tetragonismi capacem: Utpote æqualem exposito Quadrato QQ .

Idem sic aliter absolvi potest: sed majore sollicitudine.

Fig. 22.

Cum sit (ut jam ostensum est) Hemisphæricæ Superficies curvæ Quadrans DPA æqualis Semicirculo ABD ; ejusque Segmenta Planis Basi parallelis abscissa, Segmentis Axis proportionalia: Sumatur in DP Quadrantali arcu, arcus PQ graduum 60; (quod mihi Caswellus suggerit) Polo P descriptus Circulus QTS bifecabit Axem (propter Sinum Versum grad. 60 = $\frac{1}{2}R$;) adeoque Quadrantem Hemisphæricæ Superficies curvæ DPA dirimet in duo Segmenta inter se æqualia. Quorum alterum, $DQTSA$ Quadrilineum, æquat Quadrantem circulare BCD ; reliquumq; Trilineum $PQTS$ æquat Quadrantem ACD . Unde si porro auferatur $QRST$ Bilineum, æquale segmento Circuli ADE : reliquum Trilineum $PQRS$, æquabit ADC Triangulum. Taliaq; quatuor, in quatuor Quadrantibus Hemisphærii, æquabunt Quadratum Basi inscriptum. Habebitur autem illud Bilineum per ea quæ nos dudum docuimus locis modo citatis.

Idem universaliter sic fiet.

Sumpto Q ubivis in Arcu DZ , (ne major sit DQ quam DZ ;) & quanto deficit Quadrilineum $DQTSA$ à toto auferendo, tantundem suppleat Bilineum $QRST$: Reliquum æquabit ADC Triangulum.

Et quidem, si sumatur Q in D (quo evanescat Quadrilineum) sumendum erit Bilineum æquale toti auferendo. Sin sumatur Q in Z (ut Bilineo non sit opus) æquabitur Quadrilineum toti auferendo.

Eademque omnia (de Quadrilineo & Bilineo quæ simul compleant totum auferendum) pariter accommodanda erunt (mutatis mutandis) si, pro Quadrato Basi inscripto, substituatur QQ Quadratum quodvis; quod tota Superficie curva Hemisphærica non sit majus.

Postquam hæc scripta fuerant, erantque sub prelo, rescivi tandem huic eadem Problemati responsum dedisse Cl. V. D. Leibnitz, illudque in Actis Lipsicis comparere pro Mense Junio, 1692. Quod fecit ut Prelum sufflammandum curaverim per aliquot septimanas donec illud conspicerem; quod ægre tandem obtinui. Videoque Cl. Virum juxta mecum sentire, non esse Problema Determinatum, sed mille modis (nedum infinitis) solubile.

3. *Enigmatis hujus Author Problematis tandem Constructionem ingeniose admodum & expedite dedit in Tractatu Italico, de Formatione & Mensura Testudinum omnium, ad Sereniss. Etruria Principem, ubi & Nomen suum profiteri dignatus est, nempe à V. V. Postremo Galilei Discipulo, cum antea dispositis horum verborum ut in Anagrammate elementis, sub ficto nomine D. Pio Lisci Pussillo Geometra tectus latuisset.*

The Proposer's
Solution Demon-
strated, by Dr.
D. Gregory.
N. 207 p. 24.
Jan. An. 1694.

Enigma verum in sequens Problema ab Authore convertitur: *Super Hemisphaerii superficiem, assignare portionem dato quadrato aequalem: Quod sic construitur.*

Sphæra cujus Axis æqualis lateri dati Quadrati exponatur per Circulum $ACBD$ in proposita Sphæra verticalem, cujus Diameter Horizontalis est AB , centrum E . Perforetur Sphæra duobus Cylindris rectis, quorum communes Sectiones cum plano $ACBD$ sunt Circuli $BLEG$, $AHEI$, diametris EB , EA descripti. Dico factum: hoc est à quolibet Hemisphaerio *v. g.* superiori ACB ablatas esse per Cylindros perforantes quatuor Figuras bilineares, duas scilicet in parte antica & duas in postica æquales similes & similiter positas, ita ut residua superficies Hemisphærica sit æqualis Quadrato rectæ AB . Et quoniam Hemisphærica Superficies, demptis spatiis quatuor Bilinearibus prædictis, refert Velum vento Inflatum & Tensum, Testudinemve Hemisphæricam quatuor Fenestris interruptam, quæ Circulari basi AEB imposita, ipsi ad puncta A , E , E , B , innititur, hanc pro jure suo appellat *Testudinem Veliformem Florentinam Quadrabilem*.

Fig. 22.

Auctor deinceps in memorato Tractatu plurima ad Praxin attinentia profert, ut ope Torni & Terebræ Cylindricæ tam hujus quam reliquarum quinque Testudinum fiant exemplaria: Atque in hanc rem alia quædam Problemata subtilia construit quorum omnium demonstrationes ab Auctore consulto omisissæ facillime ex nunc proferendis consequentur.

Quod quatuor Fenestræ in Hemisphaerio, ut dictum est, extructæ sint Figuræ æquales similes & similiter positæ, satis liquet, reliquum est ut ostendamus reliquam superficiem Hemisphæricam Tetragonismi vere Geometrici esse capacem.

Ad planum $CADB$, in puncto E , erigi intelligatur Normalis recta æqualis EA , & super Peripheriam $ACBD$ superficies Cylindrica recta ejusdem Altitudinis. Vulgo notum est portionem Superficiei Sphæricæ inter quolibet duo plana Circulo $ACBD$ Parallela comprehensam, æqualem esse portioni Superficiei Cylindricæ inter eadem plana; & horum Annulorum similes portiones resectas à planis in erecta ex E , Normali, se mutuo inter secantibus esse etiam æquales. Si jam ducendo innumera plana Basi $ACBD$ parallela dicto modo designari intelligantur in superficie Cylindrica partes respondentibus Sphæricis æquales, quæ è regione Superficiei perforatione ablata designatur illi æqualis est. Quare patet residuam à perforatione Superficiem æqualem esse residuæ superficiei Cylindricæ dempta illa quæ è Regione ablata per dicta innumera plana designatur. Ducatur Diameter quilibet PM , secans Peripheriam AHE utcumque in H . Jungatur HA , per H ducatur RT normalis ad AB , & parallela ad CD per E ductam, occurrens Peripheria

ACBD in R & T, & Peripheriæ AIE in I. Super R T Diametro fiat Semicirculus cujus Peripheriæ occurrant H S, I Q ad R T normales S & Q. Hujus Semicirculi Planum intelligatur normaliter erectum ad Circulum ABCD. Unde Peripheria RSQT erit in Superficie Hemisphærica, rectaque HS nunc ad Planum ACBD normalis, erit Altitudo Superficiæ Cylindricæ perforantis supra Baseos punctum H. Idemq; de quolibet puncto superficiæ Cylindricæ perforantis verum est, *scil.* ejus altitudinem usque ad Superficiem Sphæræ supra quodvis in Basi punctum H, esse rectam HS, ut dictum est, genitam, sed HS æqualis est HA sinui recto Arcus MA, quoniam tam hæc quam illa est Media Geometrica inter PH & HM, altera in Circulo MAP, altera in Circulo Sphæræ etiam Maximo per puncta M, S, & P, transeunte.

Si in erecta in E ad planum ACBD normali, ab E sumatur recta æqualis HS aut HA, & ab extremo ejus puncto ducantur rectæ Parallelæ ad PM & VN, planum per illas extensum erit ad Planum ACBD parallelum, & rectæ hæc per puncta S & Q transibunt, & productæ usque ad Superficiem Cylindricam Hemisphærio circumscriptam abscindunt ex lateribus Cylindri rectas ipsi HS vel HA itidem æquales; comprehenduntque Arcus æquales & respondentes Arcubus MN & VP. Quod si alterum planum huic ad minimam distantiam parallelum similiter ductum intelligatur, hæc duo per supra ostensa designabunt in Superficie Cylindrica annuli portionem æqualem portioni inter eadem plana à Superficie Hemisphærica perforatione ablata. Quod si similis Constructio fieri supponatur ad quodlibet in Peripheria AHE, punctum portiones omnes in Superficie Cylindrica Hemisphæricæ circumscripta dicto modo genitæ & designatæ erunt æquales Superficiæ Sphæræ perforatione ablata. Quare residua Superficies Hemisphærica æqualis erit reliquæ superficiæ Cylindricæ conflata ex rectis omnibus HA ad respectiva puncta M, N, V, & P erectis, seu figuræ sinuum rectorum Semiperipheriarum ACB, ADB, hoc est, per dudum à Geometris cognita, quadruplo Quadrato Radii AE, sive denique quadrato Diametri AB. Cumque duæ integræ Figuræ comprehensæ à communi sectione prædictæ Superficiæ Cylindricæ perforantis cum Superficie Sphærica, æquales sint quatuor semissibus earundem, patet residuam superficiem Hemisphæricam ACB, ablatis quatuor spatiis Bilinearibus (ut supra in Constructione) æqualem esse quadrato Diametri AB, Q. E. D.

Si Semiperipheria AHE, ita inflectatur ut congruat cum æquali quadrante Peripheriæ ARC; punctum H incident in punctum M ob æquales Arcus AH, AM, & HS altitudo ad H superficiæ Cylindricæ super AHE insistentis congruet cum æquali HA altitudine ad M Figuræ Sinuum rectorum super AMC erectæ; idemque in reliquis punctis fiet. Unde curva quæ est communis intersectio Superficiæ Sphæræ cum Superficie Cylindrica super Basi AHE, quamvis non jaceat in eodem Plano inflexa, ut dictum est, congruet, & proinde æqualis est curvæ terminanti figuram Sinuum rectorum; hoc est communi Sectioni Superficiæ Cylindricæ supra Quadrantalem Arcum ARC erectæ cum Plano secante Planum Baseos in recta BA ad Angulos semirectos; sive quadranti curvæ Ellipseos cujus minor Axis est AB, major vero potest hujus duplum. Adeoque Perimeter Veli Quadrabilis Florentini ex hujusmodi quatuor constans æqualis est Perimetro dictæ Ellipseos.

Sed & hoc amplius adnotare non pigebit, Superficies Cylindrorum duorum perforantium intra Sphæram, æquales esse Superficie Sphære post perforationem relictæ, five duplici Velo Florentino, hoc est duplo quadrato Diametri. Atque hoc exinde patet quod Velum Florentinum æquale sit Figuris quatuor sinuum rectorum Quadrantis & Superficies perforans iisdem etiam sit æqualis, quoniam illis congruit si inflectio fiat ut supra.

Hoc tantum addam, Considerationem Figuræ Sinuum rectorum (cujus etiam partes in Quadrata facile mutantur) sufficere ad Demonstrationem eorum omnium quæ de aliis solidis Torno elaboratis vel Cylindro perforatis, eorumque Superficiebus ab Acutissimo Geometra V. V. (Vicentio Viviani, ni fallor) Dignissimo Galilæi Discipulo proferuntur; dum Fabricam & Mensuram Testudinum docet. Speciatim Superficies Testudinis Scaphoidis Romanæ [*Volta a Schifo alla Romana*] ex octo Figuris Sinuum rectorum Arcus Quadrantalibus constat, ac proinde Testudini Veliformi Florentinæ æqualis est. Unde patet quomodo æqualibus quadratis superimponi possunt duæ Testudines, quarum altera est undique clausa, altera quatuor Fenestris interrupta, utraque Quadrati Baseos dupla.

VII. 1. Drawing the straight Lines EA , and EB (cutting the Arc AB in G), and on AG , a Perpendicular EF , (which will therefore pass to the Center C , because bisecting AG at Right Angles;) the Right-lined Triangle AFE , is equal to ADE , the proposed Portion of the Lunula.

The Quadrature of the Parts of the Lunula, by Mr. J. Perks, a little vary'd by Dr. J. Wallis, N. 259. p. 411. Dec. An. 1699. Fig. 23.

The Demonstration is to this purpose; viz. ADB being a Quadrantal Arc; the Angle AGB will be three Halves of a Right Angle; (and its conjunct Angle EGB , half a Right Angle) and that Angle (being external to the Triangle AGE , is equal to the two opposite Intervals $GEA + EAG$. Whereof GEA (because an Angle in the Semicircle AEB) is a Right Angle, and therefore EAG is half a Right Angle, (as are also FEG , and FEA) and the three Triangles AFE , GFE and GEA , each of them half a Square. And AG to AE , as $\sqrt{2}$ to 1, (proportional to the respective Radii of the two Circles.) And the like Segments ADG , AE , in their respective Circles (as the Squares of their respective Radii) as 2 to 1. And therefore the Semisegment AFD , equal to the Segment AE . And consequently (one taking from the Triangle as much as the other adds to it) the Portion of the Lunula ADE , equal to the Triangle AFE . *Q. E. D.*

If the Point E chance to be in K (the middle of the Arc AEB) there will be no Intersection at G (the Points G , B , being then coincident, but without any Disturbance to the Demonstration): If it happen beyond it, toward B ; then G will be on the other side; and what is here said of EGB , must be accommodated to EGB .

The Ground of the whole Process is plainly this; The Angle ACE , being an Angle at the Center of the greater Circle; but at the Circumference of the lesser, the Line CDE (as it passeth from CA to CB) doth in the same Proportion, divide the Quadrantal Arc ADB , and the Semicircular AEB : whence all the rest doth naturally follow.

Improved by Dr.
Gregory, *ibid.*
p. 414.
Dec. An. 1699.
Fig. 24.

2. If you compleat the two Circles, whose Arcs contain the *Lunula* of *Hippocrates*; the same is true as well of the Points in the other Semicircle $A C B$, as of those in the Semicircle $A E B$; and for the same Reasons. As appears in the Scheme annex'd, wherein I have mark'd the Points in the Semicircle $A C B$, (correspondent to those of Mr. *Perky* in $A E B$;) with the correspondent small Letters of the *Roman* and *Greek* Alphabets.

If Mr. *Perky* had made his Construction universal by making both $E A$ and $E B$, meet with the greater Circle, (which he might have done by protracting these Lines and the greater Circle till they meet;) he might have found that the Portions of the Spaces $A \cdot C M$, $B H C N$, (supposing $M C N$ Parallel to $A B$) are Quadrable as well as those of *Hippocrates's Lunula*: And that $E A \gamma$ being a streight Line, the Portion $A E D$ of *Hippocrates's Lunula*, is to $A \cdot s$ (the Correspondent of $A \cdot C M$) in duplicate Proportion of $C \cdot$ to $A \cdot$. For $E R \cdot$ (at R the Center of the lesser Circle) is in this Case, a Right Angle.

Moreover, If you take any Point ϵ , in the Semicircle $A C B$, and proceed according to Mr. *Perky's* Construction Universaliz'd as above said; you will find on the one side, the Trilineum $A \cdot s$ (contain'd by the Arcs $A \cdot$, $A s$, and the streight Line ϵs) equal to the Rectilineal Triangle $A \cdot e$. And on the other side, the Trilineum contained by the Arc $B \cdot$ (the Complement of $A \cdot$ to the Semicircumference,) and the Arc $B d$ (the Complement of $A s$ to the fourth Part of the Circumference,) and the streight Line ϵd , (that is the Trilineum $B H C d$ diminished by the Segment $C \cdot$;) to be equal to the Rectilineal Triangle $B \cdot f$. And that those two Spaces $A \cdot s$, and the difference of $B H C d$, from the Segment $C \cdot$ (Parts of the *Lunula* $A C B g \gamma A$) taken together, are equal to the Triangle $A C B$, as well as to the two Spaces $A E D$ and $B E D$, Parts of the *Lunula* of *Hippocrates*.

So that upon the whole it appears, that the two Circles (containing the *Lunula* of *Hippocrates*) being compleated, this *Lunula* $A E B G A$, and the other $A C B g \gamma A$, make up one System, and are conjugate Figures.

For, drawing a streight Line $C D E$, or $C \cdot s$, or $\epsilon C d$, at Pleasure, thro' C the Center of the greater Circle, and cutting those two Circles, the Space contained within two Arcs of these two Circles, and part of the said streight Line (as $A E D$, or $A \cdot s$, or $B H \cdot d$) is equal to the rectilineal Triangle $A E F$, or $A \cdot e$, or $B \cdot f$, respectively.

And it so happens, that, if this Line going out from C , be on the same side of the Diameter $M N$, with the *Lunula* of *Hippocrates*; the aforesaid Space (which receives a perfect Quadrature) is solitary; such as are the Parts of *Hippocrates's Lunula*, and of the two Spaces $A \cdot C M$, $B H C N$, which therefore are Parts of the *Lunula* more nearly relating to one another).

But if that Line going out from C , be on the other side of $M N$; then the Space which is equal to the Rectilineal Triangle, is the difference of two Mixtilineal Figures (the one a Trilineum, the other a Segment of the lesser Circle) as is abovesaid; neither of which can be squared severally.

All these Particulars are plain from Mr. *Perky's* Demonstration; which with a little Variation (such as is usual in the different Cases of the same Theorem) is applicable to them all; tho' perhaps he was not aware of it.

The

The like was done (without any Demonstration) by M. *Tschirnhaufe* in the *Acta Lipsie* 1687, to this purpose ; If from any Point E, in the Circumference of the lesser Circle, we let fall on AB, a Perpendicular Line cutting it in L, and draw the Line CL ; the Triangle CAL, is equal to the Portion of the *Lunula* AED. (And consequently the Triangle CBL, equal to the Portion BED.) Which I shall demonstrate, so as the Demonstration may also reach the Portions of the conjugate Space ACBg > A.

For the Triangles ACB, AEF, are like Triangles, each being the half of a Square : and therefore by 19 *El.* 6. the Triangle ACB is to the Triangle AEF, in the duplicate Proportion of BA to AE, that is, by 8 *El.* 6, as BA is to AL. But, by 1 *El.* 6, the Triangle ACB is to the Triangle ACL, as BA is to AL. Therefore by 9 *El.* 5, the Triangles ACL and AEF are equal. But the Triangle AEF is (by Mr. *Perks*) proved equal to the Portion AED. And therefore the Portion AED is also equal to the Triangle ACL.

3. On the Center B, Mr. *Caswell* draws by A, a third Circle, which forms another *Lunula* than that of *Hippocrates* : and he doth (very dexterously) square the Portions of this *Lunula*. And doth thereby let us in, to a new System, which may be pursued in like manner, as Dr. *Gregory* hath done that of *Hippocrates*. By Mr. Caswell
ib. p. 417.

4. M. *Tschirnhaufe*, letting fall, from E, (on AB) a Perpendicular EL, determines the Angle ALC, equal to the Portion ADE. Which being admitted, we may thus divide the *Lunula* in any given Proportion ; if we divide AB, at L, in such given Proportion ; CL will, in the same Proportion (because of the common Altitude) divide the Triangle ACB (which is equal to the whole *Lunula*.) And LE (erected at Right Angles on ALB) will determine the Point E ; from whence if we draw to C, the straight Line EC, this will, at DE, divide the *Lunula* in the same Proportion. By Dr. Wallis
ibid. Fig. 25

Mr. *Perks*, on EDC, drawing the Perpendicular AF, determines the Semicquare AFE, equal to the proposed Portion ADE. Which Semicquare, is a like Figure, and alike situate to AE, as is ACB to AB. Fig. 26.

And therefore (because like Figures are in duplicate Proportion of their respective sides) if we so inscribe AE, as that the Square of AE be to the Square of AB in such given Proportion, the *Lunula* will at DE be so divided as is required.

And this will hold (if duly apply'd, according as the different Cases may require) tho' E be taken (in the Continuation of the Semicircle) beyond A. For, still like Figures will be in duplicate Proportion of their respective sides ; and $CE = CD \pm DE$. And the same is yet improveable much further.

VIII. If upon BC you take any two Points D, E, and draw the Perpendiculars DH, EM, meeting BA in I and L, and cutting a Portion FGMH, of the *Lunula* ; the Solid generated by the Conversion of this Portion about the Axis BC, is equal to a Prism, whose Base is ILMH, and Height the Circumference of a Circle, whose Diameter is BC ; and the Solid generated by the The Dimension of
Solids generated
by the Conversion
of Hippocrates
Lunula by M.
Ab. de Moivre,
N. 264. p. 624.
Jul. An. 1700

Fig. 17.

the Semicircle BKA, is equal to a Prism or Semicylinder, whose Base is the Semicircle BKA, and Height, the Circumference of a Circle whose Diameter is BC.

Having bisected BA in R, and BC in P, the Surface generated by the Conversion of the Arc HM about the Axis BC, is equal to

$$- \times BP \times HM + BR \times DE \text{ (supposing the Ratio of the Radius to the Cir-}$$

cumference to be as r to c) and the Surface generated by the Semicircumference BKA is equal to a Rectangle, whose Base is the Summ of that Semicircumference and Diameter BA, and Height, the Circumference of a Circle, whose Diameter is BC. As for the Surface generated by the Arc GF, 'tis well known that it is equal to a Rectangle, whose Base is the Circumference of a Circle whose Radius is BC, and Height DE; therefore the Surface generated by the Conversion of the Portion MHFG, is known.

Fig. 18.

If upon BA, you take any two Points I, L, and draw IN, LV, perpendicular to it, cutting the Quadrant in O and T, and the Circumference in N and V, the Solid generated by the Conversion of the Portion ONVT about the Axis BA, is equal to a Prism, whose Base is IOTL, and Height the Circumference of a Circle whose Diameter is BA.

Having bisected BA in R, and drawn CR, meeting the Quadrant in G, the Surface generated by the Conversion of the Arc OT about BA, is equal to $- \times CG \times IL - CR \times OT$.

Fig. 17.

Bisect DE in r , thro' the Center draw SQ, parallel to BC, meeting the Circumference BKA in S, BK parallel to AC in V, and the Lines DH, EM, in N and O; the Solid generated by the Conversion of the Portion FGMN

about the Axis AC, is $- \times \frac{1}{3} MO^3 - \frac{1}{3} NH^3 + PC \times NOMH +$
 $CY \times DNOE - \frac{1}{3} EG^3 + \frac{1}{3} DF^3$, and the Solid generated by the Segment

KBS is $- \times \frac{1}{3} VK^3 + PC \times BVKS$. Therefore the Solid generated by

the Semicircle BKA about AC is $- \times PC \times VQAK + PC \times BCQV$
 $- \frac{1}{3} AC^3 + \frac{1}{3} VK^3 + PC \times BVKS$, which by due Reduction will be found equal to the Solid generated by the Conversion of the same Semicircle about the Axis BC.

Fig. 18.

The Solid generated by the Portion ONVT, about the Axis CP, is equal to $- \times \frac{1}{3} LV^3 - \frac{1}{3} IN^3 - \frac{1}{3} QT^3 + \frac{1}{3} PO^3 + CS \times PQIL$.

Fig. 17.

From the Points M, H, drop the two Perpendiculars MZ, HW, upon CA prolong'd, if need be; the Surface generated by the Conversion

fion

fion of the Arc HM, about the Axis CA is equal to $\frac{r}{r} \times \overline{PC} \times \overline{HM}$

$-\overline{RA} \times \overline{WZ}$, when the Point Z is next to C, or $\frac{r}{r} \times \overline{PC} \times \overline{HM} + \overline{RA} \times \overline{WZ}$,

when the Point W is next to it.

Those that will think it worth their while to bestow some little Pains to find the Demonstration of this, may solve the following Problem.

Any two Conic Sections, being given, forming a Lamula by their Interfection, and a Right Line being given by Position, about which, as an Axis, this Lamula is imagined to turn, to find the Solids generated by the Conversion of any of its Parts, cut off by Lines Perpendicular to that Axis, or Parallel to it, or making any given Angle with it, as also the Surfaces made by that Conversion.

IX. Suppose DPV, to be half of an exterior Epicycloid, VB its Axis, V the Vertex, VLB half the generant Circle, E its Center; DB the Base, C its Center: Bisection the Arc of the Semicircle VB in L, and on the Center C thro' L, draw a Circle cutting the Epicycloid in P: Then, I say, the Curvilinear Triangle VLP will be $= BE \cdot q \text{ in } \frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle, will be to the Curvilinear Triangle VLP, as CB the Semidiameter of the Base to CE: Which CE in the Exterior Epicycloid is the Summ of the Semidiameters of the Base and Generant, but in the Interior Epicycloid DPV, 'tis the difference of the said Semidiameters.

The Quadrature of a Portion of the Epicycloid, by Mr. Caswell. N. 217, p. 113. Oct. An. 1695. Fig 29.

COROL. I.] In the Interior Epicycloid, if CE is $\frac{1}{2}$ CB, the Epicycloid then degenerating into a Right Line, the Quadrature of the Triangle LPV, will be in effect the same with the Quadrature of *Hypocrates Chius*.

COROL. II.] If the Semidiameter of the Base is supposed infinite, the Epicycloid then being the common Cycloid, the Area of the said Triangle will be equal to the Square of the Radius of the Generant; and so it falls in with that *Theorem* which *Lalouera* found and calls *Mirabile*.

The general Proposition from whence I deduced the above said Quadrature, is this; viz. the Segments of the generant Circle are to the Correspondent Segments of the Epicycloid, as CB, to $2 \cdot CE + CB$. For Example. Suppose FmG, the Position of part of the Generant, when the Point F of the exterior Epicycloid, was designed, then the Segment FmG is to the Segment DFmG, as CB to $2 \cdot CE + CB$. And consequently the whole Epicycloid to the whole Generant in the same Proportion; which is the only Case demonstrated by M. de la Hire.

It follows also, that in the vulgar Cycloid, its Segments are triple of the correspondent Sectors of the Generant, which was first shewn by Dr. Wallis.

X. Area.

A general Proposition for measuring all Cycloids and Epicycloids, by Mr. Edm. Halley, N. 218. p. 125.

X. *Area Cycloidis vel Epicycloidis, sive Primaria, sive Contracta vel Prolata, est ad Aream Circuli Generantis; atque etiam Area partium genitarum in iisdem Curvis, ad Areas analogorum segmentorum Circuli, ut summa dupla Velocitatis centri ac Velocitatis motus Circularis, ad Velocitatem motus Circularis.*

Fig. 30.

Demonstratio.] Describatur Epicyclois quævis $YPSQVB$, revolutione circuli VLB , super Basi circulari $YMN B$; ponatur centrum circuli generantis in c , ductaque $cM K$, infistat circulus Basi in puncto M ; sitque punctum lineans S . Jam divisis motibus transferatur primum motu circulari punctum S in R , ut augeatur arcus SM particula indivisibili RS ; deinde progrediatur centrum c in C ; hoc motu, traducto segmento $RS M$ in situm QTN , punctum Q tanget Curvam. Patet triangulum RSM esse momentum sive fluxionem Areæ segmenti Circuli: Trapezium vero $QSMN$ esse Fluxionem Areæ spatii curvilinei simul geniti. Jam cum SM , RM , QM , non nisi puncto inter se deferre intelligantur, concipe Areolam $QSMN$ constare ex tribus sectoribus RMS , RMQ , MQN ; adeoque Areolam RMS , esse ad Areolam $QSMN$, ut est angulus RMS ad summam trium angulorum $RMS + RMQ + MQN$. At anguli $RMQ + MQN$, æquantur angulis $MCN + MKN$, sive angulo cMC ; propter lineas RM , QN invicem inclinatis sub angulo ipsi MKN æquali ac propter angulum MQN ipsius MCN dimidium (per *Eucl.* 3. 20.) Proinde angulus RMS est ad angulos $RMS + cMC$, hoc est (per eadem 3. 20.) arcus $\frac{1}{2} RS$, ad duos arcus $Cc + \frac{1}{2} RS$, sive RS ad $2 Cc + RS$, ut areola RSM ad areolam $QSMN$: sive ut momentum segmenti circularis QTN , ad momentum segmenti in Epicycloide simul geniti $QSYM N$. Cumque hæc momenta semper sint in eadem illa ratione, ubicunque assumpseris punctum Q , constat Areas ipsas QTN , $QSYM N$ his momentis genitas, eandem habere constantem rationem, nempe velocitatis motus circularis RS , ad duplam velocitatem centri addito motu circulari, sive $2 Cc + RS$. Sicut etiam Aream VBZ ad Aream $QVBN$, ac proinde semicirculum VLB ad spatium Curvilineam $VQYNB$. Ergo constat propositio. Nulla autem alia est differentia in modo demonstrandi, si Circulus genitor super Arcu Basis Concavæ moveatur, nisi quod angulus cMC , hoc in casu, sit differentia angulorum MCN , MKN . Si vero Basis sit linea recta, evanescente MKN , ac ob RM , QN parallelas, etiam facilius erit probatio. In omnibus autem hujus modi Curvis portiones analogæ portionibus illis quas in Cycloide primaria perfectæ Quadraturæ capaces invenit *Cl. Wallisius*, sunt æque quadrabiles, quod quidem facile consequitur ex præmissis.

Centro K , per punctum Q duc circulem arcum QZ , ac age ZB abscindens segmentum ZLB , æquale segmento QTN , dein biseca semicirculum VB in L , ac per punctum L , centro etiam K , describe arcum PL , secantem Epicycloidem in P , circulum Genitorem in T , ac Chordas QN , ZB in y & x . Jam sit Arcus $VZ = a$, ejusque sinus $= s$, Radius Genitoris $= r$, Radius vero Basis $= R$; sitque arcus CE , sive motus centri $= m$.

Patet

Patet Sectorem CKE eam rationem habere ad spatium X γ N B, quam habet quadratum ex KE, ad differentiam quadratorum ex KL & KB; sive ut $RR + 2Rr + rr$ ad $2Rr + 2rr$; hoc est ut $R + r$ ad $2r$, vel KE ad BV; ac proinde rectangulum BE in CE sive rm æquari spatio X γ N B.

Spatium vero VZB æquale est rectang. $\frac{1}{2}ar + \frac{1}{2}sr$; adeoque juxta nostram propositionem, erit ut a ad $2m$, ita $\frac{1}{2}ar + \frac{1}{2}sr$ ad $\frac{mar + msr}{a}$,

æquale spatio Curvilineo QVZLBNQ: Ex hoc subduc spatium

X γ N B = rm , & remanebit spatium QVZX γ = $\frac{mrs}{a}$: Cumque spatia

ZXL, Q γ T æquantur inter se, spatium QVLTQ etiam æquabitur ipsi

$\frac{mrs}{a}$: Quoties itaque a ad m , sive motus circularis ad progressivum Centri,

fuerit in data ratione, dabitur etiam perfecta Quadratura spatiorum Curvilinearum QVLTQ: Totumque spatium VPL, ad Quadratum Radii BE, erit in eadem ratione motuum m ad a , hoc est, in omni Epicycloide primaria, in ratione radiorum KE ad KB, quæ est ipsa D. Caswelli Propositio. Spatia autem minora QVLTQ erunt inter se ut Sinus arcuum VZ; ac spatia Triangularia QTP, eodem argumento erunt ut Sinus Versi arcuum QT vel ZL: ac proinde etiam quadrantur. Pari modo probabuntur spatia P Λ T, pLn, p Λ T, semper esse ad Radii BE quadratum (in omnibus his Figuris) in prædicta ratione m ad a ; eorumque portiones pqt, ut Sinus Versi arcuum interceptorum qt. Residua autem segmenta, ut qtTA, qtTA, &c. erunt ut sinus recti complimentorum eorundem arcuum qt.

Componitur autem ratio velocitatum m ad a , ex ratione radiorum KE, BE, ac ratione angulorum simul æquabiliter descriptorum CKE, VEZ: ac proinde data etiam illa angulorum ratione, etiam Quadrabuntur spatia omnia Epicycloidalia prædicta.

XI. 1. *Queritur Curva ejus Proprietatis, ut duo Segmenta (lineæ rectæ à dato puncto per curvam dictæ) ad quamcunque potentiam datam elevata & simul sumpta faciant ubique unam eandemque summam.* Generalem solutionem, Analysis eruendam relinquimus.

A Problem proposed by M. J. Bernouilli. N. 224. p. 387. Jan. An. 1697.

2. Problema (si recte intellexi) sic proponi potest. Queritur Curva KIL ea lege ut sit recta PKL à dato quodam puncto P, ceu Polo utcunque ducatur, & eidem Curvæ in punctis duobus K & L occurrat, potestates duorum ejus segmentorum PK & PL, à dato illo puncto P ad occursum illos ductorum si sint æque altæ (id est vel quadrata, vel cubi vel quadrato-quadrata, &c.) datam summam PK q + PL q , vel PK $^{cub.}$ + PL $^{cub.}$ &c. (in omni rectæ illius positione) conficiant.

Solved by H. N. ibid. p. 389. Fig. 31.

SOLUTIO.] Per datum quodvis punctum A, ducatur recta quævis infinita positione data ADB, recte mobili PKL occurrens in D, & nominentur AD, x , & PR, vel PL, y , sintque Q & R quantitates ex quantitatibus quibuscunque datis & quantitate x quomodocunque constantes, & relatio inter x & y definiatur per hanc æquationem $YY + QY + R = 0$. Et si R

fit quantitas data, Rectangulum sub segmentis PK & PL dabitur. Si Q fit quantitas data, summa segmentorum illorum (sub signis propriis conjunctorum) dabitur. Si $QQ - 2R$ datur, summa quadratorum ($PKq + PLq$) dabitur. Si $Q^3 - 3QR$ data sit quantitas, summa cuborum ($PK^{cu} + PL^{cu}$) dabitur. Si $Q^4 - 4QQR + 2RR$ data sit quantitas, summa quadrato-quadratorum ($PKqq + PLqq$) dabitur. Et sic deinceps in infinitum. Efficiatur itaque ut R, Q, $QQ - 2R$, $Q^3 - 3QR$, &c. datae sint quantitates, & Problema solvetur. Q. E. F.

Ad eundem modum Curvæ inveniri possunt quæ tria vel plura abscindent segmenta similes proprietates habentia. Sit æquatio $Y^3 + QY + R + S = 0$; ubi Q, R & S, quantitates significant ex quantitatibus quibuscunque datis, & quantitate x utcunque constantes; & Curva abscindet segmenta tria. Et si S data sit quantitas, contentum solidum illorum trium dabitur. Si Q fit quantitas data, summa trium illorum dabitur. Si $QQ - 2R$ fit data quantitas, summa quadratorum ex tribus illis dabitur.

The Use of Fluxions in the Solution of Geometrick Problems, by Mr. Abr. de Moivre. N. 216. p. 52. March, Anno 1695.

XII. Habes hic Methodum de Figurarum Curvilinearum Quadraturis; de Solidorum a Rotatione Plani genitorum eorumque superficierum dimensione; de Rectificatione Curvarum, deque Centri Gravitatis Calculo. Priusquam, verum, ulterius progrediar, hoc te monitum velim me usurpare illa, quæ demonstravit Clarissimus *Newtonus*, in pag. 251, 252 & 253. *Prin. Phil.* circa Momentanea Incrementa vel Decrementa Quantitatum quæ Fluxu continuo

crescunt vel decrefcunt, præsertim quod dignitatis cujuscunque $A^{\frac{n}{m}}$, Momentum fit $\frac{n}{m} a A^{\frac{n}{m}-1}$.

Porro data Fluxione $\frac{n}{m} a A^{\frac{n}{m}-1}$ vicissim reperiri potest Quantitas Fluens

$A^{\frac{n}{m}}$, 1^o tollendo a de Fluxione, 2^o Fluxionis Indicem Unitate augendo, 3^o denique Fluxionem dividendo per Indicem sic Unitate auctum.

Curvæ abscissa designabitur deinceps per x , ejus Fluxio per $\dot{x}$, ordinatim applicata per y , ejusque Fluxio per $\dot{y}$.

His positis ut ad Quadraturas deveniamus. 1^o assumatur valor ordinatim applicatæ ope Æquationis naturam Curvæ exprimentis. 2^o Multiplicetur hic valor per Fluxionem abscissæ; rectangulum hinc ortum erit Fluxio Areae. 3^o. Data Fluxione Areae reperiat quantitas Fluens, habebitur Area quæsitæ.

Proponatur æquatio $x^n = y^n$ cujuscvis Paraboloidos naturam exprimens, valor ordinatim applicatæ y est $x^{\frac{n}{n}}$, qui si multiplicetur per x Rectangulum

$$x^{\frac{n}{n}} x$$

$x^{\frac{m}{n}}$ erit Fluxio Areæ, proindeque Area quaesita erit $\frac{n}{m+n} x^{\frac{m}{n}+1}$,

seu (posito y pro $x^{\frac{m}{n}}$) $\frac{n}{m+n} xy$.

Rursum proponatur Curva cujus Aequatio sit, $x^4 + aaxx = yy$ (illa scilicet quæ inter Exempla *Cl. Craigi* extat prima) assumpto $x\sqrt{xx+aa} = y$, Fluxio Areæ erit $xx\sqrt{xx+aa}$; cum autem sub Radicalitate involvatur, supponatur $\sqrt{xx+aa} = z$, hinc $xx+aa = z^2$, ideoque $xx = z^2 - aa$; positisque zz & z pro xx & $\sqrt{xx+aa}$, Fluxio à Surdis liberata erit z^2z , quam si ad Originem suam $\frac{1}{3}z^3$ revocaverimus, repositoque $\sqrt{xx+aa}$ pro z , habebitur $\frac{1}{3}xx+aa\sqrt{xx+aa}$ pro Area quaesita.

Sed quo magis constet quam facili negotio conficiantur hujusmodi Quadraturæ, unum amplius Exemplum proferre visum est; Aequatio Curvæ talis

fit $\frac{x^2}{x+a} = y^2$, igitur $y = \frac{x}{\sqrt{x+a}}$, ideoque $\frac{xx}{\sqrt{x+a}}$ est Fluxio

Areæ: supponatur $\sqrt{x+a} = z$, hinc $x = zz - a$, & $x = 2zz$, itaque

$\frac{xx}{\sqrt{x+a}} = 2z^2z - 2az$, ac proinde $\frac{1}{3}z^3 - 2az$ seu $\frac{1}{3}x - \frac{1}{3}a\sqrt{x+a}$ erit Area quaesita.

Verum sæpe accidit ut quædam Curvæ, quales Circulus, & Hyperbola, ejus naturæ sint, ut frustra tentaveris earum Fluxiones Surdis immunes facere; tunc valore ordinatæ in Seriem infinitam conjecto singulisque hujus Seriei Terminis Fluxionem abscissæ, ut supra, multiplicatis, reperiatur singulorum Terminorum Quantitas Fluens, orietur nova Series quæ Quadraturam Curvæ exhibebit.

Methodus hæc eadem facilitate ad Dimensionem Solidorum à Plani circumvolutione genitorum accommodatur, nempe assumendo pro eorum Fluxione productum ex Fluxione abscissæ per Circulum Basis; Ratio Quadrati ad Circulum sibi inscriptum vocetur $\frac{n}{1}$, Aequatio Circulo competens est $yy = dx - xx$,

igitur $4 \frac{dxx - x^2}{n}$ est Fluxio Portionis Sphæræ, igitur $4 \frac{\frac{1}{2}xdx - \frac{1}{2}x^2}{n}$

est Portio ipsa, huic circumscriptus Cylindrus est $4 \frac{dxx - x^2}{n}$, ideoque Ratio Portionis Sphæræ ad circumscriptum Cylindrum est ut $\frac{1}{2}d - \frac{1}{2}x$ ad $d - x$.

Rectificatio Curvarum obtinebitur si Hypothenuſa Trianguli Rectanguli cujus latera ſunt Fluxiones abſciſſæ & ordinatæ tanquam Curvæ Fluxio conſideretur, ſed curandum eſt ut in expreſſione iſtius Hypothenuſæ, alterutra Fluxionum ſolummodo ſuperſit, ac una tantum Indeterminatarum, illa ſcilicet cujus Fluxio retinetur. Res Exemplis clarior fiet.

Fig. 32.

Ex dato ſinu recto CB, arcum AC invenire, poſitis AB = x, CB = y, OA = r; ſit CE Fluxio abſciſſæ, ED Fluxio ordinatim applicatæ, CD

Fluxio Arcus CA; ex Circuli proprietate $2rx - xx = yy$, unde $2rx$

$- 2xx = 2yy$, ideoque $\dot{x} = \frac{yy}{r-x}$, ſed $CDq = \dot{y}y + \dot{x}x = \dot{y}y$

$+ \frac{y^2 \dot{y}y}{rr - 2rx + xx} = \dot{y}y + \frac{y^2 \dot{y}y}{rr - yy} = \frac{rr \dot{y}y}{rr - yy}$, igitur $CD =$

$\frac{ry}{\sqrt{rr - yy}}$, ſed $\frac{ry}{\sqrt{rr - yy}}$ factum eſt ex $\frac{1}{\sqrt{rr - yy}}$ ſeu $\frac{1}{rr - yy}^{\frac{1}{2}}$

in ry; proindeq; ſi $\frac{1}{rr - yy}^{\frac{1}{2}}$ conjiciatur in Seriem infinitam cujus ſingula membra per ry multiplicentur, & ex unoquoque producto ad Quantitatem Fluentem fiat retrogreſſus, habebitur Longitudo arcus AC.

Non abſimili modo ex dato Sinu Verſo reperietur idem arcus; Reſumatur

æquatio ſupra inventa $2xx - 2x\dot{x} = 2yy$, ſit $\dot{y} = \frac{rx - xx}{2y}$, ſed

$CDq = \dot{x}x + \dot{y}y = \dot{x}x + \frac{rrxx - 2rxx\dot{x} + x^2\dot{x}\dot{x}}{yy} =$

$\dot{x}x + \frac{rrxx - 2rxx\dot{x} + x^2\dot{x}\dot{x}}{2rx - xx}$, ſeu (omnibus ſub eodem denominatore reductis, expunctiſque iis quæ ſub diverſis ſignis continentur)

$= \frac{rxx}{2rx - xx}$, unde $CD = \frac{rx}{\sqrt{2rx - xx}}$, ideoque Longitudo arcus

AC, per ea quæ jam dicta ſunt facile obtinebitur.

Fluxio curvæ facilius interdum reperitur per comparationem inter Triangula ſimilia CED, CBO, inſtitui enim poteſt hæc proportio CB : CO ::

CE : CD, hoc eſt, pro Circulo $\sqrt{2rx - xx} : r :: x : \frac{xx}{\sqrt{2rx - xx}}$

Fig. 33.

Curva Cycloidis eadem opera cognosci poterit. Sit ALK ſemicyclois, cujus circulus genitor ADL. Aſſumpto in diametro AL quovis puncto B, ducatur BI parallela Baſi LK, peripheriæ circuli in puncto D occurrens; compleatur

pleatur rectangulum AEIB, ducaturque FH erectæ EI parallela, idemque infinite vicina, BI productam secans in G, curvamque AK in H; ponatur $AL = d$, $AB = EI = x$, $GH = \dot{x}$; Notum est rectam BG esse ubique aggregatum arcus AD & sinus recti BD, hinc manifestum est Fluxionem IG esse aggregatum Fluxionum arcus AD & sinus recti BD. Porro

Fluxio arcus AD reperta est $= \frac{\frac{1}{2} dx}{\sqrt{dx - xx}}$, Fluxio autem Sinus Recti

BD reperiatur $= \frac{dx - 2xx}{2\sqrt{dx - xx}}$, igitur IG $= \frac{dx - xx}{\sqrt{dx - xx}}$, ideoque

IHQ = IGq + GHq $= \frac{ddx - dxx}{dx - xx}$; Quamobrem IH =

$\frac{x \sqrt{dd - dx}}{\sqrt{dx - xx}} = \frac{x \sqrt{d}}{\sqrt{x}} = d^{\frac{1}{2}} x^{-\frac{1}{2}}$, proindeque AI $= 2 d^{\frac{1}{2}} x^{\frac{1}{2}} = 2 \sqrt{dx} = 2 AD$.

Hæc conclusio minimo cum labore deduci potest ex nota proprietate Tangentis, cum enim illius portiuncula IH semper sit parallela chordæ AD, fit ut Triangula IGH, ABD sint similia, unde AB : AD :: GH : IH, hoc

est $x : \sqrt{dx} :: \dot{x} : \frac{x \sqrt{dx}}{x}$, igitur IH $= \frac{x \sqrt{dx}}{x} = d^{\frac{1}{2}} x^{-\frac{1}{2}}$.

Sed nihil vetat quo minus adhibito Fluxionis IH auxilio, ipsam Cycloidis aream investigemus. Fluxio Areæ AEI est rectangulum EIG

$= \frac{dxx - x^2 \dot{x}}{\sqrt{dx - xx}} = x \sqrt{dx - xx}$, sed Fluxio portionis ABD non alia est ab illa: Itaque Area AEI, correspondensque circuli portio ABD, semper sunt æquales.

Esto AB curva Parabolæ cujus Axis AF, Parameter a; Ponatur AE = x, EB = y, AB = z, BD = \dot{x}, DC = \dot{y}, BC = z, assumpta æquatione Parabolæ naturam constituyente, videlicet $ax = yy$, fit $ax =$

$2yy$, unde $\dot{x} = \frac{2y\dot{y}}{a}$, sed BCq = BDq + CDq, hoc est $z\dot{z} =$

$\dot{x}\dot{x} + \dot{y}\dot{y} = \frac{4y^2\dot{y}\dot{y}}{aa} + \dot{y}\dot{y} = \frac{4y^2\dot{y}\dot{y} + aa\dot{y}\dot{y}}{aa}$, ideoque $\dot{z} =$

$\dot{y} \frac{\sqrt{4y^2 + aa}}{a}$ vel, quod idem est, $\dot{z} = \dot{y} \frac{\sqrt{y^2 + \frac{1}{4}aa}}{\frac{1}{2}a}$: si ergo

$\dot{y} \frac{\sqrt{y^2 + \frac{1}{4}aa}}{\frac{1}{2}a}$, in seriem infinitam transformetur, Curva AB haud difficulter innotescet.

Insuper

Fig. 34.

Insuper statim apparet dato Hyperbolico spatio curvam hanc dari, & vicissim. Nam $\frac{1}{2} a z = y \sqrt{y^2 + \frac{1}{4} a a}$, ac proinde $\frac{1}{2} a z =$ spatio cujus Fluxio est $y \sqrt{y^2 + \frac{1}{4} a a}$, sed hujusmodi spatium nihil aliud est quam Hyperbola æquilatera exterior ABEG, cujus semiaxis AB = $\frac{1}{2} a$, abscissa AE = y , ordinatim applicata EG = x .

Ad dimetiendam superficiem conversione curvæ circa suum Axem descriptam, assumi debet pro ejus Fluxione Cylindrica superficies cujus altitudo est ipsa curvæ Fluxio, cujusque distantia ab Axe est ordinatim applicata huic Fluxioni conveniens.

Fig. 32.

Sit ex. gr. AC Circuli Arcus qui circa Axem AD revolvendo superficiem Sphæricam generet, quamque dimetiri statuamus; DC arcus Fluxio jam re-

perta est $\frac{r x}{\sqrt{2 r x - x x}}$, hanc si multiplicemus per Circumferentiam ad Radium BC pertinentem, hoc est $\frac{c}{r} \sqrt{2 r x - x x}$ (posita ratione Circumferentiæ ad Radium = $\frac{c}{r}$) habebimus Fluxionem superficiæ Sphæricæ = $c x$; adeoque superficies ipsa est $c x$.

Ad centra gravitatis quod attinet, reperta Superficiæ Solidivæ Fluxione, hacque ducta in suam à Vertice distantiam, ad quantitatem Fluentem recurrendum est: qua divisa per Superficiem ipsam Solidumve ipsum, prodibit distantia Centri Gravitatis à Vertice.

Inveniendum sit Centrum Gravitatis omnium Paraboloidum horum Fluxio

sic generaliter exprimitur $x^{\frac{m}{n}} x$, hanc multiplica per x , fit $x^{\frac{m}{n} + 1} x$ cujus quantitatem Fluentem $\frac{n}{m + 2n} x^{\frac{m}{n} + 2}$, divide per Paraboloidos Aream, puta

$\frac{n}{m + n} x^{\frac{m}{n} + 1}$; prodibit $\frac{m + n}{m + 2n} x$, distantia Centri Gravitatis à Vertice.

Centrum Gravitatis in Portione Sphære eodem modo colligitur, namque il-

lius Fluxione $4 \frac{d x x - x^2 x}{n}$ in x ducta, fit $4 \frac{d x^2 x - x^3 x}{n}$, cujus quan-

titas Fluens $4 \frac{\frac{1}{3} d x^3 - \frac{1}{4} x^4}{n}$ per Portionis Soliditatem divisa, puta

$4 \frac{\frac{1}{3} d x x - \frac{1}{4} x^3}{n}$, exhibet $\frac{\frac{1}{3} d - \frac{1}{4} x}{\frac{1}{3} d - \frac{1}{4} x} x$, seu $\frac{4 d - 3 x}{6 d - 4 x} x$ Distantiam Centri Gravitatis à Vertice.

XIII. 1. Prop. I. Probl.] *Invenire relationem inter Fluxionem Axeos & Fluxionem Ordinatae in Curva Catenaria.*

The Catena, by
Dr. Dav. Gregory. n. 231.
p. 637.
Aug. An. 1697.
Fig. 36.

Sit Catena FAD ab extremitatibus F & D dependens, cujus punctum imum (seu Curvæ vertex) A, Axis AB ad horizontem erectus, eique applica BD horizonti parallela. Inveniendae est relatio inter Bb, seu Ds & d s; posito b puncto ipsi B proximo, & bd ad BD, item Ds ad BA parallela.

Ex Mechanicis constat potentias tres in æquilibrio positas eandem habere rationem cum rectis tribus ad ipsarum directionis parallelis, vel in dato angulo inclinatis, à mutuo occurfu terminatis; adeoque si Dd exponat gravitatem absolutam particulæ Dd (ut in Catena æqualiter crassa rite fit) ds representabit gravitatis partem eam quæ normaliter in Dd agit, quaque fit ut dD (ob Catenæ flexilitatem circa d mobilis) in situm verticalem se componere conatur. Adeoque si ds (five fluxio ordinatæ BD) constans sit; gravitatis actio in partes correspondentes Catenæ Dd normaliter exerta etiam constans erit five ubique eadem. Exponatur hæc per Rectam a. Porro ex supracitato Lemmate Mechanico, Ds five fluxio axeos AB, exponet vim secundum directionem ipsius dD exerendam, quæ priori conatui Lineæ gravis dD ad componendam se in situm verticalem æquipolleat, eumque impedire possit. Hæc vero vis oritur à linea gravi DA secundum directionem dD trahente; estque proinde (cæteris manentibus) lineæ DA proportionalis. Est igitur ds fluxio ordinatæ ad s D fluxionem abscissæ, sicut constans recta a ad DA curvam. Q. E. F.

COROL.] Si recta TD tangat Catenariam, & axi BA producto occurrat in T, erit BD : BT :: (ds : D ::) a : DA Curvam.

Prop. II. Theorem.] Si ad perpendicularum AB tanquam axem, vertice A, describatur hyperbola æquilatera AH, cujus semiaxis AC æqualis a; & ad eundem axem & verticem, parabola AP cujus parameter æqualis quadrupulo axi hyperbole, & producatæ semper hyperbolæ ordinatæ HB, donec HF æqualis Curvæ AP: Dico Curvam FAD, in quo puncta F & D versantur (positis BD, BF æqualibus) esse Catenariam.

Vocetur AB, x, erit Bb = x, & BH = $\sqrt{2ax + x^2}$. Unde ex Methodo Fluxionum, Fluxio ipsius BH (five mb) = $\frac{ax + x\dot{x}}{\sqrt{2ax + x^2}}$, rursus quia parabolæ AP parameter = 8a, erit BP = $\sqrt{8ax}$. Unde np (hoc est Fluxio ipsius BP) æqualis $\frac{2a\dot{x}}{\sqrt{2ax}}$. Quare Fluxio Curvæ AP (= Pp) = $\sqrt{npq + Pnq} = \frac{\sqrt{4a^2\dot{x}^2}}{2ax} + \dot{x} = \frac{\sqrt{2a\dot{x}^2 + x\dot{x}^2}}{x}$, quæ ducen-

do

do tam numeratorem quam denominatorem in $\sqrt{2ax + x^2}$, $= \frac{2ax + x\dot{x}}{\sqrt{2ax + x^2}}$.

Et cum HE sit ubique = AP, erit Fluxio HF rectæ, hoc est $mb + sf$

$= \frac{2ax + x\dot{x}}{\sqrt{2ax + x^2}}$. Sed hactenus inventa est $mb = \frac{ax + x\dot{x}}{\sqrt{2ax + x^2}}$. Unde

sf , five Fluxio ipsius BF ordinatæ ad axem Catenariæ, est æqualis

$\frac{ax}{\sqrt{2ax + x^2}}$. Et igitur Fluxio Curvæ AF (five ipsa $Ff = \sqrt{sfq + Fsq}$

$= \frac{\sqrt{a^2\dot{x}^2}}{2ax + x^2} + \dot{x}^2) = \frac{ax + x\dot{x}}{\sqrt{2ax + x^2}}$, cujus fluens modo ostensa est

$\sqrt{2ax + x^2}$. Et igitur AF = $\sqrt{2ax + x^2}$. Patetque fluxionem ordi-

natæ BF five $\frac{ax}{\sqrt{2ax + x^2}}$ esse ad x fluxionem abscissæ AB, sicut data a

ad Curvam AF, quæ est superius inventa Catenariæ proprietas. Igitur Catenariæ puncta recte determinantur, per præcedentem constructionem. Q. E. D.

COROL. 1.] Ex Constructione patet BF ordinatam Catenariæ æquari Curvæ parabolicæ AP, dempta BH correspondente ordinata hyperbolæ conterminatæ AH.

2. Ex demonstratione constat Catenariam Curvam AF æquari BH correspondenti ordinatæ conterminæ Hyperbolæ æquilateræ. Cum enim harum linearum Fluxiones æquentur & simul nascantur ipsæ lineæ, patet illas ubique esse æquales. Unde data Catena, dabitur AC five a , quippe æqualis semi-axi Hyperbolæ æquilateræ cujus vertex A, & ordinata ad abscissam AB catenæ AD est æqualis.

3. Catenariæ omnes sunt inter se similes, cum ex simili similibus, & similiter positarum figurarum constructione generentur. Undæ duæ rectæ ad Horizontem similiter inclinatæ per Catenarum vertices ductæ abscindent figuras similis & Catenarum portiones abscindentibus rectis proportionales.

4. Si Catena QAD suspendatur à punctis Q & D inæqualiter altis, Curvæ pars FAD eadem manet, ac si ex punctis æquialtis F & D esset suspensa, quoniam nihil refert utrum punctum F affixum sit vel non affixum ad planum verticale.

5. Si Catenæ vis trahens secundum directionem, dD exponatur per Dd , dividetur, ut vulgo notum in vim ut $d\delta$ secundum directionem horizontalem, & vim ut δD , secundum directionem verticalem: Igitur vis in Catenæ extremo

mo directe accedendi ad axem, est ad vim in eodem descendendi secundum perpendicularum; sive vis sustententis pars secundum directionem BD agens, est ad ejusdem partem secundum directionem $D\delta$ agentem, ut semi-axis Hyperbolæ conterminæ AH ad DA , longitudinem Catenæ usque ad verticem Curvæ: Unde data Catena ratio hæc datur. Et in eadem Catena nunc magis nunc minus laxè suspensa, vis ista Horizontalis est ut Hyperbolæ conterminæ axis, cum DA eadem maneat si extrema æquialta sint.

6. Catena in Plano verticali, sed situ inverso, figuram servat nec decedit, adeoque arcum seu fornitem facit tenuissimum: Hoc est Sphæræ minimæ rigidæ & lubricæ in inversa Curva Catenaria dispositæ, arcum constituunt, cujus nulla pars ab aliis extrorsum vel introrsum propellitur; sed manentibus infimis punctis immotis, virtute suæ figuræ sustinetur. Cum enim punctorum Curvæ Catenariæ situs, partiumque inclinatio ad Horizontem eadem sit, sive in situ FAD , sive in situ inverso, dummodo Curva sit in plano ad Horizontem recto, patet illam æque servare figuram immutatam in uno situ ac in altero. Et è converso solæ Catenariæ sunt fornices sive arcus legitimi: Et cujuscunque alterius figuræ arcus ideo sustinetur, quod in illius crassitie quædam Catenaria inclusa sit: Neque, si tenuissimus esset, partesque haberet lubricas sustineretur. Ex præcedente *Corol.* 5. colligitur, quali vi arcus muros quibus insistit extra propellit; nempe hæc eadem est cum parte vis Catenam sustententis, quæ secundum directionem Horizontalem trahit. Quæ enim in Catena introrsum trahit vis, in arcu Catenæ æquali, extrorsum propellit. Alia omnia de murorum quibus fornices imponuntur firmitate requisita, ex hac theoria Geometrice determinantur, quæ in ædificiorum extruptione præcipua sunt.

7. Si loco gravitatis alia quælibet vis similiter agens in lineam flexilem vires suas exerat, eadem produceretur linea, v.g. Si ventus æquabilis supponatur, & secundum rectas datæ positione rectæ parallelas spirans, linea vento inflata eadem erit cum Catenaria. Nam cum omnia quæ in gravitate consideravimus, in altera hac vi obtineant, patet eandem Curvam productum iri.

Prop. 3. Theor.] Si manente prædicta Hyperbola AH , per A ducatur recta GAL Axi AB normalis, & describatur Curva KR ejus naturæ, ut BK sit tertia proportionalis rectis BH & AC , & ad AC applicetur rectangulum AV æquale spatio interminato $ABKRLA$, erit F concursus rectarum HB , VG ad Catenariam.

Fig. 37.

Nam ex constructione est $BK = \frac{a^2}{\sqrt{2ax + x^2}}$, quare fluxio spatii

$$ABKRLA = (BKkb = BK \times Bb =) \frac{a^2 x}{\sqrt{2ax + x^2}} \quad \text{Cumque}$$

$BF = \text{spatio } \frac{ABKRLA}{AC}$, & AC detur, erit fluxio ipsius $BF =$

fluxioni spatii $\frac{ABKRLA}{AC} = \frac{ax}{\sqrt{2ax+x^2}}$. Sed in præcedentis *Prop.*

construptione, fluxio ordinatæ $BF = \frac{ax}{\sqrt{2ax+x^2}}$. Quare hæc constructio eodem redit cum constructione *Prop.* præcedentis, & consequenter punctum F est ad Catenariam. *Q. E. D.*

COROL.] Sicut in *Prop.* præced. Catenaria describitur ex data longitudine Curvæ Parabolicæ, ita in hac, illius descriptio pendet à quadratura spatii in quo $x^2 y^2 = a^4 - 2ax y^2$. Nam y (sive BK) = $\frac{a^2}{\sqrt{2ax+x^2}}$

Fig. 36.

Prop. 4. Theor.] *Spatium AGF sub Catenaria AF & Rectis FG, AG ad AB, BF parallelis comprehensum, æquale est rectangulo sub semi-axe AC, & DH intervallo applicatarum in Hyperbola & Catenaria.*

Nam $DH = (BH - BD =, \text{ ex } \textit{Prop. 2.} \text{ hujus, } \frac{ax+xx}{\sqrt{2ax+x^2}} - \frac{ax}{\sqrt{2ax+x^2}} =) \frac{xx}{\sqrt{2ax+x^2}}$. Quare fluxio rectanguli sub data $AC \& HD = (\frac{axx}{\sqrt{2ax+x^2}} = x \times \frac{ax}{\sqrt{2ax+x^2}} = f \times FG =) \text{ fluxioni spatii } AGF$. Cumque figuræ hæc simul nascantur, sequitur rectangulum sub $AC \& DH$ æquari spatio AGF . *Q. E. D.*

COROL.] Hinc sequitur, spatium FAD , sub Catena FAD , & recta Horizontali FD comprehensum, æquari rectangulo sub $FD \& BA$, dempto rectangulo sub Hyperbolæ AH axe alterutro, & DH excessu rectæ BH , vel Curvæ AD , supra ordinatam BD .

Fig. 35.

Prop. 5. Theor.] *Si ad rectam AL, applicetur Rectangulum LE æquale spatio Hyperbolico ALH, erit E centrum Æquilibrii Curvæ Catenaria AFD.*

Concipiatur curva gravis FA librari super axe GL . Ex Centro barycis constat momentum gravis FA exponi per superficiem Cylindrici recti super FA erecti, & reflecti plano per GL transeunte, cum plano Curvæ angulum semi-rectum faciente. Et hujus superficiei fluxio, sive $FA \times FG$, æqualis est fluxioni spatii ALH sive $BH \times HL$; quia FA , BH , item $FG \& HL$ æquantur.

æquantur. Ac propterea (cum simul nascantur) dicta superficies Cylindrici recti æqualis est spatio Hyperbolico ALH. Hoc proinde applicatum ad ipsum grave AF, vel illi æqualem rectam AL, facit latitudinem AE æqualem distantia centri gravitatis ab axe librationis GL. Unde Curvæ FAD, æqualiter ad utramque Axeos AB partem jacentis, centrum æquilibrii est E. Q. E. D.

COROL. 1.] Spatia ABHL, BAH, & AGF sunt Arithmetice

proportionalia. Nam fluxio spatii ALH = $\left(\frac{ax + x \cdot x}{\sqrt{2ax + x^2}} \right) \times x =$

$$\frac{ax + x^2 \times x}{\sqrt{2ax + x^2}} = \frac{2ax + x^2 - ax \times x}{\sqrt{2ax + x^2}} = x \sqrt{2ax + x^2} -$$

$$\frac{axx}{\sqrt{2ax + x^2}} = \text{fluxioni spatii BAH, multata Fluxione spatii AGF,}$$

per Prop. 4. hujus. Cumque hæ tres figuræ simul nascantur, erit BAH - AGF = (ALH =) BL - BAH. Quare 2. BAH = BL + AGF. Unde sequitur spatia BL, BAH & AGF esse in proportione Arithmetica.

2. Catenæ centrum gravitatis est omnium linearum ejusdem Longitudinis, eisdemque terminos habentium, infimum. Nam tantum descendet grave quantum potest. Cumque tantum descendat figura, quantum ejus centrum gravitatis descendit, se sic disponet linea gravis flexilis, ut ejus centrum gravitatis sit inferius quam si aliam quamcunque figuram indueret. Atque ex hoc Symptomate lineæ gravis flexilis, reliqua omnia facile deduci possent.

3. Si super quæcunque Curvas eandem longitudinem eisdem terminos D & F cum Catenaria FAD habentes, erecti Cylindrici recti secantur plano per DF transeunte; superficierum Cylindricarum sic resectorum maxima est quæ super Catenariam insistit. Hæ enim superficies (si angulus sub planis fuerit semirectus) ad ipsas Curvas (quæ sunt in casu præsentis longitudinis ejusdem) applicatæ, latitudines faciunt æquales distantis centrorum gravitatis Curvarum à DF recta. Cum distantia hæc sit in Catenaria maxima (ob maximum descensum centri gravitatis) erit Cylindrica superficies applicanda etiam maxima. Et quoniam superficierum Cylindricarum resectorum plano, cum plano baseos angulum quemvis continente, eadem est ratio, atque cum dictus angulus est semirectus, patet propositum universaliter.

LEMMA.] Si in cujuscunque Curvæ AFO, descriptæ evolutione alterius Curvæ KV, ordinatam quævis FB ad axem AB normalem, à correspondente in KV puncto V demittatur normalis VR ordinate occurrens in R: erunt, manente fluxione axeos AB eadem, fluxio fluxionis ordinatæ BF, fluxio Curvæ AF, & recta FR, continue proportionales.

Fig. 36.

Producatur rectula Ff , donec proximæ ordinatæ $W\phi$ occurrat in o . Et quoniam ex hypothefi $Fs = fW$, erit $of = Ff$, adeoque $o\phi$ erit fluxio ipsius fs , hoc est fluxio fluxionis ordinatæ. Porro triangula $o\phi f$, fFR sunt æquiangula, quia $o\phi f =$ alterno fFR , & $f\phi\phi = (Ffr =) FfR$, quia illorum intervallum Rfr alterutrius respectu evanescit, cum Rr præ fr nulla sit. Et igitur $o\phi : \phi f :: fF : FR$, sed $o\phi$, fF æquales sunt, cum fluxione utriusvis, tantum differant. Quare $o\phi : fF :: fF : FR$. *Q. E. D.*

Fig. 36.

Prop. 6. Probl.] *Invenire Curvam KV cujus evolutione Catenaria AFQ describitur.*

Vocetur ut prius AB , x , item BF , y . Est, ex Prop. 2. hujus, $y =$

$\frac{ax}{\sqrt{2ax+x^2}}$, sive $2ax\dot{y}^2 + x^2\ddot{y}^2 = a^2\dot{x}^2$. Quare, per satis nunc usurpatam *Newtoni* methodum, $2ax\dot{y}^2 + 4ax\dot{y}\ddot{y} + 2x\dot{x}\dot{y}^2 + 2x^2\ddot{y}\ddot{y} (= 2a^2\dot{x}\dot{x}$ quæ, propter $\dot{x} = 0$, cum constans x non fluat) $= 0$.

$$\text{Quare } \ddot{y} = \left(\frac{-ax\dot{y} - x\dot{x}\dot{y}}{2ax + x^2} \right) = \frac{a+x \times ax^2}{2ax + x^2 \times \sqrt{2ax+x^2}}$$

ponendo loco y ejus valorem $\frac{ax}{\sqrt{2ax+x^2}}$. (Nam signum $-$ quantitati y præfixum, tantum denotat locum puncti R ex F spectati, oppositum esse loco puncti F ex B spectati, cum Curva AFQ est cava versus axem AB) & Ff ,

per Prop. 2. hujus, $= \frac{a+x \times x}{\sqrt{2ax+x^2}}$: Quare per præcedens Lemma,

$$FR = \left(\frac{Ff}{y} = \frac{a+x^2 \times x^2}{2ax+x^2} \times \frac{2ax+x^2 \times \sqrt{2ax+x^2}}{a+x \times ax^2} \right) =$$

$\frac{a+x \times \sqrt{2ax+x^2}}{a}$. Rursus ob triangula rectangula Fsf , FRV , habentia angulos fFs , VFR æquales, quia VFs est utriusque comple-

mentum ad rectum, est $Fs : sf :: FR : VR$, sive $x : \frac{ax}{\sqrt{2ax+x^2}} ::$

$\frac{a+x \times \sqrt{2ax+x^2}}{a} : VR$, quæ proinde æqualis $a+x$. Hæc igitur est

natura Curvæ KV , ut si AB vocetur x , erit $FR = \frac{a+x \times \sqrt{2ax+x^2}}{a}$

& $VR = a+x$. *Q. E. D.*

COROLL.

COROL. 1.] $AC:CB::BH:FR$. Hæc enim est proprietas rectæ FR superius inventa.

2. Recta CB æqualis est rectæ BI , sive VR . Utraque enim est æqualis $a+x$.

3. Recta evolvens VF , est tertia proportionalis ipsis AC , CB . Nam ob æquiangula triangula FFI , VFR , est $F:Ff::FR:VF$. Sive

$$x : \frac{ax + xx}{\sqrt{2ax + x^2}} :: \frac{a + x \times \sqrt{2ax + x^2}}{a} : VF, \text{ quæ proinde} =$$

$$\frac{a + x}{a}^2. \text{ Unde } a : a + x :: a + x : VF, \text{ quæ præterea est radius circuli}$$

Catenæ in F æquicurvi.

4. Cum punctum F est in A , sive cum vertex evolutione describitur, id est cum $x = 0$, valor evolventis rectæ VF , quæ in hoc casu est KA , nempe

$$\frac{a + x}{a}^2 \text{ fiet } a : \text{ hoc est punctum } K, \text{ ubi Curva } VK \text{ occurrit axi, tantum}$$

extat supra Catenæ verticem A , quantum C deprimitur infra eundem. Unde diameter circuli, Catenæ ad verticem æquicurvi, æqualis est axi conterminæ Hyperbolæ AH . Adeoque Catenæ AD & Hyperbolæ AH eadem est curvatura in vertice A : Nam vulgo notum est circulum prædictum, Hyperbolæ æquilatæ AH in vertice A æquicurvum esse. Sed & hoc aliunde, ex ipsa Catenæ natura *Prop. 2.* hujus demonstrata, constat. Nam nascentis FH sive ($AP =$ nascenti $BP =$) $\sqrt{8ax}$ dupla est nascentis BH sive ($\sqrt{2ax + x^2}$, hoc est, evanescente x^2 , cum x minima sit) $\sqrt{2ax}$; Et igitur idem punctum est tam in nascente Hyperbola quam nascente Catenaria; hoc est nascentis Hyperbolæ AH cum nascente Catenaria AD coincidit, & proinde æquicurvæ sunt hæc lineæ ad verticem A .

5. Curva KV tertia proportionalis ad rectam AC & curvam AF sive rectam AL . Ex natura enim evolutionis, $KV = (VKA - KA = VF$

$$- KA = \frac{a + x}{a}^2 - a = \frac{a^2 + 2ax + x^2}{a} - a = \frac{2ax + x^2}{a}. \text{ Et}$$

igitur $a : \sqrt{2ax + x^2} :: \sqrt{2ax + x^2} : KV$. Sed $\sqrt{2ax + x^2}$, ex *Corol. 2. Prop. 2.* = AF . Unde $AC:AF::AF:KV$.

6. Recta KI dupla est ipsius AB . Cum enim $BI = (BC =) CA + AB$, erit $AI = CA + 2AB$; At $AK = AC$, per *Corol. 4.* hujus; igitur $KI = 2AB$.

7. Rectangulum sub AC & BR est æquale duplo spatio hyperbolico

$$BAH. \text{ Nam } FR \times AC = \left(\frac{a + x \times \sqrt{2ax + x^2}}{a} \right) \times a = a + ax$$

$$\sqrt{2ax + x^2}$$

$\sqrt{2ax+x^2} = x \times \sqrt{2ax+x^2} + a \times \sqrt{2ax+x^2} = AB \times BH + AC \times BH = AB \times BH + AC \times BD + AC \times DH$. Quare $FR \times AC = BD \times AC$, hoc est $BR \times AC = AB \times BH + AC \times DH$. Sed per Prop. 4. hujus, $AC \times DH = AGF$ spatio. Et igitur $BR \times AC = (ABHL + AGF = \text{per Corol. 1. Prop. 5.}) 2 BAH$.

Fig. 38.

Prop. 7. Theor.] Si in Curva Logarithmica LAG cujus data subtangens HS equalis rectæ a , Corol. 2. Prop. 2. hujus definitæ, sumatur punctum A cujus distantia ab HP asymptoto, nempe AC , equalis sit subtangenti HS , & ex punctis H & P utrunque in asymptoto sumptis à puncto C equaliter distantibus, erigantur HL , PG ordinatæ ad Logarithmicam, quarum semisumma ponatur equalis HD vel PF , erunt D & F ad Catenariam rectæ AC correspondentem.

Vocetur AB , x , adeoque CB vel DH semisumma ordinatarum HL , PG erit $a + x$; semidifferentia earundem vocetur y . Unde $HL = a + x + y$, & $PG = a + x - y$. Cumque ex natura Logarithmicæ, CA fit inter has media proportionalis, erit $a^2 + 2ax + x^2 - y^2 = a^2$. Unde $y =$

$\sqrt{2ax+x^2}$. Adeoque $HL = a + x + \sqrt{2ax+x^2}$, & $PG = a + x - \sqrt{2ax+x^2}$. Quare fluxio ipsius HL , sive ipsa lm est $\frac{ax + xx + x\sqrt{2ax+x^2}}{\sqrt{2ax+x^2}}$. Et ob æquiangula triangula lmI ,

LHS , est $LH : HS :: lm : mL$, unde mL sive $d\delta$ fluxio ipsius $BD =$

$\frac{ax}{\sqrt{2ax+x^2}}$. Hoc est Curva AD ex Logarithmica supradictæ modo genita, ejus est naturæ, ut si axis vocetur x , ejusque fluxio x , fluxio ordinatæ BD sit

$\frac{ax}{\sqrt{2ax+x^2}}$. Sed hæc ipsa est proprietas Catenariæ ad quam a pertinet, Prop. 1. hujus demonstrata. Ergo Curva FAD superius descripta est hæc ipsa Catenaria. Q. E. D.

COROL. 1.] Sicut ope Logarithmorum Catenaria describitur, vice versa ope Catenariæ per ipsam rerum naturam productæ, numeri dati vel potius rationis datæ Logarithmus invenitur. Ut si posita CA unitate, cujus Logarithmus est nihilo æqualis, quæratur Logarithmus Numeri CQ sive rationis inter CA & CQ ; Rectis CQ & CA tertia proportionalis sit CV , ipsarumque CQ , CV semisumma CB ; ex B ordinata ad Catenariam, nempe BD est Logarithmus quæsitus. Ratio ex propositione manifesta est.

2. Vicissim si dato Logarithmo CH vel CP , quæratur correspondens numerus HL vel PG , seu ratio HL ad CA , sive PG ad CA . Ex H vel P erigatur perpendicularum Catenæ occurrens in D vel F , ipsique HD vel PF hoc est CB , fiat æqualis CR ad horizontalem AR terminata; eritque AR

A R semidifferentia quæsiturum L H, G P, sicut ex supra demonstrata Catenæ natura H D vel C R est earundem semisumma: (Nam in tribus quantitativibus Geometrice proportionalibus quales sunt H L, C A, P G, quadratum semisummae extremarum multatum quadrato mediae, æquatur quadrato semidifferentiæ extremarum) adeoque $CR + AR$, & $CR - AR$ sunt numeri H L vel G P, dato Logarithmo C H vel C P congrui.

3. Ex demonstratione patet quod sicut H D semisumma Logarithmicæ ordinatarum H L, P G, ad C H normaliter applicata in H, est ordinata Catenariæ, sic semidifferentia earundem H L, P G, ad C A normaliter applicata in B est ordinata Hyperbolæ æquilateræ centro C vertice A descriptæ: ac proinde, per *Corol. 2. Prop. 2.* hujus, æqualis Catenæ A D. Nam $y = \sqrt{2ax + x^2}$. Cumque *Corol. præced.* ostensum sit, A R esse etiam semidifferentiam rectarum H L, P G, patet A R esse æqualem Catenariæ portioni A D. Unde obiter elucescit modus, data Catena A D, inveniendi C centrum Hyperbolæ conterminæ, vel punctum id asymptoto Logarithmicæ G L. Nam si sumatur A R æqualis Catenæ A D, & ex junctæ rectæ B R puncto medio erigatur ad ipsam B R normalis, hæc occurret B A Axi Catenæ in quæsito puncto C, uti patet. Nam sic erit $CR = CB$.

4. Hinc etiam sequitur si B D T, angulus fiat æqualis A C R, rectam D T tangere Catenariam in D. Nam sic fiet in triangulis æquiangulis D B T, C A R; $DB : BT :: CA : AR$ five huic æqualem A D curvam. Et igitur per *Corol. Prop. 1.* hujus, D T tangit Catenariam.

5. Sequitur etiam spatium A C H D æquari rectangulo sub C A & A R. Nam quoniam A Y D est, per *Prop. 4.* æquale triangulo sub C A & (A D. - B D =, per *Corol. 3. hujus Prop.* $AR - AY =$) Y R, patet propositum. Et quoniam C A datur, constat spatium A C H D esse sicut A D curva, illiusque fluxionem H d sicut D d fluxio hujus.

6. Si per punctum K, ubi C R secatur H D, ducatur K Z parallela P H, rectæ A C occurrens in Z, sumaturque C E æqualis semisummae ipsarum B C, C Z, erit E centrum Æquilibrii Curvæ F A D.

Intelligatur super F A D erecta superficies Cylindrici recti resecti plano per P H ad angulos semirectos cum plano Curvæ F A D; exponet hæc superficies momentum Curvæ F A D super axe P H librata; ejusque fluxio est

$$\begin{aligned}
 DH \times Dd + PF \times Ff &= 2 BC \times AD = 2 \times a + xx \times \frac{ax + xx}{\sqrt{2ax + x^2}} \\
 &= \frac{2a^2x + 4axx + 2x^2x}{\sqrt{2ax + x^2}} = \frac{a^2x}{\sqrt{2ax + x^2}} + \frac{a^2x + axx}{\sqrt{2ax + x^2}} \\
 &+ \frac{3axx + 2x^2x}{\sqrt{2ax + x^2}}, \text{ cujus fluens } a \times BD + a \sqrt{2ax + x^2} + \\
 x \sqrt{2ax + x^2} &= CA \times BD + CB \times AD. \text{ Quare } CA \times BD + \\
 CB \times AD &= (\text{quoniam simul nascitur, dictæ superficiei Cylindricæ} =) \\
 \text{momento Curvæ F A D super axe P H librata. Unde distantia centri gravi-} & \\
 &\text{tatis.}
 \end{aligned}$$

tatis Curvæ FAD a puncto C est $\frac{CA \times BD + CB \times AD}{2 AD}$ five $\frac{CA \times BD}{AD}$

+ $\frac{1}{2} CB$. Porro ob ZK parallelam AR , est $AD : BD :: (AR : ZK ::)$

$CA : CZ$, unde $CZ = \frac{CA \times BD}{AD}$, & igitur CE quæ per constructionem

est $= \frac{1}{2} BC + \frac{1}{2} CZ$, erit $= \frac{1}{2} \frac{CA \times BD}{AD} + \frac{1}{2} BC$: hoc est Curvæ

FAD centrum gravitatis, & E punctum ex constructione definitum æqualiter distant a C ; sed & in eadem recta & versus easdem partes sita sunt, ergo coincidunt illa.

Potest & coincidentia puncti E , ut supra determinati, cum centro æquilibrii, *Prop. 5.* hujus definitio, Synthetice sic ostendi. Per *Corol. 1. Prop. 5.* $2 BAX = AYD + BA \times AR$. Unde $AH + 2 BAX = (ACHD + BA \times AR =$ per præced. *Corol.*) $AR \times CA + BA \times AR$: hoc est $BD \times AC + 2 BAX = AR \times CB$; five $BD \times AC = AR \times CB - 2 BAX$. Unde $BD \times AC + AD \times BC = (AD \times BC + AR \times CB - 2 BAX = 2 AD \times BC - 2 BAX =)$ $2 AD \times AC + 2 AD$

$\times AB - 2 BAX$. Et applicando ad $2 AD$, erit, $\frac{BD \times AC}{AD} + \frac{1}{2} BC$

$= (AC + \frac{AB \times AD - BAX}{AD} =) CA + \frac{ARX}{AR}$. Sed $\frac{ARX}{AR}$ est di-

stantia centri æquilibrii Catenæ a vertice A , per *Prop. 5.* hujus determinata,

ac proinde, secundum dictam *Prop. 5.* $CA + \frac{ARX}{AR}$ est distantia puncti

E a C , & $\frac{1}{2} \frac{BD \times AC}{AD} + \frac{1}{2} BC$, est ejusdem E distantia ab eodem C ,

secundum hoc *Corol. 6.* Unde patet duas istas determinaciones puncti E eo-

dem recidere, quoniam $CA + \frac{ARX}{AR} = \frac{1}{2} \frac{BD \times AC}{AD} + \frac{1}{2} BC$.

7. Spatii $PFADH$ centrum gravitatis est in I , medio puncto rectæ CE . Cum centrum gravitatis fluxionis ipsius AD , five Dd & Ff , duplo magis distet a PH quam centrum gravitatis fluxionis ipsius $ACHD$ five DH hd & Ff ppf , & $Dd + Ff \times AC$ datam, æquale $DdhH + FfpP$ patet & fluentis FAD centrum gravitatis E duplo magis distare a PH , quam fluentis $PFADH$ centrum I . Sed libet propositum aliter & ad modum superiorum ostendere.

Intelligatur supra figura $PFADH$ erectus Cylindricus rectus & reflectus plano per PH transeunte, cum plano baseos angulum semirectum comprehendente; exponet istud solidum momentum figuræ $PFADH$ super
axe

axe PH librata: hujusque solidi sive prædicti momenti fluxio (solida nempe erecta super P F f p & H D d h) producitur, si momentum fluxionis, sive fluxio momenti ipsius AD, ducatur in $\frac{1}{2}$ AC datam. Nam per *Corol. 5.* hujus *Prop.* $H D d h = D d \times AC$: Quare ipsum momentum fluens produci- tur ducendo momentum Curvæ FAD respectu axis PH, superiore *Corol.* determinatum, nempe $CA \times BD + CB \times AD$, in $\frac{1}{2}$ AC; eritque pro- inde $\frac{1}{2} AC \times AC \times BD + \frac{1}{2} AC \times CB \times AD$. Adeoque si hoc applice- tur ad figuram librata P F A D H sive $2 CA \times AD$, per hujus *Prop. Co- rol. 5.* fiet distantia centri gravitatis figuræ PFADH ab axe PH =

$$\left(\frac{CA \times BD}{AD} + \frac{1}{2} CB \right) = \text{dimidia rectæ CE superius determinatæ.}$$

8. Si per N punctum ubi DT tangens Catenariam in D, secat AR, du- catur recta parallela ipsi BC, Occurrens rectæ per E ad AR parallela in O; erit O centrum gravitatis Curvæ AD. Nam per *Corol. 6.* centrum gra- vitatis curvæ AD est in recta EO, sed demonstrabitur illud esse in NO recta, & proinde erit ipsum O punctum. Intelligatur DA librari circa HL axem; hujus momentum est curva DA ducta in distantiam centri gravitatis ab HL: & ejus proinde fluxio = $DA \times Hb$ (Hb est fluxio distantia axis libratio-

nis à gravitatis centro) = $\sqrt{2ax + x^2} \times \frac{ax}{\sqrt{2ax + x^2}} = ax$. Ac pro- inde ipsum momentum Curvæ gravis DA circa axem HL librata = ax . Et igitur distantia centri gravitatis ab eodem axe est ax applicata ad AD, sive $\frac{AC \times DY}{AR}$. Sed quia DT tangit Catenariam, per *Corol. 4.* hujus *Prop.*

angulus BDT sive DNY = ACR, & anguli ad A & Y sunt recti, quare in triangulis æquiangulis RAC, DYN; $RA : AC :: DY : YN$. Unde $YN = \frac{AC \times DY}{RA}$, hoc est YN est distantia centri gravitatis Catenæ AD ab axe HL, sive centrum prædictum est in recta NO.

9. Si super I ducatur recta ad AR parallela, rectæ ON productæ occur-rens in W, erit W centrum gravitatis spatii ACHD. Nam per *Corol. 7.* centrum gravitatis spatii ACHD est in recta IW, sed ut mox ostendetur, est in NW, & proinde est ipsum W punctum. Eodem enim modo, quo in *Corol. præced.* fluxio momenti spatii ACHD circa HL librati ostenditur esse $ACHD \times Hb = AC \times AD \times Hb = ax \sqrt{2ax + x^2}$

$\times \frac{ax}{\sqrt{2ax + x^2}} = aax$. Ac proinde ipsum momentum spatii ACHD

circa axem HL librati, æquale est fluenti cujus $a^2 x$ est Fluxio, hoc est ipsi $a^2 x$. Hoc igitur applicatum ad ipsum spatium ACHD sive $ax \sqrt{2ax + x^2}$

dat distantiam centri gravitatis spatii ACHD ab HL = $\frac{4x}{\sqrt{24x+x^2}}$
 $= \frac{AC \times DY}{RA}$. Sed *Corol.* præced. ostensa est $YN = \frac{AC \times DY}{RA}$. Et

igitur centrum gravitatis spatii ACHD est in NW. Atque ex duobus hisce ultimis *Corol.* invenitur centrum gravitatis cujusvis portionis Catenæ etiam ad verticem A non pertingentis: vel cujusvis spatii Catenariæ portione quavis, & aliis rectis præter prædictas comprehensi.

10. Hinc mensurantur superficies & solida genita rotatione Catenæ aut spatii sub illa & rectis comprehensi, circa axes datos. Nam figura rotatione genita æquatur, uti vulgo notum, figuræ rotatæ ductæ in peripheriam à centro gravitatis inter rotandum percursum, etiam datam cum detur illius radius sive distantia centri gravitatis ab axe dato. Sic si Catena AD rotetur circa axem

AB, $\frac{\pi}{2}$ AN est peripheria à centro gravitatis O percursa, ($\frac{\pi}{2}$ denotat rationem peripheriæ circuli ad semidiametrum) adeoque superficies rotatione Catenæ AD genita = ($\frac{\pi}{2} \times AN \times AD$) = $\frac{\pi}{2} \times AN \times AR$. Hoc est

circulus cujus radius potest duplum rectangulum RAN, æquabitur superficiæ à Catenæ AD rotatione circa axem AB genitæ. Pari modo solidum genitum rotatione spatii ACHD circa AC, æquale ostendetur Cylindro cujus basis est prædictus circulus, altitudo vero æqualis AC. Similiterq; superficies & solida, ex rotatione harum figurarum circa alios quosvis datos axes facta mensurantur, Nam dato centro gravitatis hæc non latebunt.

The Animadver-
sions of
Answered by Dr.
D. Gregory.
N. 259. p. 419.
Dec. An. 1699.

2. Quæ in Animadversione ad nostras de Catenaria Demonstrationes objicit Anonymus, *Act. Lips. M. Feb. A. 1699.* sunt hæc; Quod rem ab aliis jam ante septennium inventam & publice expositam demonstrare aggressus sim, modo quodam meo. Ita quidem est; Quid vero hic redarguendum sit non capio. Celeberrimi viri *Hugenius, Leibnizius, & Bernoullius*, plurimas Catenariæ proprietates detexerunt, & ediderunt, at non demonstrarunt: Ego quod suscepi, demonstrationes pertexui.

Sed an res hæc (nempe Catenariæ Natura & proprietates primariæ) ab aliis inventa & publice exposita fuit? Certe ista Catenariæ proprietas, *Corol. 6. Prop. 2.* aliis indicta est penitus ante editas hæc demonstrationes: Cum tamen sit ni fallor inter primarias illius proprietates, & omnium longe utilissima, & ad vitæ communis usus facillime reducenda. Ab omni ævo, in ædificiis publicis Fornices arcusque tam ad firmitatem quam pulchritudinem adhibuerunt Architecti: Qualis tamen sit Fornicis figura legitima adusque editas nostras demonstrationes ignoratum est.

Primum autem quod reprehendat invenit, quod quædam ex *Mechanicis constare* dixerim, quæ distinctius enuntiare atque etiam applicare operæ pretium fuisse ait. Ego qui Geometris demonstranda Theoremata quædam susceperam, omnia minutim exequenda non credebam.

Verum

Verum ut *Animadversori* gratum faciam, *Lemma* istud (*Prop. 1.*) demonstrepo, cum distinctius enuntiare nequeam, quam est hactenus factum in hæc verba.

Potentia tres in æquilibrio posita eandem habent rationem cum rectis tribus ad ipsarum directiones parallelis, vel in dato Angulo inclinatis, à mutuo occurssu terminatis.

Putà si potentia tres trahentes, impellentes vel utcunque agentes, secundum rectas PA, PB, PC sint in Æquilibrio; & inclinentur ad has directiones tres rectæ EF, FD, DE, in angulo quovis dato, hoc est si anguli EAP, FBP, DCP, fuerint æquales, dico potentias A, B, & C, esse inter se ut rectæ FE, FD & DE.

Producantur rectæ AP, BP, CP in G, H & K.

In Quadrilatero FABP, cum Angulus externus EAP sit, ex Hypothesi, æqualis interno & opposito PBF, erunt interni duo oppositi FAP, & FBP æquales duobus rectis; cumque omnes quatuor interni quatuor rectis æquentur, erunt reliqui duo F & APB in eodem quadrilatero oppositi, duobus rectis etiam æquales. Sed APB & BPG efficiunt duos rectos, & igitur angulus F est æqualis angulo BPG. Similiter ostendentur D & BPK æquales, item E & APK.

Quoniam tres Potentiæ sunt in Æquilibrio, sunt immotæ, & igitur earum quælibet pro Hypomochlio haberi potest reliquarum duarum respectu quæ in æquilibrio manent. Si B habeatur pro Hypomochlio, per *Mechanica notissimum Theorema*, Potentia A est ad Potentiam C, sicut sinus Anguli BPK, ad sinum Anguli BPG, hoc est sinus Anguli D ad sinum Anguli F, hoc est recta FE ad rectam DE. Rursus posito C Hypomochlio, potentia A est ad potentiam B, ut sinus Anguli CPH ad sinum Anguli CPG, sive sinus Anguli BPK ad sinum Anguli APK, hoc est sinus Anguli D ad sinum Anguli E, hoc est ut recta FE ad rectam FD. Tres igitur Potentiæ A, B, & C, sunt ut rectæ FE, FD, & DE. Q. E. D.

De Applicatione hujus *Lemma* Mechanici nunc agendum. Si concipiatur (ut supra-dictum *Prop. 1.*) lineolæ dD gravitas absoluta per dD exposita, in ejus centro gravitatis M collecta, & grave hoc secundum directionem MF ad dD normalem vi gravitatis suæ descendere: Potentia secundum MD trahens quæ in æquilibrio est cum prædicto gravi, per præmissum *Lemma*, est ad ejus momentum sive potentiam trahentem secundum MF, sicut dD ad dF. Nam Angulus dDd, quo D d inclinatur ad MD, æqualis est angulo dF, quo dF inclinatur ad MF; viz. uterque complementum anguli d ad rectum. Atque hoc etiam obtinet, agnoscante *Animadversore*, si ut in vulgari *Mechanica*, prædictum grave, plano MF incumbens, interposita trochlea ad M, trahatur ab alio grave ipsi MD incumbente: Erit hoc ad illud sicut D d ad dF.

Quod si, reliquis manentibus, modus applicationis harum potentiarum mutetur, ita ut ad flexilis lineæ dD, cujus extremum d immotum, punctum medium M applicetur pondus secundum MF vires exerens, quippe arcum centro d, radio dM, in descensu descripturum: Erit ponderis hujus vis, ad flexilem lineam rectam ad M incurvandam, infinita respectu vis suæ gravitatis absolutæ; & vis secundum MD trahens ad modo descriptam incurvationem impediendam requisita, etiam infinita respectu ejus quæ prius requirebatur ad pondus M

Fig. 39.

Fig. 40.

in plano M F sustinendum. Adeo ut Potentiæ quæ in priore applicationis modo exponebantur per d , δ D, nunc exponendæ veniant per infinite majores prioribus proportionales: Nam ut prius pondus M trahit secundum directionem M F, & Potentia illud sustinens secundum M D; & hæc duo esse in æquilibrio ex partium Catenæ quiete constat. Eadem igitur manebit harum ratio quæ prius fuerat. Sed causa quæ lineam flexilem d D (cujus extremum d immotum, cujusque medio puncto M applicatur grave infinite quidem parvum, sed cujus vires per hunc applicationis modum infinite majores redduntur, & proinde in *Animadversoriis* Phrasi assignabiles fiunt) in rectam extendit, est Catenæ D A gravitas quæ est ipsius longitudini proportionalis. Hæc ergo est ad constantem & assignabilem a (constanti sed inassignabili d proportionalem) ut D δ , ad δ d. Atque sic *Animadversoriis* patere credo veram conclusionem absque assumptis erroneis fuisse probatam.

The Quadrature of Figures Geometrically Irrational, by Mr. J. Craig. N. 2. 2. p. 708. Sept. An. 1697. Fig. 41.

XIV. Sit ACF Semicirculus, cujus Diameter est AF, ADE Curva Geometrice irrationalis, cujus ordinatim applicata BD, secat Semicirculum in C. Quantitates vero sic designentur; Diameter AF = $2a$, abscissa AB = y , Arcus AC = v , ordinata BD = z : sitque $z = rvy$ æquatio generalis exprimens naturas Curvarum Geometrice irrationalium ADE, in qua r denotat quantitatem quamlibet datam & determinatam, & n exponentem indefinitum quantitatis indeterminatæ y . Dico Aream

$$ABD = \frac{rvy^{n+1}}{n+1} - qv + v \sqrt{2ry - yy} \times \frac{ra}{n+1} y^n + \frac{2nra^2 + ra^2}{n \times n + 1} y^{n-1} + \frac{aA \times 2n-1}{n-1} y^{n-2} + \frac{aB \times 2n-3}{n-2} y^{n-3} + \frac{aC \times 2n-5}{n-3} y^{n-4} + \frac{aD \times 2n-7}{n-4} y^{n-5} + \frac{aE \times 2n-9}{n-5} y^{n-6} + \&c.$$

De hac Serie Infinita hæc sunt notanda: (1.) Quod Literæ majusculæ A, B, C, D, E, &c. designent coefficientes Terminorum ipsis immediate præcedentium, scil. $A = \frac{2nra^2 + ra^2}{n \times n + 1 \times n + 1}$, $B = \frac{aA \times 2n-1}{n-1}$,

$C = \frac{aB \times 2n-3}{n-2}$, & sic porro. (2.) Quod si exponent n sit numerus

integer & positivus, aut nihilo æqualis, vel etiam si $2n$ sit numerus impar, tum Quadratura spatii ABD exhibetur per Quantitatem finitam; serie in his casibus abruptente. (3.) Quod q designet Terminum ultimo abruptentem. (4.) Quod omnes illæ Figuræ in quibus Series abruptitur, habeant unam portionem Geometrice Quadrabilem, ex ipsa Serie facillime assignabilem: Nimi-

um si capiatur abscissa $y = \frac{r}{n+1} \times \frac{1}{nq+q^{n+1}}$; erit huic abscissæ competens

competens Area Geometricae Quadrabilis. (5.) Quod solus Terminus Irrationalis $\sqrt{2ay - yy}$ in Terminos ipsum sequentes fit multiplicandus.

Exemplum 1.] Sit $z = v$, quia in hoc casu $r = 1$, $n = 0$, ideo $\frac{ra}{n+1} y^n$ est Terminus ultimo abrumpens, quare $q = a$, unde $ABD = vy - av + a\sqrt{2ay - yy}$: & proinde si (per Not. 4.) capiatur Abscissa $y = a$, id est, si ordinata transeat per Circuli Centrum, erit portio huic competens Geometricae Quadrabilis, scil. Area = aa , id est Radii, Quadrato.

Exemplum 2.] Sit $z = \frac{vy}{a}$, quia in hoc casu $r = \frac{1}{a}$, $n = 1$, ideo $\frac{anraa + raa}{n \times n + 1} y^{n-1}$ est terminus ultimo abrumpens, quare $q = \frac{3a}{4}$; unde $ABD = \frac{vy^2}{2a} - \frac{3av}{4} + \frac{y + 3a}{4} \sqrt{2ay - yy}$, & proinde si (per Not. 4.) capiatur $y = \sqrt{\frac{3aa}{2}}$, erit huic abscissae competens Area Geometricae Quadrabilis, scil. Area = $\sqrt{\frac{6aa^3}{2} - \frac{3a^3}{2} \times \frac{\sqrt{\frac{3a^3}{32} + \frac{3a}{4}}}{32} + \frac{3a}{4}}$.

Exemplum 3.] Sit $z = \frac{vy^2}{aa}$; in hoc casu $r = \frac{1}{aa}$, $n = 2$, ideo $\frac{aA \times 2n - 1}{n - 1} y^{n-2}$ est Terminus ultimo abrumpens, ergo $q = \frac{5a}{6}$; unde per seriem infinitam erit $ABD = \frac{6vy^3 - 15a^2v + 2ay^2 + 5a^2y + 15a^3}{18a} \sqrt{2ay - yy}$, & proinde si (per Not. 4.) capiatur $y = \sqrt[3]{\frac{5a^3}{2}}$ erit abscissae competens Area Geometricae Quadrabilis: scil. Area $\frac{2ay^2 + 5a^2y + 15a^3}{18a} \times \sqrt{2ay - yy}$.

Secundo.] Sit ACF Parabola, cujus Axis AE, vertex A, & latus rectum BA. Sitque ADG Curva Geometricae irrationalis, cujus ordinatim applicata BD fecat Parabolam in C. Et vocetur abscissa AB = y , Ordinata BD = z , Arcus Parabolicus AC = v . Sitque aequatio generalis, exprimens Naturas Infinitarum Curvarum irrationalium, hæc, $Z = rvy$, in qua r denotat Quantitatem datam & determinatam, & n exponentem indefinitum Quantitatis indeterminatae y . Dico Aream

Fig. 422

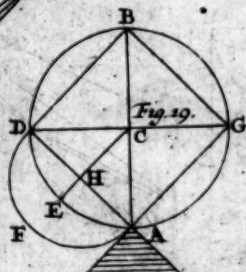
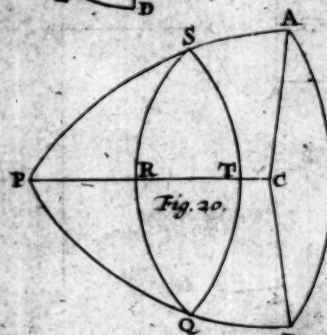
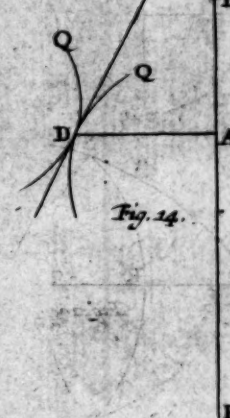
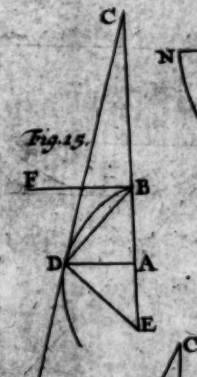
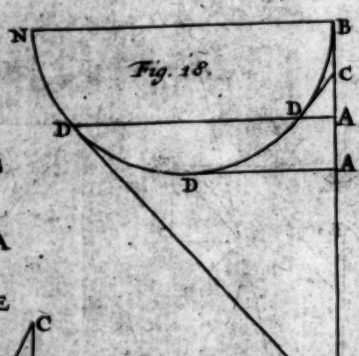
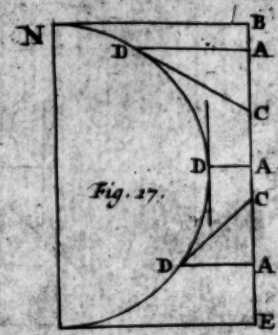
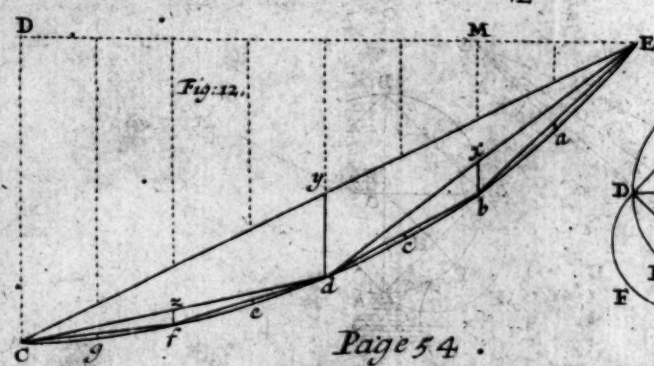
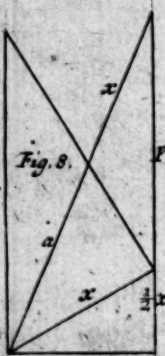
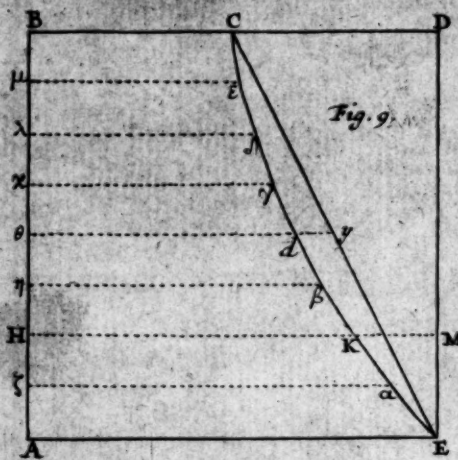
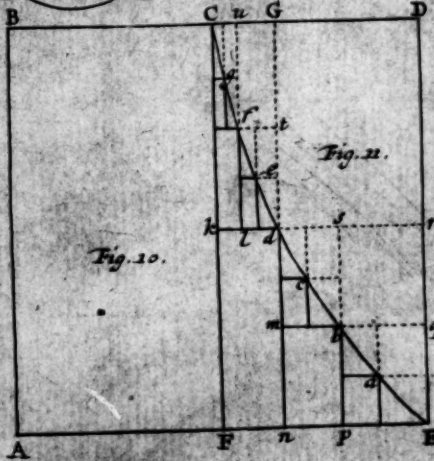
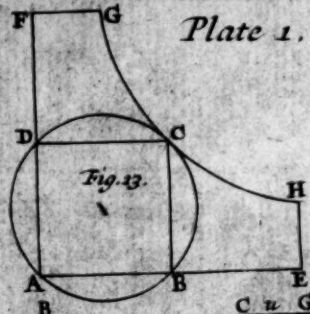
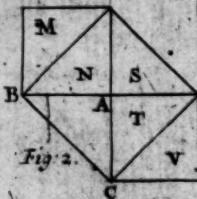
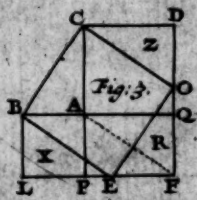
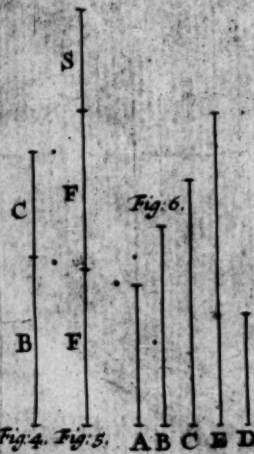
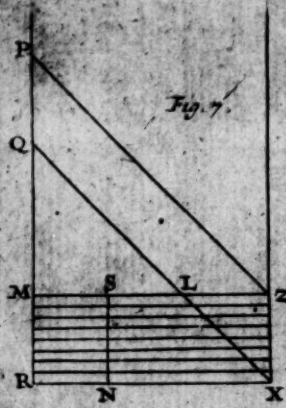
ABD

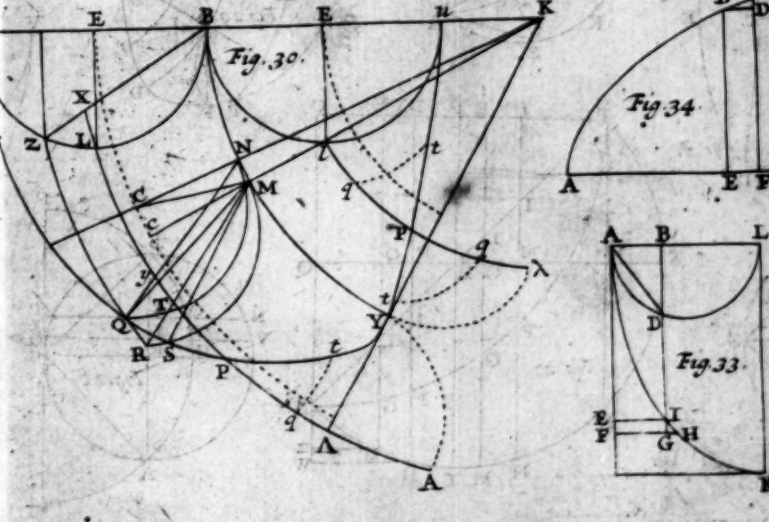
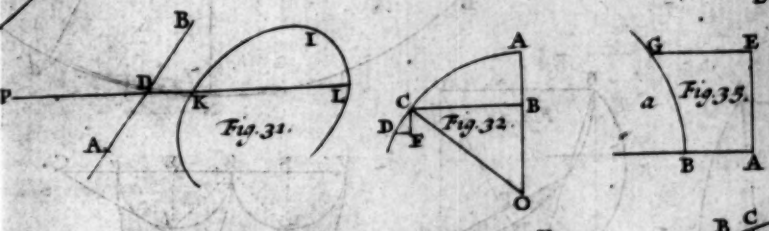
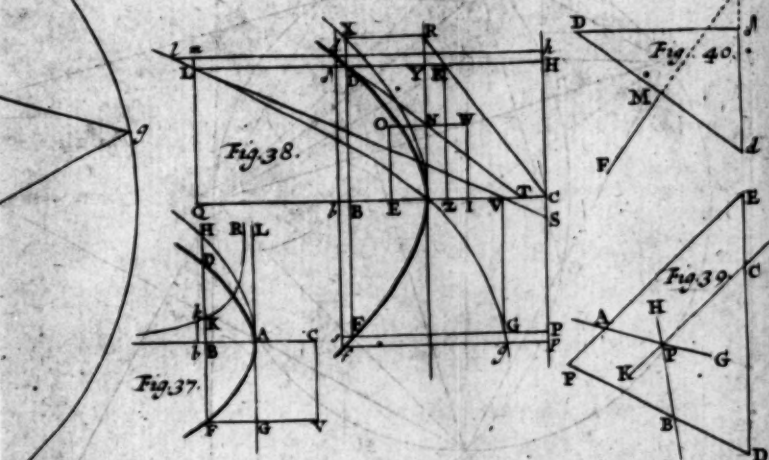
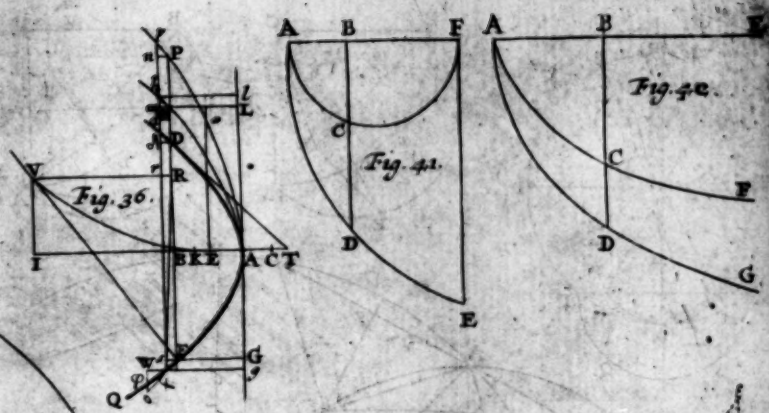
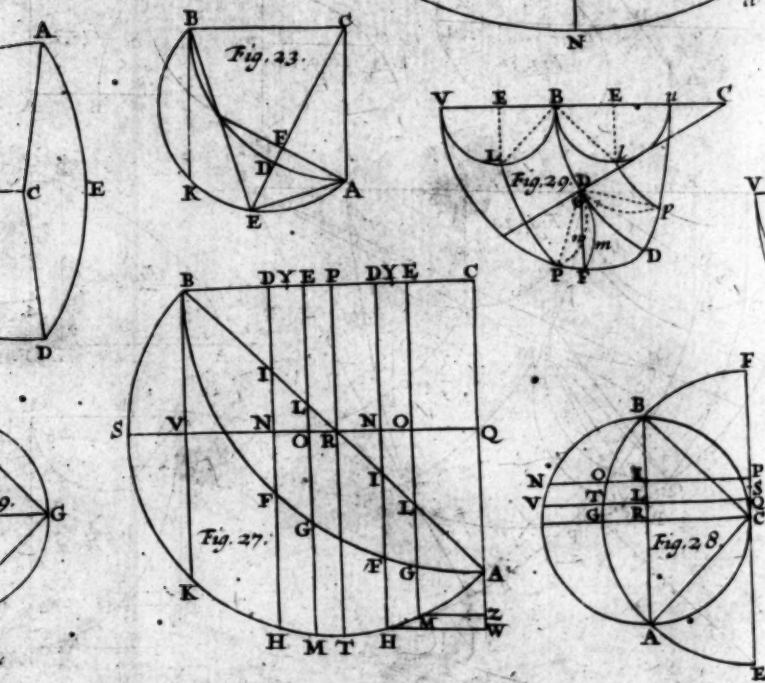
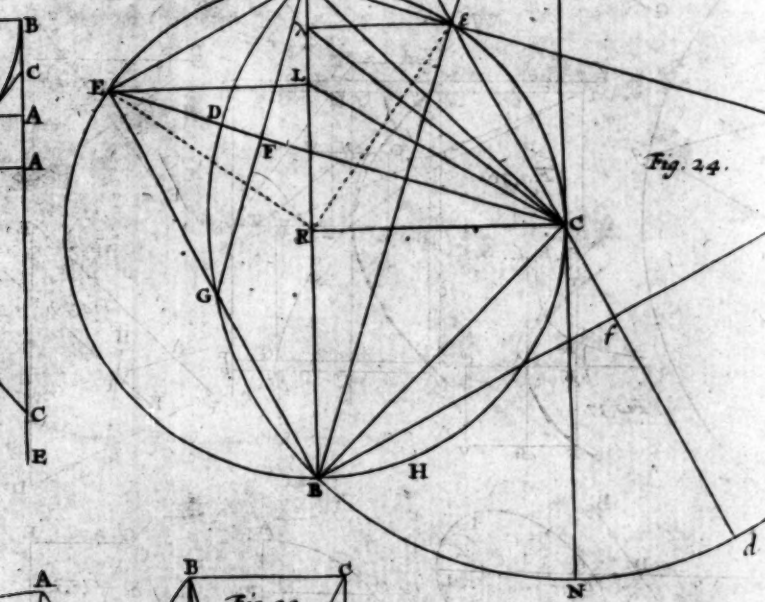
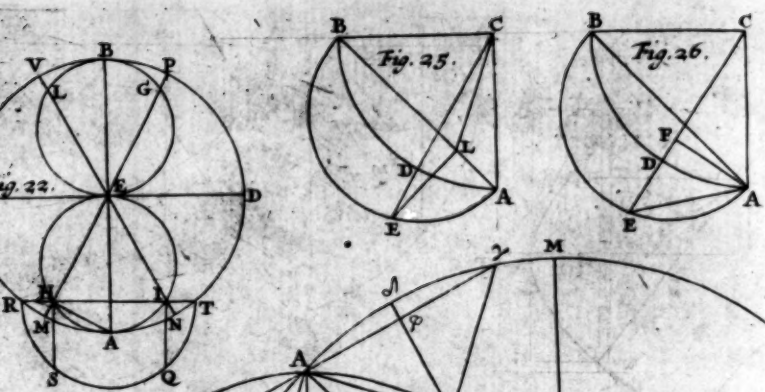
$$\begin{aligned}
 ABD = & \frac{ry^{\frac{1}{2} + r \times v}}{n+1} - qv + \sqrt{2ay + yy} - \frac{y}{n+2 \times n+1} y^{n+1} \\
 & - \frac{ra}{n+2 \times n+1} y^n + \frac{raa \times 2n+1}{n \times n+2 \times n+1} y^{n-1} - \\
 & \frac{aA \times 2n-1}{n-1} y^{n-2} + \frac{aB \times 2n-3}{n-2} y^{n-3} - \frac{aC \times 2n-5}{n-3} y^{n-4} \\
 & + \&c.
 \end{aligned}$$

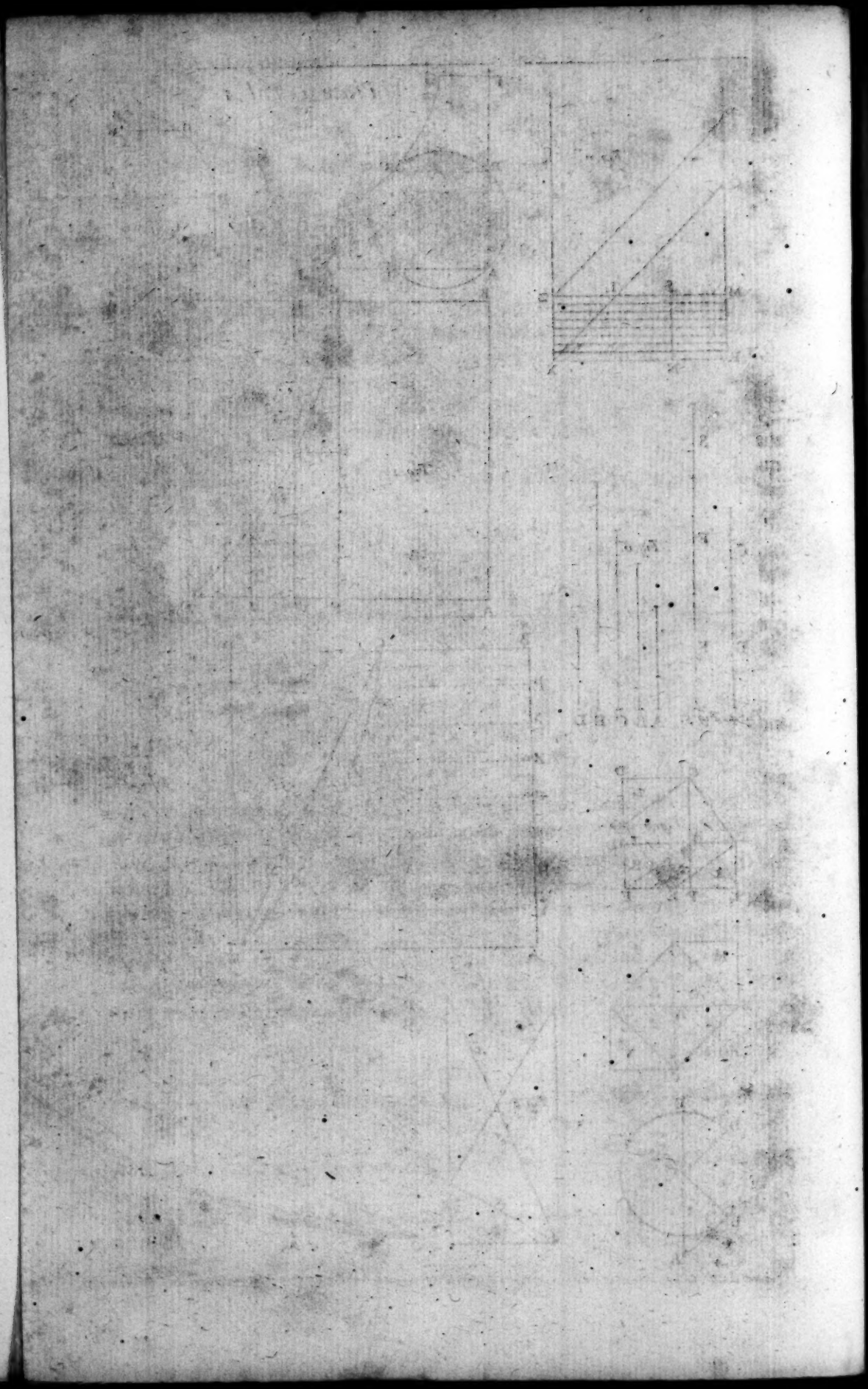
De hac Serie hæc sunt notanda : (1.) Quod Literæ majusculæ, A, B, C, &c. denotent coefficientes terminorum ipsis præcedentium. (2.) Quod si exponens n sit integer positivus aut nihilo æqualis, aut etiam si $2n$ sit numeris impar, tum Quadratura exhibeatur per numerum terminorum finitum; Serie in his casibus abrupte. (3.) Quod $+q$ sit æqualis ultimo termino abrupti. (4.) Quod ex terminis Quantitatem $\sqrt{2ay + yy}$ multiplicandis ultimo abruptus sit duplicandus. (5.) Quod omnes illæ Figuræ, in quibus n est numerus integer positivus & impar, vel generalius, omnes illæ figuræ, in quibus ultimus terminus abruptus habet signum affirmativum seu $+$, habeant unam portionem Geometricæ Quadrabilem, & ex ipsa serie facile assignabilem, sumendo abscissam, ut in Not. 4. præcedentis seriei.

Exemplum 1.] Sit $z = v$, quia in hoc casu $r = 1$, $n = 0$, ideo terminus ultimo abruptus est $-\frac{ra}{n+2 \times n+1} y^n$, unde $+q = -\frac{a}{2}$ (per Not. 3.) & quia in hoc casu $-\frac{a}{2}$ est terminus ultimo abruptus, ideo $-a$ est ultimus terminus in $\sqrt{2ay + yy}$ multiplicandus, (per Not. 4.) Adeoque $ABD = vy + \frac{av}{2} + \sqrt{2ay + yy} \times -\frac{1}{2}y - a$.

Exemplum 2.] Sit $z = \frac{vy}{a}$ quia in hoc casu $r = \frac{1}{a}$, $n = 1$, ideo terminus ultimo abruptus est $\frac{raa \times 2n+1}{n \times n+2 \times n+1} y^{n-1} = \frac{a}{4}$, unde $q = \frac{a}{4}$, & $\frac{a}{2}$ ultimus terminus in $\sqrt{2ay + yy}$ multiplicandus; adeoque $ABD = \frac{vy}{2a} - \frac{av}{4} + \sqrt{2ay + yy} \times -\frac{1}{6a} - \frac{1}{12} + \frac{a}{2}$; & si capiatur







piatur $y = \sqrt{\frac{aa}{2}}$, erit Area competens hinc Abscissæ Geometrice Quadra-

$$\text{bilis, scil. Area} = \frac{1}{12} \sqrt{2aa^2 + \frac{a^2}{2}} \times 5a - \sqrt{\frac{a^2}{2}}.$$

Tertio.] Sit ACF semicirculus, ADE Curva Geometrice irrationalis, cujus N. 235. p. 785. ordinatim applicata BD secat semicirculum in C. Quantitates vero designentur ut prius, scil. Diameter AF = 2a. Abscissa AB = y, Arcus AC = v, Ordinata BD = z; sitque $z = rv^2y^n$. Aequatio exprimens Naturas Curvarum ADE, in qua r denotat quantitatem quamlibet datam & determinatam, & n exponentem indefinitum quantitatis indeterminatæ y. Dico Aream

$$\begin{aligned} ABD = & rv^2y^{n+1} - qv^2 + v\sqrt{2ay - y^2} \times \frac{2ra}{n+1}y^n + \\ & \frac{2ra^2 \times 2n+1}{n \times n+1}y^{n-1} + \frac{aA \times 2n-1}{n-1}y^{n-2} + \frac{aB \times 2n-3}{n-2}y^{n-3} \\ & + \frac{aC \times 2n-5}{n-3}y^{n-4} + \frac{aD \times 2n-7}{n-4}y^{n-5} + \frac{aE \times 2n-9}{n-5}y^{n-6} \\ & \&c. - \frac{2ra^2}{n+1}y^{n+1} - \frac{2ra^3 \times 2n+1}{n^2 \times n+1}y^n - \frac{a^2A \times 2n-1}{n-1}y^{n-2} \\ & - \frac{a^2B \times 2n-3}{n-2}y^{n-3} - \frac{a^2C \times 2n-5}{n-3}y^{n-4} \&c. \end{aligned}$$

De hoc Theoremate hæc sunt notanda; (1.) Quod componatur ex duabus seriebus infinitis, quarum prior (signo + connexa) multiplicatur in $v\sqrt{2ay - y^2}$; termini autem posterioris (signo - affecti) sunt absoluti. (2.) Quod in priori serie literæ majusculæ, A, B, C, D, E, &c. designent coefficientes terminorum ipsis respective præcedentium; nec non in posteriori eosdem obtineant valores, quos in priori. (3.) Quod Quadratura exhibeatur per quantitatem finitam, quando n est numerus integer positivus, aut nihilo æqualis, vel etiam si 2n sit numerus impar: nam in his casibus utraque Series abrumpitur. (4.) Quod 2q sit æqualis ultimo termino abrumpenti prioris Seriei.

Exemplum I.) Sit $z = \frac{v^2}{a}$. Quia in hoc casu $n=0$, $r=\frac{1}{a}$, ideo erit

$$\text{Area ABD} = \frac{yv^2}{a} - v^2 + 2v\sqrt{2ay - y^2} - 2ay. \text{ Corol. Inte-}$$

gra figura AFE est æqualis duplo Quadrato, cujus Latus est ACF, dempto diametri Quadrato.

Exemplum

Exemplum 2.] Sit $z = \frac{y^2 v^2}{a^2}$, quia hoc in casu $n = 1$, $r = \frac{1}{a^2}$, ideo
 erit Area ABD = $\frac{y^2 v^2}{2 a^2} - \frac{1}{4} v^2 + v \sqrt{2 a y - y^2} \times \frac{v}{2 a} + \frac{3}{2} -$
 $\frac{1}{4} y^2 - \frac{3 a y}{2}$.

Exempl. 3.] Sit $z = \frac{y^2 v^2}{a^2}$, quoniam in hoc casu $n = 2$, $r = \frac{1}{a^2}$,
 ideo erit Area ABD = $\frac{y^2 v^2}{3 a^2} - \frac{1}{6} v^2 + v \sqrt{2 a y - y^2} \times$
 $\frac{2 y^2}{9 a a} + \frac{5 y}{9 a} + \frac{5}{3} - \frac{2 y^2}{27 a} - \frac{5 y^2}{18} - \frac{5 a y}{3}$.

*The Quadrature
 of the Logarith-
 mick Curve, by
 Mr. J. Craig.
 N. 244. p. 73.
 Oct. An. 1698.
 Fig. 43.*

XV. Esto ONF Curva Logarithmica, cujus Asymptotos AR, in qua
 tale sumatur punctum A, ut ejus prima ordinata AO sit subtangenti seu uni-
 tati æqualis: Quæritur spatium Curvilineum AONM à duabus ordinatis

AO, MN, Abscissa AM, & Curva Logarithmica ON comprehensum.
 Ex O ducatur OE ad AM parallela, & secans MN in E; dico quod
 rectangulum ex segmentis ME, EN sit æquale spatio quæsito.

Demonstratio.] Vocetur Ordinata MN, z ; subtangens AO seu ME, s ;
 & ad axem AR construatur alia Curva HGE, cujus æquatio $2sz = x^2$,
 ubi ejus ordinata GM = x ; dico quod sit quadratrix Logarithmicæ juxta
 Methodi meæ fundamentum; scil. ejus subnormalis est respectivæ hujus Ordi-
 natæ æqualis: ut ex calculo istius Methodi patebit; Ergo (juxta alibi à me
 exposita) si ad G ducatur GC perpendicularis & æqualis lineæ GM, nec
 non HD parallela ad GC, & lineis GM, CM occurrens in B & D; erit
 trapezium GBDC = AONM. Sed GBDC = GMC - BMD =
 $\frac{1}{2} x^2 - \frac{1}{2} BM q = SZ - \frac{1}{2} HA q$; sed HA = $\sqrt{2 AO q}$ ex natura Cur-
 væ H G Q, ergo GBDC = SZ - AO q = AO × MN - AO q
 = AO × MN - AO = ME × MN - ME = ME × EN; Ergo
 etiam AONM = ME × EN. Q. E. D.

*A Quadratrix to
 the Circle; being
 the Curve de-
 scrib'd by its E-
 quable Evolution,
 by
 N. 260. p. 445.
 Jan. An. 1700.
 Fig. 44.*

XVI. 1. By the Equable Evolution of a Circle, I mean such a gradual
 approach of its Periphery to Rectitude, as that all its Parts do together, and
 equally, evolve or unbend; or so that the same Line becomes successively
 a less and less Arc of a reciprocally greater Circle.

2. Let AHKA be the Periphery of a Circle, AE a Tangent to the Point
 A. Let this Circular Line be suppos'd, cut or divided at A, and then to unbend
 (like a Spring), its upper end remaining fixt to its Tangent AE, whilst the
 other

other parts do equally evolve or extend themselves thro' all the degrees of less Curvature (as in ABD , AMC , &c.) till they become streight in coincidence with the Tangent AE .

3. Let AMC be the Evolving Curve in any middle position between its first and last. Join the fixt end A , and the moving end C , by the Chord-line AC , intersecting the first Circle at H . I say, that AMC is a like Segment to $A \# H$, cut off in the first Circle by the Chord AH . For, by the Supposition, AMC is the Arc of a Circle, having AE a Tangent common both to it and $A \# H$, and both Arcs are terminated in the same Right-Line AC .

4. Hence the Curve $ADCE$ (describ'd by the moving end of the Periphery in its Evolution) may be thus constructed. Let the Circle $AHKA$ be by Bisections divided into any number of equal Parts. Let H be one of the Points of such division. Then say, as the Number of equal Parts in the Arc $A \# H$, is to the Number of Parts in the whole Periphery $AHKA$; so is the Chord AH , to a fourth Line, which let be AC in AH produc'd. So is C a point in the Curve $ADCE$.

5. *Dem.* Upon AC describe AMC , an Arc like to the Arc $A \# H$. Whence $AH : AC :: A \# H : AMC$. But by construction, $AH : AC :: A \# H : \text{Periph. } AHKA$, therefore is the Arc AMC equal to the whole Periphery $AHKA$, and like to the Arc $A \# H$. Consequently AMC represents the Evolving Periphery, in a Position like to the Arc $A \# H$, and C is the describing Point.

6. After the same manner may be found other Points, thro' which the Curve may be drawn. But here (as in the old *Quadratrix* of *Dionysius*) the Point E cannot be precisely determined, but the Curve may be brought so near it, that its flexure or tendency will so lead to the Point E , that AE shall be near enough to the truth for common Uses.

7. Supposing the Point E found, a Tangent to any point of the Curve may be drawn : and supposing a Tangent drawn, the point E may be determined ; the property of the Tangent being this, that supposing RT a Tangent to the Point C , and CA , CE , drawn from C to each end of the rectify'd Circle, the Angle ACT (the lesser Angle that AC makes with the Tangent) is equal to ACE the Angle made by the two Lines drawn from C .

8. Let c be a point in the *Quadratrix* indefinitely near to C ; and draw Ac intersecting $AHKA$ in b , and AMC in o . To Ac as a Chord, draw the Arc $A \# c$, like unto the Arc $A \# b$. To the point C of the Arc AMC draw the Tangent $CL = AE$, and joyn LA : so is oC an indefinitely little particle of the Arc coincident with its Tangent.

9. Because of the like Segments $A \# bA$, $AMoA$, $A \# cA$, as Chord Ac to Chord Ao ; so is Arc $A \# c$ ($= AMC$), to Arc AMo . Or, $Ac : Ao :: A \# c$ ($= AMC$) : AMo . And dividing, $Ac - Ao$ ($= co$) : $Ao :: A \# c - AMo$ ($= Co$) : AMo . That is, $co : Ao :: Co : AMo$, and alternately, $co : Co :: Ao : AMo$. Put AC for Ao , and AMC for AMo as differing indefinitely little) and then 'tis $co : Co :: AC : AMC$. But by construction $CL = AE = AMC$, whence $co : Co :: AC :$

$AC : CL$, and the Angle $LCA = Coc$, (oc being infinitely near to AC , is therefore parallel to it.) And therefore Coc , ACL , are like Triangles.

10. Because of $CL = AE$, Angle $EAC = LCA$, (CL and EA being Tangents to the two ends of the same Circular Arch AMC , make equal Angles with its Chord AC) and AC common to both, the Triangles EAC , and ACL , are like and equal: therefore are all three Coc , ACL , EAC , like Triangles. Whence it follows, That the Angle ACE (in the Triangle EAC) is equal to the Angle ocC in the Triangle coC) but $ocC = ACT$ because oc and AC are parallel; therefore the Angle $ACE = ACT$.
Q. E. D.

The Dimensions
of a Sphere and
Cylinder com-
pared, by D.
Wallis N. 263.
p. 547.
Mar. An. 1700.

XVII. In Epistola quadam mea (*Oper. Mathem.* Vol. III.) inter alias meas Methodos (quibus in Tetragonismis utor) occurrunt hæ duæ; quarum alteram appello Methodum *Convolutionis & Evolutionis*; alteram, Methodum *Complicationis & Explicationis*. Quarum ope ostendo (tum aliarum Figurarum, tum speciatim) Cycloidis dimetiendæ quis sit modus omnium simplicissimus.

Simili Artificio colligetur, tota Sphæræ cum Cylindro collatio: Quod sibi Monumentum fecit Archimedes.

Fig. 45.

Quippe si ad Basin P (Peripheriæ Circuli æqualem) sumatur altitudo R (æqualis Radio) fiet Parallelogrammum Rectangulum $= RP$. Quod ex minutis Parallelogrammis æque altis, numero infinitis, (juxta receptam Methodum Indivisibilium) conflatum intelligatur. Quorum si omnes vertices intelligentur in unicum punctum contrahi, quo ex illis minutis Parallelogrammis totidem fiant Triangula super eisdem Basibus æque alta; singula singulorum, adeoque omnia omnium, dimidia; (curvata Basi in Circuli Peripheriam) fiet Circulus (Centro C , Radio R .) Parallelogrammi dimidius $= \frac{1}{2} RP$.

Fig. 46.

Quæ est ipsa Archimedis Dimensio Circuli: æqualis utique Triangulo Rectangulo, cujus laterum (circa Angulum Rectum) æquatur alterum Peripheriæ, alterum Radio expositi Circuli. Quippe $\frac{1}{2} R$ (semi-altitudo Trianguli) in P (Basin) ducta, exhibet Magnitudinem istius Trianguli $= \frac{1}{2} RP$, circulo æqualem. Idemque accommodabitur Sectori Circulari, sumpto arcu A pro P Peripheria.

Fig. 47.

Porro; si ad illud Parallelogrammum $= RP$ (ut Basim) sumatur itidem (in ordine ad Hemisphærium) Altitudo R ; fiet Parallelepipedum $= RRP$. Quod pariter, ex minutis Parallelepipedis æque altis, numero infinitis, conflatum intelligatur (minutis areolis istius Plani insistentibus; quorum omnium communis altitudo sit R ; & Basium Aggregatum $= RP$. Quod si Parallelogrammum hoc (manente magnitudine $= RP$) intelligatur in Curvam Superficiem Cylindricam curvari (cujus Basis sit P , jam in Peripheriam circuli convoluta, Altitudo R) quo minuta illa Parallelepipeda in totidem Cuneos, seu Prismata basium triangularium, (Parallelepipedorum singula singulorum, adeoque omnia omnium, sub-dupla) redigantur; Acies seu Vertices habentia totidem C puncta (seu lineolas minutas) in Axe Cylindri constituta, eumque complentia, fiet Cylindrus (Parallelepipedi Dimidius) $= \frac{1}{2} RRP$.

Vel (in ordine ad Sphæram integram) si sumatur utrinque Altitudo R, (ut sit tota Altitudo $D = 2 R$;) fiet convolutione pariter facta) Cylindrus (ut prius) ex Cuneis seu Prismatibus numero infinitis (Vertices seu Acies habentibus in Axe Cylindri) $= R R P = \frac{1}{2} R P \times 2 R$, æqualis facto ex $\frac{1}{2} R P$ (circulari Basè) in altitudinem $2 R$: seu (quod tantundem est) $= \frac{1}{2} R \times 2 R P$, æqualis facto ex $\frac{1}{2} R$ (semisse communis Altitudinis Cuneorum) in (Basium aggregatum) $2 R P$.

Quod quidem Basium Aggregatum est ipsa Cylindrica Superficies curva $= P \times 2 R$) æqualis Facto ex Basis Circularis Peripheria P in Altitudinem $2 R$ ducta:) seu $\frac{1}{2} R P \times 4$, (æqualis quatuor Circulis in Sphæra maximis:) Quibus si accenseantur, oppositæ duæ Bases circulares; fiet Cylindri (Sphæra circumscripti) tota superficies, æqualis sex Circulis maximis, $\frac{1}{2} R P \times 6 = 3 R P$. Et Cylindri Magnitudo, $= R R P = \frac{1}{2} R P \times 2 R$, æqualis Facto ex Basè Circulari $\frac{1}{2} R P$ in altitudinem $2 R$ ducta: ut prius.

Quod si porro, Cuneorum horum omnium Vertices (Cylindri Axem Complentes) intelligantur in unum punctum contrahi: quo Cunei illi, seu Prismata, jam fiant totidem Pyramides, super iisdem Basibus æque altæ; singulæ singularum, adeoque omnes omnium, subsesqui-tertiæ, seu ut $\frac{1}{2}$ ad $\frac{1}{2}$; & Superficies, prius Curva Cylindrica, jam fiat Sphærica propter ejus omnia puncta æqualiter à Centro remota; manente, quod prius erat, Basium aggregato $= 2 R P$, (quatuor Circulis Maximis æquali,) habebitur, tum tota Sphæra Superficies $= 2 R P = \frac{1}{2} R P \times 4$ (æqualis quatuor Circulis maximis; & quidem toti Curvæ Cylindricæ æqualis & partes partibus respectivè æquales, eadem Axis partes respicientibus;) tum Sphæra magnitudo $= \frac{1}{2} R R P = \frac{1}{2} R P \times 2 R$; æquales Facto ex $\frac{1}{2} R$ (triente communis Altitudinis Pyramidum omnium) in $2 R P$ (Basium Aggregatum, jam factam superficiem Sphæricam) ducto.

Est itaque Cylindri Sphæra circumscripti tum Superficies tum Magnitudo, ad Superficiem & Magnitudinem inscriptæ Sphæra, sesqui-altera, seu ut 3 ad 2. (Illic quidem, ut sex Circuli Maximi $= 3 R P$, ad quatuor Circulos maximos $= 2 R P$: Hic vero ut $R R P$ ad $\frac{1}{2} R R P$.) quod est illud ipsum Archimedis Inventum celebre.

Idem paulo brevius haberetur; si, in Parallelepipedo illo (super plana Basè $2 R P$ (cum Altitudine R) ex minutis Parallelepipedis conflato; Horum omnium Vertices immediate censeantur in unicum C (punctum) comprimi. Quo, manente ut prius Basium Aggregato $= 2 R P$, Parallelepipeda illa in totidem Pyramides redigantur; Vertices habentes ad Sphæra Centrum cœuntes; cujus Radius R , (communis Pyramidum omnium altitudo;) & Sphærica Superficies, Basium omnium Aggregatum. Quippe $\frac{1}{2} R$ (triens communis Altitudinis) in $2 R P$ (Basium Aggregatum) exhibet Sphæra magnitudinem (ut prius) $\frac{1}{2} R R P$; & Sphæra Superficiem $= 2 R P$.

Potestque hoc itidem Sectori Sphærico accommodari. Ducto $\frac{1}{2} R$ (triente communis Altitudinis Pyramidum inibi omnium) in portionem Sphæricæ Superficiæ plano abscissam: Quæ est ad totam Superficiem Sphæricam, ut est Diametri (seu Axis) pars abscissa ad totum Diametrum; ut supra ostensum

Cujus quidem processus totius Ratio his Principiis nritur; nempe, quod figura ex Triangulis est dimidia figuræ ex Parallelogrammis, super eisdem Basibus, æque-altis: (Illam ego appello Figuram *Convolutam*; hanc *Evolutam*;) Et figura ex Pyramidibus, est, triens figuræ ex Parallelepipedis, super eisdem Basibus æque-altis: (Illam ego appello Figuram *Complicatam*; hanc *Explicatam*.) Quæ possunt mille modis accommodari Figuris Curvilineis (tum Superficialibus tum Solidis) mirum in modum perplexis.

Improvements in
England in the
Resolution of
Equations in
Numbers, by Mr.
J. Collins.
N. 46. p. 929.
Apr. An. 1669.

XVIII. 1. It hath been observed by divers of this Nation, that in any Equation, howsoever affected, if you give a Root, and find the absolute Number or Resolvend, (which *Vieta* calls *Homogeneous Comparationis*;) and again give Roots and find more Resolvends; that if these Roots, or rather rank of Roots, be assumed in Arithmetical Progression, the Resolvends, as to their first, second, or third Differences, &c. imitate the Laws of the pure Powers of an Arithmetical Progression of the same degree, that the highest Power, or first Term of the Equation is of. *E. g.* In this Equation $aaa - 3aa + 4a = N$.

$$\text{If } a \text{ be} = \begin{cases} 10 \\ 9 \\ 8 \\ 7 \\ 6 \end{cases} \left\{ \begin{array}{l} \text{Then } N \text{ or the} \\ \text{Absolutes or Re-} \\ \text{solvents will be} \\ \text{found to be} \end{array} \right\} \begin{array}{c} 740 \\ 522 \\ 352 \\ 224 \\ 132 \end{array} \begin{array}{c} 1 \text{ diff.} \\ 2 \text{ diff.} \\ 3 \text{ diff.} \end{array} \begin{array}{c} 218 \\ 170 \\ 128 \\ 92 \\ 36 \end{array} \begin{array}{c} 48 \\ 42 \\ 36 \\ 6 \\ 6 \end{array}$$

To wit, the 3d differences of those Absolutes are equal, as in the Cubes of an Arithmetical Progression.

2. To find what habitude those differences have to the Coefficient of the Equation, 'tis best to begin from an Unite.

3. In any Arithmetical Progression, if you multiply Numbers by Pairs, you shall create a rank of Numbers whose second differences are equal; and if by Ternaries, then the 3d differences of those Products shall be equal. And how to find the greatest Product of an Arithmetical Progression of any Number of Terms having any common difference assign'd, contain'd in any Number propos'd, is shewed by *Pascal* in his Tract *du Triangle Arithmetique*, where he applies it to the Extraction of the Roots of Simple Powers.

4. It appears, how this Rank may be carried easily by Addition, till you have a Resolvend either equal or greater or less than that proposed.

5. When you have a *Majus* and *Minus*, you may interpolate as many more Terms in the Arithmetical Progression as you will, that is to say, Subdivide the common difference in the Arithmetical Progression, and render it less; and then renew, and find the Resolvends, which are easily obtained out of the Powers and their Coefficients, which are supposed known, and may be readily raised from a Table of Squares and Cubes, &c. with which kind the Reader may be furnished in *Guldini Centrobaryca*, and *Babington's Fireworks*. By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more easily, and is so far improved, that after any Figure is placed in the Root, most certain Characters

are

are given to know by Aid of the Subsequent Dividend and Divisor, whether the Figure before assumed be too great or too small: or lastly it may be well concluded, that, as in Logarithms, when you propose such an one as is not absolutely given in the Canon, you do by proportional Work, using the Aid of their first Differences (when their absolute Numbers differ by Unite) find the absolute Number true to 5 or 6 Places farther than the Canon gives it (the reason whereof is, that the first Differences do likewise agree to about the same number of Places;) that I say the like may be done in Equations, after divers of the first Figures of the Roots are found; provided there be the like Agreement in the first Differences of the Interpolated Resolvends.

Moreover we ought here to take notice of a more subtle kind of Interpolation, common to all Gradual Ranks or Progreſſion of Numbers, wherein Differences happen to be equal: Of which kind the Reader may find Examples in *Briggii Arithmetica, Logarithmica & Trigonometria Britannica*, relating to Logarithms, Sines, and the Powers of an Arithmetical Progreſſion: But the Method there delivered may be rendred more easy and general, viz. by Aid of a Table of Figurate Numbers, by deriving Generating Differences sought, from those given; a Doctrine that easily flows from *Mercator's Logarithmorechnia*, and of use in the Case in hand, should we suppose these Powers and their Coefficients unknown, or a Table of Squares and Cubes wanting, and give nothing more than a few Resolvends belonging to equal Moments or Spaces. And this may likewise be of good Use in *Gauging*, when having the Contents of a Solid, for every three Inches more or less given, without knowing the Dimensions of the Figure, and even in most Cases, when the Differences are Progreſſive of one kind, without knowing the Figure it self, having nothing given but its Contents at several equal parallel Distances, each such distance may be subdivided, and made as many as you please, and the respective Contents found by this general Method of Interpolation.

After one Root is obtained, the Methods of *Huddenius* and others will depress the Equations so as to obtain more, and consequently all of them.

6. It is easy by a Table of Figurate Numbers to give the Sum of any such Rank, or any Term in it relating to a known part of the Series of Equals or Roots; but *è converso*, giving the Resolvend to find the Root, comes to an Equation as difficult as that propos'd; as in *Dr. Wallis's Chapter of Figurate Numbers*.

7. Some affirm, they can give good Approaches for the obtaining a Root of any pure Power, affected Equation, or for the finding any of the mean Proportionals in any Rank between the two Extrems given.

8. Others pretend to have found out the Method (incited thereto by an Example in *Albert Gerard's Invention Nouvelle en Algebre, a Amsterdam. 1629*) so much, by comparing of Equations, to increase or diminish the unknown Root of Equation as to render it a whole Number (or less differing therefrom than any Error assign'd,) and by *Albert Gerard's Method of Aliquot Parts* to find the same, and thereby the Root sought, altho' it be a mixt Number, Fraction, or Surd.

Probably this may Sympathize with what is promised by the Learned *Huddeus* in *Annexis Geometriae Cartesiana*, where he saith he intended not then to publish certain Rules he had ready; whereof one was to find out all the Irrational Roots both of Literal and Numeral Equations. This must be understood, when such Roots are possible; for 'tis certain there are Infinite Equations, whose Roots are no ways explicable, either in whole or mixt Numbers, Fractions, or Surds, and can be no otherwise explain'd, but by a *quaproxime*.

Ibid. p. 932.

9. The *Author* of this *Narrative* considering, that the Conick Sections may be projected from lesser Circles placed on the Sphere, and thence easily (otherwise than hitherto hath been handled) described by Points, and that by their Intersections, some Spherick Problem is determined; accordingly he found that this following Problem, according to the various Situation of the Eye and of the projecting Plan, would take in all Cases.

The Distances of an unknown Star are given from two Stars of known Declination and Right Ascension; the Declination and Right Ascension of the unknown Star is required.

And saith, He hath observed, that, admitting the Mechanism of dividing the Periphery of a Circle into any Number of equal Parts, or (which is equivalent, the use of a Line of Chords, that this Problem, wherever the Eye be placed, may be resolved by plain *Geometry*, and yet the Eye shall be so placed, as to determine it by the Intersection of the Conic Sections; consequently those Points of Intersection (the Species and Position of the Figures being given) may be found without describing any more Points than those sought; and the Lengths of Ordinates falling from thence on the Axes of either Figure Calculated by mixt *Trigonometry*, and hence likewise the Roots of all *Cubick* and *Biquadratick Equations* found by *Trigonometry*.

For giving, from the *Mesolabe* of *Slusius*, the Scheme that finds these Roots, it will then be required to fit those Sections into Cones, which have their Vertex either in the Center, or an assigned Point in the Surface of the Sphere, to which they relate as Projected, and proceed to the Resolution of the Problem proposed: And how to fit in those Sections, see the 7 Books of *Apollonius*, *Mydorgius*, the 3d Volume of *des Cartes's* Letters; *Leotaudi Geometria Practica*, *Andersonii Exercitatio Geometrica*.

As to the Problem it self, it is determined on the Sphere by the Intersections of the two lesser Circles of Distance, whose Poles are the known Stars. And this Problem hath divers Geometrick ways of Resolution.

1. By plain *Geometry* (in the Sense before mentioned; supposing a Plain to touch the Sphere or the North Pole: if the Eye be at the South Pole, projecting those Circles into the said Plain, they are still Circles (by reason of the sub-contrary Sections of the visual Cones) whose Centers fall in the sides of the Right-lined Angle, made by the Projected Meridians, that pass thro' the known Stars; and thus the Problem is easily solved in this manner.

2. If it be required to be performed by Conick Geometry; in one Case it may be done, by placing the Eye at the Center of the Sphere, and projecting

as before; to wit, when the longer Axes of the Figures being produced, concur above the Vertex: Here the Problem is determined by the Intersections of two Conick Sections (whereof a Circle cannot be one, unless its Center be in the Axis of the other Figure:) And in this second Case, these points of Intersection fall in the same Right-line or projected Meridian, they did before, but at a more remote distance from the Pole Point, to wit, in the former Supposition, the Polar distance was measured by a Right Line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident, how one Projection may beget another, yea, infinite others, altering the Scale: and how the lesser Circles in the *Stereographick* Projection help to describe the Conick Sections in the *Gnomonick* Projection: But (to reduce the matter to the common Radius) if we suppose two Spheres equal, and so placed about the same Axis, that the Pole point of the one shall pass thro' the Center of the other, and the Touch-Plane to pass thro' the said Center or Pole-Point; and that a lesser Circle hath the same Position in the one as in the other; then, if the Eye be at the South Pole of the one, it is at the Center of the other; and any Projected Meridian drawn from the Projected Pole point to pass thro' both the Projections of these lesser Circles, the distances of the Points of Intersection are the Tangents of the half and the whole Arch of the Meridian so intersected. But as to the Points of Intersection, which determine the Problem propos'd, they may be found without the aid of the former way, from a *Gnomonick* and *Stereographick* Method of measuring and setting off the Sides and Angles of Spherical Triangles in those Projections, which is necessary in what follows.

3. If the Problem is to be performed by mixt Geometry, as by a Circle, and either a Parabola, Hyperbola, or Ellipsis, the Circle may be conceived to be the sub-contrary Section of a Cone projected by the Eye at the South Pole, and any of the rest of the Sections by the Eye at the Center of the Sphere.

4. If by any of the Conick Sections however posited, the Projecting Plane may remain the same; but the Eye must be in some other Part of the Surface of the Sphere, and not in the Axis.

XIX. 1. Constructio quam tradit *Cartesius*, quæque facillime radices Aequationum omnium Cubicarum vel Biquadraticarum, ubi deficit secundus terminus, eruit, ut nota supponi potest; attamen cum cardo sit a quo subsequenter pendent, ex illius Geometria desumptam placuit Regulam adjungere, pauculis nonnullis in melius (uti reor) transpositis.

The Construction of Cubick and Biquadratic Equations, by a Parabola and a Circle, By M. Edmund Halley. N. 188. p. 334. Jul. An. 1687.

Deficiente secundo termino omnes æquationes Cubicæ reducuntur ad hanc formam $z^3. * . a p z . a a q . = 0$, ac Biquadraticæ ad hanc $z^4. * . a p z z . a a q z . a^3 r = 0$, (ubi a designat latus rectum Parabolæ cujusvis datæ, quam in Constructione adhibere licet,) vel sumendo a pro unitate, ad hanc $z^3. * . p z . q = 0$, vel ad hanc $z^4. * . p z z . q z . r = 0$.

Jam data Parabola F A G, cujus Axis sit A C D K L ac latus rectum a vel 1, fiat A C ejus dimidium ac collocetur semper a vertice A versus interiora figuræ; dein sumatur $CD = \frac{1}{2}p$ in linea illa A C continuata versus C si in æquatione fuerit $-p$, vel versus alteram partem si habeatur $+p$. Porro è puncto

Fig. 48.

puncto D, aut ex puncto C si non habeatur quantitas p , erigenda est ad axem perpendicularis DE æqualis $\frac{1}{2}q$, dextrorsum quidem si fuerit $-q$, ad alterum vero axis latus si fuerit $+q$; ac Circulus, Centro E, Radius AE descriptus, si æquatio fuerit tantum Cubica, Parabolam tot punctis F & G interfecabit quot veras habet Radices, quarum quidem affirmativæ, ut G K, erunt ad dextram Axis partem, Negativæ, ut FL, ad sinistram.

Ast si Æquatio Biquadratica fuerit, augeri vel minui debet Circuli Radius AE, addendo si fuerit $-r$, vel subducendo si sit $+r$, ex ejus quadrato rectangulum ar , seu contentum sub Latere Recto & quantitate data r ; id quod nullo fere negotio efficitur Geometrice. Hujus vero Circuli intersectiones cum Parabola omnes veras Biquadraticæ Æquationis Radicis, dimissis ad axem perpendicularis, exhibebunt; affirmativas quidem ad dextram Axis, Negativas vero ad sinistram. Totius demonstrationem *Cartesio*, ejus Inventori, relinquo.

Notandum hic me operam dare ut semper habeantur Radices affirmativæ ad dextrum Axis latus, ut evitetur confusio, à pluribus cautionibus, quarum causa minime evidens est, necessario oritura.

His præmissis, ut aditus pateat ad constructionem etiam earum æquationum ubi reperitur terminus secundus, consideranda venit Regula pro tollendo termino secundo, ac reducenda æquatione ad aliam quæ methodo præcedente construi possit. Omnes vero hujus classis æquationes cubicæ ad hanc formam, $z^3. bzz. apz. aaq = 0$, vel ad hanc, $z^3. bzz. *.aaq = 0$; Biquadraticæ vero ad hanc, $z^4. bz^3. apz. aaqz. a^3r = 0$, vel ad hanc, $z^4. bz^3. *.aaqz. a^3r = 0$, vel $z^4. bz^3. apz. *.a^3r = 0$, vel denique ad hanc, $z^4. bz^3. *. *.a^3r = 0$, reduci possunt: è quibus omnibus, prout signis $+$ & $-$ diversimode connectuntur, ingens oritur varietas; unde Regula generalis omnibus inserviens obscura ac maxime difficilis redditur, nisi methodo quam subjungimus illustrata nodisque extricata tractetur.

Tollitur in Biquadraticis secundus terminus ponendo $x = z + \frac{1}{4}b$, si fuerit $+b$ in æquatione, vel $z = z - \frac{1}{4}b$, si fuerit $-b$: hinc $x - \frac{1}{4}b$ in primo casu, & $+\frac{1}{4}b$ in altero æquatur z ; & in Æquatione quavis proposita, substituta loco z quantitate æquali, prodibit nova æquatio termino secundo carens, cujus radices omnes x data differentia $\frac{1}{4}b$ vel excedunt vel deficiunt à radice quæsita z .

$$\text{Exemp. 1.]} \quad z^4 + bz^3 - apzz - aaqz + aaar = 0. \\ \text{fit } x - \frac{1}{4}b = z \quad \text{Et erit}$$

$$xx - \frac{1}{2}bx + \frac{1}{16}bb = zz$$

$$xxx - \frac{3}{4}xxb + \frac{3}{16}xbb - \frac{1}{64}bbb = z^3$$

$$\& x^4 - bx^3 + \frac{3}{8}bbxx - \frac{1}{16}b^3x + \frac{1}{256}b^4 = z^4.$$

Hinc

$$\begin{aligned}
\text{Hinc } x^4 - bx^3 + \frac{3}{8}bbxx - \frac{1}{16}bbbbx + \frac{1}{256}b^4 &= +z^4 \\
+bx^3 - \frac{3}{4}bbxx + \frac{3}{16}bbbbx - \frac{1}{64}b^4 &= +bz^3 \\
- apxx + \frac{1}{2}apbx - \frac{1}{16}apbb &= -apxz \\
- aaqx + \frac{1}{4}aaqb &= -apqz \\
+ aaar &= +aaar
\end{aligned}$$

Harum omnium summa fit æquatio nova secundo termino carens, quæque proinde juxta Regulam *Cartesianam* construi possit, secundo loco $\frac{1}{2}p$ dimidium coefficientis termini tertii per a sive Latus rectum divisi, hoc est

$$- \frac{3}{16} \frac{bb}{a} - \frac{1}{2}p; \text{ ac Loco } \frac{1}{2}q \text{ dimidium coefficientis termini quarti per } aa \text{ divisi, sive } + \frac{1}{16} \frac{bbb}{aa} + \frac{1}{4} \frac{pb}{a} - \frac{1}{2}q.$$

Cujus partes signo $+$ notatæ sinistrorsum ab Axe, signo $-$ notatæ dextrorsum collocandæ sunt, ut habeatur centrum Circuli ad constructionem requisiti, ac cujus intersectiones cum Parabola, dimissis in axem perpendiculis, radices omnes veras x designet, affirmativas quidem ad dextram axis, negativas vero ad sinistram. Cum vero $x - \frac{1}{2}b = z$, ducendo lineam axi Parallelam, ad dextrum ejus latus & ad distantiam $\frac{1}{2}b$, perpendiculara illa ad hanc Parallelam terminata designabunt omnes radices quæsitæ z , affirmativas ad dextram, negativas vero ad sinistram. Radium circuli quod attinet, habetur ille addendo partes negativas ac auferendo partes affirmativas termini quinti aa divisi, e quadrato lineæ AE , à centro invento E ad verticem Parabolæ A ductæ: id quod maxima ex parte efficitur capiendo loco lineæ AE lineam EO , quæ ad O intersectionem Parabolæ ac parallelæ prædictæ terminatur; ejus enim quadratum omnes termini quinti partes ex ablatione termini secundi æquationi novæ ingestas completitur (uti facile probabitur): ac restat solummodo ut ipsius EO quadratum augeatur, si in æquatione habeatur $-r$, vel minuatur, si sit $+r$, additione vel subtractione rectanguli ar , unde conflatur quadratum Radii Circuli quæsitum.

Hæc est Methodus investigandi regulam centalem *D. Bakeri* omnibus cautionibus libera, ac satis facilis; ac sola differentia ex eo provenit, quod ego juxta Axem, ille vero juxta Axi parallelam circuli ejusdem centrum determinat: quodque ego semper radices affirmativas ex Axis dextro latere invenio, quas ille nunc dextro nunc sinistro constituit.

Æquationes cubicas quod attinet, eas reduci debent ad Biquadraticas, antequam eadem regula generali construi possint; id quod fit ducendo æquationem propositam in radicem suam z , unde provenit æquatio Biquadratica in qua deficit terminus ultimus sive r : quapropter sublato secundo termino & invento centro E , linea EO est radius Circuli; cum scilicet ar sit $= 0$, & in nova æquatione totus terminus quintus ex ipsa ablatione termini secundi oriatur.

Exemp. 2. $z^4 - bzz + apz + aaq = 0$: Quæ ducta in z fit
 $z^5 - bzz^2 + apzz + aaqz = 0$

Ad tollendum secundum terminum ponatur $x + \frac{1}{4}b = z$, & fiet

$$\begin{aligned} x^4 + bx^3 + \frac{3}{8}bbxx + \frac{1}{16}b^3x + \frac{1}{256}b^4 &= +z^5 \\ -bx^3 - \frac{3}{4}bbxx - \frac{3}{16}b^3x - \frac{1}{64}b^4 &= -bz^2 \\ + apxx + \frac{1}{2}apbx + \frac{1}{16}apbb &= +apzz \\ + aaqx + \frac{1}{4}aaqb &= +aaqz \end{aligned}$$

In hac nova æquatione, tertii termini semi-coefficientis per a divisa, viz.

$$-\frac{3}{16}\frac{bb}{a} + \frac{1}{2}p, \text{ loco } \frac{1}{2}p \text{ usurpanda est; ac coefficientis termini quarti di-}$$

$$\text{midium, divisum per } aa \text{ Lateris recti quadratum, viz. } -\frac{1}{16}\frac{bbb}{aa} + \frac{1}{4}\frac{pb}{a}$$

+ $\frac{1}{4}q$, vicem ipsius $\frac{1}{4}q$ in constructione *Cartesii* subit : unde centrum E determinatur. Deinde ducta Axi Parallela ad distantiam $\frac{1}{4}b$ ad sinistram ejus latus (ob $x + \frac{1}{4}b = z$) cujus intersectio cum Parabola fit O ; circulus centro E , Radio EO descriptus Parabolam secet vel tanget in tot punctis quot æquatio veras habet radices : quæ quidem radices seu z sunt perpendiculara de punctis illis in Axi parallelam demissa; ad dextram quidem Affirmativæ, Negativæ ad sinistram.

Si in æquatione defuerit terminus tertius vel quartus vel uterque, investiganda regula centrali nulla omnino observanda est methodus differentia, sed deficiente quantitate p vel q , deerunt partes illæ linearum CD ac DE ex quantitate illa aliquo modo deductæ, ac producendum est cum reliquis coefficientibus termini tertii & quarti in æquatione nova, sicut in præmissis exemplis præscriptum est.

Hactenus *Cl. Bakeri* methodum generalem pertractavimus, qua quidem nulla alia faciliior ac paratior expectanda est, assumpta ad constructionem sive Parabola, sive alia quævis linea curva, cum scilicet æquatio ad Biquadraticam ascendit. Etenim dum hæc scribo mihi occurrit regulæ Centralis Effectio Geometrica præter omnem spem expedita, ac harum rerum Curiosis abunde satisfactura.

Descripta Parabola NAM , cujus vertex A , Axis ABC , ac latus rectum a , reducatur æquatio ad hanc formam $z^4 - bzz + apz + aaqz + ar = 0$, vel ad hanc $z^4 - bzz + apz + aaq = 0$, si cubica tantum fuerit : dein ad distantiam $BD = \frac{1}{4}b$ ducatur linea DH Axi parallela, ad sinistram quidem si fuerit $-b$, ad dextram si $+q$, Parabolæ occurrens in puncto D ; de quo demittatur perpendicularum in axem BD . In linea AB continuata versus B fiat $BK = \frac{1}{4}a$, & ducatur linea DK utrinque interminata. Porro sit $KC = 2AB$ in Axe semper ultra K continuato; ac sic habeatur quantitas p signo $-$ affecta, versus eandem partes etiam sumatur $CE = \frac{1}{4}p$, vel in contrarias si habeatur $+p$, ac è puncto E erigatur Axi perpendiculum.

diculum EF (vel è puncto C si defuerit quantitas p) lineæ DK, si opus est continuatæ, occurrens in puncto F; quod quidem Circuli requisiti centrum est, si defuerit quantitas q ; ast si habeatur q , sumenda est in FE, si opus est continuata, lineæ FG = $\frac{1}{2}q$, sinistrorsum quidem si fuerit $+q$, dextrorsum si $-q$ collocanda: Et punctum G erit centrum Circuli ad constructionem propositam idonei; ejusque Radius, si defuerit quantitas r , hoc est, si tantum Cubica fuerit, erit lineæ GD; cujus quadratum in Biquadraticis augendum est, si fuerit $-r$, vel minuendum, si $+r$, additione vel subtractione rectanguli sub r & latere recto. Descripto sic Circulo, ab intersectionibus ejus cum Parabola demissis in lineam DH perpendicularis, quæ ad sinistram sunt, ut NO, radices æquationis negativas semper designant, quæ ad dextram, ut ML, affirmativas.

Aliter ac paulo simplicius Æquationes Cubicæ juxta *Schootenii* Regulam construuntur, quaque etiam radices ad Axem referuntur: quoniam vero ipse inventor nec modum inveniendi, nec demonstrationem inventi exponit, non abs re erit ejusdem fundamentum hic adjicere, simul atque Effectiorem Geometricam concinniorem reddere, atque cautionibus quibus implicatur extricare.

Hæc Regula derivatur ex eo quod omnis æquatio Cubica reduci possit ad Biquadraticam, in qua deficiet terminus secundus: Hoc fit ducendo æquationem propositam in $z - b = 0$, si fuerit $+b$ in æquatione, vel in $z + b = 0$, si fuerit $-b$; & æquatio nova producta easdem habebit radices cum Cubica, atque insuper alteram ipsi $-b$ æqualem, si fuerit $-b$ in æquatione; vel contra.

Proponatur construenda $z^3 - z^2b + apz + aaq = 0$.

Hæc ducta in $z + b$ fit $z^4 - z^3b + apz^2 + aaqz + z^3b - bbzx + abpz + aaqb$.

Hic deficit secundus terminus, ac coefficientis tertii $-bb + ap$ dat

$-\frac{1}{2}\frac{bb}{a} + \frac{1}{2}p$ loco $\frac{1}{2}p$ vel CD in Constructione *Cartesii*, & ex dimidio coefficientis

termini quarti fit $+\frac{1}{2}q + \frac{1}{2}\frac{bp}{a}$ loco $\frac{1}{2}q$ vel DE usurpanda; adeoque

determinatur centrum Circuli quæsitum: atque ob datam unam ex radicibus æquationis novæ, viz. $-$ vel $+b$, dabitur etiam punctum in circumferentia, id est Radius ejus. Denique descripto Circulo, ab intersectionibus ejus cum Parabola demissa in axem perpendicularia æquationis radices exhibebunt, affirmativas & negativas, eadem lege ac supra.

Investigatur autem centrum Circuli constructione perquam facili, cæterisque omnibus in Cubicis præferenda. Descriptæ Parabolæ AMD sit vertex A, atque Axis AF: ad distantiam ipsi b æqualem ducatur Axi parallela DK, ad dextram si fuerit $+b$ in æquatione, ad sinistram si $-b$, quæ Parabolæ occurrat in puncto D. Centris D & A describantur Radiis æqualibus arcus occulti utrinque sese interfecantes, ac per sectionum puncta ducatur linea interminata BC, quæ medio lineæ suppositæ AD perpendiculariter insi-

K 2

stat,

stat, & Axi occurrat in puncto E. Ab E, inferne quidem si in æquatione habeatur $-p$, vel superne versus A si fuerit $+p$, ponatur $EF = \frac{1}{2}p$; & ex F (vel ex E si defuerit p) educatur perpendicularum FG, linea BC occurrens in puncto G; & in GF producta fiat $GH = \frac{1}{2}q$, dextrorsum quidem si in æquatione habeatur $-q$, aliter sinistrorsum, applicanda: ac punctum H erit Centrum quæsitum, HD vero Circuli Radius, qui demissis in Axem Perpendicularis ab intersectionibus suis cum Parabola, ut LM, Radices omnes, ut prius, commonstrabit.

*The Number of
Roots in such
Equations, with
their Limits and
Signs, by Mr. E.
Halley N. 190.
p. 589.
Nov. An. 1687.*

2. Ex *Cartesio* & ex supradictis constat, tam in Cubicis quam in Biquadraticis æquationibus, Radices exponi posse demittendo perpendiculara in Axem, datamve diametrum Parabolæ datæ, ab intersectionibus Curvæ illius cum Circulo. Cum Circulus Parabolam secans, vel in quatuor vel in duobus punctis eam interfecare necesse est, constat in Biquadraticis vel duas vel quatuor radices veras, Affirmativas vel Negativas, semper haberi; uti etiam, si forte Circulus illam tangat, quo in Casu æqualitas duarum Radicum ejusdem signi concluditur. In Cubicis autem, quoniam una ex intersectionibus ad Constructionem requiritur, non nisi una vel tres reliquæ Radices designant unam vel tres; uti in Casu Contactus, unde constat duas æquales reperiri Radices, Problemaque unde resultat æquatio revera Planum esse.

Cubicæ itaque omnes quomodocunque affectæ una vel triplici Radice explicabiles sunt, utique semper possibiles, nempe si Radices Negativas pro veris admiseris: sic Biquadraticæ, quarum terminus ultimus r signo $-$ affecta est, duabus vel quatuor. Ast si habeatur $+r$ in æquatione, eaque tanta sit, ut

Fig. 49.

$\sqrt{GDq - ar}$, minor sit quam ut Circulus, eo radio ac centro G descriptus, Parabolam contingere in aliquo puncto possit, æquatio data omnino impossibilis est, nec ulla Radice Negativa vel Affirmativa explicabilis: Sed de his plura in sequentibus.

Quoniam vero tanta intercedit differentia inter casus Cubicarum & Biquadraticarum, ut simul comprehendendi nequeant; primum Cubicas deinde alteras tractabimus. Cubicæ vero infinitis Circulis in data Parabola construuntur, Biquadraticæ autem unico tantum (saltem his methodis). id adeo quia ponendo $x - e$ sive indeterminata aliqua, æqualem nihilo, æquatio Cubica reducitur ad Biquadraticam easdem Radices cum Cubica habentem, atque insuper aliam ipsi e æqualem; unde fit ut tot Circulis diversis construi possit Cubicæ, quot imaginari velis quantitates e , id est infinitis. Inter has vero Constructiones, illa quam superius (§. ult.) dedi longe facillima est. Huic tamen non multum cedit alia, quæ ad enucleationem Numeri Radicum, earumque Limitum magis accommodata videtur, quæque ortum trahit ex ablatione secundi termini, ponendo modo vulgari $x = z +$ vel $-$ tertia parte Coefficientis termini secundi. Hæc autem est. Data Parabola A B Y ejusque vertice A, Axe A E & Latere recto a , reducatur æquatio ad formam consuetam, viz. $z^3 + bz^2 + apz + aaq = 0$. Deinde ad distantiam $\frac{1}{2}b$ ducatur Axi parallela B K, dextrorsum quidem si fuerit $+b$, aliter sinistrorsum, Parabolæ occurrens in B; ac lineæ suppositæ A B erigatur perpendicularis utrinque interminata D P, Axi occurrens in puncto G. De B in Axem demitte perpendicularum B C, & ipsi A C fiat G E semper æqualis, ac versus inferiora ponatur. Ab E fiat

Fig. 51.

E H

$EH = \frac{1}{2}p$, sursum quidem, si in æquatione fuerit $+p$, deorsum vero, si $-p$, ac è puncto H (vel ex E si defuerit quantitas p) educatur perpendicularum HQ interminatæ DP occurrens in puncto O. Denique in linea HQ interminatæ, fiat $OR = \frac{1}{2}q$, ab O dextrorsum si fuerit $-q$, Sinistrorsum si $+q$, collocanda: ac Circulus centro R, radio RA descriptus, tot punctis secabit Parabolam, quot æquatio proposita veras habet radices; æque erunt perpendiculara ZY à punctis intersectionem Y in Axi parallelam BK demissa; quarum quæ ad dextram lineæ BK Affirmativæ sunt, ad Sinistram Negativæ.

Hujus Constructionis commoditas in eo consistit, quod Circulo per verticem transeunte peragitur, perinde ac si defuisset secundus terminus; ideoque ad Radicum Numerum determinandum, sufficit Loci sive Lineæ Curvæ proprietates perspectas habere, quæ spatia discriminat, ubi si ponatur Centrum Circuli qui per Parabolæ Verticem transeat, circumferentia ejus vel uno vel tribus aliis punctis eam secabit; hoc est Lineæ curvæ, in quam incidunt centra omnium Circulorum per verticem transeuntium ac deinde Parabolam tangentium, naturam definire.

Locus autem ille est Parabolois, quam cum *Cl. Wallisio* semicubicalem appellare licet, sive in qua Cubi applicatarum ad Axem sunt inter se ut Quadrata portionum Axis. Cujus Latus rectum est, $\frac{27}{8}$ Lateris Recti datæ Parabolæ, Vertex vero punctum V existente AV dimidium Lateris recti ejusdem Parabolæ.

Hoc est, si ponatur Unitas pro Latere Recto datæ Parabolæ, $\frac{8}{27}$ cui ordinatim applicatæ æquabuntur Quadrato partis diametri, sive cubus ex $\frac{1}{2}VH$ quadrato ex HR, si scilicet R sit Centrum Circuli qui per verticem Parabolæ transeat, eamque deinde contingat; Hæc est Curva illa, quam primus mortalium *Nelius* Nostras rectæ datæ æqualem demonstravit, eaque occasione apud Principes Geometras dudum celebris; ejusque proprietates *Cl. Wallisius* sub finem Libri de *Cissoide*, & *Hugenius Prop. 8 & 9, de Linearum Curvarum Evolutione*, aliique acri ingenio disquisivere, quorum scripta consultat Lector. Hæc Curva utrinque ab Axe Parabolæ descripta, viz, VNL, VPX, spatium complectitur, in quo si ponatur centrum Circuli, qui per verticem A transeat, interfecabit ille Parabolam in tribus aliis punctis; spatia vero ab Axe remotiora Centra præbent Circulis non nisi uno præter verticem puncto Parabolam secantibus.

His probe intellectis jam ad determinandum Radicum Numerum accingimur: Ac primum deficiat secundus terminus; sitque Latus Rectum 1, vel AV $= \frac{1}{2}$; in constructione VH est $\frac{1}{2}p$, HR vero $\frac{1}{2}q$; cumque si fuerit $+p$, ab V versus superiora ponenda sit $\frac{1}{2}p$, Centrum Circuli extra spatium LVX semper constituitur; ideoque una tantum Radice explicabilis est, Affirmativa si $-q$, Negativa si $+q$; quæ quidem Radices *Cardani* Regulis investigantur. Si vero fuerit $-p$, VH $= \frac{1}{2}p$ inferne ponitur, ac fieri potest ut HR, cadat inter Axem & Curvam VX vel VL, si scilicet Cubus ex $\frac{1}{2}VH$, sive ex $\frac{1}{2}p$, major sit quam quadratum ex $\frac{1}{2}q$, sive $\frac{1}{4}q^2$, major quam $\frac{1}{4}q^2$, quo in casu tres dantur Radices, duæ Negativæ, si fuerit $-q$, ac una Affirmativa earum summæ æqualis; vel si $+q$, duæ Affirmativæ unaque Negativa. Quod si $\frac{1}{27}p^3$ minor sit quam $\frac{1}{4}q^2$ una tantum reperitur Radix, Affirmativa si $-q$, Negativa

Negativa si $+q$. Atque hæc passim docentur ab iis qui hanc Geometriæ partem tractarunt.

Jam adsint omnes termini, ac primum proponatur, *e. g.* æquatio hæc $z^3 - z^2b + zp - q = 0$; cui etiam Figuram (51) adaptavimus. In hujus constructione $BC = \frac{1}{3}b$, $VG = \frac{1}{2}AC = \frac{1}{18}bb$, $VE = \frac{1}{6}bb$, $VH = \frac{1}{6}bb -$

$\frac{1}{2}p$, $GH = \frac{1}{9}bb - \frac{1}{2}p$ vel $\frac{1}{2}p - \frac{1}{9}bb$, hinc $HO = \frac{1}{27}b^3 - \frac{1}{6}bp$, vel

$\frac{1}{6}bp - \frac{1}{27}b^3$, atque HR, sive distantia Centri Circuli R ab Axe, est semper dif-

ferentia inter $\frac{1}{6}bp$ & $\frac{1}{27}b^3 + \frac{1}{2}q$; quæ si æquantur, Centrum cadit in

Axe; si $\frac{1}{6}bp$ major sit quam $\frac{1}{27}b^3 + \frac{1}{2}q$ ad Sinistram Axis, sin minor ad

Dextram. Si itaque Cubi ex $\frac{2}{3}VH$, (hoc est ex $\frac{1}{9}bb - \frac{1}{3}p$ quam nomi-

nemus d) Latus Quadratum sive $\sqrt{dd}$, majus sit quam HR, sive differen-

tia inter $\frac{1}{27}b^3 + \frac{1}{2}q$ & $\frac{1}{6}bp$; reperitur Centrum R intra spatium NPV,

Paraboloidibus VPX, VNL, ac recta interminata DNP, circumscriptum:

ac proinde Circulus Parabolam secabit in tribus punctis Y, Y, Y, ad dextram

lineæ BK sitis, atque adeo æquatio tres habet Radices Affirmativas. Centro

vero extra hoc spatium NVP constituto, non nisi una Radice Affirmativa

explicari potest. Hic obiter notandum Rectam DP Paraboloidem VPX

tangente in puncto P, existente $EP = \frac{1}{27}b^3$; alteram vero VNL secare in

puncto N, ita ut demisso in Axem Perpendiculo NF, VF sit pars quarta

ipsius EV, sive $\frac{1}{24}bb$, NF vero $\frac{1}{108}b^3$. VW autem, quæ è puncto V Axi

perpendiculariter erecta lineæ DP occurrit in W, æqualis est $\frac{1}{54}b^3$ sive $\frac{1}{2}EP$.

Hinc tuto concluditur, si in æquatione vel p major sit quam $\frac{1}{3}bb$, vel q ma-

ior quam $\frac{1}{27}b^3$, non nisi unam eamque Affirmativam Radicem reperiri; Fallit

itaque Regula Cartesii (Edit. Amst. 1659, pag. 70.) ubi tot veras dari Radi-

ces quot sunt in æquatione mutationes signorum $+$ & $-$ pronunciat, frustra

etiam in Commentariis suis Sphalma hoc excusante Schootenio; Fingi enim pos-

sunt infinite plures æquationes præcedentis formulæ tres signorum mutationes

habentis, quæ unam tantum quam quæ tres habeant Radices. *Propositi*

etiam quinta Sectionis quinta Artis Analytica Harriotti Nostri, uti Prob. 18. Nu-

merosa Potes. Resol. Vieta, vix satis firma est, cum ex Limitationibus quas ibi

posuerunt, toti Parallelogrammo PIVW id conveniat, quod soli spatio NVP

jam competere probavimus, hoc est ut centrum præbeat Circulo tribus aliis

punctis præter Verticem Parabolam secante. Quantitas

Quantitas autem q , siue terminus ult., datis, b & p ea lege ut p minor sit quam $\frac{1}{3}bb$, accurate limitatur ex præcedente æquatione $\sqrt{ddd} =$

$\frac{1}{27}b^3 + \frac{1}{6}bp$; cum scilicet Circulus Parabolam contingat. Itaque

$\frac{1}{2}q$ minor esse debet quam $\frac{1}{6}bp - \frac{1}{27}b^3 + \sqrt{ddd}$; at si p major fuerit

quam $\frac{1}{4}bb$, majorem etiam esse oportet $\frac{1}{2}q$ quam $\frac{1}{6}bp - \frac{1}{27}b^3 - \sqrt{d}$, ne

cadat centrum in spatiolo $N V W$. Atque his conditionibus æquatio semper triplici Radice explicabilis erit, aliter non nisi una. Semper vero, siue tres siue una, Affirmativæ sunt, ob positionem centri R ad dextram lineæ $D P$.

Atque hic est casus maxime difficilis, ita ut quicumque præmissa bene calculetur sequentia facili negotio intelliget. Detur jam æquatio $z^3 - bz^2 + pz + q = 0$. Hic ut tres habeantur Radices, oportet Centrum Circuli alicubi intra spatium $P N \Delta$, rectis $P N$, $P \Delta$, & curva Paraboloidis $N \Delta$, definitum,

reperiri; quapropter cum $E F$ sit $= \frac{1}{8}bb$, p minor esse debet quam $\frac{1}{4}bb$: jam ad determinationem quantitatis q , existente $d = \frac{1}{9}bb - \frac{1}{3}p$, ut antea,

$\sqrt{ddd} + \frac{1}{7}bbb - \frac{1}{6}bp$ semper major esse debet quam $\frac{1}{2}q$, ut constitua-

tur Centrum Circuli in spatio prædicto $P N \Delta$: quod cum sit æquatio talis duas habet radices Affirmativas ac unam Negativam. Si vero p major est

quam $\frac{1}{3}bb$, vel $\frac{1}{2}q$ major quam $\sqrt{ddd} + \frac{1}{27}bbb - \frac{1}{6}bp$, non nisi una eaque Negativa Radice explicabilis est.

Proponatur jam æquatio $z^3 - bz^2 - pz - q = 0$. Ut hæc æquatio tres habeat Radices, oportet Centrum Circuli alicubi inveniri in spatio indefinito, inter rectam $D P D$ & curvam Paraboloidis $P X$; hinc quantitas p non

est obnoxia limitationibus, $\frac{1}{2}q$ vero semper minor esse debet quam $\sqrt{ddd} - \frac{1}{27}bbb - \frac{1}{6}bp$, posito $d = \frac{1}{9}bb + \frac{1}{3}p$: Hæc pacto dantur Radices

Negativæ, ac una Affirmativa; aliter vero si $\frac{1}{2}q$ major sit quam $\sqrt{ddd} - \frac{1}{27}bbb - \frac{1}{6}bp$, unica tantum Affirmativa exponi potest. Quarto loco sit

æquatio $z^3 - bz^2 - pz + q = 0$, quæ duas Affirmativas habet Radices ac unam Negativam si Centrum Circuli reperjatur in spatio indefinito inter re-

ctas $P \Delta$, $P D$, ac curvam Paraboloidis ΔL ; hoc est, (posito $d = \frac{1}{9}bb +$

$\frac{1}{3}p$) si $\frac{1}{2}q$ minor sit quam $\sqrt{ddd} + \frac{1}{27}bbb + \frac{1}{6}bp$; si vero $\frac{1}{2}q$ major hac quantitate fuerit, una tantum Negativa inest Radix. Quatuor

Quatuor autem æquationes reliquæ, in quibus habeatur $+b$, quoad limitationem Numeri Radicum non differunt à prædictis, si signum termini ultimi mutetur, servato signo termini tertii; quæ vero Affirmativæ erunt Radices in illis hic fiunt Negativæ, & vice versa. Sic in æquatione, $z^3 - bz^2 + pz - q = 0$, una vel tres erant Affirmativæ Radices; in hac vero, $z^3 + bz^2 + pz + q = 0$, vel una vel tres Negativæ sunt, sub iisdem conditionibus; nulla vero omnino Affirmativa. Sic in $z^3 + bz^2 + pz - q = 0$, duæ sunt Negativæ & una Affirmativa, si p minor sit quam $\frac{1}{3}bb$, ac $\frac{1}{2}q$ minor quam

$$\sqrt{d^3 + \frac{1}{27}b^3 - \frac{1}{6}bp}, \text{ quemadmodum in } z^3 - bz^2 + pz + q = 0,$$

duæ erant Affirmativæ & una Negativa; excedentibus autem leges præscriptas p vel q , una tantum hic est Radix Affirmativa, quæ ibi Negativa erit. Pari modo in $z^3 + bz^2 - pz + q = 0$, vel duæ sunt Affirmativæ ac una Negativa, vel una Negativa tantum; denique iisdem de causis in æquatione, $z^3 + bz^2 - pz - q$ duæ sunt Negativæ & una Affirmativa, vel una Affirmativa tantum, quibus in æquatione, $z^3 - bz^2 - pz + q$, duæ erant Affirmativæ & una Negativa, vel una Negativa tantum, nempe prout

$$\frac{1}{2}q \text{ major vel minor fuerit quam } \sqrt{d^3 + \frac{1}{27}b^3 + \frac{1}{6}bp}.$$

Si defuerit terminus tertius, five pz , Centrum R semper cadit in linea I P E Δ , quocirca si fuerit $z^3 - bz^2 \cdot * - q$, vel $z^3 + bz^2 \cdot * + q$, una tantum esse potest Radix, si $-b$ Affirmativa, si $+b$ Negativa. At si fuerit $z^3 - bz^2 \cdot * + q$, vel $z^3 + bz^2 \cdot * - q$, duæ possunt esse Affirmativæ ac una Negativa in priore, vel una Affirmativa & duæ Negativæ in posteriore, cadente Centro in linea P Δ inter P ac Δ , hoc est si $\frac{1}{4}q$ minor sit quam $\frac{1}{27}b^3$; sin major fuerit, una tantum Negativa in priore, vel una

Affirmativa in posteriore, dare potest.

Hactenus Numerum Radicum in Cubicis æquationibus plenius affecuti sumus, restat ut nonnulla adjiciam de Quantitate Radicum. Hic primum Notandum quod omnis æquatio tres habens Radices ope Tabulæ Sinuum, Triflectione scilicet Anguli, satis expedite resolvi possit; Ponendo scilicet

$$\sqrt[3]{\frac{4}{9}bb - \frac{4}{3}p}, \text{ vel } \sqrt[3]{4d}, \text{ si fuerit } +p \text{ in æquatione, vel } \sqrt[3]{\frac{4}{9}bb + \frac{4}{3}p},$$

si $-p$, pro Radio Circuli; Angulum vero trifecandum qui Sinum habeat in

$$\text{Tabula Sinuum } \frac{\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d^3 + \frac{1}{27}b^3 + \frac{1}{6}bp}}; \text{ Invento hoc angulo, Sinus tertiæ par-$$

tis ejus, ut & Sinus tertiæ partis compl. ad Semicirculum, eorumque summa,

$$\text{ex Tabu'a Sinuum dabuntur. Hi vero Sinus in Radium } \sqrt[3]{\frac{4}{9}bb + \frac{4}{3}p} \text{ du-$$

rendi sunt, & habebuntur Quantitates ($y \odot, y \odot, y \odot$, in Fig.) quarum & $\frac{1}{3}b$ vel Sum-

Summa vel differentia, prout casus postulat, veras Radices Æquationis exhibebunt. Hæc omnia ex inventis *Cartesii* derivantur: Ut vero casus omnes, quantum fieri possit, breviter complectar, dico quod Centro R, in prima æquationum formula, cadente in spatio V G P, Sectiones duæ Y, Y, cadunt inter A & B, ac proinde utraq; ex Minoribus Radicibus Minor est quam $\frac{1}{3}b$, tertia autem & Major semper superat $\frac{1}{3}b$, superatur vero à b . Quod si cadat in spatio G N V, duæ majores sunt quam $\frac{1}{3}b$, minores vero quam $\frac{2}{3}b$, tertia vero est $b -$ duabus alteris, ac proinde minor quam $\frac{1}{3}b$: Sed adhibita Limitatione Quantitatis p , arctioribus terminis Radices includuntur. Maxima enim Radix minor

est quam $\sqrt{\frac{4}{9}bb - \frac{4}{3}p + \frac{1}{3}b}$, major vero quam $\sqrt{\frac{1}{4}bb - p + \frac{1}{2}b}$;

at cum $\frac{1}{4}bb$ minor est quam p , limes ille fit $\sqrt{\frac{1}{9}bb - \frac{1}{3}p + \frac{1}{3}b}$; Ra-

dix media semper minor est quam $\sqrt{\frac{1}{4}bb - p + \frac{1}{2}b}$; major vero quam $\frac{1}{3}b - \sqrt{\frac{1}{9}bb - \frac{1}{3}p}$; hunc vero litem nunquam excedit Radix Minima, sed cum Quantitate q evanescit.

In secunda formula præscriptis legibus duæ sunt Affirmativæ ac una Negativa, ac cadente Centro in spatio G P E, altera ex Affirmativis major est, altera minor quam $\frac{1}{3}b$, Major vero non excedit b , Negativa autem Major non

esse potest quam $\sqrt{\frac{1}{3}bb - \frac{1}{3}b}$, est autem differentia ipsius b & summæ Affirmativarum. Centro autem in spatio E N G Δ posito, utraq; Affirma-

tiva major est quam $\frac{1}{3}b$, minor vero quam $\sqrt{\frac{1}{3}bb + \frac{1}{3}b}$, Negativa vero semper minor est quam $\frac{1}{3}b$. Limites autem propiores ex data p evadunt Ra-

dicis quidem maximæ Affirmativæ $\sqrt{\frac{1}{4}bb - p + \frac{1}{2}b}$, qua semper minor

est, ut & major quam $\sqrt{\frac{1}{9}bb - \frac{1}{3}p + \frac{1}{3}b}$; hoc tamen limite minor est altera Affirmativa, quæ cum quantitate q minuitur. Negativa vero semper mi-

nor est quam $\sqrt{\frac{4}{9}bb - \frac{4}{3}p - \frac{1}{3}b}$, ac deficiente quantitate q evanescit.

In tertia formula duæ Negativæ sunt ac una Affirmativa: in hac, ut & in quarta, Radices non limitantur à quantitate b . Affirmativa vero

semper minor est quam $\sqrt{\frac{4}{9}bb + \frac{4}{3}p + \frac{1}{3}b}$, major tamen quam

$\sqrt{p + \frac{1}{4}bb + \frac{1}{2}b}$: Maxima vero ex Negativis semper major est quam

$\sqrt{\frac{1}{9}bb + \frac{2}{3}p - \frac{1}{3}b}$, minor vero quam $\sqrt{p + \frac{1}{4}bb - \frac{1}{2}b}$. Minor

autem ex Negativis semper minuitur cum minuta quantitate q .

In quarta formula, cadente Centro intra spatium $L \triangle PD$; si duæ sint Affirmativæ ac una Negativa, Maxima ex Affirmativis major esse nequit

quam $\sqrt{p + \frac{1}{4}bb + \frac{1}{2}b}$, nec minor quam $\sqrt{\frac{1}{9}bb + \frac{1}{3}p + \frac{1}{3}b}$; Minor

vero Radix ab hoc Limite minuitur, minuta quantitate q . Negativa autem minor

est quam $\sqrt{\frac{4}{9}bb + \frac{4}{3}p - \frac{1}{3}b}$; major vero quam $\sqrt{p + \frac{1}{4}bb - \frac{1}{2}b}$.

Notandum vero hic Radices Negativas ubique signo Affirmativo notari, quia hæ sunt Radices Affirmativæ quatuor æquationum illarum, in quibus habetur $+b$, ac q signo contrario notatur; ut supra monui. Horum omnium Demonstratio ex eo consequitur, quod ubicunque Centrum Circuli R incidit in Lineas curvas VPX , vel $V \triangle L$, circumferentia ejus Parabolam tangit in puncto, cujus distantia ab axe est $\sqrt{\frac{2}{3}} V H$, eamque fecat ex altera

Axis parte, ad distantiam $2 \sqrt{\frac{2}{3}} V H$, cum vero Centrum cadit in Lineam DPD ,

altera ex Radicibus fit $= 0$, ac proinde Cubica reducitur ad Quadraticam, sive ad $x^2 - bx + p = 0$, cujus Radices Limites designant ubi evanescit quantitas q : ac quo minor est q , eo propius ad has limites accedunt Radices. Quadratica est etiam cum Centrum cadit in Axe; hoc est, cum

$\frac{1}{2}q = \frac{1}{6}bp - \frac{1}{27}b^3$, in prima formula; vel $\frac{1}{2}q = \frac{1}{27}b^3 - \frac{1}{6}bp$, in se-

cunda; in tertia impossibile est; at in quarta cum $\frac{1}{2}q = \frac{1}{27}b^3 + \frac{1}{6}bp$, quo

in casu Minor ex Radicibus Affirmativis est $\frac{1}{3}b$, Major $\sqrt{\frac{1}{3}bb + p + \frac{1}{3}b}$;

Negativa vero $\sqrt{\frac{1}{3}bb + p - \frac{1}{3}b}$. In prima, Radices sunt $\frac{1}{3}b$, & $\frac{1}{3}b$

+ $\sqrt{\frac{1}{3}bb - p}$. In secunda vero formula, $\frac{1}{3}b$, & $\sqrt{\frac{1}{3}bb - p + \frac{1}{3}b}$,

sunt Affirmativæ: Negativa autem $\sqrt{\frac{1}{3}bb - p - \frac{1}{3}b}$.

Atque hæc in Cubicis sufficere posse videntur; ob eximium vero Usus Methodi, qua ope Tabulæ Sinuum Radices harum æquationum inveniuntur, placuit unum vel alterum Exemplum adjungere, ut Praxis illius compendium inde innotescat. Proponatur Æquatio $x^3 - 39x^2$

+ 479x

4792 - 1881 = 0; quærentur Radices $\sqrt{\frac{1}{9}bb - \frac{1}{3}p} = \sqrt{9\frac{1}{3}}$

= $\sqrt{d}$, cujus duplum $\sqrt{37\frac{1}{3}}$ Radius est Circuli; & $\frac{\frac{1}{27}b^3 + \frac{1}{3}q - \frac{1}{6}bp}{\sqrt{d}} =$

$\frac{2197 + 940\frac{1}{2} - 3113\frac{1}{2}}{2}$, five $\frac{24}{2}$ est Sinus Tabularis Anguli

$9\frac{1}{3}\sqrt{9\frac{1}{3}}$ $9\frac{1}{3}\sqrt{9\frac{1}{3}}$

hoc est, facta divisione ope Logarithmorum, Log. 9. 9251560, cui respon-
det Angulus 57 gr. 19 m. 11 s. Hujus tertia pars 19 gr. 6 m. 24 s. &
complementi 40 gr. 53 m. 36 s. Sinus dant Log. 9. 514983, & 9. 816011,

qui ducti in Rad. $\sqrt{37\frac{1}{3}}$ producant Y &, et Y &, Log. 0. 301030. = 2, et

Log. 0. 601039 = 4, tertio vero Y &, æqualis est eorum summa five 6.
Ideoque Radices sunt 13 - 4 = 9, 13 - 2 = 11, & 13 + 6 = 19, ex
quibus singulis conflatur prædicta æquatio. Ubi Notandum duas Minores

Radices non excedere $\frac{1}{6}b$ vel 13, quia Centrum R in constructione cadit

ad dextram Axis; id est $\frac{1}{6}bp$ minor est quam $\frac{1}{27}b^3 + \frac{1}{2}q$.

Exemplum alterum sit $x^3 - 15x^2 - 229x - 525 = 0$, & quæ-

rantur Radices $\sqrt{\frac{1}{9}bb + \frac{1}{3}p} = \sqrt{101\frac{1}{3}} = \sqrt{d}$, & Radius Circuli

$\sqrt{405\frac{1}{3}} \cdot \frac{\frac{1}{27}b^3 + \frac{1}{6}bp + \frac{1}{2}q}{\sqrt{d}} = \frac{125 + 572\frac{1}{3} + 262\frac{1}{3}}{101\frac{1}{3}\sqrt{101\frac{1}{3}}} =$

$\frac{960}{101\frac{1}{3}\sqrt{101\frac{1}{3}}}$ = Sinui Tabulari Arcus, cujus Log. 9. 9736426, &

Arcus ipse 70 gr. 14 m. 22 s. hujus pars tertia est 23 gr. 24 m. 47 s. & Com-
plementi 36 gr. 35 m. 12 s; quorum Sinus Log. sunt 9. 599183, &
9. 775275, quibus addito Log. $\sqrt{405\frac{1}{3}}$ fiunt Log. 0. 903089 = 8, &
Log. 1. 079181 = 12, & eorum summa = 20. Hinc concluditur 20

+ $\frac{1}{3}b$, vel 25, æquari Radici Affirmativæ, & 8 & 12 - $\frac{1}{3}b$, five 3 & 7,

Negativis. Quod si æquatio fuisset $x^3 + 15x^2 - 229x + 525 = 0$,
3 & 7 fuissent Affirmativæ; 25 vero Negativa. Cæteræ autem Cubicæ
unica tantum Radice explicabiles juxta Regulas *Cardani* resolvendæ sunt,
postquam demptus fuerit secundus terminus; nec video quo pacto
minori calculo hoc negotium peragi possit. At si desideretur Radix
hæc in Quantitatibus, b , p , q , expressa, dico eam esse in prima for-

mula, $\frac{1}{3} b +$ vel $-$ summa vel differentia Radicum Cubicarum ex

$$V \frac{1}{4} qq - \frac{1}{108} p^3 b^3 + \frac{1}{27} b^3 q - \frac{1}{6} b p q + \frac{1}{27} p^3 + \frac{1}{27} b^3 + \frac{1}{2} q - \frac{1}{6} b p: \text{viz. } +, \text{ si } \frac{1}{27} b^3 + \frac{1}{2} q \text{ major sit quam } \frac{1}{6} b p, \text{ aliter } -; \text{ Sum-$$

ma vero quoties $\frac{1}{3} b b$ major est quam p ; sin minor fuerit $\frac{1}{3} b b$, differen-
tia. Inque cæteris formulis Radix semper conflatur ex iisdem elementis, va-
riatis tamen signis $+$ & $-$, ut facile percipiet qui velit experiri.

Ope vero Tabulæ Logarithmicæ Sinuum Versorum Radices hæc satis prom-
pte inveniuntur: nempe si coefficientis Numeri sint surdi vel fracti, ac Radi-
ces Numeris ineffabiles; ut plerumque fit. Hæc autem est Regula: in prima

ac secunda formula, si $\frac{1}{3} b b$ minor sit quam p ; sit $\frac{1}{3} p - \frac{1}{9} b b = d$, &

posita differentia inter $\frac{1}{6} b p$ & $\frac{1}{27} b^3 + \frac{1}{2} q$, hoc est HR, in prima, ac

inter $\frac{1}{6} b p + \frac{1}{2} q$ & $\frac{1}{27} b^3$, in secunda, pro Radio; inveniatur Angulus

cujus Tangens est $d \sqrt{d}$. Deinde ut Co-sinus hujus Anguli, ad ejusdem Sinum

Versum: Ita differentia pro Radio habita, ad quartum; cujus Latus cubi-

cum trisacando Logarithmum habebitur: ac diviso $\frac{1}{3} p - \frac{1}{9} b b$ per hoc La-

tus Cub. è Quoto subducatur Divisor, Residuum erit quantitas Y &: Hujus Re-

sidui ac $\frac{1}{3} b$ summa, si centrum cadit ad dextram Axis, aliter differentia ea-

rundem, Radix erit quæsitæ. Quod si $\frac{1}{3} b b$ major sit quam p , posito HR

pro Radio, sit $d \sqrt{d}$, sive distantia Paraboloidis ab Axe, Sinus Arcus cujus-

dam; Hujus Sinus versus ducatur in Radium, sive $\frac{1}{6} b p - \frac{1}{27} b^3 + \frac{1}{2} q$,

ac trisecto producti Logarithmo, habebitur ejus Latus Cubicum, per quod

dividatur $\frac{1}{9} b b - \frac{1}{3} p$. Dico Quoti ac Divisoris summam eadem lege addi-

tam vel ablatam ex $\frac{1}{3} b$, Radicem quæsitam exhibere. Ac par est Ratio

in tertia ac quarta formulis, nisi quod $\frac{1}{27} b^3 + \frac{1}{6} b p + \frac{1}{2} q$ pro Radio

assumenda est, ac $\frac{1}{9} b b + \frac{1}{3} p$ in $V \frac{1}{9} b b + \frac{1}{3} p$ sive $d \sqrt{d}$ pro Sinu:

Sed hæc præcepta exemplis fortasse melius percipientur.

Sit æquatio Cubica, $z z z - 17 z z + 54 z - 350$, ac quærat^r Radix z :
 Hic $\frac{1}{3} b b$ major est quam p , sed q major est quam Cubus ex $\frac{1}{3} b$, ideoque una
 tantum Affirmativa Radice explicabilis est. Jam $\frac{289}{9} - \frac{54}{3}$ est d , ac $\frac{127}{9} \sqrt{\frac{127}{9}}$
 pro Sinu habenda est, ad Radium $\frac{49^{13}}{27} + 175 - 153$, hoc est $\frac{5507}{27}$: Ar-
 cus vero competens fit $15^{\circ} 3'. 49''$. Hujus Sinus Versi Log. 8. 5362376,
 additus Log. Radii 2. 3095913, dat 0. 8457889, cujus tertia pars 0. 2819276
 est Log. Radicis Cubicæ 1. 91394, quo Divisore diviso, $\frac{127}{9}$ sive d , fit Quo-
 tus 7. 37281; Quoti ac Divisoris summa, aucta editione $\frac{1}{3} b$, fit Radix quæ-
 sita, nempe 14. 9534, &c.

Exactis Cubicis Biquadraticas jam aggrediamur. Hæ semper vel nullam;
 vel duas, vel quatuor Radices veras habent, quarum determinatio, partim à
 Coefficientibus, partim à signo & magnitudine Numeri absoluti dati, pendet.
 In Constructione æquationis $z^4 - b z^3 + p z z - q z + r = 0$, fit $BD =$

Fig. 49.

$$\frac{1}{4} b, AB = \frac{1}{16} b b, BK = \frac{1}{2}, \text{ sive dimidio Lateris recti, } KC = 2 AB =$$

$$\frac{1}{8} b b, KE = \frac{1}{8} b b - \frac{1}{2} p, AE = \frac{1}{2} = \frac{3}{16} b b - \frac{1}{2} p, FE = \frac{1}{16} b^3$$

$$- \frac{1}{4} b p, \text{ ac } EG = \frac{1}{16} b^3 - \frac{1}{4} b p + \frac{1}{2} q; \text{ quo facto Circulus, Centro G,}$$

Radio $\sqrt{GD^2 - r}$, interfecabit Parabolam vel nullo, duobus, aut qua-
 tuor punctis, quæ perpendicularis in lineam HD Radices omnes z exhi-
 bent. Ut autem quatuor sint, evidens est Centrum Circuli alicubi con-
 stitui debere intra spatium, de cujus puncto quovis tria perpendiculara in
 Curvam Parabolæ demitti possint; atque simul Radium minorem esse
 maximo ex illis perpendicularis majorem vero medio. Quod si Centrum
 constituatur extra hoc spatium, ut non nisi una perpendicularis in Para-
 bolam demitti possit, qua major sit Radius; vel si minor sit media ex tri-
 bus perpendicularibus, major vero quam minima ex illis, duæ tantum
 possunt esse Radices; nulla vero omnino datur, quoties Radius $\sqrt{GD^2 - r}$,
 minor est minima ex tribus, vel una illa, quoties una tantum est. Jam
 quale spatium hoc sit, quibusque limitibus discernitur, ac quibus conditio-
 nibus Radius Circuli minor vel major sit prædictis perpendicularibus, nobis re-
 stat inquirendum; ac primum quo pacto perpendicularis in Parabolam de-
 mitti possit ostendendum est.

Sit ABC Parabola, AE Axis ejus, AV Semi-Latus Rectum, G punctum
 de quo demittenda est perpendicularis: Ducatur Axi perpendicularis GE, ac
 bisecetur VE in F, & erecta perpendiculari FH ad idem Axis latus, fiat
 FH = $\frac{1}{4}$ GE; dico quod Circulus, Centro H, Radio HA descriptus, Para-
 bolam

Fig. 53.

bolam interfecabit in punctis tribus, vel uno, Z; ad quæ ductæ rectæ G Z Curvæ Parabolicæ perpendiculariter insistant.

Fig. 51.

Ut autem tres sint hujusmodi intersectiones, oportet Centrum Circuli H ita collocari, ut sit intra spatium Paraboloidibus inclusum; hoc est ut FH minor sit quam $\sqrt{\frac{8}{27}} VF^{\frac{3}{2}}$, five FH² minus quam cubus ex $\frac{2}{3} VF$: atque adeo

GE = 4 FH, minor erit quam $\sqrt{\frac{8}{27}} VF^{\frac{3}{2}}$, five $4 \sqrt{\frac{1}{27}} VE^{\frac{3}{2}}$, hoc est

Quadratum ex GE minus erit quam $\frac{16}{27} VE^3$. Coincidunt itaque hi Limi-

tes cum Paraboloidibus duabus ejusdem generis cum iis quibus in Cubicis usus sumus, sed quarum Latus Rectum duplo minor est; viz. $\frac{27}{16}$ Lateris Recti Parabolæ, hoc est $\frac{27}{8}$ ipsius AV: ideoque ea ipsa est linea Curva cujus Evo-

Fig. 52.

lutione generatur Parabola, sic demonstrante *Hugenio*; quamque semper contingit linea DF, quæ Parabolæ perpendiculariter insistit in puncto D. Punctum autem P, five in quo contingit recta DF Paraboloidem, Centrum est Circuli, qui Radio DP descriptus cum Parabola in puncto D coincidit, five ejusdem Curvitatatis est; ut per se satis constat.

Fig. 52.

Descriptis itaque hujusmodi Paraboloidibus VXP, VNΔ, utrinque ab Axe; perspicuum est quod, nisi Centrum Circuli constituatur intra hos limites, non possit ille pluribus quam duobus in punctis Parabolam interfecare: unde determinare licet quibus sub conditionibus Coefficientes terminorum intermediorum coercentur, in æquationibus Biquadraticis, ut habeantur quatuor Radi-

ces. Ac prima fronte clarum est p majorem esse non posse quam $\frac{3}{8} b b$, (Gal. in formulis ubi habetur + p) nec q quam $\frac{1}{16} b^3$. Generaliter vero $\frac{1}{16} b^3$

+ $\frac{1}{4} p b + \frac{1}{2} q$, id est distantia Centri ab Axe EG, minor esse debet quam

EH = $4 \sqrt{\frac{1}{27} VE^3}$, hoc est (ob $VE = -\frac{3}{16} b b + \frac{1}{2} p$) quam $\frac{1}{4} b b +$

$\frac{2}{3} p \sqrt{\frac{1}{16} b b + \frac{1}{6} p}$; signis + & - in dubio relictis, ut secundum æqua-

tionis cujusvis naturam variari possint; quemadmodum in Cubicis superius ostensum est.

Termini autem ultimi & limitatio eadem facilitate inveniri nequit; id adeo, quia Problema sit Solidum in Curvam Parabolæ demittere perpendicularem, quodque non sine solutione æquationis Cubicæ resolvi possit. Itaque primo loco deficiat secundus terminus, vel si adfuerit tollatur, ut æquatio habeat formulam, $z^3 + p z^2 + q z + r = 0$. Ac si fuerit - r, semper duabus vel quatuor Radicibus explicari potest; ut autem quatuor sint, oportet Centrum Circuli infra Paraboloides prædictas constitui, five ut sit - p

ac

ac $q q$ minus quam $\frac{8}{27} p^3$, sive cubo ex $\frac{2}{3} p$. Deinde habeantur Radices æqua-

tionis hujus $y^3 + \frac{1}{2} p y - \frac{1}{4} q = 0$, quantitibus p & q iisdem signis annexis quibus in Biquadratica. Hæ autem Radices auxilio Tabulæ Sinuum satis expeditè inveniuntur. Inventis autem tribus illis y , (quæ sunt ordinatim applicatæ ad Axem Parabolæ, de punctis ubi incidunt perpendiculara in Curvam ejus scil. ZY) $ppp - 3y^3$ ex minore y , quantitatem maximam r designabit, si fuerit $-r$; qua si minor fuerit r , æquatio quatuor habebit Radices, aliter duas. Ast si fuerit $+r$, oportebit eam minorem esse quam $3y^3 - ppp$ ex media y , nam si major sit, non nisi duas habere potest Radices, saltem si minor sit r , quam $3y^3 - ppp$ ex maxima y . Hac vero si major sit, nulla omnino Radice vera explicabilis est æquatio. Hi vero iidem Limites aliter designantur ex quantitate q , scil. $\frac{1}{2} qy - y^3$ in primo casu, $y^3 - \frac{1}{2} qy$ in se-

Fig. 53.

cundo, ac $y^3 + \frac{1}{2} qy$ in tertio.

Fieri autem potest ut duæ minores quantitates y non longe distant ab invicem, unde evenit quod utraque ex perpendicularibus major sit quam recta GA , scil. cum $q q$ majus sit quam $\frac{4}{27} p^3$, minus vero quam $\frac{8}{27} p^3$; cadente centro intra spatium Paraboloidibus (utriusque Figuræ 51 & 52) interjectum. Hoc in casu, si fuerit $+r$, non nisi duæ possunt esse Radices, existente $y^3 + \frac{1}{2} qy$ ex maxima y , major quam r ; aliter nulla. At si $\frac{1}{2} qy - y^3$ ex minima y , major fuerit quam r signo $-$ notata, r vero major quam $\frac{1}{2} qy - y^3$ ex media y , tunc habentur quatuor radices; at duæ tantum, si vel major priore vel minor posteriore inventa sit r .

Si vero in æquatione fuerit $+p$, vel si sit $-p$ & $q q$ majus fuerit quam $\frac{8}{27} p^3$, æquatio $y^3 + \frac{1}{2} p y - \frac{1}{4} q = 0$, unica tantum explicatur Radice y ; hoc est, una tantum perpendicularis de Centro Circuli demitti potest: unde certe concluditur duas tantum Radices haberi posse in æquatione data, quarum Summa, si fuerit $-r$, cum quantitate r augetur; at si habeatur $+r$, obventa quantitate y , quantitas illa r minor esse debet quam $y^3 + \frac{1}{2} qy$; nam si ea major sit, æquatio proposita absurda & impossibilis est.

Longum & superfluum esset omnes hujus sensus æquationes percurrere, cum ex jam dictis attendenti satis evidens sit, quæ Negativæ quæ Affirmativæ sint; atque quod Radicum harum Limites ex quantitibus inventis y pe-
tantur. In exemplum vero, quod cuivis in cæteris imitari licet, proponantur indagandi Limites sive Conditiones sub quibus in Æquatione Biquadratica 4 Ra-
dices Affirmativæ dari possint. Hoc autem fit quoties Centrum Circuli G , ponitur

Fig. 54.

in spatio VPK, ac simul habetur $+r$, sive Circuli Radius minor quam GD: Unde patet æquationem de qua agitur hujus esse formulæ, $x^3 - bx + p = 0$

$-qz + r = 0$; p vero majorem esse non posse quam $\frac{3}{8}bb$, nec $\frac{1}{4}pb$, hoc

in casu, quam $\frac{1}{16}b^3 + \frac{1}{2}q$; deinde opus est ut $\frac{1}{4}bb - \frac{2}{3}p$ in

$V \frac{1}{16}bb - \frac{1}{6}p$ major sit quam $\frac{1}{16}b^3 + \frac{1}{2}q - \frac{1}{4}pb$; & ex his Limitibus

certo constabit Centrum intra spatium VPK inveniri. Ut vero definiatur quan-

titas r , solvenda primum est Cubica, $y^3 - \frac{3}{16}b^3 - \frac{1}{2}py = \frac{1}{32}b^3$

$+ \frac{1}{4}q - \frac{1}{8}pb$; & habebuntur puncta, in quæ perpendiculares de Centro

in Curvam Parabolæ cadunt.

Inventis autem tribus valoribus hujus y , r minor esse debet quam $\frac{3}{256}b^3$

$+ \frac{1}{4}bq - \frac{1}{16}bbp + 3y^3 - \frac{3}{8}b^3yy + pyy$ ex media y , major vero

quam $\frac{3}{256}b^3 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^3 - \frac{3}{8}b^3yy + pyy$ ex mini-

ma y . Hos vero Limites si excedat r , non nisi duæ Radices haberi possunt.

Deniq; si $\frac{3}{256}b^3 + \frac{1}{4}bq - \frac{1}{16}bbp + 3y^3 - \frac{3}{8}b^3yy + pyy$ ex maxi-

ma y , minor fuerit quam r , æquatio proposita impossibilis est.

Accidit etiam ut quatuor sint Affirmativæ, cum Centrum G constituitur in

spatiolo VTS, ducta scilicet RTS perpendiculari in medium suppositæ lineæ AD:

hoc autem fit cum p major est quam $\frac{5}{16}bb$, ac $\frac{1}{4}bb - \frac{2}{3}p$ $V \frac{1}{16}bb - \frac{1}{6}p$

major quam $\frac{1}{8}pb - \frac{5}{128}bbb - \frac{1}{2}q$. Quo in casu semper duæ, aliquan-

do tres, ex Radicibus fiunt majores quam $\frac{1}{4}b$.

Fig. 42.

Notandum vero hic limitem illum ex minima y productum, aliquando ne-

gativum fieri, sive minorem nihilo; quoties scilicet maxima ex tribus perpendi-

cularibus major est quam GD. Hoc, si acciderit quantitas $+r$, à Limite præ-

scripto ex media y , in nihilum minui potest. Defectus vero Limitis ex mini-

ma y monstrat quanta possit esse $-r$ in æquatione, si habeantur tres Radi-

ces Affirmativæ ac una Negativa; quam si excedat, non nisi duæ, altera Af-

firmativa, altera Negativa, dari possunt. Hæc autem omnia demonstrantur

ex eo quod prædicti Limites quantitatis r , sint differentię Quadratorum lineæ

GD & perpendicularem in Curvam Parabolæ.

Ob perplexas vero cautiones, quas parit in æquationibus hisce signorum diver-

sitas, præstat semper secundum terminum tollere, ac deinde juxta præcepta jam

tradita Radicum numerum ac signa inquirere; præsertim si quantitates illæ y non

multum

ab invicem. Ex quatuor autem hisce Radicibus Affirmativis, duæ semper sunt minores quam $\frac{1}{4}b$, duæ vero majores; nempe si DG, minor sit quam

AG, five $\frac{1}{4}pb$ quam $\frac{3}{64}b^3 + q$. Tres autem minores sunt quam $\frac{1}{4}b$ quoties perpendicularis media, five ex media y inventa, major est quam AG, five $\frac{3}{8}b, b, y$ major quam $3y^3 - pyy$ ex eadem media y ; Quarta vero & maxi-

ma Radix major est quam maxima $y + \frac{1}{4}b$; æquatur autem differentiz ipsius b & summæ cæterarum trium Radicum, ideoque minor est b . Sed jam manum de Tabula; Fortassis illi qui naturam Parabolæ penitus perspectam habent, majori compendio hæc omnia peragere valebunt; at si quantitates hæc omnes b, p, q & r , absque resolutione Cubicæ æquationis rite determinari possint, non sine causâ ambigitur; quæcunque enim æquationibus planis hæc in re fiunt, non veros Limites, sed Approximationes tantum exhibent.

XX. Regulas binas compendiosas admodum pro Approximatione Radicis Cubicæ nuper protulit *D. de Lagney*, alteram rationalem, alteram irrationalem; nem-

pe Cubi $aaa + b$ latus esse inter $a + \frac{ab}{3a^2 + b}$ ac $\sqrt[3]{\frac{1}{4}aa + \frac{b}{4} + \frac{1}{2}a}$

The Extraction of all Roots without any previous Reduction; by Mr. Ed. Halley. N. 210. p. 136. May. An. 1694.

Radicem autem potestatis Quintæ $a^5 + b$ sic exprimit $= \frac{1}{2}a +$

$\sqrt[5]{\frac{1}{4}a^4 + \frac{b}{4} - \frac{1}{4}aa}$ (non $\frac{1}{2}aa$ ut perperam legitur in libro Gallico impresso).

Demonstrantur autem Regulæ prædictæ ex Genesi Cubi & Potestatis quintæ. Posito enim Latere Cubi cujusque $a + e$, Cubus inde conflatus fit $aaa + 3aae + 3aee + eee$, adeoque si supponatur aaa numerus Cubus proxime minor dato quovis non Cubo, eee minor erit Unitate, ac residuum sive b æquabitur reliquis Cubi membris $3aae + 3aee + eee$: rejectoque eee ob parvitatem, $b = 3aae + 3aee$. Cumque

aae multo majus fit quam aee , $\frac{b}{3aa}$ non multum excedet ipsam e , po-

sitoque $e = \frac{b}{3aa}$, $\frac{b}{3aa + 3ae}$, cui proxime æquatur quantitas e , in-

venietur $= \frac{b}{3aa + \frac{b}{a}}$ five $\frac{b}{3aa + \frac{b}{a}}$: hoc est $\frac{ab}{3aaa + b} = e$, adeo-

que latus Cubi $aaa + b$ habebitur $a + \frac{ab}{3aaa + b}$, quæ est ipsa for-

mula rationalis *D. de Lagney*. Quod si aaa fuerit Numerus Cubus proxime

major dato, Latus Cubi $aaa - b$, pari ratiocinio invenietur $a - \frac{ab}{3aaa - b^2}$

atque hæc Radicis Cubicæ approximatio satis expedita ac facilis parum admodum fallit in defectu, cum scilicet e residuum Radicis hoc pacto inventum paulo minus justo sit. Irrationalis vero formula etiam ex eodem

fonte deniyatur, viz. $b = 3aaa + 3aee$, five $\frac{b}{3a} = ae + ee$; adeo-

que $\sqrt[3]{\frac{1}{4}aa + \frac{b}{3a}} = \frac{1}{2}a + e$, atque $\sqrt[3]{\frac{1}{4}aa + \frac{b}{3a}} + \frac{1}{2}a = a + e$, five Radici quæsitæ. Latus vero Cubi $aaa - b$ eodem modo habebitur

$\frac{1}{2}a + \sqrt[3]{\frac{1}{4}aa - \frac{b}{3a}}$. Atque hæc quidem formula aliquanto propius

ad scopum collimat, in excessu peccans sicut altera in defectu, ac ad praxim magis commoda videtur, cum restitutio Calculi nihil aliud sit quam continua

additio vel subductio ipsius $\frac{eee}{3a}$, secundum ac quantitas e innotescat; ita ut

potius scribendum sit $\sqrt[3]{\frac{1}{4}aa + \frac{b - eee}{3a}} + \frac{1}{2}a$ in priori casu, ac in poste-

riori $\frac{1}{2}a + \sqrt[3]{\frac{1}{4}aa + \frac{eee - b}{3a}}$. Utraque autem formula Ciphra jam

cognita in Radice extrahenda ad minimum triplicantur, quod quidem Arithmetica studiosis omnibus gratum fore confido, atque ipse Inventori abunde gratulor. Ut autem harum Regularum utilitas melius sentiat, exemplum unum vel alterum adjungere placuit.

Exemp. I. Quærat Latus Cubi dupli, five $aaa + b = 2$. Hic $a = 1$

atque $\frac{b}{3a} = \frac{1}{3}$, adeoque $\frac{1}{2} + \sqrt[3]{\frac{7}{12}}$ five 1, 26 invenietur Latus prope-

verum. Cubus autem ex 1, 26 est 2, 000376, adeoque 0, 63 +

$\sqrt[3]{0,3969 - \frac{0,000376}{3,78}}$ five 0,63 + $\sqrt[3]{0,3968005291005291} =$

1,259921049895 -; quod quidem tredecim figuris Latus Cubi dupli exhibet, nullo fere negotio, viz. una Divisione & Lateris Quadrati extractione, ubi vulgari operandi modo quantum desudasset Arithmeticus norunt experti. Hunc etiam calculum quousque velis continuari licet, augendo quadratum ad-

ditione $\frac{eee}{3a}$. Quæ quidem correctio hoc in casu non nisi unitatis in Radicis figura decima-quarta augmentum affert.

Exemp.

Exemplum II. Queratur Latus Cubi æqualis mensuræ Anglice Gallæ dictæ; uncias solidas 231 continentis, Cubus proxime minor est 216 cujus Latus $6 = a$, ac residuum $15 = b$; adeoque pro prima approximatione provenit

$3 + \sqrt[3]{9 + \frac{5}{6}} = \text{Radici.}$ Cumque $\sqrt[3]{9, 8333 \dots}$ fit 3, 1358... patet

$6, 1358 = a + e$. Supponatur jam $6, 1358 = a$, & habebimus Cubum ejus 231,000853894712, ac juxta regulam $3, 0679 +$

$\sqrt[3]{9, 41201041 - \frac{0, 000853894712}{18, 4074}}$ æquatur accuratissime Lateri Cubi

dati, id quod intra horæ spatium calculo obtinui 6, 13579243965195897, in octodecima figura justum, at deficiens in decima-nona. Hæc vero formula merito præferenda est rationali, ob ingentem divisorem, non sine magno labore tractandum; cum Lateris Quadrati extractio multo facilius procedat, ut experientia multiplex me docuit.

Regula autem pro Radice Surfolidi puri, sive Potestatis quintæ, paulo altioris indaginis est, atque etiam adhuc multo perfectius rem præstat: datas enim in Radice ciphras ad minimum quintuplicat, neque etiam multi nec operosi est calculi. Author autem nullibi inveniendi methodum ejusve demonstrationem concedit, etiam si maxime desiderari videatur: præsertim cum in Libro impresso non recte se habeat; id quod imperitos facile illudere possit. Potestas autem quinta Lateris $a + e$ conficitur ex his membris $a^5 + 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4 + e^5 = a^5 + b$, unde $b = 5a^4e + 10a^3e^2 + 10a^2e^3 + 5ae^4$, rejectio e^5 ob parvitatem suam: quo-circa $\frac{b}{5a} = a^4e =$

$2a^2e^2 + 2ae^3 + e^5$, atque utrinque addendo $\frac{1}{4}a^4$ habebimus

$$\sqrt[5]{\frac{1}{4}aaaa + \frac{b}{5a}} = \sqrt[5]{\frac{1}{4}a^4 + a^4e + 2a^2e^2 + 2ae^3 + e^5} = \frac{1}{2}aa$$

$+ ae + ee$. Dein utrinque subducendo, $\frac{1}{4}aa, \frac{1}{2}a + e$ æquabitur

$$\sqrt[5]{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}aa, \text{ cui si addatur } \frac{1}{2}a, \text{ erit } a + e = \frac{1}{2}a +$$

$$\sqrt[5]{\frac{1}{4}a^4 + \frac{b}{5a}} - \frac{1}{4}aa = \text{Radici potestatis } a^5 \pm b. \text{ Quod si fu}$$

isset $a^5 - b$, (assumpta a justo majore) regula sic se haberet, $\frac{1}{2}a +$

$$\sqrt[5]{\frac{1}{4}a^4 - \frac{b}{5a}} - \frac{1}{4}aa$$

Atque hæc regula nitrum in modum approximatur, ut vix restitutione opus sit; ac dum hæc mecum pensitavi, incidi in formularum methodum quandam generalem pro quavis potestate satis concinnam, quamque celare nequeo; cum etiam in superioribus potestatibus datas radices figuras triplicare valeant.

Hæ autem formulæ ita se habent tam rationales quam irrationales.

$$\sqrt{aa+b} = \sqrt{aa+b}, \text{ vel } a + \frac{ab}{2aa+\frac{1}{2}b}$$

$$\sqrt[3]{a^3+b} = \frac{1}{2}a + \sqrt[3]{\frac{1}{4}aa + \frac{b}{3a}}, \text{ vel } a + \frac{ab}{3a^3+b}$$

$$\sqrt[4]{a^4+b} = \frac{3}{4}a + \sqrt[4]{\frac{1}{2}aa + \frac{b}{6aa}}, \text{ vel } a + \frac{ab}{4a^4+\frac{1}{2}b}$$

$$\sqrt[5]{a^5+b} = \frac{4}{5}a + \sqrt[5]{\frac{1}{16}aa + \frac{b}{10a^3}}, \text{ vel } a + \frac{ab}{5a^5+2b}$$

$$\sqrt[6]{a^6+b} = \frac{5}{6}a + \sqrt[6]{\frac{1}{21}aa + \frac{b}{15a^4}}, \text{ vel } a + \frac{ab}{6a^6+\frac{1}{3}b}$$

$$\sqrt[7]{a^7+b} = \frac{6}{7}a + \sqrt[7]{\frac{1}{12}aa + \frac{b}{21a^5}}, \text{ vel } a + \frac{ab}{7a^7+3b}$$

Et sic de cæteris etiam adhuc superioribus. Quod si assumeretur a radice quæsitæ major, (quod cum fructu fit quoties Potestas resolvenda multo propior sit potestati Numeri integri proxime majoris quam proxime minoris) mutatis mutandis eadem radicum expressiones proveniunt.

$$\sqrt{aa-b} = \sqrt{aa-b}, \text{ vel } a - \frac{ab}{2aa-\frac{1}{2}b}$$

$$\sqrt[3]{a^3-b} = \frac{1}{2}a + \sqrt[3]{\frac{1}{4}aa - \frac{b}{3a}}, \text{ vel } a - \frac{ab}{3a^3-b}$$

$$\sqrt[4]{a^4-b} = \frac{3}{4}a + \sqrt[4]{\frac{1}{2}aa - \frac{b}{6aa}}, \text{ vel } a - \frac{ab}{4a^4-\frac{1}{2}b}$$

$$\sqrt[5]{a^5-b} = \frac{4}{5}a + \sqrt[5]{\frac{1}{16}aa - \frac{b}{10a^3}}, \text{ vel } a - \frac{ab}{5a^5-2b}$$

$$\sqrt[6]{a^6-b} = \frac{5}{6}a + \sqrt[6]{\frac{1}{21}aa - \frac{b}{15a^4}}, \text{ vel } a - \frac{ab}{6a^6-\frac{1}{3}b}$$

$$\sqrt[7]{a^7-b} = \frac{6}{7}a + \sqrt[7]{\frac{1}{12}aa - \frac{b}{21a^5}}, \text{ vel } a - \frac{ab}{7a^7-3b}$$

Atque inter hos duos terminos semper consistit vera radix, aliquanto propior irrationali quam rationali; e vero juxta formulam irrationalem inventa, semper peccat in excessu, sicut in defectu à rationali formula resul-

tans.

tans Quotus; adeoque si fuerit $+b$, Irrationalis majorem justo exhibet Radicem, rationalis minorem; è contrario vero si fuerit $-b$. Atque hæc de eliciendis Radicibus è Potestatibus puris dicta sunt, quæ quidem ad usus ordinarios sufficientes multo facilius habentur ope Logarithmorum: quoties vero ultra Tabularum Logarithmicarum vires accuratissime definienda est radix, ad hujusmodi methodos necessario recurrendum est. Præterea cum ex harum formularum inventione ac contemplatione, Universalis Regula pro Æquationibus Affectis (quam non sine fructu Geometriæ ac Algebræ studiosis omnibus usurpandam confido) mihi ipsi oblata sit, volui ipsius inventi primordia qua possim claritate aperire.

Æquationum quidem Affectarum Quadrato-quadratum non excedentium Constructionem generalem concinnam admodum ac facilem, *An. circiter* Vid. sup. Sect. XIX. 1. 1687. jam tum inventam, publici juris feci: ex quo ingens cupido animum incessit, idem Numeris efficiendi. At brevi post D. *Raphson* magna ex parte voto satisfecisse visus est, usque dum D. *de Lagney* etiam adhuc compendiosius rem peragi posse hoc suo libello mihi suggessit. Methodus autem nostra hæc est.

Supponatur Radix cujusvis æquationis z composita ex partibus $a +$ vel $-e$, quarum a ex hypothesi assumatur ipsi z quantum fieri possit propinqua (quod tamen commodum est, non necessarium) & ex quantitate $a +$ vel $-e$ formentur Potestates omnes ipsius z in Æquatione inventas, iisque affigantur Numeri Coefficientes respective: deinde Potestas Resolvenda subducatur è summa partium datarum in prima columna, ubi e non reperitur, quam Homogeneum Comparationis vocant, sitque differentia $+b$. Dein habeatur summa omnium Coefficientium ipsius lateris e in secunda Columna, quæ fit s ; denique in tertia addantur omnes coefficientes quadrati e^2 , quarum summam vocemus t , ac Radix quæsitæ z , formula rationali habebitur $= a +$ vel

$$- \frac{s b}{s s + \text{vel} - t b}; \text{ irrationali vero fiet } z = a \frac{+ \frac{1}{2} s + \sqrt{\frac{1}{4} s s + b t}}{t}, \text{ id quod}$$

exemplis illustrare fortasse operæ pretium erit. Instrumenti vero loco ad sit Tabella, Potestatum omnium ipsius $a +$ vel $-e$ Genesin exhibens, quæ si opus fuerit continuari facile possit. A septima vero incipiam cum pauca Problemata eo usque assurgere deprehendantur. Hanc Tabellam jure optimo *Speculum Analyticum Generale* appellare licet. Potestates autem prædictæ ex continua multiplicatione per $a + e = z$ ortæ, sic proveniunt, cum suis coefficientibus adjunctis.

$$\begin{aligned}
1z^7 &= 1a^7 + 7la^6e + 21la^5ee + 35la^4e^3 + 35la^3e^4 \\
kz^6 &= ka^6 + 6ka^5e + 15ka^4ee + 20ka^3e^3 + 15ka^2e^4 \\
hz^5 &= ba^5 + 5ba^4e + 10ba^3ee + 10ba^2e^3 + 5bae^4 \\
gz^4 &= ga^4 + 4ga^3e + 6ga^2ee + 4gae^3 + ge^4 \\
fz^3 &= fa^3 + 3fa^2e + 3faee + fe^3 \\
dz^2 &= da^2 + 2dae + dee + e^2 \\
ez &= ea + ee +
\end{aligned}$$

$$\begin{aligned}
&+ 21la^2e^5 + 7lae^6 + le^7 \\
&+ 6kae^5 + ke^6 \\
&+ he^5
\end{aligned}$$

Quod si fuerit $a - e = z$, ex iisdem membris conficitur Tabella, negatis solummodo imparibus Potestatibus ipsius e , ut ee^3, e^5, e^7 : & affirmatis paribus e^2, e^4, e^6 . Sitque summa Coefficientium lateris $e = s$: Summa Coefficientium Quadrati $ee = t$: Cubi $eee = u$: Biquadrati $e^4 = w$: Surfolidi $e^5 = x$: Summa vero Coefficientium Cubo-cubi $e^6 = y$: &c. Cum autem supponatur e exigua tantum pars Radicis inquirendae, omnes potestates ipsius e multo minores evadunt similibus ipsius a Potestatibus, adeoque pro prima hypothefi rejiciantur superiores (ut in potestatibus puris ostensum est) ac formata aequatione nova, substituendo $a + e = z$ habebimus ut diximus $+b = +se + tee$. Cujus rei cape exempla sequentia, quo melius intelligatur.

Exemp. I. Proponatur aequatio $z^4 - 3zz + 75z = 10000$. Pro prima hypothefi ponatur $a = 10$, ac consequenter prodibit aequatio.

$$\begin{aligned}
z^4 &= +a^4 + 4a^3e + 6a^2ee + 4ae^3 + e^4 \\
-dz^2 &= -da^2 + dae - dee \\
+cz &= +ca + ce \\
&= +10000 + 4000e + 600ee + 40e^3 + e^4 \\
&\quad - 300 + 60e - 3ee \\
&\quad + 750 + 75e \\
&\quad - 10000 \\
\hline
&+ 450 + 4015e + 597ee + 40e^3 + e^4 = 0
\end{aligned}$$

Signis $+$ ac $-$ (respectu e & e^3) in dubio relictis, usque dum sciatur an e sit Negativa vel Affirmativa; quod quidem aliquam pariet difficultatem, cum in aequationibus plures radices admittentibus, saepe augeantur Homogenia Comparationis, ut appellant, à minuta quantitate a , ac e contra ea aucta minuatur. Determinatur autem signum ipsius e ex signo quantitatis b ; sublata enim Resolvenda ex Homogenio ab a formato, signum ipsius se , ac proinde partium in ejus compositione praevalentium, semper contrarium erit signo

figno differentia b . Unde patebit an fuerit $-e$ vel $+e$, five an a major vel minor radice vera assumpta. Ipsa autem e semper æquatur

$$\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}, \text{ quoties } b \text{ ac } t \text{ eodem signo notantur; quoties vero di-}$$

verso signo connectuntur, eadem e fit $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t}$. Postquam vero

comptum sit fore $-e$, in affirmatis æquationis membris negentur e , e^2 , e^3 , &c. in negatis affirmentur; scribantur *scil.* signo contrario; si vero fuerit $+e$, affirmentur in affirmatis, negentur in negatis. Habemus autem in hoc nostro exemplo 10450 loco Resolvendæ 10000, five $b = +450$, unde constat a majorem justo assumptam, ac proinde haberi $-e$: Hinc æquatio fit 10450 - 4015 e + 597 ee - 4 e^3 + e^4 = 10000. Hoc est 450 - 4015 e + 597 ee = 0. Adeoque 450 = 4015 e - 597 ee five $b = se - tee$

cujus Radix e fit $\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{t}$, vel si mavis $\frac{s}{2t} - \frac{\sqrt{\frac{1}{4}ss - bt}}{4tt}$, id

est, in præfenti casu, $e = \frac{2007\frac{1}{2} - \sqrt{3761406\frac{1}{4}}}{597}$, unde provenit radix

quæfita prope verum 9,886. Hoc vero pro secunda Hypothesi substituto, emergit $a + e = z$ accuratissime 9,8862603936495..., in ultima figura vix

binario justum superans; nempe cum $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{t} = e$. Atque hoc

etiam, si opus fuerit, multo ulterius verificari possit, subducendo $\frac{\frac{1}{2}se^3 + \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss + tb}}$

si fuerit $+e$, vel addendo $\frac{\frac{1}{2}se^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - tb}}$, radici prius inventæ, si sit $-e$.

Cujus compendium eo pluris æstimandum quod quandoque, ex sola prima suppositione, semper vero ex secunda, iisdem conservatis coefficientibus quousque velis calculum continuare possis. Cæterum æquatio prædicta etiam negativam habet radicem, viz. $z = 10, 26 \dots \dots$ quam cuilibet accuratius expiscari licet.

Exempli.

Exemp. II. Sit $z^3 - 17z^2 + 54z = 350$; ac ponatur $a = 10$. Ex præscripto Regulæ,

$$\begin{aligned} z z z &= a a a + 3 a a e + 3 a e e + e e e \\ - d z z &= d a a - 2 d a e - d e e \\ + c z &= c a + c e \end{aligned}$$

$$\begin{array}{r} \text{Id est } + 1000 + 300e + 30ee + eee \\ - 1700 - 340e - 17ee \\ + 540 + 54e \\ - 350 \\ \hline \end{array}$$

$$\text{Sive } - 510 + 14e + 13ee + eee = 0.$$

Cum autem habere $- 510$, constat a minorem justo assumi, ac proinde e affirmativam esse, ac ex $510 = 14e + 13ee$ fit $\sqrt{b^2 + \frac{1}{4}ss - \frac{1}{2}s} = e$

$$= \frac{\sqrt{6679} - 7}{13}, \text{ unde } z \text{ fit } 15,7 \dots \text{ quæ nimia quidem est ob late sum-}$$

ptam a ; ideo supponatur secundo $a = 15$, ac pari ratiocinio habebimus $e =$

$$\frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - tb}}{f} = \frac{109\frac{1}{2} - \sqrt{11710\frac{1}{4}}}{28}; \text{ ac proinde } z = 14,954068.$$

Quod si calculum adhuc tertio restaurare velis, usque in vigesimam quintam figuram vero conformem invenies radicem: Paucioribus vero contentus, scribendo $tb + eeee$ loco tb , vel subtrahendo aut addendo radici prius inventæ

$\frac{1}{4}eee$ ad scopum statim perveniet. Æquatio vero proposita nulla $\sqrt{\frac{1}{4}ss + tb}$

alia radice explicari potest, quia Potestas Resolvenda 350 major est Cubo ex

$$\frac{17}{3} \text{ vel } \frac{1}{3} d.$$

Exemp. III. Sit Æquatio illa quam in Resolutione difficillimi Problematis Arithmetici adhibet *Clarissimus Wallisius*, Cap. LXII. *Algebra* suæ, quo radicem *Vieta* Methodo accuratissime quidem affecutus est. Eandemque exemplum Methodi suæ affert laudatus D. *Raphson*, pag. 25, 26. nempe $z^4 - 80z^3 + 1998z^2 - 14937z + 5000 = 0$. Hæc autem æquatio ejus formulæ est, ut plures habeat radices Affirmativas, ac quod difficultatem ejus augeat, prægrandes sunt Coefficientes respectu Resolvendæ datæ: Quo melius autem tractetur, dividatur, ac juxta notas punctationum regulas ponatur $-z^4 + 8z^3 - 20z^2 + 15z = c$, 5 (ubi z est $\frac{1}{10} z$ in æquatione proposita

proposita) ac pro prima Hypothesi habemus $a = 1$. Proinde $+z = 5a$
 $- zee + 4e^2 - e^3 = 0$, $5 = 0$.

Hoc est $1\frac{1}{2} = 5e + zee$; hinc $\frac{\sqrt{\frac{1}{4}ss + bt} - \frac{1}{2}s}{e} = e$ fit $\sqrt{37 - 5}$

adeoque $z = 1, 27$, unde constat $12, 7$ radicem esse æquationis propositæ
 vero vicinam. Secundo loco supponatur $z = 12, 7$, ac juxta præscriptum
 Tabellæ Potestatum oritur.

$$\begin{array}{r} b \quad s \quad t \quad u \\ - 26014, 4641 - 8193, 532e - 967, 74ee - 50, 8e^3 - e^4 \\ + 163870, 640. + 38709, 60.e + 3048, ..ee + 80, .e^3 \\ - 322257, 42.. - 50749, 2..e - 1998, ..ee \\ + 189699, 9... + 14937, ...e \\ - 5000, \end{array}$$

$$+ 298, 6559 - 5296, 132e + 82, 26ee + 29, 2e^3 - e^4 = 0$$

Adeoque $- 298, 6559 = - 5296, 132e + 82, 26ee$, cujus radix e juxta

$$\text{regulam} = \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt}}{e} \text{ fit } \frac{2648, 066 - \sqrt{6987686, 106022}}{82, 26} =$$

$$0, 05644080331.... = e \text{ minori vero : ut autem corrigatur, } \frac{\frac{1}{2}ue^3 - e^4}{\sqrt{\frac{1}{4}ss - bt}}$$

$$\text{five } \frac{0, 0026201....}{2643, 423....} \text{ fit } 0, 00000099117; \text{ ac proinde } e \text{ correctæ} =$$

$0, 05644179448$; Quod si adhuc plures radices figuras desideras, formetur ex

$$\text{correctæ } tue^3 - te^4 = 0, 43105602423..., \text{ ac } \frac{\frac{1}{2}s - \sqrt{\frac{1}{4}ss - bt} - tue^3 + te^4}{e}$$

$$\text{five } \frac{2648, 066 - \sqrt{6987685, 67496597577...}}{82, 26}$$

$0, 05644179448074402 = e$, unde $a + e = z$ radix accuratissima fit
 $12, 75644179448074402....$ qualem invenit *Cl. Wallisius* in loco citato.
 Ubi observandum, redintegrationem calculi semper triplicare notas veras in as-

sumpta a , quas prima correctio five $\frac{\frac{1}{2}ue^3 - \frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss - bt}}$ quintuplices reddit, quæque

etiam commode per Logarithmos efficitur. Altera autem correctio post pri-
 mam, etiam duplum Ciphrarum numerum adjungit, ut omnino assumptas
 septuplicet; prima tamen plerumque usibus Arithmetices abunde sufficit.
 Quæ vero dicta sunt de numero Ciphrarum in radice recte assumptarum, ita
 intelligi velim, ut cum a non nisi decima parte distet à vera radice, prima figu-
 ra recte assumatur; si intra centesimam partem, duæ primæ; si intra millesi-
 mam, tres priores rite se habeant; quæ deinde juxta nostram regulam tractatæ
 statim novem evadunt.

Restat jam ut nonnulla adjiciam de nostra formula rationali, viz. $e = \frac{sb}{ss+tb}$ quæ quidem satis expedita videbitur, nec multum cedit priori, cum

etiam datas ciphras triplicare valeat. Formata autem æquatione ex $a + e = z$, ut prius, statim patebit an a assumpta sit major vel minor vero, cum scilicet se signo semper notari debeat contrario signo differentie Resolvendæ ac Homogenii sui ex a producti. Deinde posito quod $+b + se + vel -tee = 0$. Divisor fit $ss - tb$, quoties bac et iisdem signis notantur; idem vero fit $ss + bt$, si signa diversa sint. Praxi autem magis accommodata videtur si scribere-

tur Theorema, $e = \frac{b}{s + \frac{tb}{s}}$ nempe cum una multiplicatione ac duabus di-

visionibus res peragatur, quæ tres multiplicationes ac unam divisionem alias requireret. Hujus etiam Methodi Exemplum capiamus à prædictæ Equætionis radice 12. 7. ubi 298,6559 -- 5296,132. $e + 82,26 e e$

$+b - s + e$
 $+ 29, 2 e^3 - e^4 = 0$, adeoque $\frac{tb}{s - s} = e$, hoc est, fiat ut s ad t ita b ad $+s$

$\frac{tb}{s} = 5296, 132 : 298, 6559$ in $82, 26 (4, 63875 \dots$; quocirca di-

visor fit, $s = \frac{tb}{e} = 5291, 49325 \dots : 298, 6559 (0, 056441 \dots = e$,

viz. quinque figuris veris adjectis radici assumptæ. Corrigi autem nequit hæc formula sicut præcedens irrationalis; adeoque si plures desiderentur radicis figura, præstat assumptæ nova Hypothesis calculum de integro repetere: ac novus Quotus triplicando figuras in radice cognitæ supputatori etiam maxime scrupuloso abunde satisfaciet.

A Method of
Raising an Infi-
nite Multinomi-
al, to any given
Power, by M. de
Moivre. N. 230.
p. 619.
July. An. 1697.

XXI. Theorem. $ax + bz^2 + cz^3 + dz^4 + ez^5 + fz^6$

$$+gz^7 + hz^8 + iz^9 \&c.]^m = a^m z^m + \frac{m}{1} a^{m-1} b z^{m+1} +$$

$$+ \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b^2 z^{m+2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^3 z^{m+3} +$$

$$+ \frac{m}{1} a^{m-1} c + \frac{m}{1} \times \frac{m-1}{1} a^{m-2} bc$$

$$\times \frac{m}{1} a^{m-1} d$$

$$+ \frac{m}{1}$$

$$\begin{aligned}
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times \frac{m-5}{6} a^{m-6} b^6 z^{m+6} \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} a^{m-5} b^5 c \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4} b^4 d \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} a^{m-4} b^3 c^2 \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b^2 e \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} b c d \\
& + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} b f \\
& + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} a^{m-3} c^3 \\
& + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} c e \\
& + \frac{m}{1} \times \frac{m-1}{2} a^{m-2} d^2 \\
& + \frac{m}{1} a^{m-1} g, \&c.
\end{aligned}$$

I suppose that the Infinite Number Multinomial is $az + bz^2 + cz^3 + dz^4 + ez^5, \&c.$ m is the Index of the Power, to which this Multinomial ought to be rais'd, or if you will, 'tis the Index of the Root, which is to be extracted: I say, That this Power or Root of the Multinomial is such a Series as I have express'd.

For the understanding of it, it is only necessary to consider all the Terms by which the same Power of z is Multiplied; in order thereto, I distinguish two things in each of these Terms; 1°. The Product of certain Powers of the Quantities, $a, b, c, d, \&c.$ 2°. The Unciæ (as *Oughtred* calls 'em) prefix to these Products. To find all the Products belonging to the same Power of z , to that Product, for Instance, whose Index is $m + r$ (where r may denote any Integer Number) I divide these Products into several Classes; those which immediately after some certain Power of a (by which all these Products begin) have b , I call Products of the first Class; for Example, $a^{m-4} b^5 c$ is a Product of the first Class, because b immediately follows a^{m-4} ; those which immediately after some Power of a have c , I call Products of the second Class, so $a^{m-3} c c d$, is a Product of the second Class;

sis; those which immediately after some Power of a have d , I call Products of the third Classis, and so of the rest.

This being done, I Multiply all the Products belonging to z^{m+r-1} (which preceeds immediately z^{m+r}) by b , and divide 'em all by a ; 2°. I Multiply by c , and Divide by a , all the Products belonging to z^{m+r-2} except those of the first Classis; 3°. I Multiply by d and Divide by a , all the Products belonging to z^{m+r-3} , except those of the first and second Classis; 4°. I Multiply by e and Divide by a , all the Terms belonging to z^{m+r-4} , except those of the first, second and third Classis, and so on, till I meet twice with the same Term. Lastly, I add to all these Terms, the Product of a^{r-1} into the Letter, whose Exponent is $r+1$.

Here I must take notice, that by the Exponent of a Letter, I mean the Number which expresses what Place the Letter has in the Alphabet, so 3 is the Exponent of the Letter c , because the Letter c is the 3d in the Alphabet.

It is evident by this Rule, you may easily find all the Products belonging to the several Powers of z , if you have but the Product belonging to z^m , viz. a^m .

To find the Unciæ which ought to be prefix'd to every Product, I consider the Sum of Units contain'd in the Indices of the Letters which compose it (the Index of a excepted) I write as many Terms of the Series $m \times m - 1$;

$\times m - 2 \times m - 3$, &c. as there are Units in the Sum of these Indices, this Series is to be the Numerator of a Fraction, whose Denominator in the Product of the several Series $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. the first of which contains as many Terms as there are Units in the Index of b , the second as many as there are Units in the Index of c , the third as many as there are Units in the Index of d , the fourth as many as there are Units in the Index of e , &c.

Demonstration,] To raise the Series $ax + bxz + cz^2 + dz^3$, &c. to any Power whatsoever, write so many Series equal to it, as there are Units in the Index of the Power demanded. Now it is evident, that when these Series, are also Multiplied, there are several Products in which there is the same Power of z ; thus if the Series $ax + bxz + cz^2 + dz^3$, &c. is raised to its Cube, you have the Products b^3z^6 , $abcz^6$, $aadz^6$, in which you find the same Power z^6 . Therefore let us consider what is the Condition that can make some Products to contain the same Power of z ; the first thing that will appear in relation to it is, that in any Product whatsoever, the Index of z is the Sum of the particular Indices of z in the Multiplying Terms (this follows from the Nature of the Indices): thus b^3z^6 is the Product of bz^2 , bz^2 , bz^2 , and the Sum of the Indices in the Multiplying Terms, is $2+2+2=6$; $abcz^6$ is the Product of ax , bxz , cz^2 , and the Sum of the Indices of z in the Multiplying Terms is $1+2+3=6$; $aadz^6$ is the Product of ax , ax , dz^3 , and the Sum of the Indices of z in the Multiplying Terms is $1+1+4=6$: the next thing that appears, is, that the Index of z in the Multiplying Terms is the same with the Exponent of the Let-

ter to which z is joyn'd; from which two Considerations it follows, that, to have all the Products belonging to a certain Power of z , you must find all the Products where the Sum of the Exponents of the Letters which compose them, shall always be the same with the Index of that Power. Now this is the Method I use, to find easily all the Products belonging to the same Power of z , let $m + r$ be the Index of that Power, I consider that the Sum of the Exponents of the Letters which compose the Products, must exceed by 1, those which belong to z^{m+r-1} , now because the excess of the Exponent of the Letter b , above the Exponent of the Letter a , is 1, it follows, that if each of the Products belonging to z^{m+r-1} is Multiply'd by b , and Divided by a , you will have Products, the Sum of whose Exponents will be $m + r$; Likewise the Sum of the Exponents of the Letters which compose the Products belonging to z^{m+r} exceeds by 2 the Sum of the Exponents of the Letters which compose the Products belonging to z^{m+r-1} : Now because the Exponent of the Letter a is less by 2 than the Exponent of the Letter c , it follows, that if each Product belonging to z^{m+r-1} is Multiplied by c , and Divided by a , you will have other Products, the Sum of whose Exponents is still $m + r$; Now if all the Products belonging to z^{m+r-2} were Multiplied by c and Divided by a , you would have some Products that would be the same as some of them found before, therefore you must except out of 'em those that I have called Products of the first Classis; what I have said shows why all the Products belonging to z^{m+r-3} , except those of the first and second Classis must be Multiplied by d , and Divided by a : Lastly, you see the Reason why to all these Products is added the Product of a^{m-1} by the Letter whose Exponent is $r + 1$; 'Tis because the Sum of the Exponent is still $m + r$.

As for what relates to the Unciæ; observe that when you Multiply $az +$, $bzx + cz^2 + dz^3$, &c. by $az + bzx + cz^2 + dz^3$, &c. each Letter, a, b, c, d , &c. of the second Series, is Multiplied by each of the Letters a, b, c, d , &c. of the first Series. Thus the Letter a of the second Series is Multiplied by the Letter b of the first Series, and the Letter b of the second Series is Multiplied by the Letter a of the first; therefore you may have the two Planes, ab, ab , or $2ab$; for the same Reason you have $2ac, 2ad$, &c. Therefore you must prefix to each Plane of those that compose the Square of the Infinite Series $az + bzx + cz^2$, &c. the Number which expresses how many ways the Letters of each Plane may be changed; Likewise if you Multiply the Product of the two preceeding Series by $az + bzx + cz^2$, &c. each Letter a, b, c, d , of the third Series is Multiplied by each of the Planes form'd by the Product of the first and second Series; Thus the Letter a is Multiplied by the Planes bc and cb ; the Letter b is Multiplied by ac and ca ; the Letter c is Multiplied by ab and ba ; therefore you have the six Solids, $abc, acb, bac, bca, cab, cba$, or six abc ; Therefore you must prefix to each Solid whereof the Cube of the Infinite Series is compos'd, the Number which expresses how many ways the Letters of each Solid may be changed. And generally, you must prefix to any Product, whereof any Power of the Infinite Series $az + bzx + cz^2$, &c. is compos'd, the Number which expresses how many ways the Letters of each Product may be changed.

Now

Now to find how many ways the Letters of any Product, for instance, $a^m - b^h c^p d^r$ may be changed; this is the Rule which is commonly given: write as many Terms of the Series $1 \times 2 \times 3 \times 4 \times 5$, &c. as there are Units in the Sum of the Indices, viz. $m - n + b + p + r$; let this Series be the Numerator of a Fraction, whose Denominator shall be the Product of the Series, $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. $1 \times 2 \times 3 \times 4 \times 5$, &c. whereof the first is to contain as many Terms as there are Units in the first Index $m - n$; the second as many as there are Units in the second Index b ; the third as many as there are Units in the third Index p ; the fourth as many as there are Units in the fourth Index r . But the Numerator and Denominator of this Fraction have a common Divisor, viz. the Series of $1 \times 2 \times 3 \times 4 \times 5$, &c. continued to so many Terms as there are Units in the first Index $m - n$; therefore let both this Numerator and Denominator be divided by this common Divisor, then this new Numerator will begin with $m - n + 1$, whereas the other began with 1, and will contain so many Terms as there are Units in $b + p + r$, that is, so many as there are Units in the Sum of all the Indices excepting the first; as for the New Denominator, it will be the Product of three Series only; that is, of so many as there are Indices, excepting the first. But if it happens withall, that n be equal to $b + p + r$, as it always happens in our *Theorem*; then the Numerator beginning by $m - n + 1$, and being continued to so many Terms as there are Units in $b + p + r$ or n , the last Term will be m necessarily, so if you invert the Series, and make that the first Term which was the last, the Numerator will be $m \times m - 1$

$\times m - 2 \times m - 3$, &c. continued to so many Terms as there are Units in the Sum of the Indices of each Product, excepting the first Index. And whatever is here said of Powers, whose Index is an Integer, may be adapted to Roots or Powers whose Index is a Fraction.

XXII. *Theorem*.] If $ax + bx^2 + cx^3 + dx^4 + ex^5 + fx^6$, &c. = gy The Extraction of a Root of an Infinite Equation, by Mr Abr. de Moivre. N. 247. p. 150. May. An. 1698.

$$+ hy^2 + iy^3 + ky^4 + ly^5 + my^6, \text{ \&c. then will } x \text{ be } = \frac{g}{a}y +$$

$$\frac{h - bAA}{a}y^2 + \frac{i - 2bAB - cA^2}{a}y^3 + \frac{k - bBB - 2bAC - 3cAAB - dA^3}{a}y^4 +$$

$$\frac{l - 2bBC - 2bAD - 3cABB - 3cAAC - 4dAB - eA^2}{a}y^5 + \frac{m - 2bBD - bCC - 2bAE - cB^2 - 6cABC - 3cAAD - 6dAAB - 4dAC - 5eAB - fA^2}{a}y^6, \text{ \&c.}$$

For the understanding of this Series, and in order to continue it as far as we please; it is to be observed, 1. That every Capital Letter is equal to the Coefficient of each preceding Term; thus the Letter B is equal to the Coefficient

cient $\frac{b - b A A}{A}$. 2. That the Denominator of each Coefficient is always A .

3. That the first Member of each Numerator, is always a Coefficient of the Series $g y + b y y + i y^3$, &c. viz. the first Numerator begins with the first Coefficient g , the second Numerator with the second Coefficient b , and so on. 4. That in every Member after the first, the Sum of the Exponents of the Capital Letters, is always equal to the Index of the Power to which this Member belongs: Thus considering the Coefficient

$k - b B B - 2 b A C - 3 c A A B - d A^4$, which belongs to the Power y^4 ,

we shall see that in every Member $b B B$, $2 b A C$, $3 c A A B$, $d A^4$, the Sum of the Exponents of the Capital Letters is 4, (where I must take notice, that by the Exponent of a Letter, I mean the Number which expresses what place it has in the Alphabet; thus 4 is the Exponent of the Letter D) hence I derive this Rule for finding the Capital Letters of all the Members that belong to any Power; Combine the Capital Letters as often as you can make the Sum of their Exponents equal to the Index of the Power to which they belong. 5. That the Exponents of the small Letters, which are written before the Capitals, express how many Capitals there is in each Member. 6. That the Numerical Figures or Unciæ that occur in these Members, express the Number of Permutations which the Capital Letters of every Member are capable of.

For the *Demonstration* of this; suppose $z = A y + B y y + C y^3 + D y^4$, &c. Substitute this Series in the room of z , and the Powers of this Series in the room of the Powers z , there will arise a new Series; then take the Coefficients which belong to the several Powers of y in this new Series, and make them equal to the corresponding Co-efficients of the Series $g y + b y y + i y^3$, &c. and the Coefficients A, B, C, D , &c. will be found such as I have determined them.

But if any one desires to be satisfied, that the Law by which the Coefficients are form'd, will always hold, I'll desire 'em to have recourse to the *Theorem* I have given for *Raising an Infinite Series to any Power*, or extracting any Root of the same; for if they make use of it, for taking successively the Powers of $A y + B y y + C y^3$, &c. they will see that it must of necessity be so. I might have made the *Theorem* I give here, much more general than it is; for I might have suppos'd, $a z^m + b z^{m+1} + c z^{m+2}$, &c. $= g y^m + h y^{m+1} + i y^{m+2}$, &c. then all the Powers of the Series $A y + B y y + C y^3$, &c. designed by the universal Indices, must have been taken successively; but those who will please to try this, may easily do it, by means of the *Theorem* for raising an Infinite Series to any Power, &c.

This *Theorem* may be applied to what is called the Reversion of Series, such as finding the Number from its Logarithm given; the Sine from the Arch; the Ordinate of an Ellipse from an Area given to be cut from any point in the Axis: But to make a particular Application of it, I'll suppose we have this

this Problem to solve; viz. *The Chord of an Arc being given, to find the Chord of another Arc, that shall be to the first as n to 1.* Let y be the Chord given, z the Chord required; now the Arc belonging to the Chord y , is

$$y + \frac{y^3}{6dd} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^6} \&c. \text{ And the Arc belonging to the Chord}$$

$$z \text{ is } z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6} \&c. \text{ The first of the Arcs is to the}$$

second as 1 to n ; therefore multiplying the Extreams and Means together, we shall have this Equation.

$$z + \frac{z^3}{6dd} + \frac{3z^5}{40d^4} + \frac{5z^7}{112d^6} \&c. = ny + \frac{ny^3}{6dd} + \frac{3ny^5}{40d^4} + \frac{5ny^7}{112d^6} \&c.$$

Compare these two Series with the two Series of the Theorem, and and you will find $a = 1$, $b = 0$, $c = \frac{1}{6dd}$, $d = 0$, $e = -\frac{3}{40d^4}$, $f = 0$,

$$\&c. g = n, h = 0, i = \frac{n}{6dd}, k = 0, l = -\frac{3n}{40d^4}, m = 0, \&c.$$

$$\text{hence } z \text{ will be } = ny + \frac{n-n}{6dd} y^3, \&c. \text{ or } ny + \frac{1-nn}{2 \times 3 dd} y^3 A, \&c. \text{ Sup-}$$

posing A to denote the whole preceeding Term, which will be the same Series as Mr. Newton has first found.

By the same Method, this general Problem may be solved; *The Abscissa corresponding to a certain Area in any Curve being given, to find the Abscissa, whose corresponding Area shall be to the first in a given Ratio.*

The Logarithmic Series might also be found without borrowing any other Idea, than that Logarithms are the Indices of Powers: Let the Number, whose Logarithm we enquire, be $1+z$, suppose its Log. to be $az + bz^2 + cz^3$, &c. Let there be another Number $1+y$; therefore its Logarithm will be $ay + by^2 + cy^3$, &c. Now if $1+z = \overline{1+y}^n$, it follows that $az + bz^2 + cz^3$, &c. : $ay + by^2 + cy^3$, &c. : $n : 1$. that is, $az + bz^2 + cz^3$, &c. = $nay + nbby + ncy^3$, &c. therefore we may find a Value of z exprest by the Powers of y ; again, since $1+z$

$$= \overline{1+y}^n, \text{ therefore } z = \overline{1+y}^n - 1, \text{ that is } z = ny + \frac{n}{1} \times \frac{n-1}{2} yy$$

$$+ \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} y^3, \&c. \text{ Therefore } z \text{ is doubly exprest by the}$$

Powers of y . Compare these two Values together, and the Coefficients a, b, c , &c. will be determin'd, except the first a , which may be taken at Pleasure, and gives accordingly all the different Species of Logarithms.

Doctrina of Exhaustions, by Dr. Wallis. N. 255. p. 281.

Aug. An. 1699.

The Approximations of the Ancients in the Extracting of Roots, improv'd; by Dr. Wallis. N. 215. p. 2. Jun. An. 1695.

XXIII. De Differentiis Infinitesimarum-infinitesimis explicandis, non est ut sis porro sollicitus. Nam, ut tu mihi facilis concedis, quod nihili quodvis multipulum sit adhuc nihil; eadem ego facilitate tibi permitto, ut Differentias Infinitesimas in Infinitesimas ductas, tu merito negligas, potestque id tuto fieri, modo caute; Quippe, in quovis genere Quantitatum, quæ differunt dato minus, reputanda sunt *Æqualia*. Quo nritur *Exhaustionum* Doctrina tota, Veteribus pariter & Recentioribus necessaria.

XXIV. It is agreed by all, that if a Number propos'd be not a true Square, it is in vain to hope for a just Quadratick Root thereof, explicable by rational Numbers, Integers or Fracted. And therefore in such Cases, we must content our selves with Approximations (somewhat near the truth) without pretending to Accuracy.

And so, for the Cubick Root, of what is not a perfect Cube. And the like for Superior Powers.

Now the Ancients (being aware of this) had their Methods of Approximation, which tho' scarce apply'd by them beyond the Quadratick, or perhaps the Cubick Root, yet are equally practicable (by due adjustments) to the Superior Powers also.

I shall begin with the Square Root : For which the Ancient Method is to this purpose.

From the propos'd Non-quadrate (suppose N) subtract (in the usual manner) the greatest Square in Integers therein contained (suppose A q.) The remainder (suppose $B = 2 A E + E q$) is to be the Numerator of a Fraction, for designing the near Value of E (the remaining part of the Root sought $A + E = \sqrt{N}$) whose Denominator or Divisor is to be 2 A (the double Root of the subtracted Square) or $2 A + 1$ (that double Root increased by 1) the true Value falling between these two; sometime the one, sometime the other, being nearest to the true Value. But (for avoiding of Negative Numbers) the latter is commonly directed.

This Method Monsieur *De L'agny* affirms to be more than 200 Years old : And it is so ; for I find it in *Lucas Pacciolus* (otherwise called *Lucas de Burgo*, or *de Burgo Sancti Sepulchri*) Printed at *Venice* in the Year 1494 (if not even sooner than so, for I find there have been several Editions of it.) And how much older than so, I cannot tell : For he doth not deliver it as a New Invention of his own, but as a received Practice, and derived from the *Moors* or *Arabs*, from whom they had their *Algorism*, or Practice of Arithmetick by the 10 Numeral Figures now in use.

And it is continued down hitherto in Books of Practical Arithmetick in all Languages, which teach the Extraction of the Square Root, and (therein) this Method of Approximation, in case of a Non-quadrate.

The true ground of this Rule is this ; A q being (by Construction) the greatest Integer Square contained in N, 'tis evident that E must be less than 1 ; (otherwise not A q but the Square of $A + 1$, or some greater than so, would be the greatest Integer Square contained in N.) Now if the remainder $B = 2 A E + E q$ be divided by 2 A, the result will be too great for E, (the Divisor being too little ; for it should be $2 A + E$, to make the Quotient E.) But if

(to

(to rectify this) we diminish the Quotient, by increasing the Divisor, adding 1 to it, it then becomes too little; because the Divisor is now too big. For (E being less than 1) $2A + 1$ is more than $2A + E$; and therefore too big.

As for Instance; If the Non-quadrate proposed be $N = 5$, the greatest Integer Square therein contained is $Aq = 4$ (the Square of $A = 2$;) which being subtracted, leaves $N - Aq = 5 - 4 = 1 = B = 2AE + Eq$. Which divided by $2A = 4$ gives $\frac{1}{4}$; But divided by $2A + 1 = 4 + 1 = 5$, gives $\frac{1}{5}$. That too great, and this too little, for E. And therefore the true Root ($A + E = \sqrt{N}$) is less than $2\frac{1}{4} = 2.25$, but greater than $2\frac{1}{5} = 2.2$; And this was anciently thought an Approach near enough.

If this Approach be not now thought near enough, the same Process may be again repeated; and that as oft as is thought necessary.

Take now for A, $2\frac{1}{5} = 2.2$, whose Square is $4.84 = Aq$, (now considered as an Integer in the second place of Decimal Parts.) This subtracted from 5.00, (or, which is the same, 0.84, the Excess of this Square above the former, from 1, which was then the Remainder) leaves a new Remainder

$B = 0.16$: which divided by $2A = 4.4$, gives $\frac{0.16}{4.4} = \frac{2}{55} = 0.03636 +$,

too much. But divided by $2A + 1 = 4.5$, it gives $\frac{0.16}{4.5} = \frac{8}{225} =$

$0.03555 +$, too little. The true Value (between these two) being 2.236 proxime, whose Square is 4.99696.

If this be not thought near enough, subtract the Square from 5.000000: the Remainder $B = 0.000304$, divided by $2A = 4.472$, or by $2A + 1 = 4.473$, gives either way $0.000068 -$; which added to $A = 2.236$, makes $2.236068 -$, somewhat too big; but $2.236067 +$ would be much more too little.

Which gives us the Square Root of 5, adjusted to the sixth place of Decimal parts, at three Steps. And by the same Method, if it be thought needful, we may proceed further.

For the Cubick Root the Rule is this.

From the Non-Cubick proposed, (suppose N,) subtract the greatest Cube in Integers therein contained, (suppose A^3 ;) the Remainder suppose $B = 3A^2E + 3AE^2 + E^3$, is to be the Numerator of a Fraction for designing the Value of E, (the remaining part of the Root sought $A + E = \sqrt[3]{N}$.) To this Numerator, if (for the Denominator or Divisor) we subjoin $3A^2$, the Result will certainly be too great for E, because the Divisor is too little: (For it should be $3A^2 + 3AE + E^2$, to give the true Value of E. If for the Divisor we take $3A^2 + 3A + 1$, it will certainly be too little, because the Divisor is too great. (For E by construction is less than 1.) It must therefore (between these Limits) be more than this latter. And therefore this latter Result being added to A, will give a Root whose Cube may be subtracted from the Non-Cubick proposed, in order to another Step.

But if, for the Divisor, we take $3A^2 + 3A$, (or even less than so) the Result may be too great; or in case B be small) it may be too little, (and oft is so.)

**

O 2.

Which

Which comes to pass from hence; because E (by Construction) is less than 1 ; and therefore $3 A E$ less than $3 A$; and perhaps so much as that the Addition of $E q$ will not redress it. And when it so happens $3 A q + 3 A$ is a better Divisor than $3 A q + 3 A + 1$, (or even somewhat less than either.) But because it doth not always so happen (though for the most part it doth) the Rule doth rather direct the other; as which doth certainly give a Root less than the true Value, whose Cube may always be subtracted from the Non-Cubick proposed. The Design being to have such a Cube as (being subtracted) may leave another B , to be ordered in like manner for a new Approach.

But, for the most part, $3 A q$ may be safely taken for the Divisor. For tho' the Result will then be somewhat too big, yet the Excess may be so small, as to be neglected; or, at least, we may thence easily judge what Number (somewhat less than it) may be safely taken. And if we chance to take it somewhat too big, the Inconvenience will be but this, that B for the next Step will be a Negative. Of which Case we shall speak anon.

Thus for Instance; if the Non-Cube proposed be $9 = N$. The greatest Integer Cube therein contained is $8 = A^3$, (whose Cubick Root is $A = 2$.) Which Cube subtracted, leaves $9 - 8 = 1 = B = 3 A q E + 3 A E q + E^3$. This divided by $3 A q = 12$, gives $\frac{1}{12} = 0.08333 +$, too big for E . But the same divided by $3 A q + 3 A + 1 = 12 + 6 + 1 = 19$, gives $\frac{1}{19} = 0.05263 +$, too little. Or if but by $3 A q + 3 A = 12 + 6 = 18$, it gives $\frac{1}{18} = \frac{2}{36} = 0.05555 +$, yet too little, for the Cube of $A + 0.06$, $- 2.06$, is but $8.742 -$, which is short of 9 . And so much short of it, that we may safely make 2.07 as not too big: Or perhaps 2.08 , which upon tryal will be found not too big; for the Cube of 2.08 , is but 8.998912 .

If this first Step be not near enough, this Cube subtracted from 9.000000 , leaves a new $B = 0.001088$, which divided by $3 A q = 12.9796$, gives $0.000084 -$; which will be somewhat too big, but not much. (For E is now so small, as that $3 A E$ may be safely neglected, and $E q$ much more.) So that if to 2.08 , we add $0.000084 -$, the Result 2.080084 will be too big; but 2.080083 will be more too little. (As will appear if we take the Cube of each.) So that neither of them, at the second Step, gives the true Root within an Unite in the sixth place of Decimal parts. But when I say, taking the Cube of each, (which I do that the thing may be more clearly apprehended) it is not necessary that we trouble our selves with the whole Cube. For, A^3 being already subtracted, for finding $B = 3 A q E + 3 A E q + E^3$, we have no more to try, but whether $3 A q E + 3 A E q + E^3$, be greater or less than B , according as we take 0.000084 , or 0.000083 , for E .

Which may conveniently be done in this manner: Take $3 A + E$, and multiply this by E , (or E by it) so have we $3 A E + E q$. To this add $3 A q$, and multiply the whole by E , (so have we $3 A q E + 3 A E q + E^3$) too see whether this be greater or lesser than B .

That is, in the present case, if we take $E = 0.000084$, and add to this $3 A = 6.24$, then is $6.240084 = 3 A + E$. This multiplied by $E = 0.000084$, is $3 A E + E q = 0.000524 +$. To which if we add $3 A q = 12.9792$, it is $3 A q + 3 A E + E q = 12.979724$.
Which

Which multiplied again by $E = 0.000084$, is $0.0010902 + = 3 A q E + 3 A E q + E c$, which is more than $B = 0.001088$.

But if we take $E = 0.000083$, and proceed as before, we shall have $3 A q E + 3 A E q + E c = 0.001077 +$, which is less than $B = 0.001088$. And therefore (if we Subtract that from this) the Remainder 0.000011 , will be another B for the next Step, if we please to proceed further.

Hitherto I have pursued the Method most affected by the Ancients, in seeking a Square or Cube (and the like of other Powers) always less than the just Value, that it might be Subtracted from the Number proposed, leaving B a positive Remainder; thereby avoiding Negative Numbers.

But since the Arithmetick of Negatives is now so well understood, it may in this (and other Operations of like Nature) be adviseable, to take the nearest, whether it be greater or less than the just Value.

According to this Notion, for the Square Root of 5, I would say it is $(2 +)$ somewhat more than 2; and enquire how much more? But for the Square Root of 8, I would say, it is $(3 -)$ somewhat less than 3; and enquiring how much less? Taking (in both Cases) that which is nearest to the just Value.

Thus in the Cubick Root before us; I would take for E (in the last Enquiry $0.000084 -$ (where for the next Step we have $B = - 0.000002$), rather than $0.000083 +$ (where for the next Step we should have $B = + 0.000011$.) In the latter Case we are to divide $B = + 0.000011$, by $3 A q = 12.980236 -$, to find (by the Quotient) how much is to be added to 0.000083 . In the other Case we are to divide $B = - 0.000002$, by $3 A q = 12.980248$, to find (by the Quotient) what is to be abated of

0.000084 , so have we $\frac{0.000011}{12.980236} = 0.00000085 +$ to be added to

6.240083 : Or $\frac{0.000002}{12.980248} = 0.00000015 +$ to be abated of 6.240084 .

(Or it may suffice, in either to divide by $12.98 +$, or even by $13 -$, without being incumbered with a long Divisor) either of which gives us, for the Root sought, 2.08008385 proxime. True (at the third Step) to the eighth Place of Decimal Parts. And if this be not near enough, the Cube of this, compared with the Number proposed, will give us another B for the next Step. And so onwards as far as we please.

Now, what is said of the Cube, is easily applicable to the higher Powers.

I shall omit that of the Biquadrate; because here perhaps it may be thought most adviseable, to Extract the Square Root of the Number proposed; and then the Square Root of that Root. But if we would do it at once, we are from N (the Number proposed, being not a Biquadrate) to Subtract $A q q$ (the greatest Biquadrate contained in it) to find the Remainder $B = 4 A c E + 6 A q E q + 4 A E c + E q q$. Which Remainder, if we divide by $4 A c$, the Quotient will certainly be too big for E , (though perhaps not much:) If by $4 A E + 6 A q + 4 A + 1$, it will certainly be too

little.

little (for Reasons before mentioned.) And we are to use our discretion in taking some intermediate Number. And if we chance not to hit on the nearest, the Inconvenience will be but this, that our Leap will not be so great as otherwise it might be. Which will be rectify'd by another B at the next Step.

For the Surfolid (of five Dimensions) we are, from N (the Number proposed, being not a perfect Surfolid) to Subtract Aqc (the greatest Surfolid therein contain'd) to find the Remainder $B = 5AqqE + 10AcEq + 10AqEc + 5AEqq + Eqc$. Which (as before) if we divide by $5Aqq$, the Result will be somewhat too big (because the Divisor is too little:) If by $5Aqq + 10Ac + 10Aq + 5A + 1$, the Result will certainly be less than the true E. The just Value of E being somewhat between these two, where we are to use our Discretion, what Intermediate Number to take. Which, according as it proves too great or too little, is to be rectify'd at the next Step.

But for the most part it will be safe enough (and least Trouble) to divide by $5Aqq$, which gives a Quotient somewhat too big; which we may either rectify at Discretion, by taking a Number somewhat less, or proceed to another B, (Affirmative or Negative, as the Case shall require) and so onward to what Exactness we please. Which is for Substance, in a manner co-incidental with Mr. Raphson's Method, even for Affected Equations.

Thus, in the present Case; If the Number proposed be $N = 33$, then is $Aqc = 32$, and $B = 33 - 32 = 1 = 5AqqE + 10AcEq + 10AqEc + 5AEqq + Eqc$. Which if we divide by $5Aqq = 5 \times 16 = 80$, the Result $\frac{1}{80} = 0.0125$, is somewhat too big for E, but not much. And if we examine it, by taking the Surfolid of 2.0152 , or of $2\frac{1}{8}$, we shall find a Negative B (for the next Step,) but not very considerable. Or if we think it considerable, we may proceed further to another Step, or more than so.

The like Method may be apply'd (and with more Advantage) in the higher Powers, according as the Composition of each Power requires.

And the same Method may be of use (with good Advantage) in long Numbers (if duly applied) even before we come to the Place of Units; for the same will equally hold there also. Which the Reader may easily apprehend, without a long Discourse upon it.

*The Proportion
of Infinite Quan-
tities; by Mr. E.
Halley. N. 195.
p. 556.
Oct. An. 1692.*

XXV. The very Idea of Magnitudes Infinitely great, or such as exceed any assignable Quantity, does include a Negation of Limits: yet if we nearly examine this Notion, we shall find that such Magnitudes are not equal amongst themselves, but that there are really, besides Infinite Length, and Infinite Area, three several sorts of Infinite Solidity: all of which are *Quantitates sui generis*; and that those of each Species are in given Proportions.

Infinite Length, or a Line Infinitely long, is to be considered either as beginning at a Point, and so Infinitely extended one way, or else both ways from the same Point; in which Case the one, which is a beginning Infinity, in the one half of the whole, which is the Summ of the beginning and ceasing Infinity; or as I may say, of Infinity *à parte ante* and *à parte post*, which is analogous

analogous to Eternity in Time or Duration, in which there is always as much to follow as is past, for any Point or Moment of Time: Nor doth the Addition or Subduction of Finite Length or space of Time, alter the Case either in Infinity or Eternity, since both the one or the other cannot be any part of the whole.

As to Infinite Surface or Area, any Right Line, Infinitely extended both ways on an Infinite Plane, does divide that Infinite Plane into equal Parts, the one to the Right, and the other to the Left of the said Line; but if from any Point in such a Plane, two Right Lines be infinitely extended, so as to make an Angle, the Infinite Area, Intercepted between those Infinite Right Lines, is to the whole Infinite Plane, as the Arch of a Circle on the Point of Concourse of those Lines as a Center, intercepted between the said Lines, is to the Circumference of the Circle; or as the Degrees of the Angle to the 360 Degree of a Circle. For Example, two right Lines meeting at a right Angle do include, on an Infinite Plane, a quarter part of the whole Infinite Area of such a Plane.

But if so be two Parallel Infinite Lines be supposed drawn on such an Infinite Plane, the Area intercepted between them will be likewise Infinite; but at the same time will be Infinitely less than that Space, which is intercepted between two Infinite Lines that are inclined, tho' with never so small an Angle, for that in the one Case, the given finite distance of the Parallel Lines diminishes the Infinity in one Degree of Dimension; whereas in a Sector, there is Infinity in both Dimensions: and consequently the Quantities are the one Infinitely greater than the other, and there is no Proportion between them.

From the same Consideration arise these three several Species of Infinite Space or Solidity; for a Parallelipiped, or a Cylinder Infinitely long, is greater than any Finite Magnitude how great soever; and all such Solids suppos'd to be formed on given Bases, are as those Bases, in proportion to one another. But if two of these three Dimensions are wanting, as in the Space intercepted between two Parallel Planes Infinitely extended, and at a Finite Distance; or with Infinite Length and Breadth, with a Finite Thickness, all such Solids shall be as the given Finite Distances one to another; but these Quantities, tho' Infinitely greater than the other, are yet Infinitely less than any of those wherein all the three Dimensions are Infinite. Such are the Spaces intercepted between two inclined Planes Infinitely extended; the Space intercepted by the Surface of a Cone, or the sides of a Pyramid, likewise Infinitely continued, &c. of all which notwithstanding the Proportions one to another, and to the vast Abyss of Infinite Space (wherein is the *Locus* of all things that are or can be; or to the Solid of Infinite Length, Breadth and Thickness taken all manner of ways) are easily assignable. For the Space between two Planes is to the whole, as the Angle of those Planes to the 360 Degrees of the Circle. As for Cones and Pyramids, they are as the Spherical Surface intercepted by them, is to the Surface of the Sphere, and therefore Cones are as the verted Sines of half their Angles, to the Diameter of the Circle: These three sorts of Infinite Quantity are Analogous to a Line, Surface, and

and Solid; and after the same manner cannot be compared, or have no Proportion the one to the other.

Infinitely-Infinite Fractions;
by D. R. Wood.
Ph. Col. N. 3.
p. 4^c.
Dec. An. 1681.

XXVI. Infinitely-Infinite Fractions, or all the Powers of all the Fractions whose Numerator is 1, are all of them together equal to (1) an Unit.

| | | P. | | | | |
|---|--|----|----|-----|---------|--|
| A. | R. | q. | c. | qq. | | |
| $1 = \frac{1}{1}$ | $= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &c.$ | | | | Or thus | |
| $\frac{1}{2} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} &c.$ | | | | | | |
| $\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} &c.$ | | | | | | |
| $\frac{1}{4} = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} &c.$ | | | | | | |
| $\frac{1}{5} = \frac{1}{6} + \frac{1}{36} + \frac{1}{216} + \frac{1}{1296} &c.$ | | | | | | |
| $&c. &c.$ | | | | | | |

| | | + and | |
|--------------------------|--|-------|--|
| $\frac{1}{1 \times 1} =$ | $\frac{1}{2} + \frac{1}{1 \times 2} =$ | | |
| $\frac{1}{2 \times 1} =$ | $\frac{1}{3} + \frac{1}{2 \times 3} =$ | | |
| $\frac{1}{3 \times 1} =$ | $\frac{1}{4} + \frac{1}{3 \times 4} =$ | | |
| $\frac{1}{4 \times 1} =$ | $\frac{1}{5} + \frac{1}{4 \times 5} =$ | | |
| $\frac{1}{5 \times 1} =$ | $\frac{1}{6} + \frac{1}{5 \times 6} =$ | | |

| | | + and | | + and | |
|--|--|-----------------------|--|-------|--|
| $\frac{1}{2^2} + \frac{1}{1 \times 2^2} =$ | $\frac{1}{2^3} + \frac{1}{1 \times 2^3} =$ | $\frac{1}{2^4} + &c.$ | | | |
| $\frac{1}{3^2} + \frac{1}{2 \times 3^2} =$ | $\frac{1}{3^3} + \frac{1}{2 \times 3^3} =$ | $\frac{1}{3^4} + &c.$ | | | |
| $\frac{1}{4^2} + \frac{1}{3 \times 4^2} =$ | $\frac{1}{4^3} + \frac{1}{3 \times 4^3} =$ | $\frac{1}{4^4} + &c.$ | | | |
| $\frac{1}{5^2} + \frac{1}{4 \times 5^2} =$ | $\frac{1}{5^3} + \frac{1}{4 \times 5^3} =$ | $\frac{1}{5^4} + &c.$ | | | |
| $\frac{1}{6^2} + \frac{1}{5 \times 6^2} =$ | $\frac{1}{6^3} + \frac{1}{5 \times 6^3} =$ | $\frac{1}{6^4} + &c.$ | | | |

A. Is a File or Row of absolute Numbers, or rather of all the Fractions, whose Numerator is 1; which Row is supposed to be continued in *infinitum* (downwards).

R. Is

R. Is another File or Row of all the Roots (whose Numerator is 1) of all the Powers of such Fractions; supposed likewise to be continued *in infinitum* (downwards.)

P. Are all the respective Powers of such Fractions, (*viz.* Squares, Cubes, &c. (or so many Ranks of Geometrical Proportionals; supposed to be continued *in infinitum*; both to the Right-hand, and also downwards.

Lemma.] Each of the said Ranks of Powers, together with their respective Roots, is equal to each of the several Numbers under A respectively.

Demonstration.] If from the Line ab you take (for instance) $\frac{1}{2}$ part towards a , suppose ac ; and also from the other end of the same Line ab , you take two such Parts (or $\frac{1}{2}$ parts) towards b , suppose bd , (*viz.* a Number of Parts less by two than the whole Line ab , was first supposed to be divided into) there will remain the Line $cd = ac = \frac{1}{2}$ of ab . Then again, if from cd you take $\frac{1}{4}$ part thereof towards a , suppose ce , and from the other end $\frac{1}{2}$ parts of the same cd , suppose df , there will remain only $ef = ce = \frac{1}{4}$ of cd . And if you still go on without ceasing, to take on the Side towards a , $\frac{1}{4}$ part of what was taken last before, and twice as much on the other Side towards b , there shall be found between the two Lines last taken always remaining $\frac{1}{4}$ part of the Line from which they were taken. From which $\frac{1}{4}$ part there may still after the same manner be supposed to be taken two other such Lines. But if this be supposed to be done Infinite times actually, then there will nothing more remain (between), and so the continued Division on either Side will come exactly to the Point g , supposing ag to be $\frac{1}{3}$ of ab , and $bg = 2ag$. For, because that which was taken away towards b , was always twice as much as that which was taken away towards a , the total Summ of all the Lines taken away towards b , (which all together do make up the Line bg) must be twice as much as the Line ag , (which is the total Summ of all the Lines taken away towards a) *viz.* $bg = 2ag$; and consequently $bg + ag$ (or the whole Line ab) is equal to $3ag$: And therefore $ag = \frac{1}{3}$ of ab . Q. E. D.

Fig. 14.

The like Construction and Demonstration (*mutatis mutandis*) may be made use of in taking away any other part of any quantity, and the like part again of the first mentioned part, and so *in infinitum*. The total Sum of all the parts so taken, or supposed to be taken, shall be equal to any certain Quantity, or Part, or Fraction, whose Denominator shall be less by an Unite than the Denominator of the first

mentioned part; as $\frac{1}{6} = \frac{1}{7} + \frac{1}{49} + \frac{1}{343}$ &c. $\frac{1}{9} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$ &c. And so, that which the incomparable Archimedes (in his Squaring the Parabola) has only Demonstrated in one particular Case, *viz.* $\frac{1}{3} = \frac{1}{4} + \frac{1}{16}$

$+ \frac{1}{64} + \frac{1}{256} + \frac{1}{1024}$ &c. and that too, not without an huge Apparatus of Preliminary Propositions, amounting to an whole Book, is here universally De-

monstrated in all Cases (which are Infinite) and by a very simple and easy Method (in *Des Cartes's* way) and on one single Page.

Now if each of the said Ranks of Powers, together with their respective Roots, be equal to the several Numbers of Fractions under A; (as is demonstrated by the *Lemma*) then is A the Sum of them all, or equal to them all: that is to say,

$R + P = A = R + 1$: For R is the same with A wanting $\frac{1}{1}$ or 1, (as appears upon View) or but $(\frac{1}{\infty})$ one infinite part bigger. Wherefore $P = 1$,

Q. E. D. xix. Infinites Infiniti numeri Fracti aquantur Unitati, sc. minima Radici Integra.

Corollaria.] Hinc patet,

1°. Dari progressum in infinitum.

2°. Et non modo in unum Infinitum, sed etiam in plurima; seu potius infinita Infinita, vel infinites Infinita.

3°. Et hoc fieri posse, id est, hunc Calculum institui, ab ingenio valde finito aut exiguo.

4°. Et totum hunc Progressum, vel progressus hujusmodi infinitos, posse Numerari, sive aggregari in unam summam.

5°. Et in summam, non modo non infinitam; sed adeo tantillam, ut sit minor omni numero. Patet ulterius

Infinitorum alia esse aequalia, alia inaequalia.

Et unum Infinitum equari duobus, tribus, pluribus, vel Finitis vel Infinitis.

For 1. The Infinite Powers of the first Rank are $= \frac{1}{2} = \frac{1}{1 \times 2}$, and also equal to all the Infinitely-infinite Powers of all the other Ranks.

The Infinite Powers of the second Rank are $= \frac{1}{6} = \frac{1}{2 \times 3}$ } viz. equal to the
 Those of the third Rank are $= \frac{1}{12} = \frac{1}{3 \times 4}$ } respective mean
 Those of the fourth Rank are $= \frac{1}{20} = \frac{1}{4 \times 5}$ } proportional
 Those of the fifth Rank are $= \frac{1}{30} = \frac{1}{5 \times 6}$ } Number between
 the Square Num-
 bers respectively.

e.g.
 4, 6, 9
 9, 12, 16
 16, 20, 25
 25, 30, 36
 &c. in infinitum.

&c. in infinitum.

The

2. The Infinite Powers of the two first Ranks are $= \frac{2}{3}$

Those of the three first are $= \frac{3}{4}$

Those of the four first are $= \frac{4}{5}$

Those of the five first are $= \frac{5}{6}$

&c. in infinitum.

3. All the Powers of all the Infinite Ranks, except the first, are $= \frac{1}{2}$

all, except the two first, are $= \frac{1}{3}$

all, except the three first, are $= \frac{1}{4}$

all, except the four first, are $= \frac{1}{5}$

&c. in infinitum.

The later *Corollaries* may all appear by simple Addition and Subduction; and so may many more.

XXVII. 1. That the Numeral Figures now in use, with the manner of Computation by them (and the names of *Algorism*, appropriated to that way of Computation) came to us from the *Arabs* (but somewhat altered, as to the shape of the Figures in succeeding Ages) and to them from the *Indians*, is generally agreed. But it is not so generally agreed, of what Antiquity the Use of them, in our *European* Parts, hath been.

The Antiquity of the Numeral Figures; by Dr. J. Wallis. N. 144. p. 399. Dec. An. 1683.

Jo. Gerard Vossius (De Scientiis Mathematicis) thinks they came not in Use here, till about the Year of our Lord 1300; or at the farthest, later than the Year 1250. And *P. Mabillon (De Re Diplomatica)* tells us that he hath not found them any where used sooner than the 14 Century. But I think their Use in these Parts was as old at least as the times of *Hermannus Contractus*, who lived about the Year of our Lord 1050. (that is about the middle of the 11th Century,) If not so frequently in ordinary Affairs; yet at least in Mathematical things, and especially in Astronomical Tables.

But I do not remember, that I have any where seen any Monument of them more Antient than the Mantle-tree of the Parlour Chimney at the Dwelling-House of Mr. *Will. Richards*, the Rector of *Helmdon* in *Northampton-shire*.

Fig. 55.

The sides of the Chimney, by which the Mantle-tree is supported, are of Stone; but the Mantle-tree itself is of Wood. It is all over as black as Ink, having by Age and Smoak contracted that Colour. It may yet continue many hundreds of Years; for I did not discern in it, any thing either of Worm, or Rottenness, or any tendency to it. The length of it is 5 Foot 9 Inches: Its breadth or depth at the ends, (A B) is 11 $\frac{1}{2}$ Inches, but as

the middle, as CD, somewhat less. It is also carved from end to end: and the lower part of it is abated, as in the mouldings of other Chimneys. On the Front of the upper part there is (beginning at the middle) on 3 Squares parted from each other by a deep Furrow or Channel, the Date (I suppose, when it was first made) described partly in Numeral Figures A^o Doⁱ M^o 133; on a fourth a Flower, and on a 5th the two Letters W. R. with an Escutcheon, representing (I suppose) the Name of him to whom it did then belong. Both the Letters and the Figures are of an Antique Form, agreeing well enough with that Age. They are not engraved or cut in, but prominent on their several Squares (by way of *bas-relief*) the Wood being abated round about them. The o over the A, is a round o, but that over the M is a square o.

Hence it appears, that the Use of such Figures here in *England*, even on ordinary Occasions, is at least as ancient as the Year 1133. And I judge it to have been yet somewhat ancients, because the shape of the Figures, though not come just to the shape which we now use, was even then considerably varied from the shape of the *Arabick* Figures; which argues they had then been for some time in use; such change of shape in Figures and Letters coming on gradually with time.

It need not move any scruple at all, that part of the Numbers is expressed by the Numeral Letter M (or the word *Millesimo*, of which M^o is but a Contraction) while the rest is expressed in Numeral Figures. For the like doth oft occur in old *Manuscripts*; and sometimes even at this Day. And it doth rather favour the simplicity of that Age, (not very nice in such things, especially amongst Mechanicks) than any design of Imposture.

By Mr. Tho.
Luffkin. N. 255.
p. 2. 87. N. 266.
p. 677.
Aug. An. 1699.

Fig. 96.

2. Over-against the *Market-place* in *Colchester*, stands the House of Mr. *Furley*, a Linnen Draper; some of the backermost part of which is an ancient *Roman* Building, but the Front is of lesser standing, and timbred. Upon the bottom Cell (which is almost in the form of a Triangular Prism) of one of the Windows of the Front, between two carved Lions, stands an Escutcheon, containing only these Figures 1090. The Periphery of the Cyphers, and Nine, are rather Fracted than Flected, prominent, large, and very fair; but to make them the more perspicuous, they are gilded by the Proprietor. The Window looks directly North, the Date being thereby preserved from the scorching heat of the Sun; and by its Inclination (falling from the *Vertex* or *Perpendicular* by an Angle of about 60 Degrees) from Rain, Snow, &c. If it be objected, that the second and fourth Figures, may represent that among the *Arabians*, which is with us as 5; I answer, that the o is not used with all the *Arabs* for 5, but with some for a Cypher, and so it was used by the *Moors* in *Spain*, who first brought these Figures into our Parts; nor is the Square o an *Arabick* Letter, but an *English* Letter of that Age. And the form of these Figures soon degenerated from that of the *Arabs*, into such as we now use, if not at the first reception from the *Arabs*, [or *Moors*] certainly long before 1595, as this *Construction* would make it.

The Construction
of Logarithms.
by Mr. Edm.
Halley. N. 216.
p. 98.
Mar. An 1695.

XXVIII. The old Definition of *Logarithms*, that they are *Numerorum proportionalium equi-differentes compites*, is too scanty to define them fully: For they

they may much more properly be said to be *Numeri Rationum Exponentes*; where-
in *Ratio* is consider'd as a *Quantitas sui generis*, beginning from the Ratio of
equality, or $1 \text{ to } 1 = 0$; being Affirmative when the Ratio is increasing, as
of Unity to a greater Number, but Negative when decreasing; and these Rati-
ones we suppose to be measured by the Number of the Ratiunculæ contained in
each. Now these Ratiunculæ are to be so understood as in a continued Scale
of Proportionals, infinite in Number between the two Terms of the Ratio,
which infinite Number of mean Proportionals is to that infinite Number of the
like and equal Ratiunculæ between any other two Terms, as the Logarithm
of the one Ratio is to the Logarithm of the other. Thus if there be supposed
between 1 and 10 an infinite Scale of mean Proportionals, whose Number is
100000, &c. *in infinitum*; between 1 and 2 there shall be 30102, &c. of such
Proportionals, and between 1 and 3 there will be 47712, &c. of them,
which Numbers therefore are the Logarithms of the Rationes, of 1 to 10, 1
to 2, and 1 to 3; and not so properly to be called the Logarithms of 10, 2,
and 3.

This being laid down, it is obvious that if between Unity and any
Number proposed, there be taken any infinity of mean Proportionals, the in-
finitely little Augment or Decrement of the first of those Means from Unity,
will be a Ratiuncula, that is the Momentum or Fluxion of the Ratio of U-
nity to the said Number: And seeing that in these Continual Proportionals all
the Ratiunculæ are equal, their Sum, or the whole Ratio will be as the said
Momentum is directly; that is, the Logarithm of each Ratio will be as the
Fluxion thereof. Wherefore if the Root of any infinite Power be extracted
out of any Number, the Differentiola of the said Root from Unity, shall be
as the Logarithm of that Number. So that Logarithms thus produced, may
be of as many forms as you please to assume infinite Indices of the Power
whose Root you seek: as if the Index be supposed 100000, &c. infinitely,
the Roots shall be the Logarithms invented by the Lord *Napier*; but if the
said Index were 2302585, &c. Mr. *Brigg's* Logarithms would immediately
be produced. And if you please to stop at any Number of Figures, and not
to continue them on, it will suffice to assume an Index of a Figure or two
more than your intended Logarithm is to have, as Mr. *Briggs* did, who to
have his Logarithms true to 14 Places, by continual extraction of the Square
Root, at last came to have the Root of the 140737488355328th. Power;
but how operose that Extraction was, will be easily judged by who so shall
undertake to examine his *Calculus*.

Now, tho' the Notion of an infinite Power may seem very strange, and to those
that know the difficulty of the Extraction of the Roots of High Powers, perhaps
impracticable; yet by the help of that admirable Invention of Mr. *Newton*, where-
by he determines the Unciæ or Numbers prefixt to the Members composing
Powers, (on which chiefly depends the *Doctrine of Series*) the Infinity of the In-
dex contributes to render the Expression much more easy: For if the Infinite

Power to be resolved be put (after Mr. *Newton's* Method) $p + p \cdot q$

$$\frac{p + p \cdot q}{p + p \cdot q}$$

$$\overline{p + pq^m}, \text{ or } \overline{1 + q^m}, \text{ instead of } 1 + \frac{1}{m}q + \frac{1-m}{2mm}qq - \frac{1}{6m^3}q^3 + \frac{1-6m+11mm-6m^2}{24m^4}q^4, \&c.$$

(which is the Root when m is finite,) becomes $1 + \frac{1}{m}q - \frac{1}{2m}qq +$

$$\frac{1}{3m}q^3 + \frac{1}{4m^2}q^4 + \frac{1}{5m}q^5, \&c. \text{ } mm \text{ being infinite, and con-}$$

sequently whatever is divided thereby vanishing. Hence it follows that $\frac{1}{m}$

multiplied into $q - \frac{1}{2}qq + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5, \&c.$ is the Augment of the first of our mean Proportionals between Unity and $1 + q$, and is therefore the Logarithm of the Ratio of 1 to $1 + q$; and whereas the Infinite Index m may be taken at pleasure, the several Scales of Logarithms to

such Indices will be as $\frac{1}{m}$, or reciprocally as the Indices. And if the Index

be taken 10000, &c. as in the case of *Napier's* Logarithms, they will be simply $q - \frac{1}{2}qq + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5 - \frac{1}{6}q^6, \&c.$

Again, If the Logarithm of a decreasing Ratio be sought, the Infinite Root

$$\text{of } 1 - q, \text{ or } \overline{1 - q^m} \text{ is } 1 - \frac{1}{m}q - \frac{1}{2m}q^2 - \frac{1}{3m}q^3 - \frac{1}{4m}q^4 -$$

$$\frac{1}{5m}q^5 - \frac{1}{6m}q^6, \&c. \text{ whence the Decrement of the first of our Infinite Number}$$

of Proportionals will be $\frac{1}{m}$ into $q + \frac{1}{2}qq + \frac{1}{3}q^3 + \frac{1}{4}q^4 + \frac{1}{5}q^5 + \frac{1}{6}q^6,$

&c. which therefore will be as the Logarithm of the Ratio of Unity to $1 - q$.

But if m be put 10000, &c. then the said Logarithm will be $q + \frac{1}{2}qq + \frac{1}{3}q^3 + \frac{1}{4}q^4 + \frac{1}{5}q^5 + \frac{1}{6}q^6, \&c.$

Hence the terms of any Ratio being a and b , q becomes $\frac{b-a}{a}$, or the dif-

ference divided by the lesser term, when 'tis an Increasing Ratio, or $\frac{b-a}{b}$

when 'tis Decreasing, or as b to a . Whence the Logarithm of the same Ratio may be doubly exprest, for putting x for the difference of the terms a and b , it will be either

(III)

$$\frac{1}{m} \text{ into } \frac{x}{b} + \frac{x^2}{2bb} + \frac{x^3}{3b^2} + \frac{x^4}{4b^3} + \frac{x^5}{5b^4} + \frac{x^6}{6b^5} \&c. \text{ or}$$

$$\frac{1}{m} \text{ into } \frac{x}{a} - \frac{x^2}{2aa} + \frac{x^3}{3b^2} - \frac{x^4}{4a^3} + \frac{x^5}{5a^4} - \frac{x^6}{6a^5} \&c.$$

But if the Ratio of a to b be supposed to be divided into two parts, *viz.* into the Ratio of a to the *Arithmetical Mean* between the Terms, and the Ratio of the said *Arithmetical Mean* to the other Term b , then will the Sum of the Logarithms of those two Rationes be the Logarithm of the Ratio of a to b ; and substituting $\frac{1}{2}z$ instead of $\frac{1}{2}a + \frac{1}{2}b$ the said *Arithmetical Mean*, the Logarithms of those Rationes will be by the foregoing Rule

$$\frac{1}{m} \text{ into } \frac{x}{z} + \frac{xx}{2zz} + \frac{x^3}{3z^2} + \frac{x^4}{4z^3} + \frac{x^5}{5z^4} + \frac{x^6}{6z^5} \&c. \text{ and}$$

$$\frac{1}{m} \text{ into } \frac{x}{z} - \frac{xx}{2zz} + \frac{x^3}{3z^2} - \frac{x^4}{4z^3} + \frac{x^5}{5z^4} - \frac{x^6}{6z^5} \&c.$$

the Sum whereof $\frac{1}{m} \text{ into } \frac{2x}{z} * + \frac{2x^3}{3z^2} * + \frac{2x^5}{5z^4} * + \frac{2x^7}{7z^6} \&c.$ will be

the Logarithm of the Ratio of a to b , whose difference is x , and sum z . And this Series converges twice as swift as the former, and therefore is more proper for the Practice of making Logarithms: which it performs with that expedition, that where x the difference is but the hundredth part of the Sum, the first Step $\frac{2x}{z}$ suffices to seven Places of the Logarithm, and the second Step to twelve;

but if *Briggs's* first Twenty *Chiliads* of Logarithms be supposed made, as he very carefully computed them, to fourteen Places, the first Step alone is capable to give the Logarithm of any intermediate Number, true to all the Places of those Tables.

After the same manner, may the Difference of the said two Logarithms be very aptly applied to find the Logarithms of Prime Numbers, having the Logarithms of the two next Numbers above and below them: For the difference of the Ratio of a to $\frac{1}{2}z$, and of $\frac{1}{2}z$ to b , is the Ratio of ab to $\frac{1}{4}zz$, and the half of that Ratio is that of $\sqrt{ab}$ to $\frac{1}{2}z$, or of the Geometrical Mean to the Arithmetical. And consequently the Logarithm thereof will be the half difference of the Logarithms of those Rationes, *viz.*

$$\frac{1}{m} \text{ into } \frac{xx}{2zz} + \frac{x^4}{4z^4} + \frac{x^6}{6z^6} + \frac{x^8}{8z^8} \&c.$$

Which is a *Theorem* of good Dispatch to find the Logarithm of $\frac{1}{2}z$. But the same is yet much more advantageously performed by a Rule derived from the foregoing, and beyond which in my Opinion nothing better can be hoped. For the Ratio of ab to $\frac{1}{4}zz$, or $\frac{1}{4}aa + \frac{1}{2}ab + \frac{1}{4}bb$, has the difference of its Terms, $\frac{1}{4}aa - \frac{1}{4}ab + \frac{1}{4}bb$, or the Square of $\frac{1}{4}a - \frac{1}{4}b = \frac{1}{4}xx$, which in the present case of finding the Logarithms of Prime Numbers is always

ways Unity, and calling the Sum of the Terms $\frac{1}{2} z z + a b = y y$, the Logarithm of the Ratio of $\sqrt{a b}$ to $\frac{1}{2} a + \frac{1}{2} b$, or $\frac{1}{2} z$ will be found

$$\frac{1}{m} \text{ into } \frac{1}{y y} + \frac{1}{3 y^3} + \frac{1}{5 y^5} + \frac{1}{7 y^7} + \frac{1}{9 y^9}, \&c.$$

which converges very much faster than any *Theorem* hitherto published for this purpose.

Here note that $\frac{1}{m}$ is all along applied to adapt these Rules to all sorts of

Logarithms. If m be 10000, &c. it may be neglected, and you will have *Napier's* Logarithms, as was hinted before; but if you desire *Brigg's* Logarithms, which are now generally received, you must divide your Series by 2, 30258-5092994045684017991454684364207610101488628772976033328, or multiply it by the reciprocal thereof, *viz.* 0, 434294481903251827651-128918916605082294397005803666566114454.

But to save so operose a Multiplication (which is more than all the rest of the Work) it is expedient to divide this Multiplier by the Powers of z or y continually, according to the Direction of the *Theorem*, especially where x is small and Integer, reserving the proper Quotes to be added together, when you have produced your Logarithm to as many Figures as you desire; of which Method I will give a Specimen, in the Logarithms of the first Prime Numbers under 20 to sixty places, computed by Mr. *Abraham Sharp*, as they were communicated to me by our common Friend, Mr. *Euclid Speidal*.

Num.

Logarithms,

| | | |
|-----|----|--|
| 2. | 0, | 301029995663981195213738894724493026768189881462108541310427 |
| 3. | 0, | 477121254719662437295027903255115309200128864190695864829866 |
| 7. | 0, | 845098040014256830712216258592636193483572396323965406503835 |
| 11. | 1, | 041392685158225040750199971243024241706702190466453094596539 |
| 13. | 1, | 113943352306837769206541895026246254561189005053673288598083 |
| 17. | 1, | 230448921378273028540169894328337030007567378425046397380368 |
| 19. | 1, | 278753600952828961536333475756929317951129337394497598906819 |

The next Prime Number is 23, which I will take for an Example of the foregoing Doctrine; and by the first Rules, the Logarithm of the Ratio of 22 to 23 will be found to be either.

$$\frac{1}{22} - \frac{1}{968} + \frac{1}{31944} - \frac{1}{937024} + \frac{1}{25768160}, \&c. \text{ or}$$

$$\frac{1}{23} + \frac{1}{1058} + \frac{1}{36501} + \frac{1}{1119364} + \frac{1}{32181715}, \&c.$$

As likewise that of the Ratio of 23 to 24 by a like Process.

$$\frac{1}{23} - \frac{1}{1058} + \frac{1}{36501} - \frac{1}{1119364} + \frac{1}{32181715}, \text{ \&c. or}$$

$$\frac{1}{24} + \frac{1}{1152} + \frac{1}{41471} + \frac{1}{1327104} + \frac{1}{39813120}, \text{ \&c.}$$

And this is the Result of the Doctrine of *Mercator*, as improv'd by the Learn-

ed Dr. Wallis. But by the second Theorem, viz. $\frac{2x}{2} + \frac{2x^3}{3x^3} + \frac{2x^5}{5x^5}$, &c.

the same Logarithms are obtained by fewer steps. To wit,

$$\frac{2}{45} + \frac{2}{273375} + \frac{2}{922640625} + \frac{2}{2615686171875}, \text{ \&c. And}$$

$$\frac{2}{47} + \frac{2}{311469} + \frac{2}{1146725035} + \frac{2}{3546361843241}, \&c.$$

which was Invented and Demonstrated in the Hyperbolic Spaces Analogous to the Logarithms, by the Excellent Mr. *James Gregory*, in his *Exercitationes Geometricae*, and since further prosecuted by the aforesaid Mr. *Speidall*, in a late Treatise in *English* by him Publish'd on this Subject. But the Demonstration, as I conceive, was never till now Perfected without the consideration of the Hyperbola, which in a matter purely Arithmetical as this is, cannot so properly be applied. But what follows, I think, I may more justly claim as my own, viz. That the Logarithm of the Ratio of the Geometrical Mean to the Arithmetical, between 22 and 24, or of $\sqrt{528}$ to 23, will be found to be either,

$$\frac{1}{1058} + \frac{11}{1119364} + \frac{11}{888215334} + \frac{11}{616487882248}, \text{ \&c. or}$$

$$\frac{1}{1057} + \frac{1}{3542796579} + \frac{1}{659676558485285}, \&c.$$

All these Series being to be Multiplied into 0,4342944819, &c. if you design to make the *Logarithm* of *Briggs*. But with great Advantage in respect of the Work, the said 0,4342944819, &c. is divided by 1057, and the Quotient thereof again divided by three times the Square of 1057, and that Quotient again by $\frac{1}{2}$ of that Square, and that Quotient by $\frac{1}{3}$ thereof, and so forth, till you have as many Figures of your Logarithm as you desire. As for Example, the Logarithm of the Geometrical Mean between 22 and 24 is found by the Logarithms of 2, 3, and 11, to be

| | |
|---------------|---|
| | 1,36131656126690612945009172669805 |
| 1057) | 43429 &c. (-----41087462810146814347315886368 |
| 3 in 1117249) | 41087 &c. (-----12258521544181829460074 |
| 3 in 1117249) | 12258 &c. (-----6583235184376175 |
| 3 in 1117249) | 65832 &c. (-----4208829765 |
| 3 in 1117249) | 42088 &c. (-----2930 |
| Summa | 1,3617278360175928788677711225117 |

Which is the Logarithm of 23 to Thirty two Places, and obtained by 5 Divisions with very small Divisors, all which is much less Work than simply Multiplying the Series into the said Multiplier 0,43429, &c.

From the Logarithm given to find what Ratio it expresses, is a *Problem* that has not been so much considered as the former, but which is solved with the like Ease, and demonstrated by a like Process, from the same general *Theorem* of Mr. *Newton*. For as the Logarithm of the Ratio of 1 to $1 + q$ was pro-

ved to be $1 + q \left| \frac{1}{m} \right| - 1$, and that of the Ratio of 1 to $1 - q$ to be $1 -$
 $1 - q \left| \frac{1}{m} \right|$: so the Logarithm, which we will from henceforth call L , being

given, $1 + L$ will be equal to $1 + q \left| \frac{1}{m} \right|$ in the one Case; and $1 - L$ will
 be equal to $1 - q \left| \frac{1}{m} \right|$ in the other: Consequently $1 + L \left| \frac{1}{m} \right|$ will be equal to
 $1 + q$, and $1 - L \left| \frac{1}{m} \right|$ to $1 - q$; that is, according to Mr. *Newton's* said Rule,

$1 + mL + \frac{1}{2} m^2 L^2 + \frac{1}{6} m^3 L^3 + \frac{1}{24} m^4 L^4 + \frac{1}{120} m^5 L^5, \&c.$ will
 be = to $1 + q$, and $1 - mL + \frac{1}{2} m^2 L^2 - \frac{1}{6} m^3 L^3 + \frac{1}{24} m^4 L^4 -$

$\frac{1}{120} m^5 L^5, \&c.$ will be equal to $1 - q$, m being any infinite Index what-
 soever; which is a full and general *Proposition* from the Logarithm given to
 find the Number, be the Species of Logarithms what it will. But if *Napier's*
Logarithm be given, the Multiplication by m is saved, (which Multipli-
 cation is indeed no other than the reducing the other Species to his), and the

Series will be more simple, *viz.* $1 + L + \frac{1}{2} L^2 + \frac{1}{6} L^3 + \frac{1}{24} L^4$

$+ \frac{1}{120} L^5, \&c.$ or $1 - L + \frac{1}{2} L^2 - \frac{1}{6} L^3 + \frac{1}{24} L^4 - \frac{1}{120} L^5, \&c.$

This Series, especially in great Numbers, Converges so slowly, that is were
 to be wished it could be contracted.

If one Term of the Ratio, whereof L is the Logarithm, be given, the o-
 ther Term will be had easily by the same Rule: For if L were *Napier's* *Loga-*
rithm

rithm of the Ratio of a the lesser, to b the greater Term, b would be the Product of a into $1 + L + \frac{1}{2} L^2 + \frac{1}{6} L^3$, &c. $= a + aL + \frac{1}{2} aL^2$

$+ \frac{1}{6} aL^3$, &c. But if b were given, a would be $= b - bL + \frac{1}{2} bL^2$

$- \frac{1}{6} bL^3$, &c. Whence by the help of the *Chiliads*, the Number apper-

taining to any Logarithm will be exactly had to the utmost Extent of the Tables. If you seek the nearest next Logarithm, whether greater or lesser, and call its Number a if lesser, or b if greater, then the given L , and the difference thereof from the said nearest Logarithm you call l ; it will follow that the number answering to the Logarithm L will be either a into $1 + l$

$+ \frac{1}{2} l^2 + \frac{1}{6} l^3 + \frac{1}{24} l^4 + \frac{1}{120} l^5$, &c. or else b into $1 - l + \frac{1}{2} l^2$

$- \frac{1}{6} l^3 + \frac{1}{24} l^4 - \frac{1}{120} l^5$, &c. wherein as l is less the Series will converge

the swifter. And if the first 20000 Logarithms be given to 14 places, there is rarely occasion for the three first Steps of this Series to find the Number to as many places. But for *Vlacq's* great *Canon* of 100000 Logarithms, which is made but to ten places, there is scarce ever need for more than the first step $a + al$, or $a + mal$ in one Case, or else $b - bl$, or $b - mbl$ in the other, to have the Number true to as many Figures as those Logarithms consist of.

There is another Series which is not indeed so simple and uniform, yet the first step thereof is most commodious for Practice, and exact enough for Tables

not exceeding 14 Places. It is thus: $a + \frac{al}{1 - \frac{1}{m}l}$, or $b - \frac{bl}{1 + \frac{1}{m}l}$ will be the

Number answering to the Logarithm given, differing from the Truth but by one half of the third step from the former Series. But that which renders it yet more eligible is, that with equal facility it serves for *Briggs's* or any other sort of

Logarithms, with the only variation of Writing $\frac{1}{m}$ instead of 1, that is

$a + \frac{al}{\frac{1}{m} - \frac{1}{m}l}$, and $b - \frac{bl}{\frac{1}{m} + \frac{1}{m}l}$, or $\frac{\frac{1}{m}a + \frac{1}{m}la}{\frac{1}{m} - \frac{1}{m}l}$ and $\frac{\frac{1}{m}b - \frac{1}{m}lb}{\frac{1}{m} + \frac{1}{m}l}$, which are

easily resolved into Analogies, viz.

As 43429 &c. $-\frac{1}{m}l$ to 43429 $+\frac{1}{m}l$: so is a : } to the Number sought.
Or As 43429 &c. $+\frac{1}{m}l$ to 43429 $-\frac{1}{m}l$: so is b : }

If more Steps of this Series be desired, it will be found as follows.

$$a + \frac{al}{1 - \frac{1}{2}l} - \frac{\frac{1}{2}al^2}{1 - l} + \frac{\frac{1}{2}al^3}{1 - 2l}, &c. \text{ as may easily be demonstrated by}$$

working out the Divisions in each Step, and collecting the Quotes, whose Sum will be found to agree with our former Series, which is no other than an easy Corollary to Mr. Newton's general Theorem for forming Roots and Powers.

XXIX. Papers of less General Use Omitted.

Tangents to
Curves, N. 81.
p. 4010.
Mar. An. 1672.

Rectification of
Curves, N. 98.
p. 6145, 6149.
Nov. An. 1673.

Ib. p. 6150.

Transformation
of Curves, N. 214.
p. 233.
Nov. An. 1694.

Cycloidal Spaces
perfectly Quadra-
ble N. 217. p. 111.
Oct. An. 1695.

The Cycloid con-
sider'd long ago.
N. 229. p. 561.
June An. 1697.

1. A Breviat of Dr. Wallis's two Methods of drawing Tangents; Extracted by him from his *Con. Sect.* and other Parts of his *Mathematical Works*.

2. M. Huygens in his *Hor. Oscill.* having given M. Huract the Honour of Inventing a Curve equal to a Straight Line in the Year 1659; Dr. Wallis here asserts this Invention to Mr. William Neile (Son of Sir Paul Neile), who discovered and demonstrated the Equality of a *Paraboloid* to a Straight Line two Years before. The same was soon after otherwise Demonstrated by my Lord Brouncker, and Sir Christopher Wren, in June and July 1657; and the Demonstrations inserted by Dr. Wallis, in his Tract *de Cycloide* 1659, with a fair Relation of that whole Matter. Besides, Sir Christopher Wren found a Straight Line equal to that of a *Cycloide* in the Year 1658: Yet he freely confesses Mr. Neile's Invention of a Curve capable of *Rectification* the Year before.

3. The Abbot Galloys having, in the Year 1693, asserted that Mr. James Gregory and Dr. Barrow stole their General Propositions concerning the *Transformation of Curves* from Mr. Roberval; Dr. David Gregory here fully refutes that Assertion. For Mr. Gregory Publish'd his Book at Padua 1668, and Dr. Barrow his *Lectiones Geometricae* 1674, which Mr. Roberval doubtless had a Sight of before he dy'd (which was not till October 1675), yet he never complain'd of any such Injury done him.

4. Besides that Segment of the *Semicycloidal Figure*, first observed by Sir Christopher Wren, and after him by Mr. Huygens, and a Trilinear Part of it, which are capable of being Geometrically Squared, Dr. Wallis here produces from his Tracts *de Cycloide*, and *de Motu*, some other Portions thereof equally capable of Quadrature.

5. Dr. Wallis finds among the Mathematical Works of Bovillus, Publish'd at several times between the Years 1501 and 1510, that the Curve (which is now call'd the *Cycloid*, was then consider'd. But he also finds that Bovillus was not the first who consider'd it: For Cardinal Cusanus, as appears by an Ancient Manuscript of his Works (transcrib'd by J. Scoblant in the Year 1451) has consider'd it some time before. The Figure indeed (thro' the Unskilfulness of the Transcriber) both in the MS. and the *Basl* Edition, A. 1565, is very ill drawn; but being Corrected according to the true Meaning of that Cardinal's own Words, it evidently represents the Modern *Cycloid*. From hence 'tis manifest, that this Curve was not first taken into Consideration

either

either by *Merfennius* or *Galileo*, but some Ages before, tho' never well understood till this present Age.

6. Some Papers sent by Mr. *Jo. Collins* to Dr. *Wallis*, giving his Thoughts about some Defects in Algebra, which he did not live to finish.

Defects in Algebra, N. 159. p. 575.
May An. 1682

XXX. Accounts of Books with Editions, Emendations, &c. Omitted.

1. **E**Uclidis *Elementa Geometrica, novo ordine ac Methodo demonstrata*. Lond. N. 15. p. 261. 1666.
2. *Archimedis Opera; Apollonii Perg: Conic. Libri 4; Theodosii Spherica*; N. 114. p. 314. *Methodo nova illustrata, & succincte demonstrata: ab Isa. Barrow, R. S. S. Lond.* 1675. in 4to.
3. *Ἀρχιμήδης τῷ Συρακυσίῳ Ψαμμίτης, ἡ Κύβη Μέρησις: Εὐτοκίῃ Ἀσχυλόνιτῳ* N. 123. p. 567. *εἰς αὐτῇ ὑπόμνημα, &c. Cum Versione & Notis Jo. Wallis, S. S. Th. D. Oxon.* 1676.
4. *Theon Smyrnaeus*, published at Paris by *Ismael Bulialdus* in Greek and Latin. N. 80. p. 3095.
5. *Diophanti Alexandrini Arithmeticonum Libri 6, & de Numeris Multangulis Liber unus; cum Commentariis C. G. Bacheti, & Observationibus, D. P. de Fermat, Senatoris Tholosani: cui accessit Doctrina Analytica Inventum Novum.* Tolosæ 1670. in Folio. N. 72. p. 2185.
6. The Works of Monsieur de Fermat. N. 1. p. 15.
7. *Francisci du Laurens Specimina Mathematica, duobus Libris comprehensa. Horum prior Synthetice agit de Genuinis Matheseos Principiis in genere; in specie autem de veris Geometrie Elementis hucusque nondum traditis. Posterior Analyticus de Methodo Compositionis atque Resolutionis fuse disserit, & multa nova complectitur, quæ subtilissimam Analyseos Artem mirum in modum promouent.* This Book is here Censured, some Mistakes in it Corrected, and the Censure vindicated, by Dr. *Wallis*. N. 30. p. 580. N. 34. p. 654. N. 38. p. 744. N. 39. p. 775. N. 41. p. 825.
8. *R. P. Andreae Taquet, è S. J. Opera Mathematica.* Antwerp. 1669. in Folio. N. 43. p. 869.
9. A Mathematical Compendium, Collected out of the Notes and Papers of Sir *Jonas Moore*, by *Nicholas Stevenson*. Lond. 1674. in 12mo. N. 104. p. 83.
10. *R. P. Claudii Franc. Milliet de Chales, è S. J. Cursus seu Mundus Mathematicus, universam Mathesin tribus Tomis complectens.* Lugd. 1674. in Folio. N. 110. p. 229.
11. The Mathematical Works of Dr. *Jo. Wallis*, Savilian Professor of Geometry in the University of Oxford, & F. R. S. in three Volumes in Folio. Oxon. N. 216. p. 73. N. 254. p. 259.
12. An Introduction to Algebra, Translated out of *High Dutch* into English by *Tho. Branker*, M. A. much altered and augmented by Dr. *J. Pell*. Also a Table of such odd Numbers, as are less than one Hundred Thousand, showing those that are *Incomposui*, and resolving the rest into their Factors, or Coefficients. Lond. in 4to. N. 35. p. 688.
13. *Labyrinthus Algebrae. Auth. Joh. Jac. Ferguson.* 1667. in 4to. N. 49. p. 935.

- N. 90. p. 5142. 14. The Elements of that Mathematical Science call'd *Algebra*; by *Jo. Kersey*. Lond. 1673. in *Folio*.
- N. 95. p. 6073. 1. A Treatise of *Algebra*, both Historical and Practical; by *Jo. Wallis*.
N. 108. p. 192. D. D. In the 109th Chapter there are some Numbers mistaken, which are
N. 143. p. 21. here rectify'd by the Authot.
N. 17. p. 1295.
N. 233. p. 730.
- N. 14. p. 253. 16. *De Principiis & Ratiocinatione Geometrarum; contra Fastum Professorum*
N. 16. p. 239. *Geometria*. Authore Tho. Hobbes. This Book is here Animadverted on, and
Answered; by Dr. Wallis.
- N. 48. p. 971. 17. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphære, Duplicatio Cubi,*
confutata. Auth. Jo. Wallis. S. T. D. Oxon. 1669. in *Quarto*.
- N. 55. p. 1121. 18. Thomæ Hobbes *Quadratura Circuli, Cubatio Sphære, Duplicatio Cubi,*
(secundo edita) denuo refutata Auth. Jo. Wallis. S. T. D. Oxon. 1669.
- N. 72. p. 2185. 19. *Rosetum Geometricum, cum Censura brevis Doctrinæ Wallisianæ de Motu.*
Auth. Tho. Hobbes *Malmesburiensis*. Lond. 1671. in *Quarto*. This Book is
N. 73. p. 2202. here answered by Dr. Wallis.
- N. 75. p. 2241. 20. Four Papers of Mr. Hobbs's, Published in the Months of *August* and
September 1671. which are here answered by - - - -
- N. 86. p. 5047. 21. *Lux Mathematica, Collisionibus Johannis Wallisii S. T. D. & Thomæ*
Hobbesii Malmesburiensis, excussa, multis & fulgentissimis aucta radiis. Auth.
R. R. *Adjuncta Censura Doctrinæ Wallisianæ de Libris, una cum Roseto Hobbesii.*
N. 87. p. 5067. Lond. 1672, in *Quarto*. This Book is here answered by Dr. Wallis.
- N. 97. p. 6131. 22. *Principia & Problemata aliquot Geometrica, ante desperata, nunc breviter*
explicata & demonstrata. Auth. T. H. *Malmesburiensis*. Lond. 1673. in
Quarto.
- N. 185. p. 245. 23. *Le Grand & Fameux Probleme de la Quadrature du Cercle resolu Geome-*
triquement par le Cercle & la Ligne droit, par M. Mallement de Messange.
à Paris. 1686. in *Twelves*. This Book is here Refuted by M. D. *Cluverius*.
R. S. S.
- N. 32. p. 625. 24. *Nouveaux Elemens de Geometrie: Or a Mathematical Treatise, entituled*
New Elements of Geometry. Paris. 1667. in *Quarto*.
- N. 79. p. 3064 25. *Elemens de Geometrie; par le P. Ignace Gaston Pardies, de la Comp. de J.*
à Paris 1671. in *Twelves*.
- N. 33. p. 642. 26. 1. *Vera Circuli & Hyperbole Quadratura, in propria sua Proportionis*
Specie inventa & Demonstrata, a Jac. Gregorio Scoto. Patavii. in *Quarto*. This
Subject is here further considered, and the Area of an Hyperbole explain'd;
1b. p. 641. by Mr. J. Collins.
- N. 37. p. 732. 2. M. Huygens having Publish'd Animadversions upon this Book, in the
Journal de Scauans 1668. Mr. Gregory here answers them. To this M. Huygens
N. 44. p. 882. Reply'd in a following Journal of that Year; and Mr. Gregory, further to elu-
cidate the Controversy, here returns a second Answer.
- N. 216. p. 65. 3. In the 48th Page of this Book, Mr. Halley has Discovered and Corrected
a small Mistake in the Logarithm of 10.
- N. 35. p. 685. 27. *Geometria pars Universalis, Quantitatum Curvarum Transmutationi & Men-*
sura inserviens. Auth. Jac. Gregorio Scoto. Patavii 1668. in *Quarto*.
- N. 37. p. 373. 28. *De Infinitis Spiralibus inversis, Infinitisque Hyperbolis, aliisque Geometricis.*
Auth. F. Stephano de Angelis *Veneto*. Patavii. in *Quarto*.

29. *Michaelis Angeli Ricci Exercitatio Geometrica*. Romæ. in 4to. Reprinted N. 37. p. 738. at London, and annex to *Mercator's Logarithmotechnia*.
30. Renati Franc. Slusii *Mesolabium*. Cui accessit pars altera de *Analysi*, & N. 44. p. 903. *Miscellanea*. Leodii Eburonum 1668. in 4to.
31. *Elementa Geometria Plana*. Authore Egidio Francisco de Gottignies N. 67. p. 2054. *Bruxellensi*. S. J. Romæ. 1669. in 12mo.
32. *Synopsis Geometrica; cum tribus Opusculis, de Linea Sinuatum & Cycloide; de Maximis & Minimis, Centuria; & Synopsis Geometria Plana*. Auth. Honor. Fabry. S. J. Lugduni Galliarum 1669. in 12mo.
33. *Lectiones 13. Geometrica; in quibus (praesertim) Generalia Linearum Curvarum Symptomata declarantur*, ab Isaaco Barrow. Lond. 1669. in 4to. To these Lectures the Author here adds several *Corollaries* and *Theorems*.
34. Erasmi Bartholini *Selecta Geometrica*. Hauniae. 1674. in 4to. N. 105. p. 137.
35. *Elements des Mathematiques, ou Principes Generaux de toutes les Sciences qui ont les Grandeurs pour Object*. Par J. P. à Paris 1675. in 4to. N. 126. p. 638.
36. *Nouvelle Methode en Geometrie pour les Sections des Surfaces Coniques & Cylindriques; qui ont pour Base des Cercles, ou des Paraboles, des Ellipses, & des Hyperboliques; par Ph. de la Hire*. à Paris 1673. in 4to. N. 129. p. 744.
37. *De Cycloide & Sectionibus Conicis*. Ph. de la Hire. *Ibid.* p. 746.
38. *The Geometrical Key, or Construction of all Equations Linear, Quadratick, Cubick, and Biquadratick, by a Circle and one only Parabola; by Mr. Tho. Baker*. N. 147. p. 449.
39. *Exercitatio Geometrica de Dimensione Figurarum*. Auth. Davide Gregorio. N. 163. p. 710. Edinb. 1684. in 4to.
40. *Methodus Figurarum Lineis Rectis & Curvis comprehensarum Quadraturas determinandi*, Auth. J. Craige. Lond. 1685. in 4to. To this Tract the Author here makes an Addition; and takes notice of some Remarks made on it in the *Act. Lips.* by M. Leibnitz, and M. J. Bernoulli. *Ibid.* p. 186. N. 235. p. 786.
41. *Tractatus Mathematicus de Figurarum Curvilinearum Quadraturis & Locis Geometricis*. Auth. J. Craig. Lond. 1693. in 4to. N. 209. p. 113.
42. *Tractatus de Principiis Calculi Exponentialis*. Auth. D. Bernoullio; where in a Mistake is here Discovered and Corrected, by Mr. Craig. N. 245. p. 374.
43. *Analys. Geometrica, sive nova & vera Methodus Resolvendi, tam Problemata Geometrica, quam Arithmeticas Quaestiones*. Pars prima, de Planis. Auth. D. Antonio Hugone de Omerique Saubacarense. N. 257. p. 351.
44. *Stereometrical Propositions, variously applicable, but particularly intended for Gauging; by Rob. Anderson*. Lond. 1668. in 8vo. N. 39. p. 784.
45. *Gauging promoted; being an Appendix to Stereometrical Propositions; by Rob. Anderson*. Lond. 1669. in 8vo. N. 47. p. 960.
46. *Gauging Epitomized; by Mich. Dary*. Lond. 1699. upon one Folio Page. N. 52. p. 1054.
47. *Tabula Numerorum Quadratorum decies Millium, una cum ipsorum Latibus ab Unitate incipientibus, & Ordine Naturali usque ad 10000 progredientibus*. Lond. 1672. N. 82. p. 4050.
48. *The Description and Use of two Arithmetick Instruments, &c.* by Sir Sam. Moreland. Lond. 1673. N. 95. p. 6048.

N 139. P. 980. 49. Johannis Wallisii S. T. D. *Exercitationes tres.* 1. *De Cometaeum Distantiis investigandis.* 2. *De Rationum & Fractionum Reductione.* 3. *De Periodo*

N. 38. P. 753. *Juliano.* Lond. 1678.

Ibid. p. 756. 50. *Logarithmotechnia* Nicolai Mercatoris. Lond. 1668. in *Quarto.* This Author's Method of Squaring the Hyperbola, and of finding the Sum of the

Ibid. p. 759. Logarithms is here improved by Dr. Wallis; and further Explicated by the Author himself.

CHAP. II.

Trigonometry, Surveying.

A Chronographical Problem proposed by Mr. Rich. Townley, solved by Mr. John Collins. N. 69. p. 2793. Mar. An. 1674.

Fig. 47.

Fig. 48.

Fig. 49.

I. Prob. **T**HE Distances of three Objects in the same Plain being given, as *A, B, C*; The Angles made at a fourth Place in the same Plain as at *S*, are observed: the Distances from the Place of Observation to the respective Objects, are required.

The Problem hath six Cases.

CASE 1. If the Station be taken without the Triangle made by the Objects, but in one of the Sides thereof produced, as at *S*: find the Angle *ACB*; then in the Triangle *ACS*, all the Angles and the Side *AC* are known, whence either or both the Distances *SA*, or *SC*, may be found.

Case 2. If the Station be in one of the Sides of the Triangle, as at *S*: then having the three Sides, *AC, CB, BA*, given, find the Angle *CAB*; then again in the Triangle *SAB* all the Angles, and the Side *AB*, are known, whence may be found either *AS*, or *SB*. Geometrically, if you make the Angle *CAD* equal to the observed Angle *CSB*, and draw *BS* parallel to *DA*, you determine the Point of Station *S*.

Case 3. If the three Objects lie in a right Line as *ACD* (suppose it done), and that a Circle passeth through the Station *S*, and the two Exterior Objects *A, B*: then is the Angle *ABD*, equal to the Observed Angle *ASC* (by 21.3.E.) as Insisting on the same Arch *AD*: And the Angle *BAD* in like manner equal to the observed Angle *CSB*: By this means the point *D* is determined. Joyn *DC*, and produce the same, then a Circle passing through the Points *A, B, D*, intersects *DC*, produced at *S*, the place of Station.

Calculation] In the Triangle *ABD*, all the Angles and the Side *AB*, are known, whence may be found the Side *AD*.

Then in the Triangle *CAD*, the two Sides *CA*, and *AD*, are known, and their contained Angle *CAD* is known; whence may be found the Angles *CDA*, and *ACD*, the Complement whereof to a Semicircle is the Angle *SCA*: in which Triangle the Angles are now all known, and the Side *AC*: whence may be found either of the Distances *SC*, or *SA*.

Case

Case 4. If the Station be without the Triangle made by the Objects, the Sum of the Angles observed is less than four Right Angles. The *Construction* is the same as in the last *Case*, and the *Calculation* likewise; saving that you must make one Operation more, having the three Sides AC , CB , BA , thereby find the Angle CAB , which add to the Angle EAD , then you have the two Sides, *viz.* AC , being one of the Distances, and AD , (found as in the former *Case*) with their contained Angle CAD given, to find the Angles CDA , and ACD , the Complement whereof to a Semicircle is the Angle SCA : Now in the Triangle SCA , the Angle at C being found, and at S observed, and given by Supposition, the other at A is likewise known, as being the Complement of the two former to a Semicircle, and the Side AC given; hence the Distances CS , or AS , may be found.

Fig. 60.

Case 5. If the Place of Station be at some Point within the Plain of the Triangle, made by the three Objects, the *Construction* and *Calculation* is the same as in the last, saving only that instead of the observed Angle ASC , the Angle ABD is equal to the Complement thereof to a Semicircle, to wit, it is equal to the Angle ASD ; both of them insisting on the same Arch AD : And in like manner the Angle BAD is equal to the Angle DSB , which is the Complement of the observed CSB ; and in this *Case* the Sum of the three Angles observed, is equal to four Right Angles.

Fig. 61.

In these three latter *Cases* no use is made of the Angle observed between the two Objects, as A and B , that are made the Base-line of the *Construction*; yet the same is of ready use for finding the third Distance or last Side sought; as in the Triangle SAB , there is given the Distance AB , its opposite Angle equal to the Sum of the two observed Angles, and the Angle SAB attained, as in fourth *Case*: Hence the third Side, or last Distance SB , may be found.

Fig. 60.

And here it may be noted, that the three Angles CAS , ASB , $SB C$, are together equal to the Angle ACB ; for the two Angles CSB and CBS are equal to ECB , as being the Complement of SCB to two Right Angles; and the like in the Triangle on the other Side. *Ergo*, &c.

Case 6. If the three Objects be A , B , C , and the Station at S , as before, it may happen, according to the former *Constructions*, that the Points C and D may fall close together, and so a right Line joyning them, shall be produced with Uncertainty; in such *Case* the Circle may be conceived to pass through the Place of Station at S , and any two of the Objects, as through B and C , wherein making the Angle DBC equal to the observed Angle ASC , and BCD equal to the Complement to 180 deg. of both the observed Angles in DSB ; thereby the Point D is determined, through which, and the Points C , B , the Circle is to be described, and joyning DA , (produced when need requireth) where it intersects the Circle, as at S , is the Place of Station sought.

Fig. 62.

This *Problem* may be of good use for the due Situation of Sands and Rocks, that are within sight of three Places upon Land, whose Distances are well known; or for *Chorographical Uses*, &c. especially now there is a Method of observing Angles nicely accurate by the Aid of a Telescope.

Three Choreographic Problems
 solved by a Member of the Philo-
 sophical Society
 at Oxford.
 N. 177. p. 1231.
 Dec. An. 1685.

II. The three following Problems may occur at Sea, in finding the Distance and Position of Rocks, Sands, &c. from the Shore; or in Surveying the Sea-Coast; when only two Objects, whose Distance from each other is known, can be seen at one Station: But especially they may be useful to one, that would make a Map of a Country by a Series of Triangles derived from one or more measured Bases; which is the most exact Way of finding the Bearing and Distance of Places from each other, and thence their true Longitude and Latitude; and may consequently occur to one that would in that Matter measure a Degree on the Earth.

Fig. 63.

Prob. 1.] There are two Objects *B* and *C*, whose Distance *BC* is known, and there are two Stations at *A* and *E*, where the Objects *B*, *C*, being visible, and the Stations one from another, the Angles *BAC*, *BAE*, *AEB*, *AEC*, are known by Observation, (which may be made with an ordinary Surveying Semicircle, or Cross-Staff; or if the Objects beyond the View of the naked Eye, with a Telescopic Quadrant) to find the Distances or Lines *AB*, *AC*, *AE*, *EC*.

Fig. 64.

Construction.] In each of the Triangles, *BAE*, *CAE*, two Angles at *A*, *E*, being known, the third is also known; then take any Line *ae* at pleasure, on which constitute the Triangles, $\beta a e$, $a e \gamma$, respectively equiangular to the Triangles *BAE*, *AEC*, joyn $\beta \gamma$. Then upon *BC* constitute the Triangles *BCA*, *BCE*, equiangular to the correspondent Triangles $\beta \gamma a$, $\beta \gamma e$, joyn *AE*, and the thing is manifestly done.

The Calculation.] Assuming *ae*, of any Number of Parts, in the Triangles $\beta a e$, $a e \gamma$, the Angles being given, the Sides $a \beta$, $a \gamma$, $e \beta$, $e \gamma$, may be found by Trigonometry: Then in the Triangle $\beta a \gamma$, having the Angle $\beta a \gamma$, and the Legs $a \beta$, $a \gamma$, we may find $\beta \gamma$. Then $\beta \gamma : BC :: \beta a : BA :: \beta e : BE :: \gamma a : CA :: \gamma e : CE$.

Fig. 65.

Prob. 2.] Three Objects *B*, *C*, *D*, are given, or (which is the same) the Sides and consequently Angles, of the Triangle *BCD*, are given; also there are two Points or Stations *A*, *E*, such that at *A* may be seen the three Points *B*, *C*, *E*, but not *D*, and at the Station *E*, may be seen *A*, *C*, *D*, but not *B*; that is the Angles *BAC*, *BAE*, *AEC*, *AED*, (and consequently *EAC*, *AEC*) are known by Observation: To find the Lines *AB*, *AC*, *AE*, *EC*, *ED*.

Fig. 66.

Construction.] Take any Line *ae* at pleasure, and at its Extremities make the Angles $a \gamma$, $e a \beta$, $a \gamma$, $a e \delta$, equal to the correspondent observed Angles *EAC*, *EAB*, *AEC*, *AED*. Produce βa , δe , till they meet in ϕ , joyn $\phi \gamma$; then upon *CB* describe (according to 33. 3. E) a Segment of a Circle, that may contain an Angle $= \gamma \phi \beta$; and upon *CD* describe a Segment of a Circle capable of an Angle $= \gamma \phi \delta$: Suppose *F* the common Section of these 2 Circles; joyn *FB*, *FC*, *FD*; then from the point *C*, draw for the Lines *CA*, *CE*, so that the Angle *FCA* may be $= \phi \gamma a$, and *FCE* $= \phi \gamma e$; so *A*, *E*, the common Sections of *CA*, *CE*, with *FB*, *FD*, will be the Points required, from whence the rest is easily deduced.

Calculation.

Calculation.] Assuming α of any Number, in the Triangles $\alpha\gamma, \alpha\delta, \alpha\epsilon$, all the Angles being given, with the Side α assum'd the Sides $\alpha\gamma, \alpha\delta, \alpha\epsilon$, will be known; then in the Triangle $\gamma\alpha\epsilon$, the Angle $\gamma\alpha\epsilon$, with the Legs $\alpha\gamma, \alpha\epsilon$, being known, the Angles $\alpha\gamma\epsilon, \alpha\epsilon\gamma$, with the Side $\gamma\epsilon$, will be known: Then as for the rest of the Work, the Triangle BCD having all its Sides and Angles known, and the Angles BFC, BFD, being equal to the found $\beta\gamma\delta, \beta\delta\epsilon$; how to find FB, FC, FD, by *Calculation* (and also *Protraction*) has been already shewn above by Mr. Collins, as to all its Cases.

See. 1.

But it must here be noted, that if the Sum of the observed Angles BAE, AED, is 180 deg. then AB, and ED, cannot meet, because they are parallel, and consequently the given Solution cannot take place; for which Reason I here subjoin another.

Another Solution.] Upon BC describe a Segment BAC, of a Circle, so that the Angle of the Segment may be equal to the observed Angle $\beta\alpha\gamma$, (which is shewn 33. 3. E.) and upon CD describe a Segment CED, of a Circle, capable of an Angle equal to the observed CED; from C draw the Diameters of these Circles CG, CH; then upon CG describe a Segment of a Circle GFC, capable of an Angle equal to the observed Angle ABC; likewise upon CH, describe a Circle's Segment CFH, capable of an Angle equal to the observed Angle CAE: suppose F the common Section of the two last Circles HFC, GFC; joyn FH, cutting the Circle HEC in E; joyn also FG, cutting the Circle GAC in A: I say, that A, E, are the Points required.

Fig. 67.

Demonstration.] For the Angle BAC is $= \beta\alpha\gamma$, by *Construction* of the Segment; also the Angles CEH, CAG, are right, because each exists in a Semicircle: Therefore a Circle being described upon CF, as a Diameter, will pass thro' E, A, therefore the Angle CAE = CFE = CFH (by *Construction*) to the observed Angle $\gamma\alpha\epsilon$. In like manner the Angle CEA = CFA = CFG = observ'd Angle $\gamma\alpha\delta$.

If the Stations A, E, fall in a right Line with the Point C; the Lines GA, HE, being parallel, cannot meet; but in this Case the Problem is indeterminate, and capable of infinite Solutions. For, as before, upon CG, describe a Segment of a Circle capable of the observed Angle $\gamma\alpha\epsilon$, and upon CH, describe a Segment capable of the observed Angle $\gamma\alpha\delta$: then through C draw a Line any way, cutting the Circles in A, E, these Points will answer the Question.

Problem 3.] Four Points, B, C, D, F, or the four Sides of a Quadrilateral, with the Angles comprehended, are given; also there are two Stations A and E, such, that at A, only B, C, E, are visible, and at E, only A, D, F; that is, the Angles BAC, BAE, AED, DEF, are given, to find the Places of the two Points A, E; and consequently the Lengths of the Lines AB, AC, AE, ED, EF.

Fig. 68.

Construction.] Upon BC (by 33. 3. E.) describe a Segment of a Circle, that may contain an Angle equal to the observed Angle BAC, then from C draw the Chord CM, or a Line cutting the Circle in M, so that the Angle BCM, may be equal to the Supplement of the observed Angle BAE, i. e. its Residue

to 180 deg. In like manner on D F describe a Segment of a Circle, capable of an Angle equal to the observed D E F, and from D draw the Chord D N, so that the Angle F D N may be equal to the Supplement of the observed Angle A E F; joyn M N, cutting the two Circles in A, E: I say, A, E, are the two Points required.

Demonstration.] Joyn A B, A C, E D, E F, then is the Angle M A B = B C M, (by 21. 3. E.) = Supplement of the observed Angle B A E, by *Construction*; therefore the constructed Angle B A E, is equal to that which was observ'd. Also the Angle B A C, of the Segment, is, by *Construction* of the Segment, equal to the observed Angle B A C. In like manner the constructed Angles A E F, and D E F, are equal to the Correspondent observed Angles A E F, D E F; therefore A, E, are the Points required.

Calculation.] In the Triangle B C M, the Angle B C M, (= Supplement of B A E) and Angle B M C, (= B A C) are given, with the Side B C; thence M C may be found; in like manner D N, in the Triangle D N F, may be found. But the Angle M C D (= B C D - B C M) is known, with its Legs M C, C D, therefore its Base M D, and Angle M D C, may be known. Therefore the Angle M D N (= C D F - C D M - E D N) is known, with its Legs M D, D N; thence M N, with the Angles D M N, D N M, will be known. Then the Angle C M A (= D M C + D M N) is known, with the Angle M A C (= M A B + B A C) and M C, before found; therefore M A, and A C, will be known. In like manner in the Triangle E D N, the Angles E, N, with the Side D N, being known, the Sides E N, E D, will be known; therefore A E (= M N - M A - E N) is known. Also in the Triangle A B C, the Angle A, with its Sides B C, C A, being known, the Side A B will be known, with the Angle B C A; so in the Triangle E F D, the Angle E, with the Sides E D, D F, being known, E F will be found, with the Angle E D F. Lastly, in the Triangle A C D, the Angle A C D (= B C D - B C A) with its Legs A C, C D, being known, the Side A D will be known; and in like manner E C, in the Triangle E D C.

Note. That in this *Problem*, as also in the first and second, if the two Stations fall in a right Line with either of the given Objects, the *Locus* of A or E being a Circle, the particular Point of A or E cannot be determined from the Things given.

As to the other *Cases* of this third *Problem*, wherein A and E may shift Places, *i. e.* only D, F, E, may be visible at A, and only A, B, C, at E; or wherein B, D, E, may be visible at A, and only C, F, A, at E; or wherein A may be on one side of the Quadrilateral, and E on the other; or one of the Stations within the Quadrilateral, and the other without it; I presume that the *Surveyor* will easily direct himself, by what has been already said.

The *Solution* of this third *Problem* is general, and serves also for both the precedent. For suppose C, D, the same Point in the last Figure, and it gives the *Solution* of the second *Problem*; but if B, C, be supposed the same Points with D, F, by proceeding as in the last, you may directly solve the first *Problem*.

III. The

III. The Variation of the magnetick Needle is so commonly known, that I need not insist much on the Explication thereof; 'tis certain that the true solar Meridian, and the Meridian shewn by a Needle, agree but in very few Places of the World; and this too, but for a little Time (if a Moment) together; the Difference between the true Meridian and magnetick Meridian perpetually varying and changing in all Places, and at all Times; sometimes to the Eastward, and sometimes to the Westward.

On which Account 'tis impossible to compare two *Surveys* of the same Place, taken at distant Times, by magnetick Instruments (such as the Circumferentor, by which the *Down Survey*, or Sir William Petty's *Survey of Ireland* was taken) without due Allowance be made for this Variation. To which purpose, we ought to know the Difference between the magnetick Meridian and true Meridian, at that Time of the *Down Survey*, and the said Difference at the Time when we make a *New Survey* to compare with the *Down Survey*.

But here I would not be understood, as if I proposed hereby to shew, that a Map of the same Place, taken by magnetick Instruments at never so distant Times, should not at one Time give the same Figure and Contents as at another Time. This certainly it will do most exactly, the Variation of the Needle having nothing to do either in the Shape or Contents of the *Survey*. All that is affected thereby, is the Bearings of the Lines run by the Chain, and the Boundaries between Neighbours. And how this may cause a considerable Error (unless due Allowance be made for it) is what I shall prove most fully.

In order to which, let us suppose that about the Year 1657 (at which Time the *Down Survey* was taken) the magnetick Meridian and true Meridian did agree at *Dublin*, or pretty nigh all over *Ireland*; that is to say, that there was no Variation. And indeed by Experiment it was at that Time found, as I am well assured, that at *Dublin* it was hardly half a Degree.

Let us suppose, that in the Year 1695 the Variation was 7 Degrees from the North to the Westward: That it was really so, I believe I am pretty well assured, from an Experiment made by *my self* with all Diligence. But this is not material; let us now only suppose it.

Let A, B, represent the Survey of two Town Lands, one in the Possession of A, and the other in Possession of B, taken by the *Down Survey*, Anno 1657, when there was no Variation.

Let the Line N S, running thro' the Point P, be the true Meridian, and consequently the magnetick Meridian also at that Time, because of the supposed no Variation; and let this Line N S, be also the Boundary between the two Town Lands A, and B.

In the Year 1695, when the Variation is 7 Degrees from the North to the Westward, B having a Map of the *Down Survey*, and being suspicious that his Neighbour A had encroached on him by a Ditch P Q, employs a Surveyor to enquire into the Matter: The Surveyor finds by his Map, that the Boundary between B and his Neighbour A, run from the Point P, through a Meadow, directly according to the magnetick Meridian S P N; but observing the Ditch P Q cast up much to the Eastward of the present magnetick Meridian, he concludes that A has encroached upon B, and that the Ditch ought to have been

*An Error of
Common Survey-
ors, in comparing
Surveys taken at
long Intervals of
Time with the
magnetick Need-
le, demonst-
rated; by Mr.
William Moly-
neux. N. 230.
p. 625.
July, An. 1697.*

Fig. 69.

been cast up along the Line Pq , the Angle $Q P q$ being an Angle of 7 deg. that is, the present Variation of the Needle, and the Line Pq , the present magnetick Meridian; for which Variation not making any Allowance, he positively determines that B has all the Land in the Triangle $Q P q$, more than he ought to have; and that his Ditch ought to run along the Line Pq .

'Tis true indeed, if the *Surveyor* go the whole Surround of the Land A, and B, he will find their Figure and Contents exactly agreeable to the Map here expressed. But then the Bearings of the Lines are all 7 deg. different from the Bearings in the Map, and they will run in and out upon the adjacent neighbouring Lands, and cause endless Differences between their Possessors; as is manifest from the Figure: Wherein the prick'd Lines represent the Disagreement in the Bearings of the Lines, protracted from the Point P; and we see A encroaching upon his Neighbours on the Westward, as he encroaches upon B, and B's Eastward Neighbours encroaching on him, and so forward and clear round. Whereas by a due Allowance for the Variation of the Needle, all this Confusion and Disagreement is avoided, and every thing hits right.

Thus, for Instance, in the Case before us, knowing that the magnetick Variation has caused the present magnetick Meridian to fall in the Line $nqPs$, 7 deg. from the North to the Westward; to reduce this to the magnetick Meridian at the Time of the *Down Survey*, I must make the Meridian of my Map to fall 7 deg. to the Eastward of my magnetick Meridian; as we see the Line PQ , falls 7 deg. to the Eastward of the Line Pq .

What is here said on Supposition that the Magnet had no Variation at the Time of the first Survey taken, and that it had 7 deg. Variation Westward at the Time of the second Survey, may easily be accommodated to the Supposal of any other Variations at the first and second Surveys, *Mutatis Mutandis*, for knowing the Variations we know their Difference; and if we know their Difference, this gives us the Angle $Q P q$, by which we reduce them to each other. The best Way therefore to make Maps invariable, constant and everlasting, were for the Surveyors, who use magnetick Instruments, to make always Allowance for the magnetick Variation, and to protract and lay down Plats by the true Meridian.

Perhaps it may be objected, That Surveys may be taken without magnetick Instruments, and that therefore this Error arising from the magnetick Variation, and Change of the Bearing of Lines, may be avoided. To which I answer, first, That granting a Survey may be taken without magnetick Instruments, this is nothing against what we have laid down, relating to Surveys that are taken with magnetick Instruments, as the *Down Survey* actually was, and most Surveys at present actually are taken therewith. Secondly, Tho' a Survey may be taken truly without magnetick Instruments, so as to shew the exact Angles and Lines of the Plat, and consequently the true Contents, yet this will not give the true Bearings of the Lines, or shew my Position in relation to my Neighbours, or other Parts of the Country. This must be supplied by the Magnet, or something equivalent thereto, as finding a true meridian Line on your Land by celestial Observation. And I doubt not but the antient *Egyptians*, before the Discovery of the Magnet, were forced to some such Expedient in their Surveys, and

and Applotments of Lands, between Neighbour and Neighbour, after the *Inundations* of the Nile; which, we are told, gave the first Original to *Geometry* and *Surveying*: Absolute Necessity and Use having introduced these, as Delight and Diversion introduced *Astronomy* amongst the *Chaldeans*.

And this brings me to another Objection, which may be made against the Instance before laid down: It may be said, That certainly the Surveyor which B employed was very ignorant, who would chuse to judge of the Line P Q rather by its Bearing, than by determining the Point Q by measuring from H and G. To this I answer, What if both the Points H and G were vanished since the *Down Survey* was taken? What if the whole Face of the Country were changed, save only the Point P, and the Line P Q? How shall the Surveyor then judge of the Line P Q, but by its Bearing? That this is no extravagant Supposition, we have an Example in *Egypt* above-mentioned, where the Nile lays all flat before it, and so uniformly covers all with Mud, that there is no Distinction. In such a Case your Bearing must certainly help you out, there is no other Way.

But I answer, secondly, To say that the Surveyor might have determined the Point Q by Admeasurement from G and H, or any other adjoining noted Points, as from F, K, I, &c. 'Tis very true; but then 'tis against our *Supposition*. I am upon shewing an Error that arises from judging of the Line P Q by *magnetick Bearing*; and to tell me that this might be avoided by another Way, is to say nothing. I my self shew how it may be avoided, by allowing for the Variation; but still it is an Error till it be avoided.

But, thirdly, if B's Surveyor do not allow for the Variation of the Needle, he will never exactly determine even the Points G, F, H, K, &c. or any other Points in the Plat; but instead thereof, will fall on the Points *g, h, f, k*.

From what has been laid down, we may see the absolute Necessity of allowing for the Variation of the Magnet, in comparing old Surveys with new ones; for want of which, great Disputes may arise between neighbouring Proprietors of Lands: And it were to be wished, that our honourable and learned Judges would take this Matter into their Consideration, whenever any Business of this Kind comes before them.

IV. I have invented a *Level* with a Tube, with Glasses and a Thread, hanging between four Points, with a Weight in a Box so contrived, that as soon as the Instrument is set down, you have your *Point* of Horizon with a great deal of Exactness. I am making another, which playeth on one steel Point, standing on a Diamond.

A new Level;
by Mr. Butterfield. N. 141.
p. 1026.
Sept. An. 1678.

V. *An Account of a Book omitted; viz. The Art of Levelling; by M. N. 74. p. 2217. Mariotte.*

C H A P. III.

O P T I C K S.

*A new Theory
about Light and
Colours; by Mr.
Iſaac Newton.
N. 80. p. 3075.
Feb. An. 1672.*

IN the Year 1666. (at which time I applied my ſelf to the Grinding of Optick Glaſſes of other Figures than Spherical) I procured me a Triangular Glaſs-Prism, to try therewith the celebrated Phænomena of Colours. And in order thereto, having darkned my Chamber, and made a ſmall Hole in my Window-ſhuts, to let in a convenient Quantity of the Sun's Light, I placed my Prism at its Entrance, that it might be thereby refracted to the oppoſite Wall. It was at firſt a very pleaſing Divertiſement, to view the vivid and intenſe Colours produced thereby; but after a while applying my ſelf to conſider them more circumſpectly, I became ſurpriſed to ſee them in an oblong Form; which, according to the receiv'd Laws of Refractions, I expected ſhould have been circular. They were terminated at the Sides with ſtreight Lines, but at the Ends, the Decay of Light was ſo gradual, that it was difficult to determine juſtly what was their Figure, yet they ſeemed Semicircular.

Comparing the Length of this colour'd *Spectrum* with its Breadth, I found it about five times greater; a Diſproportion ſo extravagant, that it excited me to a more than ordinary Curioſity of examining from whence it might proceed. I could ſcarce think, that the various Thickneſs of the Glaſs, or the Termination with Shadow or Darkneſs, could have any Influence on Light to produce ſuch an Effect; yet I thought it not amiſs, firſt to examine thoſe Circumſtances, and ſo try'd what would happen by transmitting Light through Parts of the Glaſs of divers Thickneſſes, or through Holes in the Window of divers Bigneſſes, or by ſetting the Prism without, ſo that the Light might paſs through it, and be refracted, before it was terminated by the Hole: But I found none of thoſe Circumſtances material. The Faſhion of the Colours was in all theſe Caſes the ſame.

Then I ſuſpected, whether by any Unevenneſs in the Glaſs, or other contingent Irregularity, theſe Colours might be thus dilated. And to try this, I took another Prism like the former, and ſo placed it, that the Light paſſing through them both, might be refracted contrary ways, and ſo by the latter returned into that Courſe from which the former had diverted it: For by this Means I thought the regular Effects of the firſt Prism would be deſtroy'd by the ſecond Prism, but the irregular Ones more augmented, by the Multiplicity of Refractions. The Event was, that the Light, which by the firſt Prism was diffuſed into an oblong Form, was by the ſecond reduced into an orbicular One, with as much Regularity as when it did not at all paſs through them.

N. 83. p. 4061.

Fig. 70.

That this Experiment may be better apprehended; let E G, deſign the Window; F, the Hole in it, thro' which the Light arrives at the Prisms; A B C, the firſt Prism, which refracts the Light towards P T, painting there the Colour in an
Oblong,

and $\alpha \beta \gamma$, the second Prism, which refracts back again the Rays to Q, where the long Image P T is contracted into a Round one. I suppose the Plane $\alpha \gamma$ parallel to B C, and $\beta \gamma$ to A C, that the Rays may be equally refracted contrary ways in both Prisms. The Prisms also must be placed very near to one another; for if their Distance be so great, the Colours begin to appear in the Light, before its Incidence on the second Prism, those Colours will not be destroyed by the contrary Refractions of that Prism. And if a *Lens* be placed in the Hole F, or immediately after the Prisms, so that its *Focus* be at the Image Q, or P T, the Perimeter of the Image Q, and the straight Sides of the Image P T, will become much better defined than otherwise. So that, whatever was the Cause of that Length, 'twas not any Contingent Irregularity.

N. 80. p. 3076.
Feb. An. 1672.

I then proceeded to examine more critically, what might be effected by the Difference of the Incidence of Rays coming from divers Parts of the Sun; and to that end, measured the several Lines and Angles belonging to the Image. Its Distance from the Hole or Prism was 22 Foot; its utmost Length $13 \frac{1}{2}$ Inches; its Breadth $2 \frac{1}{2}$; the Diameter of the Hole $\frac{1}{4}$ of an Inch; the Angle which the Rays, tending towards the middle of the Image, made with those Lines, in which they would have proceeded without Refraction, was 44 deg. 56 min. And the Vertical Angle of the Prism, 63 deg. 12 min. Also the Refractions on both sides the Prism, that is of the Incident and Emergent Rays, were, as near as I could make them, equal, and consequently about 54 deg. 4 min. And the Rays fell perpendicularly upon the Wall. Now subducting the Diameter of the Hole from the Length and Breadth of the Image, there remains 13 Inches in the Length, and $2 \frac{1}{2}$ the Breadth, comprehended by those Rays, which passed through the Center of the said Hole, and consequently the Angle of the Hole, which that Breadth subtended, was about 31 min. answerable to the Sun's Diameter; but the Angle which its Length subtended, was more than 5 such Diameters, namely 2 deg. 49 min.

Having made these Observations, I first computed from them the refractive Power of that Glass, and found it measured by the Ratio of the Sines, 20 to 31; and then by that Ratio I computed the Refractions of two Rays flowing from opposite Parts of the Sun's *Discus*, so as to differ 31 min. in their Obliquity of Incidence, and found that the emergent Rays should have comprehended an Angle of about 31 min. as they did before they were incident.

But because this Computation was founded on the Hypothesis of the Proportionality of the Sines of Incidence and Refraction, which though by my own Experience I could not imagine to be so erroneous, as to make that Angle but 31 min. which in reality was 2 deg. 49 min. yet my Curiosity caused me again to take my Prism. And having placed it at my Window, as before, I observed, that by turning it a little about its Axis to and fro, so as to vary its Obliquity to the Light, more than an Angle of 4 or 5 degrees, the Colours were not thereby sensibly translated from their Place on the Wall, and consequently by that Variation of Incidence, the Quantity of Refraction was not sensibly varied. By this Experiment therefore, as well as by the former

Computation, it was evident, that the Difference of the Incidence of Rays, flowing from divers Parts of the Sun, could not make them after Decussation diverge at a sensibly greater Angle, than that at which they before converged; which being, at most, but about 31 or 32 min. there still remained some other Cause to be found out, from whence it could be 2 deg. 49 min.

Then I began to suspect, whether the Rays, after their Trajection through the Prism, did not move in curve Lines, and according to their more or less Curvity tend to divers Parts of the Wall. And it increased my Suspicion, when I remember'd that I had often seen a Tennis-Ball struck with an oblique Racket, describe such a curve Line. For, a circular as well as a progressive Motion being communicated to it by that Stroke, its Parts on that Side, where the Motions conspire, must press and beat the contiguous Air more violently than on the other, and there excite a Reluctancy and Re-action of the Air proportionably greater. And for the same Reason, if the Rays of Light should possibly be globular Bodies, and by their oblique Passage out of one Medium into another, acquire a circulating Motion, they ought to feel the greater Resistance from the ambient Æther, on that Side, where the Motions conspire, and thence be continually bowed to the other. But notwithstanding this plausible Ground of Suspicion, when I came to examine it, I could observe no such Curvity in them. And besides (which was enough for my purpose) I observed, that the Difference betwixt the Length of the Image, and the Diameter of the Hole, through which the Light was transmitted, was proportionable to their Distance.

The gradual Removal of these Suspicions at length led me to the *Experimentum Crucis*, which was this; I took two Boards, and placed one of them close behind the Prism at the Window, so that the Light might pass through a small Hole, made in it for the purpose, and fall on the other Board, which I placed at about 12 Feet distance, having first made a small Hole in it also, for some of that incident Light to pass through. Then I placed another Prism behind this second Board, so that the Light trajected through both the Boards, might pass through that also, and be again refracted before it arrived at the Wall. This done, I took the first Prism in my Hand, and turned it to and fro slowly about its Axis, so much as to make the several Parts of the Image, cast on the second Board, successively pass through the Hole in it, that I might observe to what Places on the Wall the second Prism would refract them. And I saw by the Variation of those Places, that the Light, tending to that End of the Image, towards which the Refraction of the first Prism was made, did in the second Prism suffer a Refraction considerably greater than the Light tending to the other End. And so the true Cause of the Length of that Image was detected to be no other, than that *Light* is not similar or homogenous, but consists of *difform Rays, some of which are more refrangible than others*; so that without any Difference in their Incidence on the same Medium, some shall be more refracted than others; and therefore that, according to their *particular Degrees of Refrangibility*, they were transmitted through the Prism to divers Parts of the opposite Wall.

Ibid. p. 3081.

I shall

I shall now proceed to acquaint you with another more notable *Difformity* in its Rays, wherein the Origin of Colours is unfolded: Concerning which I shall lay down the Doctrine first; and then, for its Examination, give you an Instance or two of the Experiments, as a Specimen of the rest.

The Doctrine you will find comprehended and illustrated in the following Propositions.

1. As the Rays of Light differ in Degrees of Refrangibility, so they also differ in their Disposition to exhibit this or that particular Colour. Colours are not Qualifications of Light, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed) but original and connate Properties, which in divers Rays are divers. Some Rays are disposed to exhibit a Red Colour, and no other; some a Yellow, and no other; some a Green, and no other; and so of the rest. Nor are there only Rays proper and particular to the more eminent Colours, but even to all their intermediate Gradations.

2. To the same Degree of Refrangibility ever belongs the same Colour, and to the same Colour ever belongs the same Degree of Refrangibility. The least refrangible Rays are all disposed to exhibit a Red Colour; and contrarily, those Rays which are disposed to exhibit a Red Colour, are all the least refrangible: So the most refrangible Rays are all disposed to exhibit a deep Violet Colour; and contrarily, those which are apt to exhibit such a Violet Colour, are all the most refrangible: And so to all the intermediate Colours in a continued Series belong intermediate Degrees of Refrangibility. And this Analogy betwixt Colours and Refrangibility is very precise and strict; the Rays always either exactly agreeing in both, or proportionally disagreeing in both.

3. The Species of Colour, and Degree of Refrangibility proper to any particular Sort of Rays, is not mutable by Refraction, nor by Reflection from natural Bodies, nor by any other Cause that I could yet observe. When any one Sort of Rays hath been well parted from those of other Kinds, it hath afterwards obstinately retained its Colour, notwithstanding my utmost Endeavours to change it. I have refracted it with Prisms, and reflected it with Bodies, which in Day-light were of other Colours; I have intercepted it with the coloured Film of Air, interceeding two compressed Plates of Glass, transmitted it through coloured Mediums, and through Mediums irradiated with other Sorts of Rays, and diversly terminated it, and yet could never produce any new Colour out of it. It would by contracting or dilating become more brisk, or faint, and by the Loss of many Rays, in some Cases very obscure and dark; but I could never see it changed in Specie.

4. Yet seeming Transmutations of Colours may be made, where there is any Mixture of divers Sorts of Rays: For in such Mixtures, the component Colours appear not; but, by their mutual allaying each other, constitute a midling Colour. And therefore, if by Refraction, or any other of the aforesaid Causes, the difform Rays, latent in such a Mixture, be separated, there shall emerge Colours different from the Colour of the Composition. Which Colours are not new generated, but only made apparent by being parted; for

if they be again intirely mix'd and blended together, they will again compose that Colour, which they did before Separation. And for the same Reason, Transmutations made by the convening of divers Colours are not real; for when the difform Rays are again severed, they will exhibit the very same Colours which they did before they enter'd the Composition; as you see Blue and Yellow Powders, when finely mix'd, appear to the naked Eye, Green, and yet the Colours of the component Corpuscles are not thereby really transmuted, but only blended. For when viewed with a good Microscope, they still appear Blue and Yellow interspersedly.

5. There are therefore two sorts of Colours. The one Original and Simple; the other compounded of these. The original or primary Colours are, Red, Yellow, Green, Blue, and a Violet-Purple, together with Orange, Indico, and an indefinite Variety of intermediate Gradations.

6. The same Colours in Specie with these primary Ones, may be also produced by Composition. For a Mixture of Yellow and Blue makes Green; of Red and Yellow makes Orange; of Orange and Yellowish Green makes Yellow. And in general, if any two Colours be mix'd, which in the Series of those generated by the Prism are not too far distant one from another, they by their mutual Alloy compound that Colour, which in the said Series appeareth in the Midway between them. But those which are situated at too great a Distance, do not so. Orange and Indico produce not the intermediate Green, nor Scarlet and Green the intermediate Yellow.

7. But the most surprizing and wonderful Composition was that of Whiteness. There is no one sort of Rays which alone can exhibit this. 'Tis ever compounded; and to its Composition, are requisite all the aforesaid primary Colours, mix'd in a due Proportion. I have often with Admiration beheld, that all the Colours of the Prism being made to converge, and thereby to be again mix'd, as they were in the Light before it was incident upon the Prism, reproduced Light, intirely and perfectly White, and not at all sensibly differing from a direct Light of the Sun, unless when the Glasses, I used, were not sufficiently clear; for then they would a little incline it to their Colour.

8. Hence therefore it comes to pass, that Whiteness is the usual Colour of Light; for Light is a confused Aggregate of Rays indued with all sorts of Colours, as they were promiscuously darted from the various Parts of luminous Bodies. And of such a confused Aggregate, as I said, is generated Whiteness, if there be a due Proportion of the Ingredients; but if any one predominate, the Light must incline to that Colour; as it happens in the blue Flame of Brimstone; the yellow Flame of a Candle; and the various Colours of the Fixed Stars.

9. These things considered, the manner how Colours are produced by the Prism is evident. For, of the Rays, constituting the incident Light, since those which differ in Colour proportionally differ in Refrangibility, they by their unequal Refractions must be severed and dispersed into an oblong Form in an orderly Succession, from the least refracted Scarlet, to the most refracted Violet. And for the same Reason it is, that Objects when looked upon thro' a Prism, appear coloured. For the difform Rays, by their unequal Re-

Refractions, are made to diverge towards several Parts of the *Retina*, and there express the Images of things coloured, as in the former Case they did the Sun's Image upon a Wall. And by this Inequality of Refractions, they become not only coloured, but also very confused and indistinct.

10. Why the Colours of the Rainbow appear in falling Drops of Rain, is also from hence evident. For those Drops which refract the Rays, disposed to appear Purple, in greatest Quantity to the Spectator's Eye, refract the Rays of other sorts so much less, as to make them pass beside it; and such are the Drops on the Inside of the primary Bow, and on the Outside of the secondary or exterior One. So those Drops, which refract in greatest Plenty the Rays, apt to appear Red, toward the Spectator's Eye, refract those of other sorts so much more, as to make them pass beside it; and such are the Drops on the exterior Part of the primary, and interior Part of the secondary Bow.

11. The odd Phenomena of an Infusion of *Lignum Nephriticum*, Leaf-Gold, Fragments of coloured Glass, and some other transparently coloured Bodies, appearing in one Position of one Colour, and of another in another, are on these Grounds no longer Riddles. For those are Substances apt to reflect one sort of Light, and transmit another; as may be seen in a dark Room, by illuminating them with similar or uncompounded Light. For then they appear of that Colour only, with which they are illuminated; but yet in one Position more vivid and luminous than in another, accordingly as they are disposed more or less to reflect or transmit the incident Colour.

12. From hence also is manifest the Reason of an unexpected Experiment, which Mr. *Hook*, somewhere in his *Micrography*, relates to have made with two Wedge-like transparent Vessels, filled the one with a Red, the other with a Blue Liquor: namely, that though they were severally transparent enough, yet both together became opaque; for if one transmitted only Red, and the other only Blue, no Rays could pass through both.

13. I might add more Instances of this Nature, but I shall conclude with this general One. That the Colours of all natural Bodies have no other Origin than is, that they are variously qualified, to reflect one sort of Light in greater plenty than another. And this I have experimented in a dark Room, by illuminating those Bodies with uncompounded Light of divers Colours. For by that means any Body may be made to appear of any Colour. They have there no appropriate Colour, but ever appear of the Colour of the Light cast upon them, but yet with this Difference, that they are most brisk and vivid in the Light of their own Day-light Colour. Minium appeareth there of any Colour indifferently, with which it is illustrated, but yet most luminous in Red; and so Bise appeareth indifferently of any Colour, with which its illustrated, but yet most luminous in Blue: and therefore Minium reflecteth Rays of any Colour, but most copiously those endowed with Red, and consequently when illustrated with Day-light; that is, with all sorts of Rays promiscuously blended, those qualified with Red shall abound most in the reflected Light, and by their Prevalence cause it to appear of that Colour. And for the same Reason Bise, reflecting Blue most copiously, shall appear Blue by the Excess of those Rays in its reflected Light; and the like of other Bodies. And that this is the nature

ture and adequate Cause of their Colours, is manifest, because they have no Power to change or alter the Colours of any sort of Rays incident apart, but put on all Colours indifferently, with which they are enlightned.

These things being so, it can be no longer disputed, whether there be Colours in the Dark, or whether they be the Qualities of the Objects we see, no nor perhaps, whether Light be a Body. For, since Colours are the Qualities of Light, having its Rays for their intire and immediate Subject, how can we think those Rays Qualities also, unless one Quality may be the Subject of, and sustain another; which, in effect, is to call it Substance. We should not know Bodies for Substances, were it not for their sensible Qualities; and the Principal of those being now found due to something else, we have as good Reason to believe that to be a Substance also.

Besides, Who ever thought any Quality to be a heterogeneous Aggregate, such as Light is discovered to be? But to determine more absolutely, what Light is, after what Manner refracted, and by what Modes or Actions it produceth in our Minds the Phantasms of Colours, is not so easy: And I shall not mingle Conjectures with Certainties.

Reviewing what I have written, I see the Discourse itself will lead to divers Experiments sufficient for its Examination: And therefore I shall not trouble you further than to describe one of those which I have already insinuated.

In a darkned Room make a Hole in the Shut of a Window, whose Diameter may conveniently be about a third Part of an Inch, to admit a convenient Quantity of the Sun's Light: And there place a clear and colourless Prism, to refract the entring Light towards the further part of the Room; which, as I said, will thereby be diffused into an oblong coloured Image. Then place a Lens of about 3 Foot Radius (suppose a broad Object-Glass of a three Foot Telescope) at the Distance of about 4. or 5 Foot from thence, through which all those Colours may at once be transmitted, and made by its Refraction to convene at a further Distance of about 10 or 12 Feet. If at that Distance you intercept this Light with a Sheet of white Paper, you will see the Colours converted into Whiteness again by being mingled. But it is requisite that the Prism and Lens be placed steddily, and that the Paper, on which the Colours are cast, be moved to and fro; for by such Motion you will not only find at what Distance the Whiteness is most perfect, but also see how the Colours gradually convene and vanish into Whiteness; and afterwards, having crossed one another in that Place where they compound Whiteness, are again dissipated and severed, and in an inverted Order retain the same Colours which they had before they entred the Composition. You may also see, that if any of the Colours at the Lens be intercepted, the Whiteness will be changed into the other Colours. And therefore, that the Composition of Whiteness be perfect, Care must be taken that none of the Colours fall besides the Lens. Thus, in the Design of this Experiment, A B C expresseth the Prism set endwise to sight, close by the Hole F, of the Window E G. Its vertical Angle A C B may conveniently be about 60 deg. M N designeth the Lens. Its Breadth 2 $\frac{1}{2}$, or 3 Inches. S F, one of the straight Lines, in which *difform* Rays may be conceived to flow successively from the Sun. F P, and F R, two
of

of those Rays unequally refracted, which the Lens makes to converge towards Q, and after Decussation to diverge again. And HI, the Paper, at divers Distances, on which the Colours are projected; which in Q constitute Whiteness, but are Red and Yellow in R, r , and p , and Blue and Purple in P, p , and π .

If you proceed further to try the Impossibility of changing any uncompounded Colour (which I have asserted in the third and thirteenth Propositions) 'tis requisite that the Room be made very dark, lest any scattering Light, mixing with the Colour, disturb and allay it, and render it compound, contrary to the Design of the Experiment. 'Tis also requisite, that there be a perfect Separation of the Colours, than, after the Manner above described, can be made by the Refraction of one single Prism; and how to make such further Separations, will scarce be difficult to them that consider the discovered Laws of Refractions. But if Tryal shall be made with Colours not thoroughly separated, there must be allowed Changes proportionable to the Mixture. Thus if compound Yellow Light fall upon the Blue Bise, the Bise will not appear perfectly Yellow, but rather Green; because there are in the Yellow Mixture many Rays endued with Green, and Green being less remote from the usual Blue Colour of Bise than Yellow, is the more copiously reflected by it.

In like manner, if any one of the prismatic Colours, suppose Red, be intercepted, on Design to try the asserted Impossibility of reproducing that Colour out of the others which are pretermitted, 'tis necessary, either that the Colours be very well parted before the Red be intercepted, or that together with the Red, the neighbouring Colours, into which any Red is secretly dispersed, (that is, the Yellow, and perhaps Green too) be intercepted; or else, that Allowance be made for the emerging of so much Red out of the Yellow-Green, as may possibly have been diffused, and scatteringly blended in those Colours. And if these Things be observed, the new Production of Red, or any intercepted Colour, will be found impossible.

II. 1. To contract the Beams of the Sun without the Hole of the Window, and to place the Prism between the Focus of the Lens and the Hole.

2. To cover over both Ends of the Prism with Paper at several Distances from the Middle; or with moveable Rings, to see how that will vary or divide the Length of the Figure.

3. To move the Prism so, as the End may turn about, the Middle being steady.

4. To move the Prism by shoving it, till first the one Side, then the Middle, then the other Side pass over the Hole, observing the same Parallelism.

2. I suppose the Design of the Proposer of these Experiments is, to have their Events expressed, with such Observations as may occur concerning them. Touching the first, I have observed, that the solar Image falling on a Paper placed at the Focus of the Lens, was by the interposed Prism drawn out in Length proportional to the Prism's Refraction or Distance from that Focus. And the chief Observable here, which I remember, was, that the streight Edges of the oblong Image were distincter than they would have been without the Lens.

Con-

Some Experiments proposed in relation to this Theory.
N. 83. p. 4059.
May, An. 1672.

Observations on this Proposal by Mr. Newton.
N. 83. p. 4060.
May, An. 1672.

Considering that the Rays coming from the Planet *Venus*, are much less inclined one to another, than those which come from the opposite Parts of the Sun's Disk; I once tried an Experiment or two with her Light. And to make it sufficiently strong, I found it necessary to collect it first by a broad Lens, and then interposing a Prism between the Lens and its Focus, at such Distance that all the Light might pass through the Prism, I found the Focus, which before appeared like a lucid Point, to be drawn out into a long splendid Line by the Prism's Refraction.

Concerning the *second* Experiment, I have occasionally observed, that by covering both Ends of the Prism with Paper at several Distances from the Middle, the Breadth of the solar Image will be increased or diminished as much, as is the Aperture of the Prism, without any Variation of the Length: Or, if the Aperture be augmented on all Sides, the Image on all Sides will be so much and no more augmented.

Of the *third* Experiment I have occasion to speak in my Answer to another Person; where you will find the Effects of two Prisms, in all cross Positions of one to another, described. But if one Prism alone be turned about, the coloured Image will only be translated from Place to Place, describing a Circle, or some other conick Section on the Wall, on which it is projected, without suffering any Alteration in its Shape, unless such as may arise from the Obliquity of the Wall, or casual Change of the Prism's Obliquity to the Sun's Rays.

The Effect of the *fourth* Experiment I have already insinuated, telling you that Light passing through Parts of the Prism of divers Thicknesses, did still exhibit the same Phenomena.

*The Genuine
Method of examining this
Theory; by Mr.
Newton. N. 84.
p. 5024.
July, An. 1672.*

III. I cannot think it effectual for determining Truth, to examine the several Ways by which Phenomena may be explained, unless where there can be a perfect Enumeration of all those Ways. You know, the proper Method for enquiring after the Properties of Things, is to deduce them from Experiments. And I told you, that the *Theory* which I propounded, was evinced to me, not by inferring 'tis thus, because 'tis not otherwise; that is, not by deducing it only from a Confutation of contrary Suppositions, but by deriving it from Experiments concluding positively and directly. The Way therefore to examine it is, by considering, Whether the Experiments which I propound do prove those Parts of the *Theory* to which they are applied, or by prosecuting other Experiments which the *Theory* may suggest for its Examination? And this I would have done in a due Method; the Laws of Refraction being thoroughly enquired into and determined, before the Nature of Colours be taken into Consideration. It may not be amiss to proceed according to the Series of these Queries; which I could wish were determined by the Event of proper Experiments, declared by those that may have the Curiosity to examine them.

1. Whether Rays, that are alike incident on the same Medium, have unequal Refractions? And how great are the Inequalities of their Refractions at any Incidence?

2. What

2. What is *the Law* according to which each Ray is *more or less Refracted*; whether it be that the same Ray is ever Refracted according to the *same Ratio* of the Sines of Incidence and Refraction; and divers Rays, according to divers Ratio's; or that the Refraction of each Ray is greater or less without any certain Rule? That is, whether each Ray have a certain degree of Refrangibility, according to which its Refraction is performed; or is Refracted without that Regularity?

3. Whether *Rays*, which are endued with particular degrees of Refrangibility, when they are by any means separated, have particular *Colours constantly belonging to them*; viz. the least Refrangible, Scarlet; the most Refrangible, deep Violet; the middle, Sea-Green; and others, other Colours? And on the contrary?

4. Whether *the Colour* of any sort of *Rays apart* may be changed by Refraction?

5. Whether *Colours* by coalescing do really *Change* one another to produce a *New Colour*, or produce it by *Mixing* only?

6. Whether a *due Mixture of Rays*, indued with all variety of Colours, produces *Light perfectly like that of the Sun*, and which hath all the same properties, and exhibits the same Phenomena?

7. Whether the *Competent Colours* of each Mixture be really *Changed*; or be only *Separated*, when from that Mixture various Colours are produced again by Refraction?

8. Whether there be any *other Colours* produced by Refraction than such, as ought to result from the *Colours* belonging to the *diversly Refrangible Rays*, by their being separated or mixed by *that Refraction*?

To determine by Experiments these and such like Queries, which involve the propounded Theory, seems the most proper and direct way to a Conclusion. And therefore I could wish all Objections were suspended, taken from Hypotheses or any other Heads than these two; Of shewing the insufficiency of Experiments to determine these Queries, or prove any other parts of my Theory, by assigning the flaws and defects in my Conclusions drawn from them; or of producing other Experiments which directly contradict me, if any such may seem to occur. For if the Experiments, which I urge, be Defective, it cannot be difficult to shew the Defects; but if Valid, then by proving the Theory, they must render all Objections Invalid.

IV. 1. Istæ tam extraordinaria Hypothesis, quæ Dioptricæ fundamenta evertit, præterque hæcenus institutas inutiles reddit, tota nititur illo Experimento Prismatici Crystallini, ubi Radii per foramen fenestræ intra obscurum Cubiculum ingressi, ac deinde in Parietem impacti, aut in charta recepti, non in rotundum conformati, ut *Cl. Newtono*, ad Regulas Refractionum receptas, attendenti, expectandum videbatur, sed in Oblongam figuram extensi apparuerunt: Unde conclusit, Oblongam ejusmodi figuram ex eo esse, quod nonnulli Radij minus, nonnulli magis Refringerentur.

Sed mihi quidem videtur juxta communes & receptas Dioptricæ Leges figuram illam, non Rotundam, sed Oblongam esse oportere. Cum enim Radij

*Animadversiones
upon this Theo-
ry, by R. P.
Ign. Galton
Pardies. N. 84.
p. 4087.
June. An. 1672.*

ex oppositis disci Solaris partibus procedentes, variam habeant in ipso transitu Prismatis Inclinationem, varie quoque Refringi debent; ut cum unorum Inclinatione 30 saltem minutis major sit Inclinatione aliorum, major quoque evadat illorum Refractio.

Igitur Radii oppositi, ex altera superficie Prismatis Emergentes magis Divergunt & Divaricantur, quam si nullatenus, aut saltem æqualiter, omnes Infracti processissent. Refractio autem ista Radiorum fit solummodo versus eas partes, quæ fingi possunt in planis ad Axem Prismatis rectis; nulla autem Refractionis inæqualitas contingit versus eas partes, quæ intelliguntur in planis Axi parallelis; ut facile demonstrari potest: Superficies enim duæ Prismatis censi possunt inter se Parallelæ, ratione habita ad Inclinationem Axis, cum singulæ ipsi Axi Parallelæ sint. Refractio autem per duas Parallelas planas superficies nulla computatur, quia quantum à prima superficie Radius in unam partem torquetur, tantum ab altera in oppositam partem detorquetur. Igitur cum Radii Solares è foramine per Prisma transmissi ad latera quidem non frangantur, procedunt ulterius, perinde ac si nulla Prismatis superficies obstitisset, (habita, inquam, ratione solum ad lateralem illam Divaricationem;) at vero cum iidem Radii ad superiores seu inferiores partes, alij quidem magis, alij vero minus, utpote inæqualiter Inclinati, Infringantur; necesse est eos magis inter se Divaricari, adeoque & in Longiorem Figuram extendi.

Quin si Calculus rite obeatur; ut Radii laterales inventi sunt à *Cl. Newtono* in ea latitudine quæ subtendit Arcum 31. min. qui Arcus respondet Diametro Solis; ita nullus dubito, quin illa inventa quoque Altitudo Imaginis quæ 2 gr. & 49 min. subtendit, sit illa ipsa quæ eidem Diametro Solis post inæquales Refractiones in illo ipso casu respondeat.

Fig. 72.

Et revera, posito Prismate ABC, cujus Angulus A sit 60 gr. Radio DE, qui faciat cum perpendiculari EH, Angulum 30 grad. invenio illum, dum Emergit per FG, facere cum perpendiculari FI Angulum 76 gr. 22 min. At vero posito alio Radio dE, qui cum perpendiculari EH, faciat Angulum 29 gr. 30 min. invenio illum, dum Emergit per fg, facere cum perpendiculari fi, Angulum 78 gr. 45 min. Unde isti duo Radii DE, dE, qui procedere supponuntur ex oppositis partibus disci Solaris, faciuntque inter se Angulum 30 min. iidem dum Emergunt per Lineas Fg, fg, ita Divergunt ut constituent Angulum inter se 2 gr. 23 min. Quod si duo alii Radii assumerentur magis accedentes ad perpendicularem EH, (v. g. qui cum eadem perpendiculari facerent, unus quidem Angulum, 29 gr. 30 min. alter vero, 29 gr. 0 min.) tunc iidem Radij emergentes magis adhuc Divergerent, constituerentque Angulum majorem etiam aliquando plus quam trium Graduum. Et præterea augetur ulterius ista intercapedo Refractorum Radiorum ex eo, quod duo Radii DE, dE, concurrentes in E, illico incipiunt Divaricari, atque impingunt in duo puncta disjuncta alterius superficie, nempe in F & in f. Quapropter non sufficit ab obeundum rite Calculum, & Longitudine Imaginis impactæ in chartam subtrahere magnitudinem Foraminis Fenestræ; quandoquidem etiam posito Foramine Indivisibili E, adhuc fieret aliud veluti Foramen Latum in alia superficie, nempe Ff.

Quod

Quod etiam vocat *Experimentum Crucis*, mihi quidem videtur quadrare cum vulgaribus & receptis Refractionum Regulis. Nam, ut modo ostendi, Radii Solares qui accedentes & Convergentes faciunt Angulum 30 min. Egre-
dientes deinde etiam post Indivisibile Foramen, Divergunt in Angulum duo-
rum & trium Graduum. Quapropter non mirum, si isti Radii, figillatim im-
pingentes in alterum Prisma, perexiguo item apertum Foramine, inæqualiter
Infringantur, cum sit inæqualis illorum Inclinatio. Neque refert, quod isti
Radii attollantur aut deprimantur per conversionem primi Prismatis, manente
immoto secundo Prismate, (quod tamen in omni casu fieri non potest) vel
quod manente primo Immobili, secundum moveatur, ut successive Radios
Coloratos totius Imaginis excipiat, & per proprium Foramen transmittat; utro-
libet enim modo necesse est Radios illos extremos, hoc est, Rubrum & Vio-
laceum, incidere in secundum Prisma sub inæquali Angulo, adeoque eorun-
dem Refractionem esse inæqualem, ut Violaceorum sit major.

Cum igitur manifesta causa appareat Oblongæ ejusmodi Figuræ Radium, causaque illa ex ipsa natura Refractionis oriatur; non videtur necesse recurrere ad aliam Hypothesin, aut admittere diversam illam Radium Frangibili-
tatem.

Quod deinde excogitavit de *Coloribus*, illud quidem egregie consequitur ex præcedente Hypothesi; veruntamen nonnullas & ipsum patitur difficultates. Nam quod ait, nullum Colorem, sed potius Candorem apparere, ubi omnes omnium Colorum Radii promiscue confunduntur, id vero non videtur con-
forme omnibus Phænomenis. Certe quæ Variationes cernuntur in permistio-
ne diversorum corporum, diversis Coloribus imbutorum, eadem omnino ob-
servantur in permistione diversorum Radium diversis item Coloribus imbu-
torum: Atque optime ipse advertit, quod quemadmodum ex Flavio & Cæru-
leo corpore exsurgit Viridis Color; ita ex Flavio & Cæruleo Radio Viridis
item Color efficitur. Quare si omnes omnium Colorum Radii simul confun-
derentur, necesse esset in ista Hypothesi, ut ille Color appareret, qui revera ap-
paret in permistione omnium Pigmentorum. Atqui si ista, hoc est, Rubrum
simul & Flavum una cum Cæruleo & Purpureo aliisque omnibus, si quæ sint,
conterantur & confundantur, non jam Candidus, sed Obscurus & Satur Color
exsurget. Ergo similis Color appareret in Lumine Ordinario, quod constaret
ex aggregatione omnium Colorum.

Præterea nihil primo aspectu magis Ingeniosum magisque aptum videtur,
quam quod ait Circa Experimentum Acutissimi *Hookii*, quo duo diversi
Liquores, quorum alter Rubeus, alter Cæruleus, uterque figillatim Pelluci-
dus, simul permixti, Opaci evadunt. Id autem ait Clarissimus *Newtonus* ex
eo oriri, quod unus Liquor solos Rubeos natus sit transmittere, alter vero
solos Flavos; unde permixti nullos transmittent. Hoc, inquam, videtur sta-
tim valde oppositum; nihilominus tamen ex eo conficeretur, quod similis opa-
citas fieret in permistione quorumcunque Liquorum qui essent diversi Coloris;
quod tamen verum non est.

2. Refractiones à diversa parte Prismatis quantum potest Inæquales statuit,
R. P. *Pardies*, cum tamen ego tum in Experimentis, tum in Calculo de Ex-
perimentis istis inito, *Æquales* adhibuerim. Si autem A B C, Prismatis Se-

Answered by
Mr. Newton,
N. p. 84. 4291.
June. An. 1672

Fig. 73.

ctio ad Axem ejus Perpendicularis, FL, & KG, Radii duo in x (medio Foraminis) decussantes, & in Prisma illud Incidentes ad G, & L; sinque eorum Refracti GH, & LM, ac denuo HI, & MN. Et cum Refractiones ad Latus AC, æquales esse Refractionibus ad Latus BC, quam proxime supposuerim; si AC, & BC, statuantur æqualia, similis erit Radiorum GH, & LM, ad AB, basin Prismatis Inclinatio; adeoque Ang. CLM = Ang. CHG, & Ang. CML = Ang. CGH. Quare etiam Refractiones in G, & M, æquales erunt, ut & in L, & H; atque adeo Ang. KGA = Ang. NMB, & Ang. FLA = Ang. BHI; & proinde Refractorum HI, & MN, eadem erit ad invicem inclinatio ac est Incidentium Radiorum FL, & KG. Sit ergo Angulus FxK, 30 min. æqualis nempe Solari Diametro, & erit Angulus, quem HI, & MN, comprehendunt, etiam 30 min. si modo Radii FL, & KG, æqualiter Refrangibiles statuantur. At mihi Experimenti prodiit Angulus ille circiter 2 gr. 49 min. quem Radius HI, extremum Violaceum Colorem, & MN, Cæruleum exhibens, constituere; ac proinde Radios illos diversimode Refrangibiles esse, sive Refractiones secundum Disparem sinuum Incidentiæ & Refractionis Rationem peragi necessario concedendum est.

Addit præterea R. P. quod non sufficit ad obeundum rite Calculum, ex Longitudine Imaginis impacta in Chartam subtrahere magnitudinem Foraminis Fenestræ; quandoquidem etiam posito Foramine Indivisibili, adhuc fieret aliud veluti Foramen Latum in posteriori superficie Prismatis. Mihi tamen videtur, his non obstantibus, quod Refractiones Radiorum, in anteriori æque ac posteriori superficie Prismatis decussantium, ex adhibitis Principiis possint rite computari. Sed si res secus esset latitudo hiatus in posteriori superficie, quod ad instar Foraminis est, haud efficeret errorem duorum minorum secundorum; & in rebus practicis non operæ pretium duco ad minutias istas attendere.

Illi insuper *Experimento*, quod *Crucis* vocaveram, nihil adversatur R. P. dum contendit, inæquales Radiorum, diversis Coloribus imbutorum, Refractiones ex Inæqualibus Incidentiis effectas fuisse. Nam Radius pro duo admodum parva, ab invicem distantia, & immota Foramina, transeuntibus, Incidentia illæ, prout ergo Experimentum institui, omnino Æquales erant, & tamen Refractiones liquido Inæquales. Sin ille de Experimentis nostris dubitet, oro, ut Radiorum diversis Coloribus præditorum Refractiones ex Incidentiis paribus mensuret, & sentiet inæquales esse. Si modus ille, quem ego ad hoc negotium adhibui minus placeat (quo tamen nullus potest esse Luculentior, facile est alios excogitare; sicut & alios ipse haud paucos cum fructu expertus sum.

Contra *Theoriam* de Coloribus objicitur, quod Pulveres diversorum Colorum permisti, non Candidum sed Subobscurum & Fuscum Colorem exhibent. Mihi vero Albus, Niger & omnes intermediæ Fusci, qui ab Albo & Nigro permistis componi possunt, non specie Coloris sed Quantitate Lucis tantum differre videntur. Et cum in mixture Pigmentorum, singula corpuscula non nisi proprium Colorem Reflectant, adeoque maxima pars Lucis incidentis supprimatur & retineatur; Lux Reflexa Subobscura evadet, & quasi cum tenebris permista, adeo ut non intensum Alborem, sed qualem Nigredinis permistio conficit, hoc est Fuscum, exhibere debeat.

Objicitur

Objicitur deinde, quod à Liqueoribus quibuscunque diversi Coloris in eodem vase commistis, æque ac in diversis vasis contentis, opacitas oriri debet; quod tamen, ait, verum non esse. Sed non video consequentiam. Nam plurimi Liqueores agunt in se invicem & novam sibi mutuo partium contexturam secreto inducunt; unde Opaci, Diaphani, vel variis Coloribus, ex Coloribus permistorum nullo modo oriundis, præditi evadere possunt. Et hac de causa Experimenta hujusmodi minus apta semper existimavi, à quibus conclusiones deduci possint. Subnoto tamen, quod ad hoc Experimentum requiruntur Liqueores saturis & intensis Coloribus præditi, qui per paucos nisi proprii Coloris Radios transmittant; quales raro occurrunt, ut videbitur illuminando Liqueores cum diversis Coloribus Prismatis in Obscurato Cubiculo. Nam pauci reperientur, qui in propriis Coloribus satis Diaphani appareant, inque alienis Opaci. Convenit præterea, ut adhibiti Colores sint inter se oppositi, quales existimo fore Rubrum & Cæruleum, vel Flavum & Violaceum, vel etiam Viridem & Purpureum illum qui Coccineo affinis est. Et ex hujusmodi Liqueoribus nonnulli (quorum partes tingentes non congregiuntur) fortasse permisti evadunt Opaciores. Sed de eventu nihil sum sollicitus, tum quod luculentius est Experimentum in Liqueoribus seorsim existentibus, tum quod Experimentum illud (sicut & Iridis, Tincturæ Nephriticæ, & aliorum Corporum naturalium Phænomena) non ad Probandam sed ad illustrandam tantum Doctrinam proposui.

Quod R. P. *Theoriam* nostram *Hypothesin* vocat, amice habeo, siquidem ipsi nondum constet. Sed alio tamen consilio proposueram, & nihil aliud continere videtur quam proprietates quasdam Lucis, quas jam inventas probare haud difficile existimo, & quas si non veras esse cognoveram, pro futuri & inani speculatione mallet repudiare, quam pro mea *Hypothesi* agnoscere.

3. In ea *Hypothesi*, quam fuse explicat noster *Grimaldus*, in qua supponitur Lumen esse Substantia quædam rapidissime mota, posset fieri aliqua diffusio Luminis post transitum Foraminis, & decussationem Radium; Item in ea *Hypothesi*, qua Lumen ponitur progredi per certas quasdam Materię Subtilis Undulationes, ut explicat Subtilissimus *Hookius*, possunt explicari Colores per certam quandam *diffusionem atque Expansionem Undulationum, qua fiat ad latera Radium ultra Foramen, ipso Contagio ipsaque materia continuatione*. Certe ego talem adhibeo *Hypothesin* in Dissertatione de *Motu Undulationis*, quæ est *Sexta Pars meorum Mechanicorum*; ut ponam, Colores istos apparentes fieri ex sola illa Communicatione Motionis, quæ ab Undulationibus directe procedentibus ad latera effundatur: Ut si Radii intrantes per Foramen *a*, progrediantur versus *b*, Undulationes quidem directe terminari deberent (habendo rationem ad motum Rectum & Naturalem) ad lineam rectam *ab*; nihilominus tamen, propter continuitatem materiæ, sit aliqua communicatio commotionis versus *Latera &c.*, ubi tremula quædam & crispans succussio excitatur: Atque si in illa laterali crispatione consistere Colores supponatur, existimo omnia Phænomena Colorum explicari posse, ut fusius in ea, quam dixi, dissertatione expono. Quibus item positis apparet etiam, cur ultra quam ferat Radium ipsorum Divaricatio, expandi Colorum Latitudinem necesse sit.

Circa *Experimentum Crucis*, nequaquam dubito, quo minus in suo Experimento talem situm adhibuerit, in quo æqualis Inclinatio fuerit Radium inci-

Some further
Objections by
R. P. Pardies.
N. 85. p. 5012.
July. An. 1672.

Fig. 74.

dentium,

dentium, quandoquidem id ita à se præstitum expresse affirmat. Verum id non ego poteram conijcere ex iis quæ superius Legeram; ubi ponuntur duo exigua & maxime distantia Foramina, & unum Prisma prope primum Foramen quod est in Fenestra; per quod Prisma Radii Colorati erumpentes, Incidunt in alterum distans Foramen: Addebatur autem, quod ad hoc, ut omnes illi Radii successive Inciderent in Secundum illud Foramen, convertebatur Primum Prisma supra Axem: Atqui hoc modo necesse est mutari Inclinationem Radiorum, qui Incidunt in secundum Foramen: Atque indicavi ego, quod perinde sese res haberet, siue manente Primo Prismate Immobili, Secundum Foramen attolleretur aut deprimeretur, ut possit successive Radios omnes depictæ Imaginis Solaris excipere; siue manente isto Secundo Foramine Immobili, Primum Prisma converteretur, ut ita eadem Imago situm mutaret, atque in Foramen impingere Secundum omnes successive partes posset. Sed alias sine dubio adhibuit cautiones Solertissimus *Newtonus*.

Quæ circa Colores objeceram, optime soluta existimo. Quod autem *Theoriam* istam, appellarim *Hypothesin*, id certe ego nullo adhibito consilio feci; atque nomen usurpavi quod primum occurrit: quapropter velim ut ne per contemptum adhibitam vocem ejusmodi existimet.

Answered by Mr.
Newton. *Ibid.*
p. 5014.
July. An. 1672.

4. Ait R. P. quod absq; varia Diverforum Radiorum Refrangibilitate, possibile sit explicare Longitudinem Colorum; puta ex Hypothesi *P. Grimaldi*, per diffusionem Luminis, quod supponitur esse substantia quædam rapidissime mota, vel ex Hypothesi *Hookij* nostri, per diffusionem vel expansionem Undulationum, quas statuit in æthere à Lucidis corporibus excitatus quaquaversum propagari. Addo quod ex Hypothesi *Cartesiana* potest etiam effingi consimilis diffusio conatus vel pressionis Globulorum, perinde ut in explicatione Caudæ Cometæ supponitur. Et eadem Diffusio vel Expansio juxta aliam quamvis Hypothesin, in qua Lumen statuitur esse Vis, Actio, Qualitas, vel Substantia quælibet à Luminosis corporibus undique emissa, effingi potest.

Ut his respondeam, animadvertendum est, quod Doctrina illa, quam de Refractione & Coloribus explicui, in quibusdam Lucis proprietatibus solummodo constitit, neglectis Hypothesibus per quas proprietates illæ explicari debent. Quamobrem ab Hypothesium contemplatione, tanquam improprio Argumentandi loco, hic abstinendum esse censui, & Vim Objectionis abstrahendam, ut plenior & magis generalem responsionem accipiat.

Itaque per Lumen intelligo quodlibet Ens vel Entis potestatem (siue sit Substantia, siue quævis ejus Vis, Actio, vel Qualitas) quod à corpore lucido recta pergens aptum sit ad excitandam Visionem; & per Radios Luminis intelligo minimas vel quælibet indefinite parvas ejus partes quæ ab invicem non dependent, quales sunt illi omnes Radii, quos Lucentia corpora vel simul vel successive secundum Rectas Lineas emittunt. Nam illæ tum collaterales tum successive partes Luminis sunt independentes; siquidem unæ absque aliis intercipi possint, & in quælibet plagis seorsim Reflecti vel Refringi. Et hoc præcognito, Objectionis Vis omnis in eo sita erit; quod Colores per aliquam Luminis ultra Foramen diffusionem, quæ non oritur ab inæquali diverforum Radiorum (seu Luminis independentium partium Refrangibilitate, in Longum diduci possint.

Quod

Quod autem non aliunde Oblongentur superius monstravi : & ut omnia summe confirmarem, adjeci *Experimentum* illud quod jam nomine *Crucis* passim insignitur : de cujus conditionibus cum R. P. dubitaverit, placuit jam designare Schemate. Sit BC Anterior Tabula, cui Prisma A, immediate præfigitur, sitque DE Altera Tabula, quasi 12 pedibus abinde distans, cui suffigitur alterum Prisma F. Tabulæ autem ad x & y, ita perforentur, ut aliquantulum Lucis ab anteriori Prismate Refractæ trajici possit per utrumque Foramen ad Secundum Prisma, inque eo denuo Refringi. Jam Prisma anterius circa Axem reciproco motu convertatur, & Colores in Tabulam Posterio-rem DE procidentes, per vices attollentur ac deprimuntur, eoque pacto alius atque alius Color successive pro arbitrio trajici potest per Foramen ejus y, ad Posterius Prisma, dum cæteri Colores in Tabulam impingunt : Et videbis, Radios diversis Coloribus præditos diversam pati Refractionem in illo posteriori Prismate, ex eo quod ad diversa loca parietis vel cujusvis obstaculi GH, pedibus aliquot ulterius remoti, allabentur; puta Violacei Radii ad H, Rubri ad G, & Intermedii ad loca intermedia : & tamen propter determinatam Positionem foraminum necesse est ut Similis sit Incidentia Radiorum cujusque Coloris per utrumque trajecti. Atque ita ex mensura constat Radios, diversis Coloribus affectos, habere diversas Leges Refractionum.

Fig. 71.

Sed suspicor unde adductus sit R. P. in dubitationem; nempe videtur collocasse Primum Prisma A, post Tabulam BC, atque ita convertendo circa Axem, verisimile est Inclinationem Radiorum qui interjacent Foramina propter Intermediam Refractionem fuisse mutatam. At, ex descriptione prius exposita, debuit Tabula illa collocari post Prisma, ut Radii inter Foramina in directum jacerent, quemadmodum, ex verbis, *I took two Boards and placed one of them close behind the Prism at the Window*, constare potest. Et usus Experimenti idem innuit.

Ex abundanti placet observare, quod in hoc Experimento Colorata Lux ob Refractionem Secundi Prismatis longe minus diffunditur ac divaricat, quam cum Alba existit, adeo ut Imago ad G, vel H, sit pene Circularis; præsertim si Prismata statuuntur parallela, & in contrario situ Angulorum, prout in Schemate designantur. Quinetiam, si præterea diameter foraminis y, adæquet Latitudinem Colorum, nulla erit ejusdem Coloratæ lucis in Longum diffusio; sed Imago, quæ à quopiam Colore ad G, vel H, effingitur, (positis Circularibus foraminibus, & Refractione posterioris Prismatis non majori quam prioris, Radiisque ad obstaculum quam proxime perpendicularibus) erit plane Circularis. Id quod arguit diffusionem, de qua supra egimus, non ex contagione vel continuitate materiæ undulantis aut celerrime motæ vel similibus causis ortam esse, sed ex certa Refractionum cujusque Generis Radiorum Lege. Cur autem Imago illa in uno casu sit Circularis, & in aliis nonnihil Oblongata, & quomodo diffusio lucis in Longitudinem in quolibet casu pro arbitrio minui possit, à Geometris determinandum, & cum experientia conferendum, relinquo.

Postquam proprietates Lucis his & similibus Experimentis satis exploratæ fuerint, spectando Radios tanquam ejus sive collaterales sive successivas partes, de quibus experti sumus per independentiam quod sint ab invicem distinctæ;

Hypotheses

Hypotheses exinde dijudicandæ sunt, & quæ non possunt conciliari rejiciendæ. Sed levissimi negotij est accommodare Hypotheses ad hanc Doctrinam. Nam si quis Hypothesin Cartesianam defendere velit, dicendum est, Globulos esse inequales; vel pressiones Globulorum esse alias aliis fortiores, & inde diversimode Refrangibiles, & aptas ad excitandam sensationem diversorum Colorum. Et sic juxta Hypothesin Cl. *Hookii* dicendum est, Undulationes *Ætheris* esse alias majores sive crassiores aliis. Atque ita in cæteris. Hæc enim videtur esse summe necessaria Lex & Conditio Hypothesium, in quibus naturalia corpora ponuntur constare ex quamplurimis corpusculis acervatim contextis, ut à diversis Lucentium corpusculis, vel ejusdem corpusculi diversis partibus (prout Motu, Figura, Mole, aut aliis Qualitatibus differunt) inæquales pressiones, motiones aut mota corpuscula per *Æthera* quaquaversum trajiciantur, ex quibus confuse mistis, Lux constitui supponetur. Et nihil durius esse potest in istis Hypothesibus quam contraria suppositio.

Ex Apertura sive Dilatatione Lucis in posteriori facie Prismatis, quam R. P. dixit esse veluti Foramen, sufficit, quod error non emerget sensibilis, si modo aliquis emergeret. Quod si Calculus juxta observationes præcise ineatur, error erit nullus. Nam diametro foraminis à Longitudine Imaginis subducta, restabit Longitudo quam Imago haberet, si modo foramen ante Prisma esset indivisibile, idque non obstante præfata Lucis dilatatione in posteriori facie Prismatis; ut facile ostenditur. Deinde ex data illa Longitudine Imaginis, ac distantia à foramine indivisibili, ut & positione & forma Prismatis, & ad id inclinatione Incidentium Radiorum, ac angulo, quem Refracti Radii ad medium imaginis tendentes, cum à centro Solis incidentibus constituunt, cætera omnia determinantur. Et quæ determinant Refractiones & Positiones Radiorum, sufficiunt ad Calculum istarum Refractionum rite ineundum. Sed res non tanti esse videtur ut moram inferat.

Quod R. P. Doctrinam nostram Hypothesin vocaverit, non aliunde factum esse credo, quam quod vocabulum usurpavit quod primum occurrit; siquidem mos obtinuit ut quicquid exponitur in Philosophia dicatur Hypothesis. Et ego sanè non alio Consilio vocabulum istud reprehendi, quam ut ne invalesceret appellatio quæ recte Philosophantibus præjudicio esse posset.

To the Satisfaction of P. Ig. Galt. Pardies. N. 85, p. 5018. July, An. 1672.

5. Omnino mihi satisfecit novissima responsio, à D. *Newtono* ad meas Instantias data. Novissimus Scrupulus, qui mihi hærebat circa *Experimentum Crucis*, penitus fuit exemptus. Atque nunc plane ex Figura ipsius intelligo, quod non intellexeram ante. Experimentum peractum cum fuerit isto modo, nil habeo quod in eo desiderem amplius.

Some Considerations upon this Theory; by Mr. Newton. N. 88, p. 5086. Nov. An. 1672.

V. The Consideration on my Theories consist in ascribing an Hypothesis to me, which is not mine; in Asserting an Hypothesis, which, as to the principal Parts, is not against me; in Granting the greatest part of my Discourse, if explicated by that Hypothesis; and in Denying some things, the truth of which would have appear'd by an Experimental Examination.

Of these particulars I shall discourse in order. And first of the Hypothesis, which is ascribed to me in these Words: *But grant this first Supposition, that Light is a Body, and that as many Colours or Degrees as there may be, so many Bodies there*

there may be; all which compounded together would make White, &c. This, it seems, is taken for my Hypothesis. 'Tis true, that from my *Theory* I argue the Corporeity of Light; but I do it without any absolute Positiveness, as the Word *perhaps* intimates; and make it at most but a very plausible Consequence of the Doctrine, and not a fundamental Supposition, nor so much as any part of it; which was wholly comprehended in the precedent Propositions. And I somewhat wonder, how the Objector could imagine, that, when I had asserted the *Theory* with the greatest Rigour, I should be so forgetful as afterwards to assert the fundamental Supposition itself with no more than a *perhaps*. Had I intended any such Hypothesis, I should somewhere have explained it: But I knew, that the Properties, which I declared of Light, were in some measure capable of being explicated, not only by that, but by many other mechanical Hypotheses; and therefore I chose to decline them all, and to speak of Light in general Terms, considering it abstractly, as something or other propagated every way in streight Lines from luminous Bodies, without determining what that thing is; whether a confused Mixture of different Powers, or Beings whatsoever. And for the same Reason I chose to speak of Colours according to the Information of our Senses, as if they were Qualities of Light without us. Whereas by that Hypothesis, I must have consider'd them rather as Modes of Sensation, excited in the Mind by various Motions, Figures, or Sizes of the Corpuscles of Light, making various mechanical Impressions on the Organ of Sense; as I express'd it in that Place, where I spake of the Corporeity of Light.

But supposing I had propounded that Hypothesis, I understand not why the Objector should so much endeavour to oppose it: For certainly it has a much greater Affinity with his own Hypothesis, than he seems to be aware of; the Vibrations of the *Æther* being as useful and necessary in this, as in his; for assuming the Rays of Light to be small Bodies, emitted every way from shining Substances, those, when they impinge on any refracting or reflecting Surfaces, must as necessarily excite Vibrations in the *Æther*, as Stones do in Water when thrown into it. And supposing these Vibrations to be of several Depths or Thicknesses, accordingly as they are excited by the said corpuscular Rays of various Sizes and Velocities; of what Use they will be for explicating the Manner of Reflection or Refraction, the Production of Heat by the Sun-Beams, the Emission of Light from Burning, Putrifying, or other Substances, whose Parts are vehemently agitated, the Phænomena of thin transparent Plates and Bubbles, and of all natural Bodies, the Manner of Vision, and the Difference of Colours, as also their Harmony and Discord; I shall leave to their Consideration, who may think it worth their Endeavour, to apply this Hypothesis to the Solution of Phænomena.

In the *second Place*, I told you, That the Objector's Hypothesis, as to the fundamental Part of it, is not against me. That fundamental Supposition is; *That the Parts of Bodies, when briskly agitated, do excite Vibrations in the Æther, which are propagated every way from those Bodies in streight Lines, and cause a Sensation of Light by beating and dashing against the Bottom of the Eye,*

Something after the manner, that Vibrations in the Air cause a Sensation of Sound by beating against the Organs of Hearing. Now, the most free and natural Application of this Hypothesis, to the Solution of Phænomena, I take to be this : That the agitated Parts of Bodies, according to their several Sizes, Figures and Motions, do excite Vibrations in the Æther of various Depths, or Bignesses, which being promiscuously propagated through that Medium to our Eyes, effect in us a Sensation of Light of a white Colour; but if by any means those of unequal Bignesses be separated from one another, the largest beget a Sensation of a red Colour; the least or shortest, of a deep Violet; and the intermediate ones, of intermediate Colours; much after the manner that Bodies, according to their several Sizes, Shapes and Motions, excite Vibrations in the Air of various Bignesses, which, according to those Bignesses, make several Tones in Sound: That the largest Vibrations are best able to overcome the Resistance of a refracting Superficies, and so break through it with least Refraction; whence the Vibrations of several Bignesses, that is, the Rays of several Colours, which are blended together in Light, must be parted from one another by Refraction, and so cause the Phænomena of Prisms, and other refracting Substances: And that it depends upon the Thickness of a thin transparent Plate or Bubble, whether a Vibration shall be reflected at its further Superficies, or transmitted; so that, according to the Number of Vibrations interceding the two Superficies, they may be reflected or transmitted for many successive Thicknesses. And since the Vibrations which make Blue and Violet, are supposed shorter than those which make Red and Yellow, they must be reflected at a less Thickness of the Plate: Which is sufficient to explicate all the ordinary Phænomena of those Plates or Bubbles, and also of all natural Bodies, whose Parts are like so many Fragments of such Plates.

These seem to be the most plain, genuine, and necessary Conditions of this Hypothesis. And they agree so justly with my Theory, that if the Animadversor think fit to apply them, he need not, on that Account, apprehend a Divorce from it. But yet how he will defend it from other Difficulties, I know not; for to me the fundamental Supposition itself seems impossible, namely, that the Waves or Vibrations of any Fluid, can, like the Rays of Light, be propagated in streight Lines, without a continual and very extravagant spreading and bending every way into the quiescent Medium, where they are terminated by it. I mistake, if there be not both Experiment and Demonstration to the contrary. And as to the other two or three Hypotheses, which he mentions, I had rather believe them subject to the like Difficulties, than suspect the Animadversor should select the worst for his own.

What I have said of this, may be easily applied to all other mechanical Hypotheses, in which Light is supposed to be caused by any Pression or Motion whatsoever, excited in the Æther by the agitated Parts of luminous Bodies; for it seems impossible that any of those Motions or Pressions can be propagated in streight Lines, without the like spreading every way into the shadowed Medium on which they border. But yet, if any Man can think it possible, he must at least allow, that those Motions, or Endeavours to Motion, caused in the Æther by the several Parts of any lucid Body, that differ

fer in Size, Figure, and Agitation, must necessarily be unequal: Which is enough to denominate Light an Aggregate of *Difform Rays*, according to any of those Hypotheses. And if those original Inequalities may suffice to difference the Rays in Colour and Refrangibility, I see no reason, why they, that adhere to any of those Hypotheses, should seek for other Causes of these Effects, unless (to use the Objector's Argument) *they will multiply Entities without Necessary.*

The *third* Thing to be consider'd is, the Condition of the Animadverfor's Concessions, which is, That I would explicate my Theories by his Hypothesis: And if I could comply with him in that Point, there would be little or no Difference between us: For he grants, That, without any respect to a different Incidence of Rays, there are different Refractions; but he would have it explicated, not by the different Refrangibility of several Rays, but by the splitting and rarefying *Æthereal Pulses*. He grants my third, fourth, and sixth Propositions; the Sense of which is, That uncompounded Colours are unchangeable, and that compounded Ones are changeable only by resolving them into the Colours, of which they are compounded; and that all the Changes, which can be wrought in Colours, are effected only by variously mixing or parting them: But he grants them on Condition, that I will explicate Colours by the two Sides of a split Pulse, and so make but two Species of them, accounting all other Colours in the World to be but various Degrees and Dilutings of those two. And he further grants, that Whiteness is produced by the Convention of all Colours; but then I must allow it to be not only by Mixture of those Colours, but by a farther Uniting of the Parts of the Ray supposed to be formerly split.

If I would proceed to examine these his Explications, I think it would be no difficult Matter to shew, that they are not only insufficient, but in some respects to me (at least) unintelligible: For though it be easy to conceive, how Motion may be dilated, and spread, or how parallel Motions may become diverging; yet I understood not, by what Artifice any linear Motion can by a refracting Superficies be infinitely dilated and rarefied, so as to become superficial: Or, if that be supposed, yet I understand as little, why it should be split at so small an Angle only, and not rather spread and dispersed through the whole Angle of Refraction. And further, though I can easily imagine, how unlike Motions may cross one another; yet I cannot well conceive, how they should coalesce into one uniform Motion, and then part again, and recover the former Unlikeness; notwithstanding that I conjecture the Ways, by which the Animadverfor may endeavour to explain it. So that the direct, uniform, and undisturb'd Pulses should be split and disturbed by Refraction; and yet the oblique and disturbed Pulses persist without splitting, or further Disturbance, by following Refractions, is (to me) as unintelligible, and there is as great a Difficulty in the Number of Colours; as you will see hereafter.

But whatever be the Advantages or Disadvantages of this Hypothesis, I hope I may be excused from taking it up, since I do not think it needful to explicate my Doctrine by any Hypothesis at all: For if Light be consider'd

abstractedly without respect to any Hypothesis, I can as easily conceive, that the several Parts of a shining Body may emit Rays of different Colours and other Qualities, of all which Light is constituted, as that the several Parts of a false or uneven String, or of unevenly agitated Water in a Brook or Cataract, or the several Pipes of an Organ inspir'd all at once, or all the Variety of sounding Bodies in the World together, should produce Sounds of several Tones, and propagate them thro' the Air confusedly intermix'd. And if there were any natural Bodies that could reflect Sounds of one Tone, and stifle or transmit those of another; then, as the Echo of a confused Aggregate of all Tones would be that particular Tone, which the echoing Body is disposed to reflect; so, since (even by the Animadverfor's Concessions) there are Bodies apt to reflect Rays of one Colour, and stifle or transmit those of another; I can as easily conceive, that those Bodies, when illuminated by a Mixture of all Colours, must appear of that Colour only which they reflect.

But when the Objector would insinuate a Difficulty in these Things, by alluding to Sounds in the String of a musical Instrument before Percussion, or in the Air of an Organ-Bellows before its Arrival at the Pipes; I must confess, I understand it as little, as if one had spoken of Light in a Piece of Wood before it be set on Fire, or in the Oyl of a Lamp before it ascend up the Match to feed the Flame.

You see therefore how much it is besides the Business in Hand to dispute about Hypotheses. For which Reason I shall now, in the last Place, proceed to abstract the Difficulties in the Animadverfor's Discourse, and without having regard to any Hypothesis, consider them in general Terms. And they may be reduced to these three Queries:

1. *Whether the unequal Refractions, made without respect to any Inequality of Incidence, be caused by the different Refrangibility of several Rays; or by the splitting, breaking, or dissipating the same Ray into diverging Parts?*
2. *Whether there be more than two Sorts of Colours?*
3. *Whether Whiteness be a Mixture of all Colours?*

The first of these *Queries* you may find determined above by an Experiment: The Design of which was to shew, that the Length of the colour'd Image proceeded not from any Unevenness in the Glass, or any other contingent Irregularity in the Refractions. Amongst other Irregularities, I know not, what is more obvious to suspect, than a fortuitous Dilating and Spreading of Light after some such manner, as *Des Cartes* has described in his *Aethereal Refractions* for explicating the Tail of a Comet; or as the Animadverfor now supposes to be effected by the splitting and rarefying of his aethereal Pulses. And to prevent the Suspicion of any such Irregularities, I told you, that I refracted the Light contrary ways, with two Prisms successively, to destroy thereby the regular Effects of the first Prism by the second, and to discover the irregular Effects by augmenting them with iterated Refractions. Now, amongst other Irregularities, if the first Prism had spread and dissipated every Ray into an indefinite Number of diverging Parts, the second should in like manner

manner have spread and dissipated every one of those Parts into a further indefinite Number, whereby the Image would have been still more dilated, contrary to the Event. And this ought to have happen'd, because those linear diverging Parts depend not on one another for the manner of their Refraction, but are every one of them as truly and compleatly Rays, as the whole was before its Incidence; as may appear by intercepting them severally.

The Reasonableness of this Proceeding, will, perhaps, better appear by acquainting you with this further Circumstance. I sometimes placed the second Prism in a Position transverse to the first, on Design to try if it would make the long Image become four-square, by Refractions crossing those that had drawn the round Image into a long one. For if, amongst other Irregularities, the Refraction of the first Prism, did, by splitting, dilate a linear Ray into a superficial; the cross Refractions of that second Prism ought, by further splitting, to dilate and draw that superficial Ray into a pyramidal Solid. But, upon Tryal, I found it otherwise; the Image being as regularly oblong as before, and inclined to both the Prisms at an Angle of 45 degrees.

I tried also all other Positions of the second Prism, by turning the Ends about its middle Part; and in no case could observe any such Irregularity. The Image was ever alike inclined to both Prisms, its Breadth answering to the Sun's Diameter, and its Length being greater or less, according as the Refractions more or less agreed or contradicted one another.

And by these Observations, since the Breadth of the Image was not augmented by the cross Refraction of the second Prism, that Refraction must have been performed without any splitting or dilating of the Ray; and therefore at least the Light incident on that Prism must be granted an Aggregate of Rays *Unequally Refrangible* in my Sense. And since the Image was equally inclined to both Prisms, and consequently the Refractions alike in both, it argues, that they were performed according to some constant Law, without any Irregularity.

To determine the *second Query*, The Animadverfor refers to an Experiment made with two Wedge-like Boxes, recited in the *Micrography* of the Ingenious Mr. Hook, *Observ.* 10. *Pag.* 73. The Design of which was to produce all Colours out of a Mixture of two. But there is, I conceive, a double Defect in this Instance: For it appears not, that by this Experiment all Colours can be produced out of two; and if they could, yet the Inference would not follow.

That all Colours cannot by that Experiment be produced out of two, will appear by considering, that the Tincture of Aloes, which afforded one of those Colours, was not all over of one uniform Colour, but appear'd Yellow near the Edge of the Box, and Red at other Places where it was thicker; affording all Variety of Colours, from a pale Yellow to a deep Red or Scarlet, according to the various Thickness of the Liquor. And so the Solution of Copper, which afforded the other Colour, was of various Blues and Indicoes. So that instead of two Colours, here is a great Variety made use of for the Production of all others. Thus, for Instance, to produce all Sorts of Greens, the several Degrees of Yellow and pale Blue must be mix'd; but to compound Purples, the Scarlet and deep Blue are to be the Ingredients. Now

Now if the Animadverfor contend, that all the Reds and Yellows of the one Liquor, or Blues and Indicoes of the other, are only various Degrees and Dilutings of the same Colour, and not divers Colours, that is a begging of the Question: And I should as soon grant, that the two Thirds or Sixths in Musick are but several Degrees of the same Sound, and not divers Sounds. Certainly it is much better to believe our Senses, informing us, that Red and Yellow are divers Colours; and to make it a philosophical Query, Why the same Liquor doth, according to its various Thickness, appear of those divers Colours, than to suppose them to be the same Colour, because exhibited by the same Liquors? For if that were a sufficient Reason, then Blue and Yellow must also be the same Colour, since they are both exhibited by the same Tincture of Nephritick Wood. But that they are divers Colours, you will more fully understand by the Reason, which in my Judgment is this: The Tincture of Aloes is qualified to transmit most easily the Rays indued with Red, most difficultly the Rays indued with Violet, and with intermediate Degrees of Facility, the Rays indued with intermediate Colours. So that where the Liquor is very thin, it may suffice, to intercept most of the Violet, and yet transmit most of the other Colours; all which together must compound a middle Colour, that is, a faint Yellow. And where it is so much thicker, as also to intercept most of the Blue and Green, the remaining Green, Yellow and Red, must compound an Orange. And where the Thickness is so great, that scarce any Rays can pass through it besides those indued with Red, it must appear of that Colour, and that so much the deeper, and obscurer, by how much the Liquor is thicker. And the same may be understood of the various Degrees of Blue, exhibited by the Solution of Copper, by reason of its Disposition to intercept Red most easily, and transmit a deep Blue or Indico Colour most freely.

But supposing that all Colours might, according to this Experiment, be produced out of two by Mixture; yet it follows not, that those two are the only original Colours; and that for a double Reason: First, Because those two are not themselves original Colours, but compounded of others; there being no Liquor, nor any other Body in Nature, whose Colour in Day-light is wholly uncompounded. And then, because though those two were original, and all others might be compounded of them, yet it follows not that they cannot be otherwise produced. For I said, that they had a double Origin, the same Colours to Sense being in some Cases compounded, and in others uncompounded; and sufficiently declared in my *third* and *fourth Propositions*, and in the *Conclusion*, by what Properties the one might be known and distinguish'd from the other. But because I suspect, by some Circumstances, that the Distinction might not be rightly apprehended, I shall once more declare it, and further explain it by Examples.

That Colour is primary or original, which cannot by any Art be changed, and whose Rays are alike refrangible: And that compounded, which is changeable into other Colours, and whose Rays are not alike refrangible. For Instance, To know whether the Colour of any green Object be compounded or not, view it through a Prism, and if it appear confus'd, and the Edges tinged

tinged with Blue, Yellow, or any Variety of other Colours, then is that Green compounded of such Colours as at its Edges emerge out of it: But if it appear distinct, and well defined, and entirely Green to the very Edges, without any other Colours emerging, it is of an original and uncompounded Green. In like manner, if a refracted Beam of Light being cast on a white Wall exhibit a green Colour, to know whether that be compounded, refract the Beam with an interposed Prism, and if you find any Difformity in the Refractions, and the Green be transformed into Blue, Yellow, or any Variety of other Colours, you may conclude that it was compounded of those that emerge: But if the Refractions be uniform, and the Green persist without any Change of Colour, then is it original and uncompounded. And the Reason why I call it so is, because a Green indued with such Properties cannot be produced by any mixing of other Colours.

Now if two green Objects may to the naked Eye appear of the same Colour, and yet one of them through a Prism seem confused and variegated with other Colours at the Edges, and the other distinct and entirely green; or if there may be two Beams of Light, which falling on a white Wall, do to the naked Eye exhibit the same green Colour, and yet one of them, when transmitted through a Prism, be uniformly and regularly refracted, and retain its Colour unchanged, and the other be irregularly refracted, and made to divaricate into a Multitude of other Colours: I suppose these two Greens will in both Cases be granted of a different Origin and Constitution. And if by mixing Colours, a Green cannot be compounded with the Properties of the unchangeable Green, I think I may call that an uncompounded Colour, especially since its Rays are alike refrangible, and uniform in all Respects.

The same Rule is to be observed in examining, whether Red, Orange, Yellow, Blue, or any other Colour, be compounded or not. And, by the way, since all white Objects through the Prism appear confused and terminated with Colours, Whiteness must, according to this Distinction, be ever compounded; and that the most of all Colours, because it is the most confused and changed by Refractions.

There now remains the *third* Query to be consider'd, which is, Whether Whiteness can be an uniform Colour, or a dissimilar Mixture of all Colours? The Experiment which I brought to decide it, the Animadversor thinks may be otherwise explained, and so concludes nothing. But he might easily have satisfied himself by trying what would be the Result of a Mixture of all Colours. And that very Experiment might have satisfied him, if he had pleased to examine it by the various Circumstances. One Circumstance I there declared, of which I see no Notice taken; and it is, That if any Colour at the Lens be intercepted, the Whiteness will be changed into other Colours: If all the Colours but Red be intercepted, that Red alone in the Concourse or crossing of the Rays, will not constitute Whiteness, but continues as much Red as before; and so of the other Colours. So that the Business is not only to shew how Rays, which before the Concourse exhibit Colours, do in the Concourse exhibit White; but to shew, how, in the same Place, where the several Sorts of Rays apart exhibit several Colours, a Confusion of all together make

make White. For Instance, If Red alone be first transmitted to the Paper at the Place of Concourse, and then the other Colours be let fall on that Red, the Question will be, Whether they convert it into White by mixing with it only, as Blue falling on a Yellow Light is supposed to compound Green? Or whether there be some further Change wrought in the Colours by their mutual Acting on one another, until, like contrary *peripatetick Qualities*, they become assimilated? And he that shall explicate this last Case mechanically, must conquer a double Impossibility. He must first shew, that many unlike Motions in a Fluid can by clashing so act on one another, and change each other, as to become one uniform Motion; and then, that an uniform Motion can of itself, without any new unequal Impressions, depart into a great Variety of Motions regularly unequal. And after this he must further tell me, why all Objects appear not of the same Colour; that is, why their Colours in the Air, where the Rays that convey them every way are confusedly mixt, do not assimilate one another, and become uniform before they arrive at the Spectator's Eye?

But if there be yet any doubting, 'tis better to put the Event on further Circumstances of the Experiment, than to acquiesce in the Possibility of any hypothetical Explications. As for Instance, by trying what will be the Apparition of these Colours in a very quick Consecution of one another. And this may be easily performed by the rapid Gyration of a Wheel with many Spokes or Cogs in its Perimeter, whose Interstices and Thicknesses may be equal, and of such a Largeness, that, if the Wheel be interposed between the Prism and the white Concourse of the Colours, one half of the Colours may be intercepted by a Spoke or Cogg, and the other half pass through an Interstice. The Wheel being in this Posture, you may first turn it slowly about, to see all the Colours fall successively on the same Place of the Paper, held at their aforesaid Concourse; and if you then accelerate its Gyration, until the Consecution of those Colours be so quick, that you cannot distinguish them severally, the resulting Colour will be a Whiteness perfectly like that, which an unrefracted Beam of Light exhibits, when in like manner successively interrupted by the Spokes or Cogs of that circulating Wheel. And that this Whiteness is produced by a successive Intermixture of the Colours, without their being assimilated, or reduced to any Uniformity, is certainly beyond all doubt, unless Things that exist not at the same time may notwithstanding act on one another.

There are yet other Circumstances, by which the Truth might have been decided; as by viewing the white Concourse of the Colours through another Prism placed close to the Eye, by whose Refraction that Whiteness may appear again transformed into Colours: And then, to examine their Origin, if an Assistant intercept any of the Colours at the Lens before their Arrival at the Whiteness, the same Colours will vanish from amongst those, into which that Whiteness is converted by the second Prism. Now, if the Rays which disappear be the same with those that are intercepted, then it must be acknowledged, that the second Prism makes no new Colours in any Rays, which were not in them before their Concourse at the Paper. Which is a plain Indication, that the Rays of several Colours remain distinct from one another in the Whiteness, and that from their previous Dispositions are derived the Colours, of the
second

second Prism. And, by the way, what is said of their Colours may be applied to their Refrangibility.

The aforefaid Wheel may also here be made use of; and, if its Gyration be neither too quick nor too slow, the Succession of the Colours may be discern'd thro' the Prism, whilst to the naked Eye of a By-stander they exhibit Whiteness.

There is something still remaining to be said of this Experiment. But this, I conceive, is enough to enforce it, and so to decide the Controversy. However I shall now proceed to shew some other Ways of producing Whiteness by Mixtures, since I persuade my self, that this Assertion above the rest appears paradoxical, and is with most Difficulty admitted. And because the Animadversor desires an Instance of it in Bodies of divers Colours, I shall begin with that. But in order thereto it must be consider'd, that such colour'd Bodies reflect but some part of the Light incident on them; as is evident by the 13th Proposition: And therefore the Light reflected from an Aggregate of them will be much weaken'd by the Loss of many Rays. Whence a perfect and intense Whiteness is not to be expected, but rather a Colour between those of Light and Shadow, or such a Grey or dirty Colour as may be made by mixing White and Black together.

And that such a Colour will result, may be collected from the Colour of Dust found in every Corner of an House, which hath been observ'd to consist of many colour'd Particles. There may also be produced the like dirty Colour, by mixing several Painters Colours together. And the same may be effected by painting a Top (such as Boys play with) of divers Colours. For when it is made to circulate by whipping it, it will appear of such a dirty Colour.

Now the compounding of these Colours is proper to my purpose, because they differ not from Whiteness in the Species of Colour, but only in a Degree of Luminousness: Which (did not the Animadversor concede it) I might thus evince. A Beam of the Sun's Light being transmitted into a darkened Room, if you illuminate a Sheet of white Paper by that Light, reflected from a Body of any Colour, the Paper will always appear of the Colour of that Body, by whose reflected Light it is illuminated. If it be a Red Body, the Paper will be Red; if a Green Body, it will be Green; and so of the other Colours. The Reason is, that the Fibres or Threads, of which the Paper consists, are all transparent and specular; and such Substances are known to reflect Colours without changing them. To know, therefore, to what Species of Colour a Grey belongs, place any Grey Body (suppose a Mixture of Painters Colours) in the said Light, and the Paper being illuminated by its Reflection, shall appear White. And the same thing will happen, if it be illuminated by Reflection from a Black Substance.

These therefore are all of one Species; but yet they seem distinguish'd not only by Degrees of Luminousness, but also by some other Inequalities, whereby they become more harsh or pleasant. And the Distinction seems to be, that Greys, and perhaps Blacks, are made by an uneven Defect of Light, consisting as it were of many little Veins or Streams, which differ either in Luminousness, or in the unequal Distribution of diversly colour'd Rays; such as ought to be

caused by Reflection from a Mixture of White and Black, or of diversly coloured Corpuscles. But when such imperfectly mix'd Light is by a second Reflection from the Paper more evenly and uniformly blended, it becomes more pleasant, and exhibits a faint or shadowed Whiteness. And that such little Irregularities as these may cause these Differences, is not improbable, if we consider, how much Variety may be caused in Sounds of the same Tone, by irregular and uneven Jarrings. And besides, those Differences are so little, that I have sometimes doubted, whether they be any at all, when I have considered, that a Black and White Body being placed together, the one in a strong Light, and the other in a very faint Light, so proportioned that they might appear equally luminous, it has been difficult to distinguish them, when viewed at Distance, unless when the Black seemed more Bluish, and the White Body in a Light still fainter, hath, in Comparison of the Black Body, itself appeared Black.

This leads me to another Way of compounding Whiteness; which is, that, if four or five Bodies of the more eminent Colours, or a Paper painted all over in several Parts of it with those several Colours in a due Proportion, be placed in the said Beam of Light, the Light reflected from those Colours to another White Paper, held at a convenient Distance, shall make that Paper appear White. If it be held too near the Colours, its Parts will seem of those Colours that are nearest them; but by removing it further, that all its Parts may be equally illuminated by all the Colours, they will be more and more diluted, until they become perfectly White. And you may further observe, that if any of the Colours be intercepted, the Paper will no longer appear White, but of the other Colours which are not intercepted. Now that this Whiteness is a Mixture of the severally coloured Rays, falling confusedly on the Paper, I see no reason to doubt of; because, if the Light became uniform and similar before it fell confusedly on the Paper, it must much more be uniform, when at greater Distance it falls on the Spectator's Eye; and so the Rays, which come from several Colours, would in no Qualities differ from one another, but all of them exhibit the same Colour to the Spectator, contrary to what he sees.

Not much unlike this Instance it is, that if a polish'd Piece of Metal be so placed that the Colours appear in it as in a Looking-Glass, and then the Metal be made rough, that by a confused Reflection those apparent Colours may be blended together, they shall disappear, and by their Mixture cause the Metal to look White.

But further to enforce this Experiment: If instead of the Paper, any White Froth consisting of small Bubbles, be illuminated by Reflexion from the aforesaid Colours, it shall to the naked Eye seem White, and yet through a good Microscope the several Colours will appear distinct on the Bubbles, as if seen by Reflexion from so many spherical Surfaces. With my naked Eye, being very near, I have also discerned the several Colours on each Bubble; and yet at a greater Distance, where I could not distinguish them apart, the Froth hath appeared entirely White. And at the same Distance, when I look'd intently, I have seen the Colours distinctly on each Bubble; and yet by straining my

my Eyes as if I would look at something afar off beyond them, thereby to render the Vision confused, the Froth has appeared without any other Colour than Whiteness. And what is here said of Froths, may easily be understood of the Paper or Metal, in the foregoing Experiments. For their Parts are specular Bodies, like these Bubbles; and perhaps with an excellent Microscope, the Colours may also be seen intermixedly reflected from them.

In proportioning the several colour'd Bodies to produce these Effects, there may be some Niceness; and it will be more convenient, to make use of the Colours of the Prism, cast on a Wall, by whose Reflection the Paper, Metal, Froth, and other white Substances may be illuminated. And I usually made my Trials this Way, because I could better exclude any scattering Light from mixing with the Colours to dilate them.

To this Way of compounding Whiteness, may be referred that other, by mixing Light after it hath been trajected through transparently colour'd Substances. For Instance, if no Light be admitted into a Room, but only through colour'd Glass, whose several Parts are of several Colours in a pretty equal Proportion; all white Things in the Room shall appear white, if they be not held too near the Glass: And yet this Light, with which they are illuminated, cannot possibly be uniform; because, if the Rays, which at their Entrance are of divers Colours, do in their Progress through the Room suffer any Alteration to be reduced to an Uniformity, the Glass would not in the remotest Parts of the Room appear of the very same Colour, which it doth when the Spectator's Eye is very near it: Nor would the Rays, when transmitted into another dark Room through a little Hole in an opposite Door or Partition-Wall, project on a Paper the Species or Representation of the Glass in its proper Colours.

And, by the by, this seems a very fit and cogent Instance of some other Parts of my *Theory*, and particularly of the 13th *Proposition*. For in this Room all natural Bodies whatever appear in their proper Colours. And all the Phænomena of Colours in Nature, made either by Refraction or without it, are here the same as in the open Air. Now the Light in this Room being such a dissimilar Mixture, as I have described in my *Theory*, the Causes of all these Phænomena must be the same that I have here assign'd. And I see no reason to suspect, that the same Phænomena should have other Causes in the open Air.

The Success of this Experiment may be easily conjectured by the Appearances of Things in a Church or Chapel, whose Windows are of colour'd Glass; or in the open Air, when it is illustrated with Clouds of various Colours.

There are yet other Ways by which I have produced Whiteness; as by casting several Colours from two or more Prisms upon the same Place; by refracting a Beam of Light with two or three Prisms successively, to make the diverging Colours converge again; by reflecting one Colour to another, and by looking through a Prism on an Object of many Colours; and (which is equivalent to the above-mention'd Way of mixing Colours by concave Wedges fill'd with colour'd Liquors) I have observ'd the Shadows of a painted Glass-Window to

become White, where those of many Colours have at a great Distance interferred. But yet for further Satisfaction, the Animadverſor may try, if he pleaſe, the Effects of four or five of ſuch Wedges fill'd with Liquors of as many ſeveral Colours.

Befides all theſe, the Colours of Water-bubbles, and other thin pellucid Subſtances, afford ſeveral Inſtances of Whitenefs produc'd by their Mixture; with one of which I ſhall conclude this Particular. Let ſome Water, in which a convenient Quantity of Soap or Waſhball is diſſolv'd, be agitated into Froth, and after that Froth has ſtood a while without further Agitation, 'till you ſee the Bubbles, of which it conſiſts, begin to break, there will appear a great Variety of Colours all over the Top of every Bubble, if you view them near at hand; but if you view them at ſo great a Diſtance that you cannot diſtinguiſh the Colours one from another, the Froth will appear perfectly White.

Thus much concerning the Deſign and Subſtance of the Animadverſor's Conſiderations. There are yet ſome Particulars to be taken notice of, before I conclude; as the Denial of the *Experimentum Crucis*. On this I choſe to lay the whole Streſs of my Diſcourſe; which therefore was the principal Thing to have been objected againſt. But I cannot be convinced of its Inſufficiency by a bare Denial, without aſſigning a Reaſon for it. I am apt to believe, it has been miſunderſtood; for otherwiſe it would have prevented the Diſcourſes about rarefying and ſplitting of Rays; becauſe the Deſign of it is to ſhew, that Rays of divers Colours conſider'd apart, do at equal Incidences ſuffer unequal Refractions, without being ſplit, rarefy'd, or any ways dilated.

Some Conſiderations upon this Doctrin of Colours; from Paris by N. 96. p. 6086. July, An. 1673.

VI. 1. Methinks that the moſt important Objection which is made againſt Mr. *Newton* by way of Query, is that, Whether there be more than two ſorts of Colours? For my part. I believe, that an Hypotheſis, that ſhould explain Mechanically, and by the Nature of Motion, the Colours Yellow, Green and Blue, would be ſufficient for all the reſt, in regard that thoſe others, being only more deeply charged, (as appears by the Priſms of Mr. *Hook*.) do produce the dark or deep Red and Blue; and that of theſe four all the other Colours may be compounded. Neither do I ſee why Mr. *Newton* doth not content himſelf with the two Colours, Yellow and Blue; for it will be much more eaſy to find any Hypotheſis by Motion, that may explicate theſe two Differences, than for ſo many Diverſities as there are of other Colours. And 'till he hath found this Hypotheſis, he hath not taught us, what it is wherein conſiſts the Nature and Difference of Colours, but only this Accident (which certainly is very conſiderable) of their *different Refrangibility*.

As for the Composition of White made by all the Colours together, it may poſſibly be, that Yellow and Blue might alſo be ſufficient for that: Which is worth while to try; and it may be done by the Experiment which Mr. *Newton* propoſeth, by receiving againſt a Wall of a darkned Room the Colours of the Priſm, and to caſt their reflected Light upon white Paper. Here you muſt hinder the Colours of the Extremities, *viz.* the Red and Purple, from ſtriking againſt the Wall, and leave only the intermediate Colours, Yellow, Green and Blue, to ſee whether the Light of theſe alone would not make the

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Paper appear White, as well as when they all gave Light. I even doubt, whether the lightest Place of the Yellow Colour may not all alone produce that Effect, and I mean to try it at the first Conveniency; for this Thought never came into my Mind but just now. Mean time you may see, that if these Experiments do succeed, it can no more be said, that all the Colours are necessary to compound White, and that 'tis very probable, that all the rest are nothing but Degrees of Yellow and Blue, more or less charged.

2. It seems to me, that *N.* takes an improper Way of examining the Nature of Colours, whilst he proceeds upon compounding those that are already compounded. Perhaps he would sooner satisfy himself by resolving Light into Colours, as far as may be done by Art, and then by examining the Properties of those Colours apart, and afterwards by trying the Effects of re-joining two or more, or all of those; and lastly, by separating them again, to examine what Changes that Re-conjunction had wrought in them. I have formerly shew'd, That all Colours cannot practically be deriv'd out of the Yellow and Blue, and consequently that those Hypotheses are groundless which imply they may. If you ask what Colours cannot be derived out of Yellow and Blue? I answer, None of those which I defined to be Original; and if he can shew by Experiment how they may, I will acknowledge my self in an Error. Nor is it easier to frame an Hypothesis by assuming only two original Colours, rather than an indefinite Variety; unless it be easier to suppose that there are but two Figures, Sizes, and Degrees of Velocity or Force of the *Æthereal* Corpuscles or Pulses, rather than an indefinite Variety; which certainly would be a harsh Supposition. No Man wonders at the indefinite Variety of Waves of the Sea, or of Sands on the Shore; but, were they all but two Sizes, it would be a very puzzling Phenomenon. And I should think it as unaccountable, if the several Parts or Corpuscles, of which a shining Body consists, which must be supposed of various Figures, Sizes and Motions, should impress but two Sorts of Motion on the adjacent *Æthereal* Medium, or any other way beget but two Sorts of Rays. But to examine how Colours may be explain'd hypothetically, is besides my purpose. I never intended to shew wherein consists the Nature and Difference of Colours, but only to shew, that *de facto* they are original and immutable Qualities of the Rays which exhibit them; and to leave it to others to explicate by mechanical Hypotheses, the Nature and Difference of these Qualities; which I take to be no difficult Matter. But I would not be understood, as if their Difference consisted in the *different Refrangibility* of those Rays; for that *different Refrangibility* conduces to their Production no otherwise, than by separating the Rays whose Qualities they are. Whence it is, That the same Rays exhibit the same Colours when separated by any other Means; as by their *different Reflexibility*, a Quality not yet discoursed of.

In the next Particular, where *N.* would shew, that it is not necessary to mix all Colours for the Production of White; the Mixture of Yellow, Green and Blue, without Red and Violet, which he propounds for that End, will not produce White, but Green; and the brightest Part of the Yellow will afford no other Colour but Yellow, if the Experiment

Answer'd, by
Mr. Newton:
N. 97. p. 6108.
Aug. An. 1673.

ment be made in a Room well darkned, as it ought ; because the colour'd Light is much weakned by the Reflection, and so apt to be diluted by the mixing of any other scattering Light. But yet there is an Experiment or two formerly mention'd, by which I have produced White out of two Colours alone, and that variouſly ; as out of Orange and a full Blue, and out of Red and pale Blue, and out of Yellow and Violet, as alſo out of other Pairs of intermediate Colours. The moſt convenient Experiment for performing this, was that of caſting the Colours of one Priſm upon thoſe of another, after a due Manner. But what *N.* can deduce from hence, I ſee not ; for the two Colours were compounded of all others, and ſo the reſulting White (to ſpeak properly) was compounded of them all, and only decomposed of thoſe two. For Inſtance, the Orange was compounded of Red, Orange, Yellow, and ſome Green ; and the Blue, of Violet, full Blue, light Blue, and ſome Green, with all their intermediate Degrees ; and conſequently the Orange and Blue together made an Aggregate of all Colours to conſtitute the White. Thus if one mix Red, Orange, and Yellow Powders to make an Orange ; and Green, Blue, and Violet Colours to make a Blue ; and laſtly, the two Mixtures to make a Grey ; that Grey, though decomposed of no more than two Mixtures, is yet compounded of all the ſix Powders, as truly as if the Powders had been all mix'd at once.

This is ſo plain, that I conceive there can be no further Scruple ; eſpecially to them who know how to examine, whether a Colour be ſimple or compounded, and of what Colours it is compounded ; which having explain'd in another Place, I need not now repeat. If therefore *N.* would conclude any thing, he muſt ſhew how White may be produced out of two uncompounded Colours : which when he hath done, I will further tell him, why he can conclude nothing from that. But I believe there cannot be found an Experiment of that Kind ; becauſe, as I remember, I once try'd, by gradual Succeſſion, the Mixture of all Pairs of uncompounded Colours ; and though ſome of them were paler, and nearer to White than others, yet none could be truly call'd White. But it being ſome Years ſince this Tryal was made, I remember not well the Circumſtances, and therefore recommend it to others to be try'd again.

A Reply, by Monſieur N.
N. 97. p. 6112.

3. Seeing that Mr. *Newton* maintains his Opinion with ſo much Concern, I liſt not to diſpute. But what means it, I pray, that he ſaith, *Though I ſhould ſhew him, that the White could be produced only of two uncompounded Colours, yet I could conclude nothing from that?* And yet he hath affirm'd, that to compoſe the White, all primitive Colours are neceſſary.

Answered, by Mr. Newton.
N. 96. p. 6087.
July, An. 1673.

4. In my ſaying, that when *Monſieur N.* hath ſhewn how White may be produced out of two uncompounded Colours, I will tell him, why he can conclude nothing from that ; my Meaning was, That ſuch a White (were there any ſuch) would have different Properties from the White which I had reſpect to, when I deſcribed my *Theory*, that is, from the White of the Sun's immediate Light, of the ordinary Objects of our Senſes, and of all white Phænomena that have hitherto fallen under my Obſervation. And thoſe different Properties would evince it to be of a different Conſtitution : Inſomuch, that ſuch a Production

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of White would be so far from contradicting, that it would rather illustrate and confirm my Theory; because by the Difference of that from other Whites it would appear, that other Whites are not compounded of only two Colours like that. And therefore if *Monsieur N.* would prove any thing, it is requisite that he do not only produce out of two primitive Colours a White, which to the naked Eye shall appear like other Whites, but also shall agree with them in all other Properties.

But to let you understand wherein such a White would differ from other Whites, and why from thence it would follow that other Whites are otherwise compounded, I shall lay down this Position.

That a compounded Colour can be resolved into no more simple Colours than those of which it is compounded.

This seems to be self-evident, and I have also tried it several Ways, and particularly by this which follows: Let *a* represent an oblong Piece of White Paper about $\frac{1}{2}$ or $\frac{3}{4}$ of an Inch broad, and illuminated in a dark Room, with a Mixture of two Colours cast upon it from two Prisms, suppose a deep Blue and Scarlet, which must severally be as uncompounded as they can conveniently be made; then, at a convenient Distance, suppose of six or eight Yards, view it through a clear Triangular Glass or Crystal Prism held parallel to the Paper, and you shall see the two Colours parted from one another in the Fashion of two Images of the Paper, as they are represented at β and γ , where suppose β the Scarlet, and γ the Blue, without Green, or any other Colour between them.

Fig. 76.

Now from the aforesaid Position I deduce these two Conclusions; 1. That if there were found out a Way to compound White of two simple Colours only, that the White would be again resolvable into no more than two. 2. That if other Whites, as that of the Sun's Light, &c. be resolvable into more than two simple Colours (as I find by Experiment that they are) then they must be compounded of more than two.

To make this plainer, suppose that *A* represents a white Body, illuminated by a direct Beam of the Sun transmitted through a small Hole into a dark Room, and *a* such another Body, illuminated by a Mixture of two simple Colours; which, if possible, may make it also appear of a white Colour exactly like *A*: Then, at a convenient Distance, view these two Whites through a Prism, and *A* will be changed into a Series of all Colours, Red, Yellow, Green, Blue, Purple, with their intermediate Degrees succeeding in Order from *B* to *C*. But *a*, according to the aforesaid Experiment, will only yield those two Colours of which it was compounded, and those not conterminate like the Colours at *B C*, but separate from one another, as at β and γ , by means of the different Refrangibility of the Rays to which they belong. And thus by comparing these two Whites, they would appear to be of a different Constitution, and *A* to consist of more Colours than *a*. So that what *Monsieur N.* contends for, would rather advance my Theory by the Access of a new Kind of White, than conclude against it. But I see no Hopes of compounding such a White.

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Vid. Suppl. p.
149. & seq.

As for *Monsieur N.* his Expression, *That I maintain my Doctrine with some Concern*, I confess it was a little ungrateful to me, to meet with Objections which had been answer'd before, without having the least Reason given me why those Answers were insufficient. Those Answers were to shew, that there are other simple Colours besides Blue and Yellow; I instanced in a simple or homogeneous Green, such as cannot be made by mixing Blue and Yellow, or any other Colours. And I also shew'd why, supposing that all Colours might be produced out of two, yet it would not follow that those two are the only original Colours. The Reasons I desire you would compare with what hath now been said of White. And so the Necessity of all Colours to produce White, might have appear'd by that Experiment, where I say, That if any Colour at the Lens be intercepted, the Whiteness (which is compounded of them all) will be changed into (the Result of) the other Colours.

However, since there seems to have happen'd some Misunderstanding between us, I shall endeavour to explain my self a little further in these Things, according to the following Method.

Definitions.] 1. I call that Light homogeneous, similar or uniform, whose Rays are equally refrangible.

2. And that heterogeneous, whose Rays are unequally refrangible.

Note. There are but three Affections of Light, in which I have observed its Rays to differ, *viz.* *Refrangibility*, *Reflexibility*, and *Colour*; and those Rays which agree in Refrangibility agree also in the other two, and therefore may well be defined homogeneous, especially since Men usually call those Things homogeneous, which are so in all Qualities that come under their Knowledge, tho' in other Qualities, that their Knowledge extends not to, there may possibly be some Heterogeneity.

3. Those Colours I call simple, or homogeneous, which are exhibited by homogeneous Light.

4. And those compound or heterogeneous, which are exhibited by heterogeneous Light.

5. Different Colours, I call not only the more eminent Species, Red, Yellow, Green, Blue, Purple, but all other the minutest Gradations; much after the same manner that not only the more eminent Degrees in Musick, but all the least Gradations are esteemed different Sounds.

Propositions.] 1. The Sun's Light consists of Rays differing by indefinite Degrees of Refrangibility.

2. Rays which differ in Refrangibility when parted from one another, do proportionally differ in the Colours which they exhibit. These two *Propositions* are Matter of Fact.

3. There are as many simple or homogeneous Colours, as Degrees of Refrangibility: For to every Degree of Refrangibility belongs a different Colour, by *Prop.* 2. and that Colour is simple, by *Def.* 1. and 3.

4. Whiteness

4. Whiteness, in all Respects like that of the Sun's immediate Light, and of all the usual Objects of our Senses, cannot be compounded of two Simple Colours alone: For such a Composition must be made by Rays that have only two Degrees of Refrangibility, by *Def.* 1, and 3; and therefore it cannot be like that of the Sun's Light, by *Prop.* 1; nor, for the same Reason, like that of ordinary white Objects.

5. Whiteness, in all Respects like that of the Sun's immediate Light, cannot be compounded of Simple Colours, without an indefinite Variety of them: For to such a Composition there are requisite Rays endued with all the indefinite Degrees of Refrangibility, by *Prop.* 1. And those infer as many Simple Colours, by *Def.* 1, and 3. and *Prop.* 2, and 3.

To make these a little plainer, I have added also the *Propositions* that follow.

6. The Rays of Light do not act on one another in passing through the same Medium. This appears by several former Passages, and is capable of further Proof.

7. The Rays of Light suffer not any Change of their Qualities from Refraction.

8. Nor afterwards from the adjacent quiet Medium. These two Propositions are manifest *de facto* in homogeneal Light, whose Colour and Refrangibility is not at all changeable, either by Refraction, or by the Contermination of a quiet Medium. And as for heterogeneous Light, it is but an Aggregate of several Sorts of homogeneal Light; no one Sort of which suffers any more Alteration than if it were alone, because the Rays act not on one another, by *Prop.* 6. and therefore the Aggregate can suffer none. These two Propositions also might be further proved apart by Experiments, too long to be here described.

9. There can no homogeneal Colours be educed out of Light by Refraction, which were not commixt in it before; because by *Prop.* 7 and 8, Refraction changeth not the Qualities of the Rays, but only separates those which have diverse Qualities, by means of the different Refrangibility.

10. The Sun's Light is an Aggregate of an indefinite Variety of homogeneal Colours; by *Prop.* 1, 3, and 9. And hence it is, that I call homogeneal Colours also primitive or original.

VII. 1. I doubt not of what Mr. *Newton* affirms; and have my self sometimes in like Circumstances observed the like Difference between the Length and Breadth of the coloured *Spectrum*; but never found it so when the Sky was clear and free from Clouds, near the Sun: But then only appeared this Difference of Length and Breadth, when the Sun either shined thro' a white Cloud, or enlightned some such Clouds near unto it. And then indeed it was no marvel, the said *Spectrum* should be longer than broad; since the Cloud or Clouds so enlightned, were in order to those Colours like to a great Sun, making a far greater Angle of Intersection in the Hole, than the true Rays of the Sun do make; and therefore are able to enlighten the whole Length of the Prism, and not only some small part thereof as we see enlightned by the true Sun-Beams coming thro' the same little Hole. And thus we behold also in the

*Animadversions
on this Theory of
Light and Co-
lours; by Mr.
Fr. Linus.
N. 110. p. 217.
Jan. An. 1675.*

true Sun-Beams, when they enlighten the whole Prism; for altho' in a clear Heaven, the Rays of the Sun passing thro' the said Hole, never make a Spectrum longer than broad, because they then can occupy but a small Part of the Prism; yet if the Hole be so much bigger as to enlighten the whole Prism, you shall presently see the Length of the Spectrum much exceed its Breadth; which Excess will be always so much the greater, as the Length of the Prism exceeds its Breadth. From whence I conclude, That the Spectrum, this *Learned Author* saw, much longer than broad, was not affected by the true Sun-Beams, but by Rays proceeding from some bright Cloud, as is said; and by Consequence, that the *Theory of Light* grounded upon that *Experiment* cannot subsist.

What I have here said, needs no other Confirmation than meer Experience, which any one may quickly try: neither have I only tried the same upon this Occasion, but near 30 Years ago shew'd the same, together with divers other Experiments of Light, to that worthy Promoter of Experimental Philosophy, Sir *Kenelm Digby*, who coming into these Parts to take the *Spaw-Waters*, resorted oftentimes to my darkned Chamber *, to see the various Phænomena of Light, made by divers Refractions and Reflections, and took Notes upon them; which Industry if they also had us'd, who endeavour to explicate the aforesaid Difference between the Length and Breadth of this colour'd Spectrum, by the received Laws of Refraction, would never have taken so impossible a Task in Hand.

* At Leige.

Answered by

N. 110. p. 219.
Jan. An. 1675.

2. These Animadversions seem to need no other Answer but this, that you would be pleased to consider the Scheme in Mr. *Newton's* second Answer to *P. Pardies*, and rest assured, that the Experiment, as 'tis represented, was tried in clear Days, and the Prism placed close to the Hole in the Window, so that the Light had no room to diverge, and the coloured Image made not parallel (as in that Conjecture) but transverse to the Axis of the Prism.

A Reply by Mr.
Fr. Linus.
N. 121. p. 499.
Jan. An. 1676.

3. If these Assertions be admitted, they do indeed directly cut off what I said of Mr. *Newton's* being deceived by a bright Cloud. But if we compare them with Mr. *Newton's* first Relation of the Experiment, it will evidently appear, they cannot be admitted, as being directly contrary to what is there delivered. For there he tells us, *The Ends of the coloured Image, he saw on the opposite Wall, near five times as long as broad, seemed to be semicircular.* Now these semicircular Ends are never seen in a clear Day, as Experience shews. From whence follows against the first Assertion, that the Experiment was not made in a clear Day. Neither are those semicircular Ends ever seen when the Prism is placed close to the Hole; which contradicts the second Assertion. Neither are they ever seen when the Image is transverse to the Length or Axis of the Prism, which directly opposes the third Assertion. But if in any of these three Cases, the Image be made so much longer than broad (as easily it may, by turning the Prism a little about its Axis, near five times as long as broad) then the one End thereof will run out into a sharp Cone or Pyramis like the Flame of a Candle, and the other into a Cone somewhat more blunt; both which are far from seeming semicircular: Whereas, if the Image be made not in a clear Day but with a bright Cloud, and the Prism not placed

placed close to the Hole but in a competent Distance from the same, then these semicircular Ends always appear, with the Sides thereof, streight Lines, just as Mr. *Newton* describes them. Neither is the Length of the Image transverse, but parallel to the Length of the Prism. Out of all which evidently follows, that the Experiment was not made in a clear Day; nor with the Prism close to the Hole; nor yet with the Image transverse. but by a bright Cloud and a parallel Image (as I conjectured); and I hope you will also now say, I had good Reason so to conjecture, since it so well agrees with the Relation. And Experience will also shew you, if you please to make Tryal, as it was made in a dark Chamber, and observe the Difference between such an Image made by a bright Cloud, and another made by the immediate Rays of the Sun: For the former you shall always find parallel, with the Ends semicircular; but the latter you shall find transverse, with the Ends pyramidical, as aforesaid, whensoever it appears so much longer than broad.

More might be said out of the same Relation, to shew that the Image was not transverse. For if it had been transverse, Mr. *Newton*, so well skill'd in *Opticks*, could not have been surpris'd (as he says he was) to see the Length thereof so much to exceed the Breadth; it being a Thing so obvious and easy to be explicated by the ordinary Rules of Refraction. That other Place also (where he says the *Incident* Refractions were made in the Experiment *equal to the Emergent*) proves again that the said oblong Image was not transverse, but parallel. For it is impossible the transverse Image should be so much longer than broad, unless those two Refractions be made very unequal, as both the Computation according to the common Rules of Refraction and Experience testify.

4. What it is that imposes upon Mr. *Line* I cannot imagine; but I suspect he has not try'd the Experiment since he acquainted himself with my Theory, but depends upon his old Notions, taken up before he had any Hint given to observe the Figure of the coloured Image. I shall desire him therefore, before he returns any Answer, to try it once more for his Satisfaction, and according to this Manner.

Answered by
Mr. *Newton*.
N. 121. p. 401.
N. 123. p. 556.
Mar. An. 1676.

Let him take any Prism, and hold it so that its Axis may be perpendicular to the Sun's Rays, and in this Posture let it be placed as close as may be to the Hole through which the Sun shines into a dark Room, which Hole may be about the Bigness of a Pea. Then let him turn the Prism slowly about its Axis, and he shall see the Colours move upon the opposite Wall, first towards that Place to which the Sun's direct Line would pass, if the Prism were taken away, and then back again. When they are in the Middle of these two contrary Motions, that is, when they are nearest that Place to which the Sun's direct Ray tends, there let him stop; for then are the Rays equally refracted on both Sides the Prism. In this Posture of the Prism let him observe the Figure of the Colours, and he shall find it not round, as he contends, but oblong, and so much the more oblong as the Angle of the Prism, comprehended by the refracting Plains, is bigger, and the Wall, on which the Colours are cast, more distant from the Prism; the Colours Red, Yellow, Green, Blue, Purple, succeeding in Order, not from one Side of the Figure to the other, as in Mr. *Line's* Conjecture, but from one End to the

other ; and the Length of the Figure being not parallel but transverse to the Axis of the Prism. After this manner I used to try the Experiment ; and it will not succeed well if the Day be not clear, and the Prism placed close to the Hole, or so near at least, that all the Sun's Light that comes from the Hole may pass through the Prism also, so as to appear in a round Form, if intercepted by a Paper immediately after it has passed the Prism.

When Mr. *Line* has try'd this, I could wish he would proceed a little further, to try that which I call'd the *Experimentum Crucis*. For when he has try'd them (which by his denying them, I know he has not done yet as they should be try'd) I presume he will rest satisfied. It may be try'd (tho' not so perfectly) even without darkning a Room, or the Expence of any more Time than half a Quarter of an Hour.

The Experiment
of Mr. *Line*
affirmed by
Mr. *Gascoigne*,
N. 121. p. 503.
marg.
Jan. An. 1676.

5. Mr. *Linus* (now deceased) try'd the Experiment again and again, and called divers on purpose to see it, nor ever made Difficulty to shew it to any one, who either by Chance came into his Chamber as he was doing it; or shewed the least Desire to see the same; so that for Point of Experience, Mr. *Newton* cannot be more confident on his Side, than we are here on the other; who are fully persuaded, that, unless the Diversity of placing the Prism, or the Bigness of the Hole, or some other such Circumstance, be the Cause of the Difference betwixt them, Mr. *Newton's* Experiment will hardly stand.

Answered by
Mr. *Newton*,
Ibid. N. 123.
p. 456.
Mar. An. 1676.

6. By Mr. *Gascoigne's* Letter one might suspect, that Mr. *Linus* try'd the Experiment some other Way than I did; and therefore I shall expect, till his Friends have try'd it according to my late Directions. In which Tryal it may possibly be a further Guidance to them, to acquaint them that the Prism casts from it several Images. One is, that oblong One of Colours which I mean; and this is made by two Refractions only. Another there is made by two Refractions and an intervening Reflection; and this is round and colourless, if the Angles of the Prism be exactly equal; but if the Angles at the reflecting Base be not equal, it will be colour'd, and that so much the more, by how much unequal the Angles are, but yet much more unround, unless the Angles be very unequal. A third Image there is, made by one single Reflection, and this is always round and colourless. The only Danger is, in mistaking the *Second* for the *First*. But they are distinguishable not only by the Length and lively Colours of the *First*, but by its different Motion too: For, whilst the Prism is turned continually the same Way about its Axis, the *Second* and *Third* move swiftly, and go always on the same Way, till they disappear; but the *First* moves slow, and grows continually slower, till it be Stationary, and then turns back again, and goes back faster and faster, till it vanish in the Place where it began to appear.

If, without darkning their Room, they hold the Prism at their Window in the Sun's open Light, in such a Posture that its Axis be perpendicular to the Sun-Beams, and then turn it about its Axis, they cannot miss of seeing the first Image; which having found, they may double up a Paper once or twice, and make a round Hole in the middle of it, about $\frac{1}{2}$ or $\frac{3}{4}$ of an Inch broad, and hold the Paper immediately before the Prism, that the Sun may shine on the Prism through that Hole; and the Prism being stayed, and held

held steady in that Posture which makes the Image stationary; if the Image then fall directly on an opposite Wall, or on a Sheet of Paper placed at the Wall, suppose 15 or 20 Foot from the Prism, or further off; they will see the Image in such an oblong Figure as I have described, with the Red at one End, the Violet at the other, and the bluish Green in the Middle: And if they obscure their Room as much as they can, by drawing Curtains, or otherwise, it will make the Colours the more conspicuous.

This Direction I have set down, that no Body, into whose Hands a Prism shall happen, may find Difficulty and Trouble in trying it. But when Mr. *Linus*'s Friends have try'd it thus, they may proceed to repeat it in a dark Room, with a less Hole made in their Window-shut. And then I shall desire that they will send a full and clear Description how they try'd it. I should be glad too, if they will favour me with a Description of the Experiment as it hath been hitherto try'd by Mr. *Linus*, that I may have an Opportunity to consider what there is in that which makes against me.

7. Mr. *Gascoigne* wanting Convenience to make the Experiment, according to the fresh Directions from Mr. *Newton*, requested me to supply this Want.

Exceptions by
Mr. Lucas.
N. 128. p. 692.
Aug. An. 1676.

The vertical Angle of my Prism was 60 Degrees; the Distance of the Wall, whereon the coloured *Spectrum* appeared, from the Window, about 18 Foot; the Diameter of the Hole in the Window-shuts about $\frac{1}{2}$ Inch, which, upon Occasions, I contracted to half the said Diameter, but still with equal Success as to the main of the Experiment. The Refractions on both Sides the Prism were, as near as I could make them, equal, and consequently about 48 Deg. 40 Min. the refractive Power of Glass being computed according to the Ratio of the Sines 2 to 3. The Distance of the Prism from the Hole in the Shuts was about 2 Inches; the Room darkned to that Degree as to equal the darkest Night, while the Hole in the Shuts was covered.

Now as to the Issue of my Tryals; I constantly found the Length of the coloured Image (transverse to the Axis of the Prism) considerably greater than its Breadth, as often as the Experiment was made on a clear Day; but if a bright Cloud were near the Sun, I found it sometimes exactly as Mr. *Line* wrote to you, namely, Broader than Long, especially while the Prism was placed at a great Distance from the Hole: Which Experiment will not, I conceive, be question'd by Mr. *Newton*, it being so agreeable to the received Laws of Refractions. And indeed the Observations of these two Learned Persons, as to this Particular, are easily reconcileable to each other, and both to Truth; Mr. *Newton* contending only for the Length of the Image (transverse to the Axis of the Prism) in a very clear Day; whereas Mr. *Line* only maintained the Excess of Breadth, parallel to the same Axis, while the Sun is in a bright Cloud. Tho' as to what is further deliver'd by Mr. *Newton*, and oppos'd by Mr. *Line*, namely, that the Length of the coloured Image was five times the Diameter of its Breadth; I never yet have found the Excess above thrice the Diameter, or at most $\frac{3}{2}$, while the Refractions on both Sides the Prism were equal. So much as to the Matter of Fact.

Now as to Mr. *Newton*'s Theory of Light and Colours, I confess his neat Sett of very Ingenious and Natural Inferences, was to me, upon the first Perusal,

Perusal, a strong Conjecture in Favour of his new Doctrine; I having formerly observed the like Chain of Inferences upon Search into natural Truths. But since several Experiments of Refractions remain still untouched by him, I conceived a further Search into them would be very proper, in Order to a further Discovery of the Truth of his Assertion. For accordingly as they are found, either agreeing with, or disagreeing from his new Theory, they must needs much strengthen, or wholly overthrow the same. The Experiments I pitched upon for this purpose are as follow :

1. Having frequently observed, that the Form of Objects viewed in the Microscope (or rather of the Microscope itself) consists almost in an indivisible Point, I concluded, two very small Pieces of Silk, the one Scarlet, the other Violet Colour, placed near together, should, according to Mr. *Newton's* Theory, appear in the Microscope in a very different Degree of Clarity, in regard their unequal Refrangibility must cause the Scarlet Rays, or Species, to over-reach the Retina, while placed in the due Focus of the Violet ones, and consequently must occasion a sensible Confusion in the Vision of the former, one and the same Point of the Scarlet Object affecting several Nerves in the Retina. Yet upon frequent Tryals I have not been able to perceive any Inequality in this Point.

2. The Second Experiment I made in Water. I took a Brass Ruler, and fastning thereunto several Pieces of Silk, Red, Yellow, Green, Blue, and Violet, I placed it at the Bottom of a Square Vessel of Water: Then I retired from the Vessel so far as not to be able to see the aforesaid Ruler and coloured Silks, otherwise than by the Help of the refracted Ray. Now, did Mr. *Newton's* Doctrine hold, I conceived I should not see all the mentioned Colours in a streight Line with the Ruler, in regard the unequal Refrangibility of different Rays must needs displace some more than others. Yet in Effect, upon many Tryals, I constantly found them in as streight a Line, as the bare Ruler had appeared in.

3. To advance this Experiment, I adjoined a second Refraction to the former of Water, by placing my Prism so as to receive perpendicularly the refracted Species of the Silk and Ruler; whereby only the emergent Species suffered a second Refraction. But still with equal Success, as to their appearing in a streight Line to the Eye placed behind the Prism.

4. To these two Refractions I further added a third, by receiving the coloured Species obliquely upon the Prism; whereby both incident and emergent Species suffered their respective Refractions. But still with the same Success as formerly, as to the streight Line they appeared in.

For further Assurance in this Experiment, lest Prepossession, occasion'd from previous Knowledge of the Silk's Situation in a streight Line, might possibly prejudice the Judgment of the Eye (as sometimes I have observ'd to happen to the Judgment the Eye passeth upon the Distance of Objects) I call'd into the Room some unconcern'd Persons, wholly ignorant of what the Experiment aim'd at; and demanding whether they saw not the coloured Silks and Ruler in a crooked Line, they answer'd in the Negative.

5. The

5. The next Experiment I made in uncompound Colours (as Mr. *Newton* terms them, *Prop. 5. & 13.*) as follows. Having cast two coloured Images upon the Wall, so as the Scarlet Colour of the one did fall in a streight Line (parallel to the Horizon) with the Violet of the other: I then looked upon both through another Prism, and found them still appear in a streight Line parallel to the Horizon, as they had formerly done to the naked Eye. Now according to Mr. *Newton's* Assertion of different Refrangibility in different Rays, I conceive the Violet Rays should suffer a greater Refraction in the Prism at the Eye, than the Scarlet ones; and consequently both Colours should not appear in a streight Line parallel to the Horizon.

6. Another Experiment I made, in order to some further Discovery of that surprizing Phenomenon of the coloured Image, which occasioned Mr. *Newton's* ingenious Theory of Light and Colours, as also of his excellent Invention of the reflecting Telescope and Microscope. Having then sometimes suspected, that not only the direct Sun-Beams, but also other extraneous Light, might possibly influence the coloured *Spectrum*, I hoped to discover the Truth of this Suspicion by means of the Sun-Spots, made to appear in the coloured Image, by placing a Telescope behind the Prism. But my Endeavours proving ineffectual herein, by reason of some intervening Difficulties, I thought at length of a more feasible Method, in order to the designed Discovery; as in the following Experiment.

I fastned a very white Paper-Circle (about an Inch in Diameter) upon my Window-shuts; and beholding it thro' my Prism, I found a coloured Image painted thereby upon my Retina, answerable in almost all respects to the former of the Sun-Beams upon the Wall, especially when the Paper-Circle was indifferently well illuminated. This Image indeed appeared contrary to the former, as to the Situation of Colours, that is, the Scarlet appearing above, the Violet below, tho' but faint. But this I was not surprized at, having observed upon dissecting the Eye, that Objects are painted on the Retina after a contrary Posture to what they appear to sight. Having thus rendered the coloured Image much more tractable than formerly it was, I conceived good Hopes of some further Discovery in the Point mentioned.

In pursuance then of my former Suspicion, having fixed my Prism in a steady Posture, I caused the Paper C to be applied close up to the Paper-Circle *a b d*; whereupon the former Violet *d*, and the Scarlet Colour of C, vanished into Whiteness. Next I removed the mentioned Circle from the Shuts, and placed it in the open Window, supported only by the Edge *d*: whereupon, to my Astonishment, all the former Colours exchanged Postures in the Retina, the Scarlet now appearing below, the Violet above, the intermediate Colours scarce discernable. And here, on the by, 'tis very remarkable, that during this Observation I clearly perceived both Blue and Scarlet Light to be transparent, I being able to discern several Objects thro' both, namely Steeples opposite to my Window. Whence it follows, that these Colours do in great Part arise from the neighbouring Light. Lastly, I placed the Paper-Circle anew, so as the one half *b* was fastned to the Shuts, the other Semicircle *a* being exposed to the open Air. Whereupon the Semicircle *a* be-
came

Fig. 77.

came bordered with Violet above, Scarlet below ; but the other Semicircle *b*, quite contrary. Hence I make the following Inferences.

First, That not only the Light reflected from the Paper-Circle, but also from the ambient Air, hath great Influence upon the coloured Image, especially as to the Violet and Scarlet Colours. Whence, perchance, it will not hereafter seem strange, that the coloured Spectrum on the Wall is so long, but only that the Breadth is not greater. Secondly, Were there a more luminous Body behind the Sun, we should in all likelihood have the Colours of the Spectrum in a contrary Situation to what they appear in at present ; Whence (Thirdly) it seems to follow, that the present Situation and Order of Colours ariseth not from any intrinsic Property of *Refrangibility*, (as maintained by Mr. *Newton*) but from contingent and extrinsic Circumstances of neighbouring Objects : For accordingly as the Body behind the Paper-Circle was more or less illuminated than the Circle itself, all the several Colours changed their Situation.

8. The next Experiment was made in Order to Mr. *Newton's* Doctrine of primary Colours, as *Prop. 5*. Having covered the Hole in the Window-shuts with a thin Slice of Ivory, the transmitted Light appeared Yellow ; but upon adding three, four, or more Slices, it became Red. Whence it seems to follow, that Yellowness of Light is not a primary Colour, but a Compound of Red, &c.

9. The last Experiment was made in reference to Mr. *Newton's* 12th *Prop.* where from his own Principles he renders a very plausible Reason of a surprising Phenomenon, related by Mr. *Hooke* ; namely, of two Liquors, the one Blue, and the other Red, both severally transparent ; yet both, if placed together, became opaque. The Reason whereof, saith Mr. *Newton*, is, because if one Liquor transmitted only Red, the other only Blue, no Rays could pass thro' both.

In reference then to this Point, I filled two small Glasses with flat polished Bottoms, the one with *Aqua fortis* deeply died Blue, the other with Oyl of Turpentine died Red, both to that Degree as to represent all Objects thro' them respectively Blue or Red : Then placing the one upon the other, I was able to discern several Bodies thro' both. Whereas, according to Mr. *Newton's* Theory, no Object should appear thro' both Liquors ; because if one transmit only Red, the other only Blue, no Rays can pass thro' both.

P. S. *Just upon the Close of the adjoynd Letter, I received from Mr. Gascoigne yours of May the 4th ; wherein you are pleased to favour us with an exact Account of the famous Experiment of the coloured Spectrum, lately exhibited before the Royal Society. I was much rejoiced to see the Tryals of that Illustrious Company agree so exactly with ours here, tho' in somewhat ours disagree from Mr. Newton.*

*Answered by
Mr. Newton.
Ibid. p. 698.*

8. The Things opposed by Mr. *Line* being upon Tryals found true and granted me, I begin with the new Question about the Proportion of the Length of the Image to its Breadth : And it is no wonder, that Mr. *Lucas* found the
Image

Image shorter than I did, seeing he tryed the Experiment with a less Angle.

The Angle indeed which I used was but about 63 degrees, 12 min. and his is set to 60 degrees: the difference of which from mine, being but 3 deg. 12 min. is too little to reconcile us, but yet it will bring us considerably nearer together. And if this Angle was not exactly measured, but the round number of 60 degrees set down by guess, or by a less accurate Measure (as I suspect by the conjectural Measure of the Refraction of his Prism, by the Ratio of the Sines to 2 to 3, set down at the same time, instead of an Experimental one,) then might it be two or three degrees less than 60 deg. if not still less: And all this, if it should be so, would take away the greatest part of the difference between us.

But however it be, I am well assured my own Observation was exact enough. For I have repeated it divers times since the receipt of Mr. *Lucas's* Letter, and that without any considerable difference of my Observations, either from one another, or from what I wrote before. And that it might appear experimentally, how the increase of the Angle increases the length of the Image, and also that no body, who has a Mind to try the Experiment exactly, might be troubled to procure a Prism which has an Angle just of the Bigness assigned by me; I tryed the Experiment with divers Angles, and have set down my Tryals in the following Table; where the first Column expresses the Angles of two Prisms which I used, which were measured as exactly as I could by applying them to the Angle of a Sector; and the second Column expresses in Inches the length of the Image made by each of those Angles; its Breadth being two Inches, its Distance from the Prism 18 Feet and 4 Inches, and the Breadth of the Hole in the Window-shut $\frac{1}{4}$ of an Inch.

| <i>Angles</i> | | <i>Lengths</i> | <i>Angles</i> | | <i>Lengths</i> |
|-----------------|----------|------------------|------------------|---------|------------------|
| The first Prism | 56°. 10' | 7 $\frac{1}{2}$ | The second Prism | 54°. 0' | 7 $\frac{1}{2}$ |
| | 60 . 24 | 9 $\frac{1}{2}$ | | 62 . 12 | 10 $\frac{1}{2}$ |
| | 63 . 26 | 10 $\frac{1}{2}$ | | 63 . 48 | 10 $\frac{1}{2}$ |

You may perceive, that the Length of the Images, in respect of the Angles that made them, are something greater in the second Prism than in the first; but that was because the Glass, of which the second Prism was made, had the greater Refractive Power.

The Days in which I made these Tryals were pretty clear, but not so clear as I desired; and therefore afterwards meeting with a Day as clear as I desired, I repeated the Experiment with the second Prism, and found the Lengths of the Image made by its several Angles, to be about $\frac{1}{4}$ of an Inch greater than before, the Measures being those set down in the Table.

| <i>Angles</i> | | <i>Lengths</i> |
|------------------|---------|------------------|
| The second Prism | 54°. 0' | 7 $\frac{1}{2}$ |
| | 62 . 12 | 10 $\frac{1}{2}$ |
| | 63 . 48 | 11 |

The reason of this difference, I apprehend, was, that in the clearest Days the Light of the White Skies, which dilutes and renders invisible the faintest Colours at the Ends of the Image, is a little diminished in a clear Day, and so gives leave to the Colours to appear to a great Length; the Sun's Light at the same time becoming Brisker, and so strengthening the Colours, and making the faint ones at the two ends more conspicuous. For I have observed, that in Days something Cloudy, whilst the Prism has stood unmoved at the Window, the Image would grow a little longer or a little shorter, accordingly as the Sun was more or less obscur'd by thin Clouds which passed over it; the Image being shortest when the Cloud was brightest, and the Sun's Light faintest. Whence 'is easie to apprehend, that if the Light of the Clouds could be quite taken away, so that the Sun might appear surrounded with Darknefs, or if the Sun's Light were much stronger than it is, the Colours would still appear to a greater Length.

In all these Observations the Breadth of the Image was just two Inches. But observing that the Sides of the two Prisms I used were not exactly plain, but a little Convex, (the Convexity being about so much as that of a double Convex-Glass of a sixteen or eighteen Foot Telescope) I took a third Prism, whose Sides were as much Concave as those of the other were Convex; and this made the breadth of the Image to be two Inches and a third part of an Inch, the Angles of this Prism, and the Lengths of the Image made by each of those Angles, being those exprest in this Table.

| <i>Angles.</i> | <i>Lengths.</i> |
|----------------|-----------------|
| 58°. | 8½ |
| 59½ | 9 |
| 62½ | 10½ |

In this case you see, the Concave figure of the sides of the Prism by making the Rays Diverge a little, causes the Breadth of the Image to be greater in proportion to its Length than it would be otherwise. And this I thought fit to give you notice of, that Mr. Lucas may examine, whether his Prism have not this fault. If a Prism may be had with sides exactly plain, it may do well to try the Experiment with that; but its better, if the sides be about so much Convex as those of mine are, because the Image will thereby become much better defined. For this Convexity of the sides does the same effect, as if you should use a Prism with sides exactly plain, and between it and the Hole in the Window-shut, place an Object-Glass of an 18 Foot Telescope, to make the round Image of the Sun appear distinctly defin'd on the Wall when the Prism is taken away, and consequently the long Image made by the Prism to be much more distinctly defined (especially at its streight sides) than it would be otherwise.

One thing more I shall add: That the utmost length of the Image, from the faintest Red at one End, to the faintest Blue at the other, must be measured. For in my first Letter about Colours, where I set down the Length to be five times the Breadth, I called that Length the utmost Length of the

the Image; and I measured the utmost Length, because I account all that Length to be caused by the immediate Light of the Sun, seeing the Colours (as I noted above) become visible to the greatest Length in the clearest Days, that is, when the Light of the Sun transcends most the Light of the Clouds. Sometimes there will happen to shoot out from both Ends of the Image a glaring Light a good way beyond these Colours, but this is not to be regarded, as not appertaining to the Image. If the Measures be taken right, the whole Length will exceed the Length of the streight Sides by about the Breadth of the Image.

By these things set down thus circumstantially, I presume Mr. *Lucas* will be enabled to accord his Tryals of the Experiment with mine; so nearly at least that there shall not remain any very considerable difference between us. For if some little difference should still remain, that need not trouble us any further, seeing there may be many various Circumstances which may conduce to it; such as are not only the different Figures of Prisms, but also the different Refractive Power of Glasses, the different Diameters of the Sun at divers Times of the Year, and the little Errors that may happen in Measuring Lines and Angles, or in placing the Prism at the Window; though for my part, I took Care to do these things as exactly as I could. However Mr. *Lucas* may make sure to find the Image as long or longer than I have set down; if he take a Prism whose Sides are not hollow ground, but plain, or (which is better) a very little Convex, and whose Refracting Angle is as much greater than that I used, as that he hath hitherto tried it with is less; that is, whose Angle is about 66 or 67 Degrees, or (if he will) a little greater.

Concerning Mr. *Lucas's* other Experiments, I am much obliged to him that he would take these things so far into Consideration, and be at so much Pains for Examining them; and I thank him so much the more, because he is the first that hath sent me an Experimental Examination of them. But yet it will conduce to his more speedy and full Satisfaction, if he a little change the Method which he has propounded, and instead of a Multitude of Things try only the *Experimentum Crucis*. For it is not Number of Experiments but Weight to be regarded; and where one will do, what need many?

The main thing he goes about to examine is, the *Different Refrangibility of Light*; and this I demonstrated by the *Experimentum Crucis*. Now if this Demonstration be good, there needs no further Examination of the thing; if not good, the Fault of it is to be shewn: For the only way to examine a Demonstrated Proposition is, to examine the Demonstration. Let that Experiment therefore be examined in the first place, and that which it proves be acknowledged, and then if Mr. *Lucas* wants my Assistance, to unfold the difficulties which he fancies to be in the Experiments he has propounded, he shall freely have it. At present I shall say nothing in Answer to his Experimental Discourse, but this in general, that it has proceeded partly from some misunderstanding of what he writes against, and partly for want of due caution in trying Experiments; and that amongst his Experiments there is one, which when duly try-

ed, is, next to the *Experimentum Crucis*, the most conspicuous Experiment, I know, for proving the different Refrangibility of Light, which he brings it to prove against.

By the Post-script of Mr. *Lucas's* Letter, one not acquainted with what has passed, might think, that he quotes the Observation of the *Royal Society* against me; whereas the Relation of their Observation, which you sent to *Liege*, contained nothing at all about the just Proportion of the Length of the Image to its Breadth according to the Angle of the Prism, nor any thing more (so far as I can perceive by your last) than what was pertinent to the things then in dispute, *viz.* that they found them succeed as I had affirmed. And therefore, since Mr. *Lucas* has found the same Success, I suppose, that when he expressed, That he much rejoiced to see the Tryals of the R. Society agree so exactly with his, he meant only so far as he agreed with mine.

P. S. I had like to have forgotten to advise, that the *Experimentum Crucis*, and such others as shall be made for knowing the Nature of Colours, be made with Prisms that Refract so much, as to make the Length of the Images five times its Breadth, and rather more than less; for otherwise Experiments will not succeed so plainly with others as they have done with me.

An Optical Experiment, by Mr. Steph. Gray. N. 221. p. 286. June. An. 1696.

VIII. I took a stiff piece of brown Paper, and pricking a small Hole therein, I held it at a little distance from me: then applying a Needle to my Eye, I was surprized to see the Point of it inverted. The nearer the Needle was to the Hole, it was so much the more magnify'd, but less distinct; and if it were so held, as that its Image was near to the Edge of the Hole, its Point seemed crooked. So that, it seems these small Holes, or somewhat in them, perform the Effects of a Concave Speculum, and so I take leave to call them *Aerial Speculums*.

A Problem of Alhazen, solved by Mr. Chr. Hugen, N. 97. p. 6119. Oct. An. 1673. Fig. 78.

IX. 1. Mitto tibi hac occasione Constructionem Problematis *Alhazeni* nuper à me inventam, & à Collegis meis felicem satis judicatam. Problema est; Dato Speculo Cavo aut Convexo, itemque Oculo & Puncto Rei visa, invenire Punctum Reflexionis.

Esto Speculum ex Sphæra quæ Centrum habeat A Punctum, Oculus vero sit in B, & Punctum Visibile in C, Planumque ductum per A, B, C, faciat in Sphæra Circulum D d, in quo invenienda sint Reflexionis Puncta. Per tria Puncta A, B, C, describatur Circuli Circumferentia, cujus sit Centrum Z; occurrat autem ei producta A E, Perpend. B C, in R, & sit duabus R A, O A, tertia Proportionalis N A, eritque N M, Parallela B C, altera Asymptoton. Rursus sint Proportionales E A, $\frac{1}{2}$ A O, A I, & Summa I Y æquali I N, ducatur Y M Parallela A Z; eaque erit altera Asymptotos. Denique sumptis I X, I S, quæ singulæ possint dimidium Quadratum A O, una cum Quadrato A I; erunt Puncta x & z in Hyperbola, aut Sectionibus oppositis D d, ad inventas Asymptotos describendis, quarum Intersectiones cum Circumferentia D O, ostendent Puncta Reflexionis quæsita. Constructio hæc, in omni Casu, quo Problema solidum est, locum habet, præterquam in uno, ubi non Hyperbola

sed

sed Parabola describenda est; cum nimirum Circumferentia per Puncta A, B, C, descripta, tangit Rectam A E.

2. Cum Nobilissimi *Hugenii* Constructionem ad Calculos revocarem, ean- By M. Slufius,
dem omnino mecum Analyfin secutum esse deprehendi; sed cum ex illa duæ ibid.
nascantur Effectiones, utraque per Hyperbolam circa Asymptotos; Ille unam,
Ego alteram, uti faciliorem, selegeram. Evidens est autem, nihil aliud quæri
hoc Problemate (si illud ad Terminos mere Geometricos revocemus) nisi
in dato Circulo, cujus Centrum A, Radius A P) Punctum aliquod ut P, à
quo ductis ad Puncta data E, B, inæqualiter à Centro A distantia, Rectis
P E, P B, Recta A P, producta bisecet Angulum E P B. Quod quidem va-
rios Casus recipit. Vel enim Normalis ex A in Rectum E B, nimirum A O,
cadit inter E & B, vel ultra B. Si ultra, vel Rectangulum E O B æquale est
Quadrato A O, vel majus vel minus. De casu æqualitatis videbimus infra;
nunc vero tres alios Casus eadem fere Constructione complectemur. Per tria
puncta A, E, B, transeat Circulus, ad cujus Circumferentiam producat A O Fig. 79, 80, 81
in D. Ac si quidem punctum O cadat inter E & B, Recta A O versus O pro-
ducenda erit; sin autem ultra B, sitque Rectangulum E O B majus Quadra-
to A O, producenda erit versus A; at si Rectangulum quadrato minus fuerit,
Circulus in ipso puncto D, rectam A O secabit. Tum ducta A X, parallela
E B, secante Circulum datum in N, fiat ut Rectangulum D A O ad quadra-
tum A N, ita $\frac{1}{2}$ A X ad A H, quæ sumenda erit versus X, si O cadat inter E
& B, aut Rectangulum E O B minus sit quadrato O A; at ex parte contraria,
si sit majus. Ponatur nunc O Q æqualis A H (in directum E B primo & se-
cundo casu, tertio vero versus E:) tum fiant proportionales X A, N A,
H K, sumenda omni casu versus X: sectaque A O in V, ut sit eadem Ratio
K A ad A V, quæ A D ad A X; jungatur K V, ac producat donec occur-
rat recta E M Parallela O A, indefinite productæ, in puncto L; erunt omni
casu K L & Q L Asymptoti Hyperbolæ, quæ per punctum O descripta, pro-
posito satisfaciunt: hoc tantum discrimine, quod primo & secundo casu Hyper-
bola per O, Problema solvet in speculo Convexo, Sectio vero ei opposita in
Concavo; at tertio casu contra, Hyperbola per O serviet Concavo, ejus opposita
Convexo. Atque id quidem, cum punctum V cadit inter A & O; nam si
ultra O caderet, unica Hyperbola inter easdem Q L, K L, descripta, tam spe-
culo Convexo quam Concavo satisfaceret. Cæterum si V caderet in ipsum
punctum O; Problema tunc Planum esset, & ipsæ Rectæ L Q, L K, illud ab-
solverent. Unde patet Problematis hujus dari Casus infinitos, qui per Locum
Planum solvi possunt: quo magis venia digni videntur ii, qui illud per eun-
dem Locum univèrse solvi posse censuerunt, quod ipsis aliquoties Calculus feli-
citer cecidisset. Nulla enim dari potest trium punctorum A, E, B, posito, (de
casu æqualitatis Rectanguli E O B, & Quadrati O A, mox videbimus,) quæ non
admittat Circulum aliquem ex Centro A describendum, ad cujus Circumfe-
rentiam Problema per Locum Planum solvi queat. Hujus autem Circuli
Radius, si tanta est, ita invenietur: in primo & secundo casu superioris Con-
structionis fiat ut Quadratum A X una cum duplo rectangulo O A D, ad du-
plum Quadratum A D; ita Quadratum A O, ad Quadratum A N, erit A N
Radius quæsitus. At in tertio casu, faciendum est, ut Quadratum A X minus
Duplo

Duplo Rectangulo OAD , ad Duplum Quadratum AD ; ita Quadratum AO , ad Quadratum AN .

Construendus nunc superest alius casus, æqualitatis nempe Rectanguli EOB , & Quadrati AO , sive in quo Circulus, per puncta A, B, E , descriptus, tangit Rectam AO . Recte autem monuit Clarissimus *Hugenius* hoc casu describendam esse Parabolam; quod tamen non ita intelligendum est quasi per Hyperbolam solvi non possit, cum & Hyperbolam & Ellipsin, imo Infinitas (si quis Methodo nostra uti velit) admittat; sed quod Parabolam quoque recipiat, quem alii casus respuunt. Eadem ratione temperandum est quod ait; Constructionem suam omni casu quo Problema solidum est; locum habere; intelligit enim, levi mutatione semper inveniri Hyperbolam quæ proposito serviat: quod casus à nobis superius Constructos cum ejus Constructione comparanti planum fiet. Ut autem ad casum æqualitatis redeam, & nequid temere asseruisse videar, Ecce tibi non unam, sed duas Parabolas, ac præterea Hyperbolas Oppositas quæ Propositum absolvunt. Sint, ut prius, puncta data E, B , Circulus ex Centro A , ac alius per tria puncta A, E, B , cujus Tangens sit AO , Centrum D . Ducta Diametro NAD , fiant tres Proportionales XA, NA, ZA , cujus dimidium sit AL . Fiant iterum tres Proportionales $2OA, NA, IA$, cujus dimidium sit KA , & perficiatur Rectangulum $LAOV$; productaque LV in S , donec VS sit tertia Proportionalis ipsarum AI, OV ; Axe SL , Latere Recto AI , Vertice S , describatur Parabola; hæc enim Circulum secabit in punctis P, P , quæsitis. Tantundem faciet alia, sic perfecto Rectangulo $DAKC$, & producta KC in T , ita ut CT sit tertia Proportionalis ipsarum AZ, DC , describatur circa Axem TK , Vertice T , Latere Recto, ZA : occurret enim Circulo in iisdem punctis P, P . Facilior adhuc est Constructio per Sectiones Oppositas; factis enim, ut prius, tribus Proportionalibus XA, NA, ZA , demittatur ZI Normalis, tertia Proportionalis Duplæ AO , & AN . Erit itaque ZI major ZA , cum Dupla AO minor sit XA : tum in puncto I , inclinetur utrinque Angulo Semirecto ad lineam IZ , rectæ IQ, IM , & ab utraque parte indefinite producantur; demum circa illas tanquam Asymptotos describatur per A Hyperbola, & alia ipsi Opposita; hæc enim satisfaciet Problemati in Speculo Convexo, illa in Concavo. Cum vero, ut ostendimus, ZI semper major sit Recta ZA , Recta IM nunquam transibit per A . Non dabitur itaque Casus, quo ex hac Constructione, velut in præcedentibus, Problema per ipsas Asymptotos solvi possit: & tamen hoc quoque aliquando Locum Planum admittit; cum scilicet accidit, ut Recta XO ducta ad Centrum D , Tangat Circulum NPP ; ipsum enim punctum Contactus quæstionem solvit. Et hæc quidem de Problemate, quod hactenus multorum ingenia exercuit, & cujus solutionem ante aliquot Annos absolvi.

Otherwise, by M. Sluſius. *ibid.*

p. 6123.

Fig. 84.

Accipe quæ circa *Alhazeni* Problema, curis secundis meditatus sum.

Datus sit Circulus, cujus Centrum A ; puncta data sunt D & d . Supponatur factum quod quæritur; sitque Radius Incidens DE , Reflexus Ed ; & ex Puncto Reflexionis E cadat in junctam DA , Normalis EI , & in eandem, ex d , Normalis dN , occurrantque eidem Tangens EC & Radius dE , productis in B . Sit nunc $DA = z$. $AI = a$. $NA = n$. $EI = e$. $dN = b$. $BA = y$. $AE = q$. $CA = x$. Igitur, cum Anguli, DEC, CEB , sint æquales, &

Angulus

Angulus CEA, Rectus, ex Hypothefi erunt tres, DA, CA, BA, Harmonice Proportionales (hoc enim facile ostenditur). Erit itaque ut DA, ad BA, ita DC, ad CB; five in terminis Analyticis, $z:y::z-x:x-y$; & $2zy$

$-xy = zx$, five $\frac{2zy}{z+y} = x$. Cum autem Rectangulum CAI, five xa ,

fit æquale Quadrato AE, five qq , erit $x = \frac{qq}{a}$, & per consequens $\frac{2zy}{z+y} =$

$\frac{qq}{a}$, five $\frac{2qq}{2za - qq} = y$. Porro, est ut dN, ad EI: ita NB, ad IB;

five $b:c::y-n:y-a$. Itaque $yc - nc = by - ba$; & $y = \frac{ba - nc}{b - c}$.

Igitur $\frac{2qq}{2za - qq} = \frac{ba - nc}{b - c}$ five $2zbaa - 2znac - qqba + qqnc$

$= bzqq - zqqc$. Quæ æquatio est ad Hyperbolam circa Asymptotos, cujus Constructio cum Circulo dato, Problemari satisfaciit. Cum vero, ob Circulum, sit $qq = aa + ee$, si loco $2zbaa$, ponatur ejus Valor $2bzqq - 2bzee$, habebitur alia pariter ad Hyperbolam circa Asymptotos $bzqq - 2bzee - 2znac - qqba + qqnc = -zqqc$. Et hac Methodo, atque illa, quam in Libello nostro de *Analyfi* exposuimus, prodibunt infinitæ Æquationes ad Hyperbolas & Ellipses, quæ cum Circulo dato Problema absolvent; nisi quod Effectiones plerumque intricatiores evadunt quam ut operæ pretium sit illas aggredi: construi tamen poterunt eo modo, quo ibi usi sumus in Ellipsi.

Retulimus, ut vides, Calculi nostri Summam ad lineam DA, sed satis animadvertis, non majori difficultate referri potuisse ad dA (quæ pariter data est) ductis scilicet lineis, quas in Schemate punctis adumbravimus. Verum novo Calculi labore non est opus. Si enim Rectæ dA, ejusque partibus, eosdem ac prius terminos Analyticos adhibeas, h. e. si ipsam dA facias æqualem z. $Dn = b.n$, $EA = n$, $AI = a$, $IE = c$. &c. prodibit eadem Æquatio quæ prius; & infinitas alias Hyperbolas & Ellipses obtinebis, quæ cum Circulo dato Problemari satisfaciunt. *¶* Desidero, essem si singulos Casus prosequi vellem, cum illorum Æquationes sola Signorum + & - variatione discernantur. Unum tamen excipio, nimirum cum Angulus dAD est Rectus; ejus enim æquatio habetur, ex punctis à priori æquatione partibus, in quibus n (quæ in nihilum abit) invenitur: nempe hæc, $2zbaa - qqba = bzqq - zqqc$, vel (pro $2zbaa$, posito ejus valore) $zbqq - qqba = 2zbee - zqqc$.

Sed animadvertendum est, quod licet referendo Analysin ad Rectam DA, statim sese offerant in æquatione duæ Hyperbolæ; & aliæ totidem à prioribus diversæ, cum refertur ad Rectam dA; easdem tamen omnino Parabolas haberi, ad utramvis Rectarum dA, vel DA, referatur Analysis: cujus rei ratio levi consideratione tibi occurret.

Fig. 85.

Patere nunc, *V. Cl.* ut superiorem Analysin omnibus, quæ circa Speculorum Sphæricorum Reflectionem proponi solent, Problematibus applicem, novo facto Schemate. Sit igitur, ut prius, Circulus, cujus Centrum A, Punctum D datum, & ab eo Radius Incidens DE, cujus Reflexus sit EQ. Juncta DA, ducatur ad illam Tangens EC, & Normalis EI; & producat ad eandem, Recta QEB; Denominentur partes ut prius. $DA = z$. $CA = x$. $AE = q$. $BA = y$. $AI = a$. $IE = c$. Igitur, propter tres DA, CA, BA, Harmonice Proportionales,

& tres CA, AE, AI, Geometrice, semper habebitur æquatio $y = \frac{zqq}{2za - qq}$,

in quodcumque Circuli punctum cadat Radius DE. Itaque si quæretur punctum E, in quod si Radius DE Incidat, Reflectatur *παράλλως* Diametro LAV normali ad DA; Reflexus QE, productus transibit per I, ut patet;

& I ac B coincident. Igitur $a = y = \frac{zqq}{2za - qq}$; five, $aa - \frac{1}{2} \frac{qq}{z}$

$= \frac{1}{2} qq$; & Problema per Plana solvetur.

Si quæretur punctum, à quo Radius Reflectatur parallelus alteri cuilibet lineæ, ut AK (ductæ ex Centro A;) ducatur ad illam, ex puncto I, Tangens KL = d. Evidens est, Triangula AKL, EIB, fore similia, cum omnia Latera unius parallela sint Lateribus alterius, &c. Itaque AL, ad LK; ut EI,

ad IB, five $q : d :: c : a - y$; & $\frac{qa - de}{q} = y = \frac{zqq}{2za - qq}$; & $zq' =$

$2qzaa - 2zdae - q'a + qqde$; five, pro aa posito $qq - ee$, $zq' = 2zq' - 2zqee - 2zdae - q'a + qqde$. Utraque autem æquatio est ad Hyperbolicam circa Asymptotos, quæ cum Circulo dato Problema absolvit.

Proponatur nunc efficere, ut Radius Reflexus transeat per datum punctum N, (ut in Problemate *Alhazeni*) vel ut productus versus punctum Reflexionis E, occurrat dato puncto N. Ex N cadat in AL Normalis NO = n, sitque AO = b. Patet esse, ut AO, ad differentiam ipsarum ON, AB; ita EI,

ad IB, $b.e.b : n - y :: c : a - y$; vel $b : y - n :: c : y - a$. Igitur $\frac{ba - ne}{b - e}$

$= y = \frac{zqq}{2za - qq}$. Unde $2zbaa - 2znae - qqba + qqne = bzqq$

$- zqqc$; nim. illa ipsa æquatio Problematis *Alhazeni* quam supra innui-

mus: Vel, secundo casu, $\frac{ba + ne}{b + e} = y = \frac{zqq}{2za - qq}$, five $2zbaa +$

$2znae - qqba - qqne = zbqq + zqqc$.

Atque hæc sunt Problemata, quæ circa punctum Reflexionis proponi solent, in quibus tamen Finitam puncti D dati distantiam supposuimus. Sed facilius erit Analysis, si supponamus Infinitam. Secta enim CA bifariam in G, constat ex proprietate trium, DA, CA, BA, Harmonice Proportionalium, tres DG, CG, BG, fore Geometrice Proportionales, supposita quacunque puncti

puncti D distantia. Itaque, si supponatur Infinita, B G abibit in nihilum, & punctum B cum puncto G coincidet. Igitur A B erit perpetuo æqualis B C; erit itaque C A = 2y, & Rectangulum C A I, æquale Quadrato A E, dabit, in

terminis Analyticis, $2ay = qq$, five $y = \frac{qq}{2a}$: Cumque distantia puncti D

supponatur infinita, erit E D Parallela A C. Itaque, si quærat RADIUS Reflexus parallelus A L, quoniam eo casu a & y coincidunt, erit a = y =

$\frac{qq}{2a}$, five aa = $\frac{1}{2}qq$: Si quærat ut Parallelus sit A K, erit rursus q:d::e:a-y;

& $\frac{qa - de}{q} = y = \frac{qq}{2a}$, five $2qaa - 2dae = q^3$. Si petatur ut tran-

seat per N, erit, ut supra, $\frac{ba + ne}{b + e} = y = \frac{qq}{2a}$, & $2baa + 2nae = bqq$

+ qqe: Quæ æquationes sunt quoque ad Hyperbolas circa Asymptotos, nisi N punctum esse supponatur in A L; nam cum tunc n abeat in nihilum, sublatis ab æquatione partibus, in quibus n continetur, residuæ dant æquationem ad Parabolam, ut supra quoque monuimus.

Non expectas, V. Cl. ut cum Specula Concava hætenus in Exemplum adduxerim, nunc agam de Convexis. Scis enim, eandem esse prorsus Analysin, & Æquationes sola Signorum + & - variatione distingui. Scis Parabolam vel Ellipsin quæ uni satisfacit, satisfacere alteri; & si Hyperbola in Convexo Problema absolvat, ejus Oppositam paria facere in Concavo. His itaque omissis, addo tantum, eadem Analyfi haberi in Speculis Concavis Focos & Spatia, quæ Radii occupant in Axe, data qualibet Puncti Lucentis distantia: Sed mira facilitate, cum Radii supponuntur paralleli; quod tamen nonnullis circuitu à quibusdam demonstrari vidi. Nam in Speculo Concavo E E, cujus Centrum A, si Radius extremus Reflecti intelligatur ad Axem A R in B, ducta Tangente E C, erit C B = B A. Bisecetur Semi-axis A R in Q; erit itaque Q Focus. Et Q B Spatium quæsitum. Est autem Q B dimidia C R (ob æquales A Q, Q R, A B, B C,) h.e. dimidia excessus secantis Arcus E R supra Sinum totum. Igitur si Arcus E R sit

Fig. 86.

(e.g.) grad. 9, erit A C, 101246, & B Q $\frac{623}{100000}$ ipsius A R.

4. Compendium, quod eodem tempore inveni circa primam Constructionem, ab initio tibi communicatam, tale est: Ducta linea A T, parallela C B, eaque bisecta in V, punctum hoc est illud, per quod transire debet una Hyperbolarum Oppositarum, quarum Asymptoti inventæ fuerunt Y M, M N.

Otherwise, by
M. HUGENS.
N. 98. p. 6143.

Fig. 87.

Sed en Tibi bonam illam Constructionem, quæ in omnibus Casibus obtinet. Sit Circulus datus E D, cujus Centrum est A; Puncta data, B & C.

Fig. 88.

Ductis Lineis A B, A C, fiant Proportionales B A (Radius Circuli) & F A: Eodem modo C A, (Radius Circuli) & G A. Tum jungatur F G, eaque bisecetur in H; & per hoc punctum ducantur Lineæ L H K, M H N,

se invicem intersecantes ad Angulos Rectos, quarumque LHK sit Parallela ei quæ bisecat Angulum BAC. Hæ sunt duæ Asymptoti Hyperbolarum describendarum per puncta F & G, & quarum una transibit etiam per Centrum A, quarum Intersectiones cum Circuli Peripheria notabunt puncta Reflexionis quæsitæ.

Further considered, by Mr. Slu-fius. ib. p. 614t.

5. Videt hic Nob. *Hugenius* qua ratione ad omnes Casus extendi posset Hyperbola Æqualium Laterum, quam in casu Anguli Recti sese statim offerre præcedentibus meis insinuaveram. Posset quoque ex infinitis Ellipsis, quæ adhiberi possunt, una feligi non difficilis Constructionis: sed piget tamdiu in eodem Problemate hære. Superest tamen aliquid, quod contemplationem habet non injucundam; *nim.* cum Sectiones, quæ cum Circulo dato ad Problematis solutionem adhibentur, illum in quatuor punctis secant, quorum duo tantum Reflexioni serviunt, quæri posset, quodnam Problema solvent duo reliqua; & quam verborum forma concipienda sit Propositio, ut quatuor illos Casus complectatur. Deinde annon etiam iidem quatuor Casus occurrant cum Puncta data æqualiter distant à Centro?

Again. ib.

Clar. *Hugenius* non alia utitur Analyfi quam mea, quæ Parabolam uno tantum casu admittit. Quod ut evidentius tibi constet, Æquationem quam Construxit hic adscribam. Repete memoria, si placet, quæ secundis curis ad te scripsi, & invenies, me duas æquationes, Problemati per Hyperbolam circa Asymptos solvendo idoneas, assignasse has nimirum;

$$2zbaa - 2znac - qqba + qqnc = bzqq - zqqe,$$

$$\text{Et } bzqq - 2znac - qqba + qqnc = 2zbee - zqqe;$$

ac subjecisse, levi mutatione, (substituendo, ex gr. pro qq , ejus Valorem $aa + ee$) invenire posse infinitas Hyperbolas & Ellipses, quæ cum Circulo dato Problema solverent. Nunc in priori ex his æquationibus pro $bzqq$ ponatur ejus Valor, fiet

$$zbaa - 2znac - qqba + qqnc = bzee - zqqe;$$

$$\text{Sive } aa - \frac{qqa}{z} = ee - \frac{qqe}{b} + \frac{znac}{b} - \frac{qqnc}{zb}.$$

Atque hæc est Æquatio, quam magno ingenii acumine, ac pari facilitate, construxit Vir Doctissimus.

Again. ib.

Fig. 89.

Incidit nuper in sequentem Constructionem, qua brevior cum dari posse vix credam, committere nolui, quin eam judicio ac censuræ tuæ submitterem. Sint igitur Puncta data EB, Circulus cujus Centrum A; junctis EA, BA, Secantibus Circulum in F & C; fiant tres Proportionales EA, FA, VA, & tres iterum BA, CA, XA: Tum juncta VX, ac producta utcunque, (Vertite X, Latere Transverso VX, ac Recto ipsi æquali) describatur Hyperbola XP, cujus applicatæ ad Diametrum VYG, Parallelæ sint Rectæ AB: Illa enim satisfacit proposito in casu Speculi Convexi, ut ejus Opposita in casu Concavi. Si Asymptotos desideres, facile reperiri possunt, producta VX, donec cum EB, pariter producta, concurrat in L; deinde bisecta VX in I, ac sumpta LD aquali LI; juncta enim DI erit Asymptoton una, in quam alia normaliter incidit ad punctum I.

Sed

Sed fortasse ingratum tibi non erit intelligere, qua via ad hanc Constructionem pervenerim. Scias itaque, me ex priori mea Analyfi deduxisse hoc modo. Datis iisdem quæ prius, cadat in E B, Normalis A O, sitque punctum quæsitum P, ex quo in A O cadat Normalis P R. Si A O fit b ; E O, z ; O B, d ; A P, q ; P R, e ; A R, a ; facile colligitur hæc Æquatio,

$$\frac{2zdae + 2bbac - 2bqqe}{zb - bd} + ee = aa - \frac{qq^2}{b}, \text{ quæ mutari}$$

potest in has, $\frac{zdae + bbac - bqqe}{zb - bd} = aa - \frac{1}{2}qq - \frac{\frac{1}{2}qq^2}{b}$; Et

$$\frac{zdae + bbac - bqqe}{zb - bd} + ee = \frac{1}{2}qq - \frac{\frac{1}{2}qq^2}{b}. \text{ Hujus ultimæ Constru-}$$

ctionem olim ad te misi; alterius vero, Cl. *Hugenius*. Primam autem, licet se statim in conspectum dedisset; ferme neglexeram, quod difficilioris Constructionis esse præsumerem. Sed me vano timore delusum agnovi, cum in hanc, quam ad te mitto, Constructionem desinere nuper sum expertus. Sit enim,

brevioris Calculi causa, $z - d = k$, $z d + b b = b m$; fiet $ee = \frac{-2qqe + 2mae}{k}$

$$= aa - \frac{qq^2}{b}. \text{ Et additis utrinque } \frac{q^4 + m m a a - 2 q q m a}{k k}, \text{ erit}$$

$$ee + \frac{mae - 2qqe}{k} + \frac{q^4 + m m a a - 2 q q m a}{k k}, \text{ (hoc est quadratum}$$

$$\text{ex } e - \frac{qq + ma}{k} -), \text{ æquale } aa - \frac{qq^2}{b} + \frac{q^4 + m m a a - 2 q q m a}{k k}.$$

Fiet igitur, *analogiæ* $kk : kk + mm :: aa - \frac{kkqq^2}{bkk + bmm} -$

$$\frac{2qqma + q^4}{kk + mm}; \text{ \& quadratum } e - \frac{qq + ma}{k}; \text{ qui ad æquationem facilio-}$$

rem reduci potest, si, posito $kk + mm = pp$, fiat $\frac{ky}{p} = a$; sit enim tan-

$$\text{dem, quadratum ex } e - \frac{qq}{k} + \frac{my}{p} = yy - \frac{qqky}{bp} - \frac{2qqmy}{kp} +$$

$$\frac{q^4}{kk}; \text{ quam Æquationem superiori Constructioni respondere animadvertes,}$$

si Calculos applicueris; ac simul observabis, ad quamcunque linearum E A, A B, B E, referatur Analyseos summa, easdem semper habere posse Sectiones, quamvis longiori circuitu & æquationibus valde diversis.

Fig. 91.

Ex hac Constructione, κατ' ἀνάλογίαν deducere licet alterius Problematis effectiōnem, cum scil. quæritur Punctum, à quo Radius Reflexus Parallelus sit cuilibet lineæ datæ; ut, si dato Puncto Luminoso B, Circulo ex Centro A, quæreretur Radius Reflexus parallelus rectæ AE. Idem enim est, ac si in alio Problemate, distantia punctorum A & E supponeretur infinita; quo casu Tertia Proportionalis ipsarum EA, FA, abiret in nihilum, & puncta A & V. Coinciderent: Itaque VX esset æqualis AX, & AE parallela PE. Applica igitur superiore Constructionem & Problema absolves. Descripta scil. (Vertice X, Latere Transverso VX, vel AX, & Recto ipsi æquali,) Hyperbola XP, cujus Applicatæ ad Diametrum AX, parallelæ sint rectæ AE.

By M. Hu-
gens. *Ibid.*
p. 614.

6. Verum est, quin imo mirandum, Constructionem quam antehac ad te misi inveniri quoque per Calculum quem Dn. *Simus* de ea instituit post mutationem qq in $aa + ee$; at hoc videtur fieri casu, nec ibi apparet Constructionis simplicitatis nisi postquam eam peragere satagemus.

Fig. 92.

Problema Alhazeni. Dato Circulo, cujus Centrum A, Radius AD, & punctis duobus B, C; invenire punctum H in Circumferentia Circuli dati, unde ducta HB, HC, faciant ad Circumferentiam Angulos æquales.

Ponatur Inventum, ductaque AM recta, quæ bifariam secet Angulum BAC, ducatur ei Perpendicularis HF, itemque BM, CL. Jungatur porro AH, cui Perpendicularis sit HE, Rectisque BH, HC, occurrat AM in punctis K, G.

$$\begin{array}{l|l} \text{Sit jam } AM = a & \text{Quia ergo æquales Anguli KHE \& CHZ, sive EHG;} \\ MB = b & \text{estque EHA Angulus Rectus, erit ut KE, ad EG; ita} \\ AL = c & \text{KA, ad AG. Quia vero BM, ad MD; ut HF, ad} \\ LC = n & \text{FK, erit,} \\ & \text{ut BM + HF ad HE, ita MF ad FK,} \\ \text{Radius AD = d} & \text{i. e. } b + y : y :: a - x : \frac{ay - xy}{b + y}. \text{ add. FA = x fit} \\ AF = x & \\ FH = y & \text{KA = } \frac{ay + bx}{b + y}. \end{array}$$

Rursus, quia CL ad LG, ut HF ad FG, erit permutando & dividendo CL

$$- HF \text{ ad } -HF, \text{ ut LF ad FG, } n - y : y :: c - x : \frac{cy - xy}{n - y}, \text{ qua ablata}$$

$$\text{ab AF = x, fit GA = } \frac{nx - cy}{n - y}. \text{ Est autem EA = } \frac{dd}{x}, \text{ quia Pro-}$$

$$\text{portionales FA, AH, AE: Ergo EA - GA, hoc est, EG, = } \frac{dd}{x} -$$

$$\frac{nx + cy}{n - y}. \text{ Et KA - EA, hoc est, KE = } \frac{ay + bx}{b + y} - \frac{dd}{x}.$$

Sed

Sed diximus, quod KE ad EG, ut KA ad AG; i.e. $\frac{ay+bx}{b+y} = \frac{dd}{x}$:

$$\frac{dd}{x} = \frac{nx+cy}{n-y} :: \frac{ay+bx}{b+y} : \frac{nx-cy}{n-y}.$$

Unde invenitur $2anxxy + 2bnx^2 - ddbnx - ddnxy = naddy + nbddx$
 $- 2acxyy - 2bcxxy + ddbcyy + ddcyy - addyy - bddxy.$

Et quia $n = \frac{bc}{a}$, fit $\frac{2bbc}{a} x^3 - \frac{bbddcx}{a} - \frac{2bbcyyx}{a}$, quia $xx =$

$dd - yy$. Est autem $\frac{2bbc}{a} x^3 = \frac{2bbcdx}{a} - \frac{2bbcyyx}{a}$, quia $xx =$

$dd - yy$. Ergo $\frac{-2bbcxxy}{a} - \frac{ddbcyx}{a} - 2acxyy + ddcyy = -$
 $addyy - bddxy.$

Et divisus omnibus per y & ductis in a

$$\begin{aligned} -2bbcxxy - ddbcx - 2aacxy + ddca y &= -aaddy - bddax \\ abddx - cbddx + acddy + aaddy &= 2aacxy + 2bbcxxy \\ \frac{abddx - cbddx + acddy + aaddy}{2aac + 2bbc} &= xy, \text{ quæ æquatio est} \end{aligned}$$

ad Hyperbolam.

$$\text{Vel quia } bc = na, \frac{abdd - anddx + acddy + aaddy}{2aac + 2bbc} = xy.$$

$$\text{Sit } \frac{add}{aa+bb} = p; \text{ Ergo } \frac{pbx - pnx + pcy + pay}{2c} = xy.$$

Unde porro non difficulter invenitur sequens Constructio: Jungantur BA, AC, & applicato feorsim ad utramque Quadrato Radii AD, fiant inde AP, AQ; & juncto PQ, dividatur ipsa bifariam in R, & per punctum R ducantur RD, RN, sese ad Rectos Angulos secantes, quorumque RD, sit parallela AD, quæ dividet bifariam Angulum BAC. Erunt jam RD, RN Asymptoti Oppositarum Hyperbolarum, quarum altera per Centrum A transire debet, quæque secabunt Circumferentiam in punctis H quæsitis. Transibunt autem Hyperbolæ per Puncta P, G. Fig. 93.

Ratio Constructionis apparet, ductis P γ , & Q ζ , perpendicularibus in

A.M. Fit enim A $\gamma = \frac{add}{aa+bb}$ five p ; & A $\zeta = \frac{ap}{c}$. Item P $\gamma = \frac{pn}{c}$,

& Q $\zeta = \frac{pb}{c}$. Quare AO = $\frac{pc+pa}{2c}$, & OR = $\frac{pb-pn}{2c}$. Un-

de Cætera facilia.

By Mr. Sluſius
ib. p. 6145.

Fig. 94.

7. Mirari deſine, Vir Clariffime, eandem in *Alhazeniano Problemate* Conſtructionem ex diverſis *Æquationibus* deduci, quandoquidem illæ omnes, quibus hætenus uſi ſumus in una eademque generali Analyſi contineantur. Quod ut oſtendam, datus ſit Circulus cujus Centrum A, puncta H & I; ſitque Punctum quaſitum K, ad quod ex punctis I & H ducantur Rectæ HK, IK, & Tangens KD. Tum ex A ducatur quaelibet AG, occurrens HK in E, IK in B, Tangenti KD in D (iis nim. productis, quas produci eſt opus). His poſitis evidens eſt, ob Angulos EKD, DKB, æquales, & Angulum AKD Rectum, tres AE, BE, DE fore ſemper Harmonice proportionales. Itaque ductis ad AE Normalibus KC, IF, HG, ac denominatis partibus, $AK = q$ habebitur, methodo, quam in ſecunda hujus Problematum Analyſi olim adhibui, hæc generalis *Æquatio*,

$$AC = a \quad \begin{array}{l} ndaa - bzaa - nqqa + bqqa = ndee - zbee + \\ CK = c \quad 2bnae + 2zdae - dqge - qqge. \end{array}$$

FG = b Finge nunc, AG eſſe perpendicularem ad HI, nihil varietatis erit in æquatione; niſi quod AF & AG, hoc eſt, d & z, erunt æquales. Poſito itaque d pro z, fiet

$$FA = z \quad \begin{array}{l} ndaa - bdaa - nqqa + bqqa = ndee - dbec + \\ FI = n \quad 2bnae + 2ddae - 2dqge. \end{array}$$

Sive applicatis omnibus ad $nd - db$,

$$aa \frac{-qqe}{d} = ee \frac{+ 2bnae + 2ddae - 2dqge}{nd - bd};$$

Eadem nempe, quam ex prima mea Analyſi, licet alia via, deduxeram, & quam nuper, modo facili conſtructam, ad te miſi.

Pone deinde, AG coincidere cum AH; abibit igitur HG, five b, in nihilum. Expunctis itaque ab æquatione partibus, in quibus b reperitur, remanebit, $ndaa - nqqa = ndee + 2zdae - dqge - qqze$. Hanc autem, ſi meministi, curis ſecundis inveni, & aliam huic ſimilem, in Caſu pro Recta AG tranſire intelligitur per I.

Supponamus demum, Rectam AG, ſecare bifariam Angulum HAI; erit ob ſimilitudinem Triangulorum HAG, IAF, ut HG ad GA, ita IF ad FA, five ut b ad d, ita n ad z, & $nd = bz$. Ablatis igitur æqualibus, ſit $bqqa - nqqa = 2bnae + 2zdae - dqge - qqze$: Illa ipſa, quam ut ex literis tuis nuper intellexi, Cl. *Hugenius* conſtruxit.

Intelligatur tandem eadem Recta HG, ſecare bifariam Rectam HI; erunt igitur æquales HG, IG, hoc eſt, $b = n$; fietque, ablatis æqualibus, $bdaa - bzaa = bdee - bzee + 2bbac + 2zdae - dqge - qqze$; quam licet non admodum difficilem, nemo noſtrum hætenus conſtruxit. Hæ autem, ut & ipſa Generalis *Æquatio*, in duas alias dividi poſſunt, poſito, ut noſti, pro aa vel ee, ejus valore, $qq - ee$, vel $qq - aa$.

Vides igitur, quicquid hætenus præſtitum eſt, in eandem Analyſin reſolvi; quæ & infinitas alias Conſtructiones per Circulum datum & Hyperbolam complectatur. Sed eas inveſtigare non eſt tanti, cum in hoc Problemate, ut olim fortassis inopia, ſic nunc copia laboremus. Ad dam tantum Conſtructionem per Parabolam, idque via duplici; quæ licet

et aliis per Hyperbolam operosior videatur, Lineæ tamen simplicitate, quæ Parabola inter reliquas Sectiones commendatur, operam compensat.

Iisdem igitur Datis, Jungatur A I, & producat in S, donec A S fiat æqualis A H, junctaque H S, & bisecta I S in M, ducatur per M Recta R M Q normalis ad H S, in quam cadat ex A Normalis A Q, & cui Parallelus ducatur Radius A C. Tum factis tribus Proportionalibus I A, A C, A E, fiat ut S A ad A E, ita M Q ad A D, & R S ad A P (in recta A Q versus Q;) & in eadem ab alia parte sumatur D O æqualis D C. Demum bisecta P D in X, inclinetur per X, Angulo Semi-recto ad A X. Recta V X L, occurrens Normali in D erectæ in puncto V, & in quam ex O cadat Normalis O B. Aio, si fiat ut V X ad X B, ita X B ad B L, punctum L esse Verticem, L V, Axem, X V Latus Rectum Parabolæ, quæ Problemati satisfacit omni casu; Secans nimirum Circulum datum in punctis K, quorum supremum & ultimum ad *Problema Alhazenianum* pertinent, reliqua ad aliud.

Fig. 95.

Datur, ut supra indicavi, alia quoque Parabola, quæ cum hac paria facit, & cujus descriptio ex hac adeo facile deducitur, ut nova non sit opus. Sumatur enim A δ , in Directum D A, & ipsi æqualis, & in Directum O A, ipsi quoque æqualis A ν . Tum bisecta P δ in ξ , ducatur per ξ Recta $\nu \xi \rho$, Normalis ad X B, concurrens cum $\delta \nu$, Normali ad O A, in ν , & in quam cadat Normalis $\nu \beta$; ac fiat ut $\nu \xi$ ad $\xi \rho$, ita hæc ad $\beta \lambda$: Erit λ Vertex, $\lambda \xi$ Axis, $\nu \xi$ Latus Rectum Parabolæ, quæ in iisdem cum priore punctis Circulum datum secabit.

X. Let B E β be a double Convex Lens, C the Center of the Segment E B, and K the Center of the Segment E β , B β the Thickness of the Lens, D a Point in the Axis of the Lens; and it is required to find the Point F, at which the Beams proceeding from the point D, are collected therein, the Ratio of Refraction being as m to n . Let the distance of the Object D B = D A = d , (the point A being supposed the same with B, but taken at a distance therefrom, to prevent the Coincidence of so many Lines) the Radius of the Segment towards the Object C B, or C A = r , and the Radius of the Segment from the Object K β , or K α = ρ , and let B β , the thickness of the Lens, be = t , and then let the Sine of the Angle of Incidence D A G, be to the Sine of the Refracted Angle H A G, or C A ϵ , as m to n ; and in very small Angles the Angles themselves will be in the same Proportion; whence it will follow, that, As d to r , so the Angle at C to the Angle at D, and $d + r$ will be as

To find the Principal Focus of Optick Glasses universally; by Mr. Edm. Halley. N. 209. p. 960.

Fig. 96.

the Angle of Incidence G A D; and again, as m to n , so $d + r$ to $\frac{dn + rn}{m}$,

which will be as the Angle G A H = C A ϵ . This being taken from A C D,

which is as d , will leave $\frac{m - nd - nr}{m}$ Analogous to the Angle A ϕ D,

and the Sides being in this Case proportional to the Angles they subtend, it will

will follow, that as the Angle $A \phi D$, is to the Angle $A D \phi$, so is the Side AD , or BD , to $A \phi$, or $B \phi$: That is, $B \phi$ will be $= \frac{m dr}{m - n d - nr}$,

which shews in what Point the Beams proceeding from D would be collected by means of the first Refraction; but if nr cannot be subtracted from $m - n d$, it follows, that the Beams after Refraction do still pass on Diverging, and the Point ϕ is on the same side of the Lens beyond D . But if nr be equal to $m - n d$, then they proceed Parallel to the Axis, and the Point ϕ is infinitely distant.

The Point ϕ being found as before, and $B \phi - B \beta$ being given, which we will call δ , it follows by a Process like the former, that βF , or the Focal

Distance sought, is equal to $\frac{\delta e n}{m - n \delta + m \phi} = f$. And in the room of δ sub-

stituting $B \phi - B \beta = \frac{m dr}{m - n d - nr} - t$, putting p for $\frac{n}{m - n}$, after due Re-

duction this Equation will arise, $\frac{m p d r \phi - n d \phi t + n p r t}{m d r + m d \phi - m p r \phi - m - n d t + n r t} = f$.

Which *Theorem*, however it may seem Operose, is not so, considering the great number of Data that enter the Question, and that one half of the Terms arise from our taking in the thickness of the Lens, which in most Cases can produce no great Effect; however, it was necessary to consider it, to make our Rule perfect. If therefore the Lens consist of Glass, whose Refraction is as

3 to 2, 'twill be $\frac{6 d r \phi - 2 d \phi t + 4 r \phi t}{3 d r + 3 d \phi - 6 r \phi - d t + 2 r t} = f$. If of Water, whose Refraction is as 4 to 3, the *Theorem* will stand thus:

$\frac{12 d r \phi - 3 d \phi t + 9 r \phi t}{4 d r + 4 d \phi - 12 r \phi - d t + 3 r t} = f$. If it could be made of Diamant, whose Refraction is as 5 to 2, it would be

$$\frac{\frac{10}{3} d r \phi - 2 d \phi t + \frac{5}{3} r \phi t}{5 d r + 5 d \phi - \frac{10}{3} r \phi - 3 d t + 2 r t} = f.$$

And this is the Universal Rule for the Foci of Double Convex Glasses exposed to Diverging Rays. But if the thickness of the Lens be rejected as

not sensible, the Rule will be much shorter, viz. $\frac{p d r \phi}{d r + d \phi - p r \phi} = f$;

or in Glass, $\frac{2 d r \phi}{d r + d \phi - 2 r \phi} = f$. All the Terms wherein t is found be-

ing omitted, as equal to nothing. In this Case, if d be so small, as that $2 r \phi$ exceed $d r + d \phi$, then will it be $-f$, or the Focus will be Negative,

gative, which shew that the Beams after both Refractions still proceed diverging.

To bring this to the other Cases, as of Converging Beams, or of Concave Glasses, the Rule is ever composed of the same Terms, only changing the Signs of + and - ; for the distance of the Point of Concourse of Converging Beams, from the Point B, or the first Surface of the Lens, I call a Negative Radius, or - r if it be the first Surface, and - e if it be the second Surface. Let then Converging Beams fall on a double Convex of

Glass, and the *Theorem* will stand thus
$$\frac{-2dre}{-dr - de - 2rp} = +f,$$
 which shews, that in this Case the Focus is always Affirmative.

If the Lens were a *Meniscus* of Glass, exposed to diverging Beams, the

Rule is,
$$\frac{-2dre}{-dr + de + 2rp} = f.$$
 Which is Affirmative, when $2rp$ is

less than $dr - de$, otherwise Negative: But in the Case of Converging

Beams falling on the same *Meniscus*, 'twill be
$$\frac{+2dre}{+dr - de + 2rp} = f.$$

And it will be $+f$ whilst $de - dr$ is less than $2rp$, but if it be greater than $2rp$, it will always be found Negative or $-f$. If the Lens be double Concave,

the Focus of Converging Beams is Negative, where it was Affirmative in the Case of Diverging Beams on a double Convex, viz.
$$\frac{-2dre}{+dr + de - 2rp} = f.$$

which is Affirmative only when $2rp$ exceeds $dr + de$: But Diverging Beams passing a double Concave, have always a Negative Focus, viz.

$$\frac{2dre}{+dr + de + 2rp} = -f.$$

The *Theorems* for Converging Beams are principally of use to determine the Focus resulting from any sort of Lens placed in a Telescope, between the Focus of the Object-Glass and the Glass itself, the distance between the said Focus of the Object-Glass and the interposed Lens being made $= -d$.

In case the Beams are Parallel, as coming from an infinite Distance, (which is supposed in the case of Telescopes) then will d be supposed infinite, and in

the *Theorem*
$$\frac{pdr}{dr + de - pr}$$
 the Term pr vanishes, as being finite,

which is no part of the other infinite Terms, and dividing the Remainder by

the infinite part d , the *Theorem* will stand thus
$$\frac{pe}{r + e} = f,$$
 or in Glass

$$\frac{2re}{r + e} = f.$$

In case the Lens were Plano-Convex exposed to diverging Beams, instead of

$\frac{p dr}{dr + d\epsilon - pr\epsilon}$, r being infinite, it will be $\frac{p d\epsilon}{d - p\epsilon} = f$, or $\frac{2 dr}{d - 2\epsilon} = f$, if the Lens be Glass

If the Lens be Double-Convex, and r be equal to ϵ , as being formed of Segments of equal Spheres, then will $\frac{p d\epsilon r}{dr + d\epsilon - pr\epsilon}$ be reduced to

$\frac{p dr}{2 d - pr} = f$; and in case d be infinite, then it will be yet farther con-

tracted to $\frac{1}{2} pr$, and p being $= \frac{n}{m - n}$, the Focal Distance in Glass will be

$= r$, in Water $1\frac{1}{2}r$, but in Diameter, r .

This is not only useful to discover the Focus from the other proposed Data, but from the Focus given, we may thereby determine the Distance of the Object, or from the Focus and Distance given we may find of what Sphere it is requisite to take another Segment, to make any given Segment of another Sphere cast the Beams from the Distance d to the Focus f . As likewise from the Lens, Focus, and Distance given, to find the Ratio of Refraction, or of m to n , requisite to answer those Data. All which, it is obvious, are fully determined from the Equation we have hitherto used, viz. $p d\epsilon r = drf + d\epsilon f - pr\epsilon f$. For to find d , the Theorem is,

$\frac{pr\epsilon f}{rf + \epsilon f - pr\epsilon} = d$, the Distance of the Object; for p , the Rule is

$\frac{drf}{p dr + df + prf} = \epsilon$; but for p , it will be $\frac{drf + d\epsilon f}{d\epsilon r + f\epsilon r} = p$; which

latter determines the Ratio of Refraction, m being to n as $1 + p$ to p .

I shall not expatiate on these Particulars, but leave them for the Exercise of those that are desirous to be informed in Optical Matters, which I am bold to say are comprehended in these three Rules, as fully as the most Inquisitive can desire them, and in all possible Cases, Regard being had to the Signs $+$ and $-$, as in the former Cases of finding the Focus. I shall only shew two considerable Uses of them; the one to find the Distance whereat an Object being placed shall by a given Lens be represented in a Species as large as the Object itself, which may be of singular Use in drawing Faces, and other things in their true Magnitude, by transmitting the Species by a Glass into a dark Room, which will not only give the true Figure and Shades, but even the Colours themselves, almost as Vivid as the Life. In this Case d is equal to f and substituting d for f in the Equation, we shall have $p dr = d dr + dd - dp$; r , and dividing all by d , $pr = dr + d\epsilon - pr$, that is,

$2 pr =$

$\frac{2pr}{r+s} = d$; but if the two Convexities be of the same Sphere, so as $r=s$,

then will the Distance be $= pr$, that is, if the Lens be Glass $= 2r$, so that if an Object be placed at the Diameter of the Sphere distant, in this Case the Focus will be as far within as the Object is without, and the Species represented thereby will be as big as the Life; but if it were a Plano-Convex, the same distance will be $= 2pr$, or in Glass to four times the Radius of the Convexity.

A second Use is to find what Convexity or Concavity is required to make a vastly distant Object be represented at a given Focus, after the one Surface of the Lens is formed; which is but a *Corollary* to our *Theorem* for finding s , having p , d , r , and f , given; for d being Infinite, the Rule becomes

$\frac{rf}{pr-f} = s$, that is in Glass $\frac{rf}{2r-f} = s$, whence, if f be greater than $2r$,

s becomes Negative, and $\frac{rf}{f-2r}$ is the Radius for the Concave sought.

But to return to their first *Theorem*, which accounting for the Thickness of the Lens, we will here again resume, viz.

$$\frac{mpdr - ndst - npr \cdot t}{m dr + m ds - m pr - m - ndt - nr t} = f.$$

And let it be required to find the Focus where a whole Sphere will collect the Beams proceeding from an Object at the distance d . Here t is equal to $2r$, and $r=t$; and after due Reduction, the *Theorem* will stand thus,

$\frac{mpdr - 2ndr + 2npr}{2nd + 2nr - mpr} = f$. But if d be Infinite, it is contracted to

$\frac{mpr}{2n} - r = \frac{2n-m}{2m-2n} r = f$; wherefore a Sphere of Glass collects the Sun's

Beams at half the Semidiameter of the Sphere without it, and a Sphere of Water at a whole Semidiameter. But if the Ratio of Refraction m to n be as 2 to 1, the Focus falls on the opposite Surface of the Sphere, but if it be of greater Inequality, it falls within.

Another Example shall be when a Hemisphere is exposed to Parallel Rays, that is, d and p being Infinite, and $t=r$, and after due Reduction, the *The-*

orem results $\frac{n}{m-mn} r = f$; that is, in Glass it is at $\frac{2}{3} r$, in Water at

$\frac{1}{2} r$, but if the Hemisphere were Diamant, it would collect the Beams at $\frac{1}{3}$ of the Radius beyond the Center.

Lastly, As to the Effect of turning the two sides of a Lens towards an Object; it is Evident, that if the thickness of the Lens be very small, so as that you neglect it, or account $t = 0$, then in all Cases the Focus of the same Lens, to whatsoever Beams, will be the same, without any difference upon the turning the Lens: But if you are so curious to consider the thickness, (which is seldom worth accounting for) in the case of Parallel Rays falling on a Plano-Convex of Glas, if the Plain-side be towards the Object, t does occasion no difference, but the Focal distance $f = 2r$. But when the Convex side is towards the Object, it is contracted to $2r - \frac{1}{2}t$, so that the Focus is nearer by $\frac{1}{2}t$. If the Lens be Double Convex, the difference is less; if a Meniscus, greater. If the Convexity on both sides be equal, the Focal Length is about $\frac{1}{2}t$ shorter than when $t = 0$. In a Meniscus, the Concave side towards the Object encreases the Focal Length, but the Convex towards the Object diminishes it. A general Rule for the difference arising on turning the Lens, where the Focus is Affirmative, is this $\frac{2at - 2pt}{3r + 3 - t}$, for double Convexes of different Spheres. But for *Menisci* the same difference becomes $\frac{2rt + 2st}{3r - 3p + t}$; of which I need give no other Demonstration, but that by a due Reduction it will so follow from what is premised, as will the *Theorems* for all sorts of Problems relating to the Foci of Optick Glasses.

The Generation
of an Hyperboli-
cal Cyliindroide;
by Sir Christop-
her Wren.
N. 48. p. 951.
Jun. An. 1669.

Fig. 97.

XI. 1. Sint Hyperbolæ Oppositæ D B, E C, quarum Axis Transversus est B C, Centrum A, & una ex Asymptotis G P; item per Centrum sit O M ducta & Angulos Rectos ipsi B C. Quare si circumducantur Hyperbolæ circa Axin O M, manifestum est, ex ea Revolutione generari Corpus Cyliindroides Hyperbolicum, cujus bases, sectionesque Basi parallelæ sunt Circuli. Dico insuper, si idem Corpus secetur per Asymptoton G P, erit sectio Parallelogrammum.

Secetur per Axin Transversum sectione Circulari B N C; item per O & M in Circulos æquales & æqualiter à Centro distantes; item per Axin in figuram Genetricem cujus semissis est B D E C, in cujus Plano erit Asymptotos G P, per quam ad Rectos Angulos planum B D E secetur in Plano F H P; jungantur denique H O.

Quoniam Triangulum O G H est Rectangulum, Ergo quadratum O H, sive O D, minus quadrato O G, est æquale quadrato G H: & quoniam D O parallela est ipsi B A, Asymptoton secatur in G, erit, (ex proprietatibus Hyperbolæ, quæ in Conicis demonstrantur) quadratum O G una cum quadrato B A, æquale quadrato O D, *b. e.* Quadratum O D minus quadrato O G, æquale quadrato A B, sive quadrato A N. Ergo quadratum G H, æquale est quadrato A N. Quare G H & A N æquantur, & sunt ad Angulos Rectos ipsi G A; idemque demonstratur de omnibus aliis sectionibus Basi parallelis. Quare Cyliindroides Hyperbolica rite secatur per Asymptoton in Parallelogrammum. *q. e. d.*

Corol.]

Corol.] Hinc patet, in superficie Cyliindroidis, quamvis è duplici flexura constet, rectas nihilominus innumeras duci posse: Pater etiam, aliam esse huius Corporis Generationem, nimir. ex revolutione Parallelogrammi circa Axin manente Angulo ad Axin æquali $G A O$, vel denique manente Linea Generatrice $H R$ immobili, & massam Volubilem formante aut secante.

Et si acies Dolabri acutissima & rectissima ita disponatur ad Axin, sicut se habeat Linea Generatrix, rotante interim Mamphure, manifestum est, Torno tam accuratas posse elaborari Hyperbolas quam Circulos, cum nihil aliud requiratur ad formandam Cyliindroidem quam ad Cyliindrum, nisi quod in Cyliindris acies Dolabri est Axi Parallela, hic vero inclinata.

Itaque notandum est, pro Inclinatione Anguli $G A O$, variari speciem Hyperbolæ; adeoque facilius accommodatur ad datam Hyperbolam quam ut demonstratione opus habeat: At si manente Angulo Generatrix magis ad Centrum accedat, exsurgit inde minor Hyperbola, sed priori prorsus similis.

2. Sint tria Corpora terendo idonea, P , Q , R ; quorum P & Q sint æqualia & Columnari forma, R vero Corpus Lenti-forme. P rotetur circa Axin $A B$; Q , circa $C D$; & R , circa $E G$. Sint autem $A B$, & $C D$, in diversis Planis; ita tamen ut $E G$ producta sit ad Rectos Angulos utrique $A B$ & $C D$: accedant denique ad se invicem Corpora, prout opus fuerit, servata tamen eadem Inclinatione & situ Axium.

The Application thereof to the Grinding of Hyperbolic Optick Glasses, by Sir Christ. Wren. N. 53. p. 1059. Nov. An. 1669. Fig. 98.

Dico ex Revolutione & mutua attritione Corporum prius positorum exsurgere nova Corpora Geometrica, quorum P & Q erunt Cyliindroidea Hyperbolica æqualia, R vero Conoides Hyperbolicum, specie & magnitudine datum.

Demonstrationem in promptu habemus, nec non modulum ipsius Machinæ; terendis Lentibus Hyperbolicis destinata; quam operosa Pictura & prolixa Explicatione describere, mihi & Artifici magis fuerit molestum, quam Dædalo cuiusvis sagaci similem adinvenire. Postquam enim exposita jam sunt Principia Geometrica, facile erit conjicere, quale sit Instrumentum; nempe, tres sunt Tabulæ Oblongæ, Planæ, Validæ, Labiles, & sibi invicem Impositæ: Infima & Media sustinent inæqualia Capitula (sive Anfas Mamphur sustinentes) alternatim posita; id postulat utriusque Mamphuris Obliquitas & quasi decussatio: Summæ Tabulæ æqualia sunt Capitula in longum Tabulæ disposita; & perforato citimo Capitulo Mamphur transmittitur. Omitto Rotas, Rotulas, Lora, Pondera, Cochleas, & reliqua ad motum expeditum & Machinæ Firmitudinem necessaria. P pertinet ad infimam Tabulam; Q ad mediam; R ad summam. R Lens est vitrea; Q Modulus Lentem terens; P Formula Modulum corrigens; quæ dum motu obliquo, & diverso à motu tam Lentis quam Moduli, fertur, delet continuo & deterit, quicquid Vitii imprimitur in Modulum ex Lentis & Matæriæ attritione.

Quare, cum adeo simplex & spontanea sit ista Hyperbolici Conoidis genitura, ex solis nempe Motibus Circularibus; cumque Motus sit duplex & varius, credibile est, Lentes Hyperbolicas ex hisce Principiis vel nullis fore explicandas.

XII. This Phænomenon appears very easily explicable, from the Consideration of placing Glasses in a Tube, which is thus; After the Object-Glass, the first Eye-glass is placed so much distant (towards the Eye) from the Focus of the Object-Glass, as is the Focus of the Eye-Glass; then the middle Eye-Glass.

Why five Convex Glasses in a Telescope, shew Objects Erect; by Mr. William Molineux. N. 183. p. 169. July An. 1686.

Glas is placed so much distant from the Focus of the first Eye-Glas, as is the Focus of the middle Eye-Glas; Lastly, the nearest Eye-Glas is placed so much distant from the Focus of the middle Eye-Glas, as is the Focus of this nearest Eye-Glas; and the Eye looking through them all is placed in the Focus of this nearest Eye-Glas.

I say therefore, 1. That one single Convex-Glas, cannot properly be said by itself to shew Objects Erect or Reverse, but in respect of the placing of the Eye that looks through it. For if the Eye, that looks through a single Convex-Glas, be placed nigher thereto than the Glas's Focus, the Objects are Erect; if the Eye be placed just in the Focus, the Objects are neither Erect nor Reversed, but all in Confusion between both; and if the Eye be placed further from the Glas than the Focus, the Objects are Reversed. I mean here distant Objects, the Rays flowing from any Point whereof may be counted to come Parallel towards the Object-Glas.

2. The Object-Glas of a Telescope reverses the Object, both to the Eye-Glas and the Eye that looks through it: For the Eye-Glas is placed farther from the Object-Glas than is the Focus of the Object-Glas. But the Eye-Glas does nothing towards the Rectification, or Reversion; the Eye being placed just in its Focus.

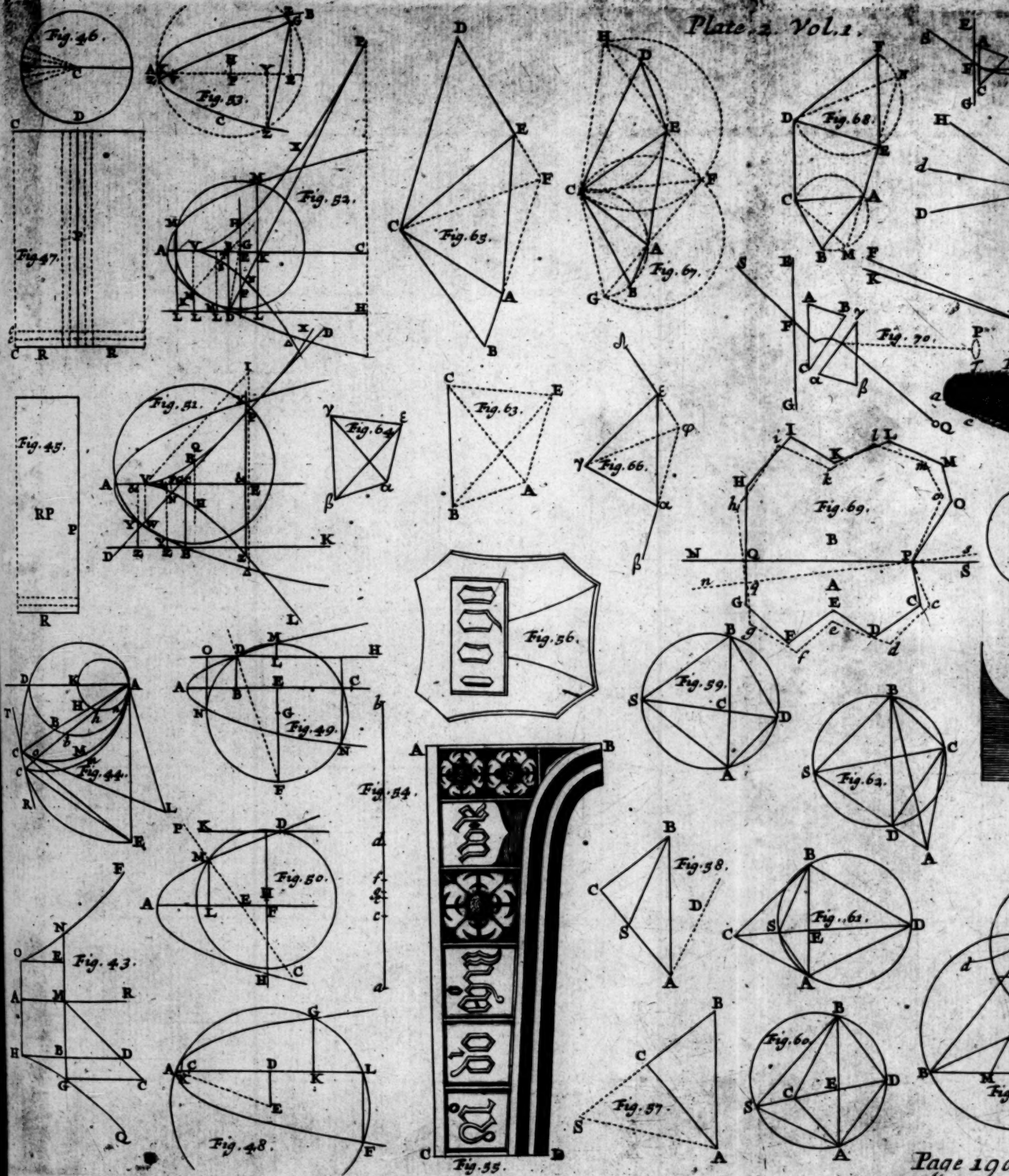
3. If the second Eye-Glas (the first being that next the Object-Glas) be placed, as it ought, in a Telescope, place the Eye nearer in this middle Eye-Glas than its Focus, and it sees the Object inverted and confused: Place the Eye in the Focus, and it sees the Objects all in Confusion, neither Erect nor Reversed; for here again there is a distinct Representation of the Objects to be received on a piece of Paper, as in the Focus of the Object-Glas, and the Eye being placed at any time at this place (which is usually call'd the distinct Base) sees all in Confusion: But then let the Eye be placed farther from this middle Glas than its Focus, and it perceives the Objects Erect and Confused.

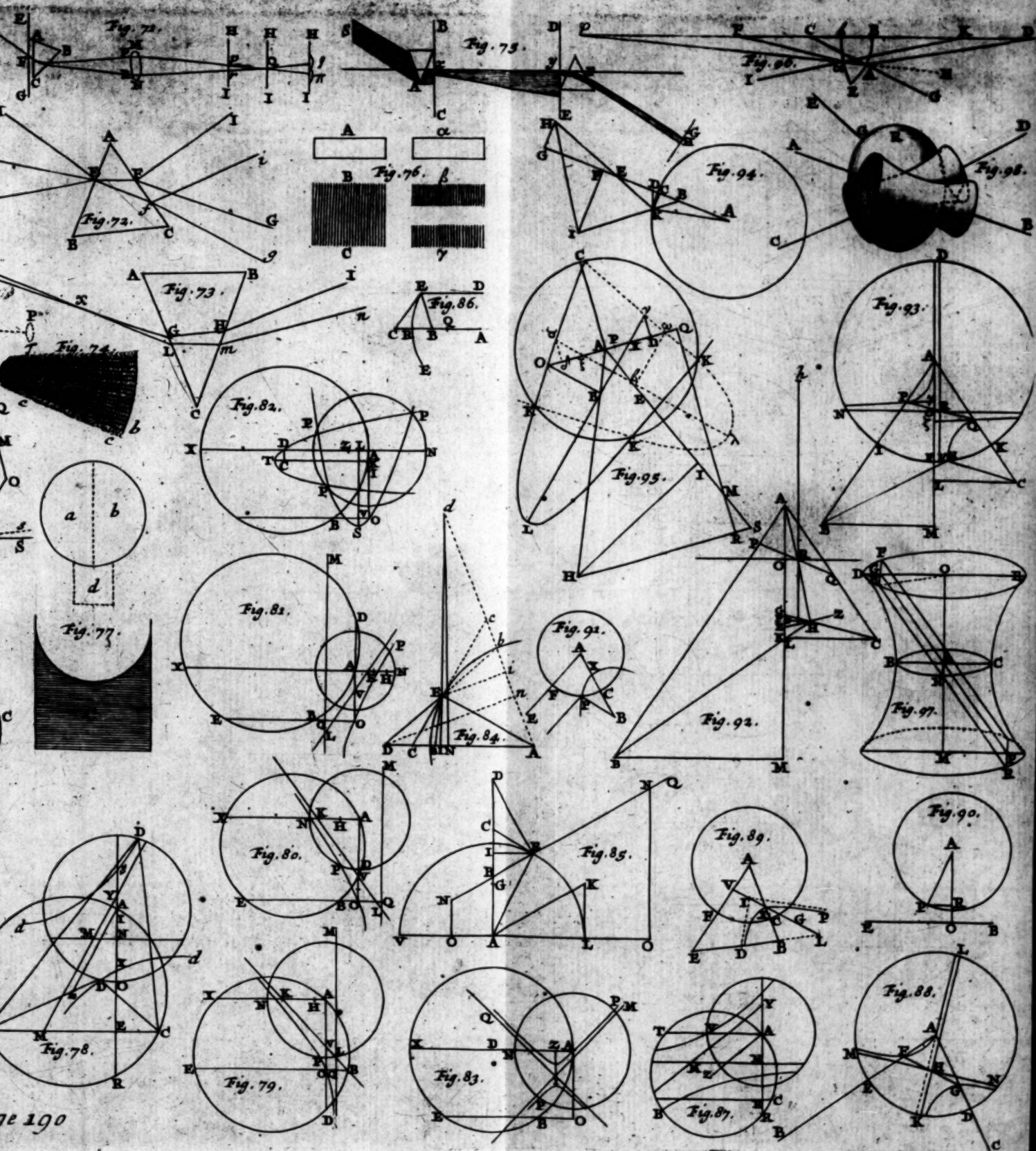
Lastly, The third, or immediate Eye-Glas, does nothing towards the Erecting or Reversing the Species, which it receives Erect from the middle Eye-Glas; no more than in a Telescope of two Convex-Glasses, the Eye-Glas does to the Species it receives from the Object-Glas, as we have shewn before. The Reason that this last or immediate Eye-Glas has nothing to do in the Erecting or Reversing the Species is the same, as in the Telescope of two Convex Glasses, *viz.* The Eye placed in its Focus, and therefore sees the Species as 'tis represented in the distinct Base; that is, the Species is inverted in the distinct Base of the Object-Glas, and therefore a single Convex Eye-Glas brings it to the Eye Inverted; but in the distinct Base of the middle or second Eye-Glas the Species is Erect, and therefore the third or immediate Eye-Glas brings it to the Eye Erect.

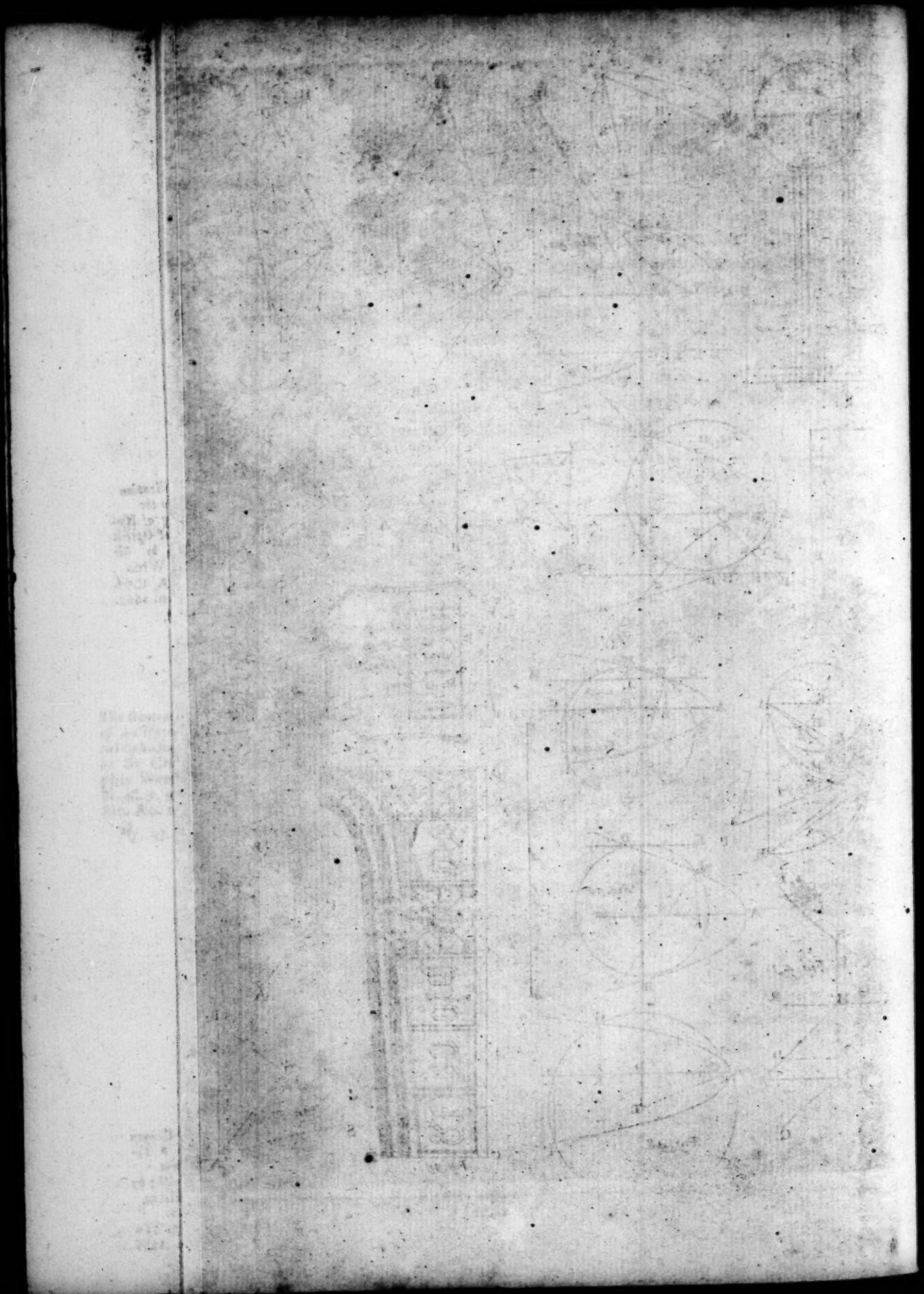
Wherefore we are to consider the Telescope consisting of an Object-Glas and three Eye-Glasses, as two Telescopes, each consisting of two Convex Glasses. The first consists of the Object-Glas, and first Eye-Glas, and this inverts the Species; that is, the Species is Inverted in the distinct Base of the Object-Glas, and so brought into the Eye. The second Telescope consists of two immediate Eye-Glasses, and this Erects what the former Inverted; that is, the Species in the distinct Base of the middle Eye-Glas, is Erect,

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Erect, and is so brought into the Eye by the Eye-Glass; the Eye-Glasses themselves in neither Case having any thing to do with the Erecting or Inverting, but merely in representing the same Posture the Species immediately before them. So that one Convex-Glass, as posited in a Telescope, Inverts; the second (that is the first Eye-Glass) does nothing towards the Erecting or Reversing, but represents the Image as it is in the distinct Base of the Object-Glass before it, that is Inverted; the third Glass Erects, or rather Restores what was before Inverted; the fourth represents the Image as it receives it from the distinct Base of the third, that is, Erect.

XIII. 1. Mr. *Auzout* has found that the Apertures, which Optick-Glasses can bear with Distinctness, are in about a subduplicate Proportion to their Lengths: And accordingly he hath made the following Table.

*The Apertures
of Telescopes.
By M. Auzout.
N. 4. p. 11.
June. An. 1665.*

| Lengths of Glasses. | | | | Apertures for | | | | Lengths of Glasses. | | | | Apertures for | | | |
|---------------------|-----|-----|------|---------------|-------|-----------|------|---------------------|-----|-----|------|---------------|-------|-----------|------|
| | | | | Excellent. | Good. | Ordinary. | | | | | | Excellent. | Good. | Ordinary. | |
| Feet. | In. | In. | Lin. | In. | Lin. | In. | Lin. | Feet. | In. | In. | Lin. | In. | Lin. | In. | Lin. |
| | 4 | | 4 | | 4 | | 3 | 25 | 3 | 4 | 2 | 10 | 2 | 4 | |
| | 6 | | 5 | | 5 | | 4 | 30 | 3 | 8 | 3 | 2 | 2 | 7 | |
| | 9 | | 7 | | 6 | | 5 | 35 | 4 | 0 | 3 | 4 | 2 | 10 | |
| 1 | 0 | | 8 | | 7 | | 6 | 40 | 4 | 3 | 3 | 7 | 3 | 1 | |
| 1 | 6 | | 9 | | 8 | | 7 | 45 | 4 | 6 | 3 | 10 | 3 | 2 | |
| 2 | 0 | | 11 | | 10 | | 8 | 50 | 4 | 9 | 4 | 0 | 3 | 4 | |
| 2 | 6 | 1 | 0 | | 11 | | 9 | 55 | 5 | 0 | 4 | 3 | 3 | 6 | |
| 3 | 0 | 1 | 1 | 1 | 0 | | 10 | 60 | 5 | 2 | 4 | 6 | 3 | 8 | |
| 3 | 6 | 1 | 2 | 1 | 1 | | 11 | 65 | 5 | 4 | 4 | 8 | 3 | 20 | |
| 4 | 0 | 1 | 4 | 1 | 2 | 1 | 0 | 70 | 5 | 7 | 4 | 10 | 4 | 0 | |
| 4 | 6 | 1 | 5 | 1 | 3 | 1 | 1 | 75 | 5 | 9 | 5 | 0 | 4 | 2 | |
| 5 | 0 | 1 | 6 | 1 | 4 | 1 | 1 | 80 | 5 | 11 | 5 | 2 | 4 | 5 | |
| 6 | | 1 | 7 | 1 | 5 | 1 | 2 | 90 | 6 | 4 | 5 | 6 | 4 | 7 | |
| 7 | | 1 | 9 | 1 | 6 | 1 | 3 | 100 | 6 | 8 | 5 | 9 | 4 | 10 | |
| 8 | | 1 | 10 | 1 | 8 | 1 | 4 | 120 | 7 | 5 | 6 | 5 | 5 | 3 | |
| 9 | | 1 | 11 | 1 | 9 | 1 | 5 | 150 | 8 | 0 | 7 | 0 | 5 | 11 | |
| 10 | | 2 | 1 | 1 | 10 | 1 | 6 | 200 | 9 | 6 | 8 | 0 | 6 | 9 | |
| 12 | | 2 | 4 | 2 | 0 | 1 | 8 | 250 | 10 | 6 | 9 | 2 | 7 | 8 | |
| 14 | | 2 | 6 | 2 | 2 | 1 | 9 | 300 | 11 | 6 | 10 | 0 | 8 | 5 | |
| 16 | | 2 | 8 | 2 | 4 | 1 | 11 | 350 | 12 | 6 | 10 | 9 | 9 | 0 | |
| 18 | | 2 | 10 | 2 | 6 | 2 | 1 | 400 | 13 | 4 | 11 | 5 | 9 | 8 | |
| 20 | | 3 | 0 | 2 | 7 | 2 | 2 | | | | | | | | |

Consider'd by
Dr. Hook. *ib.*
p. 67.

This Theory of Apertures seems to me not very clear. For the same Glass will endure greater or lesser Apertures, according to the lesser or greater Light of the Object: If it be for looking on the Sun and *Venus*, or for seeing the Diameters of the fixed Stars, then smaller Apertures do better; if for the Moon in the Day-light, or on *Saturn*, or *Jupiter*, or *Mars*, then the largest. Thus I have often made use of a 12 Foot Glass to look on *Saturn* with an Aperture of almost 3 Inches, and with a single Eye-Glass of 2 inches double Convex; but, when with the same Glass I looked on the Sun or *Venus*, I used both a smaller Aperture, and shallower Charge.

To measure Di-
stances at one
Station; by M.
Auzout. N. 7.
p. 125.
Dec. An. 1665.

XIV. I have found long since a Way to measure, with a great Telescope, the Distance of Objects upon the Earth from one Station. The Practice indeed does not altogether answer the Theory, because that the length of the Telescopes admits of some Latitude; yet one comes near enough, and perhaps as just as by most of the ways ordinarily used with Instruments. That which I am proposing, I doubt not but Mr. *Hook* will soon understand, and see the Determination of all Cases possible. I shall only say, That if we look upon the sole Theory, we may make use of an ordinary Telescope, whereof the Eye-Glass is to be Convex: For by putting the Glasses at a little greater Distance, than they are, proportionably to the Distance for which it is to serve, and by adding to it a new Eye-Glass, the Object will be seen distinct, though Obscure; and if the Eye-Glass be Connex, the Object will appear Erect. They may be done two manner of ways; either by leaving the Telescope in its ordinary Situation, the Object-Glass before the Eye-Glass; or by inverting it, and putting this before that. But if any will make use of two Object-Glasses, whereof the Focus's are known, the Distances of them will be known. If it be supposed, that the Focus of the first be B, and that of the second C, and the Distance given, $B + 2D$, and that $D - C$ be equal to F; for this Distance will be equal to $B + C + F - r F' C'$. And if you have the Focus of the first Object-Glass, equal to B, the Distance where you will put the second Glass equal to $B + C + D$; the Focus of the second

Glass will be found equal to $\frac{CD}{C+D}$. And if you will that the Object

shall be magnified as much with these two Glasses, as it would be with a single one, whereof the Focus should be of the distance given, having the Focus of the Object-Glass given equal to B, and the distance given to $B + D$; the

distance between the first and second Glass will be equal to $\frac{2B^2 + 2BD}{2B + D}$,

whence subtracting B (the Focus of the Object given) there remains

$\frac{BD}{2B + D}$: and if this Sum be composed equal to C, we shall easily know by the precedent Rule, the Focus of the second Glass.

XV. Prepare

XV. Prepare two Glasses, the one exactly flat on both sides, the other flat on the one side, and convex on the other, of what Sphere you please. Let the flat Glas be a little broader than the other. Then let there be made a Cell or Ring of Brass, very exactly turned, into which these two Glasses may be so fastned with Cement, that the plain Surfaces of them may lie exactly parallel, and that the Convex side of the Plano-convex Glas may lie inward; but so, as not to touch the Flat of the other Glas. These being cemented into the Ring very closely about the Edges, by a small Hole in the side of the Brass Ring or Cell, fill the interposed Space between these two with Water, Oyl of Turpentine, Spirit of Wine, Saline Liquors, &c. then stop the Hole with a Skrew: And according to the differing Refraction of the interposed Liquors, so shall the Focus of this Compound Glas be longer or shorter.

To make a Plano-convex Glas of a small Sphere, collect the Rays at a great Distance; by Dr Hook. N. 4. p. 66. N. 12. p. 202. June, An. 1666. May, An. 1666.

But this I would have only look'd upon as one Instance of many (for there may be others) of the Possibility of making a Glas, ground in a smaller Sphere, to constitute a Telescope of a much greater Length: Though (not to raise too great Expectation) I must add, That, of Spherical Object-Glasses, those are the best which are made of the greatest Sphere, and whose Substance hath the greatest Refraction.

XVI. 1. *S. Campani* pretends to have found a Way to work great Optick-Glasses with a Turn-tool, without any Mould: And that he useth three Eye-Glasses for his great Telescopes, without finding any Rainbow Colours.

Telescopes and other Optick-Glasses; by Campani and Divini. N. 1. p. 2. Mar. An. 1665. N. 8. p. 131. Jan. An. 1666.

The Great Duke of *Tuscany*, and Prince *Leopold* his Brother, upon Tryal made of the Glasses of *Campani* and *Divini*, have found that those of *Campani* excel the other; and with them they have been easily able to distinguish People at 4 Leagues distance.

But *Eustachio Divini* pretends, that in all the Tryals made with them, his great Glasses have performed better than those of *Campani*; and that *Campani* was not willing to do what was necessary for well comparing one with the other, viz. to put equal Eye-Glasses in them, or to exchange the same Glasses.

N. 12. p. 209.

2. 'Tis now above ten Years since I invented a peculiar Way of Grinding Optick-Glasses, and reduced it also into Practice; by which 'tis easy, without any considerable Danger of failing, to make and polish Optick-Glasses of any Conick Section, and that (which is most notable) in any Dish of any Section of a Sphere. I have already made several Glasses by it, which many Learned Men have seen and try'd.

By M. Hevelius. N. 6. p. 93. Nov. An. 1665.

M. Huygens also intends very shortly to try something in that kind.

By M. Huygens. Ibid.

3. *M. du Sons* doth at present employ himself in *London*, to bring Telescopes to Perfection, by grinding Glasses of a Parabolical Figure. I have seen two Eye-Glasses of that Shape, about one Inch and a half deep, and one Inch and a quarter broad, wrought by this eminent Artist with a rare Steel Instrument of his own Contrivance and Workmanship, and by himself also polish'd to Admiration. And certainly it will be wondred at by those, who shall see these Glasses, how they could be truly wrought to such a Figure, with

By M. du Sons. Ibid. p. 99. N. 7. p. 119. Dec. An. 1666.

such a Cavity ; and yet more, when they shall hear the Author undertake to excavate other such Eye-Glasses to above 2 Inches, and Object-Glasses of 5 Inches Diameter. He hath likewise already begun his Object-Glasses for the mentioned two ocular ones, of the same Figure of about 2 Inches Diameter, which are to be left all open, yet without causing any Colours.

By M. Burattini. N. 19.
p. 348. N. 21.
p. 374.

4. The Optick-Glasses of M. *Burattini* in *Poland*, are perfectly well wrought and polish'd. He hath sent two to *Paris*, but they are only the one of 10, the other of 8 Foot. They bear a great Aperture in respect of their Length.

By Mr. Francis Smethwick. N. 33. p. 631.

5. Mr. *Fr. Smethwick* having found a Way of grinding Glasses *not Spherical*, produced before the Royal Society (*Feb. 27. 1667.*) certain *Specimina* of that Invention ; which were, a Telescope, a Reading, and two Burning-Glasses.

The Telescope was about 4 Foot long, furnished with four Glasses, whereof the three ocular ones, Plano-convex, were of this newly invented *not Spherical* Figure, and the fourth a Spherical Object-Glass. This being compared with a common, yet very good Telescope, longer than it by about 4 Inches, and turn'd to several Objects, was found, by those of the said Society that look'd through them both, to exceed the other in Goodness, by taking in a greater Angle, and representing the Objects more exactly in their respective Proportions, and enduring a greater Aperture free from Colours.

The Reading-Glass of the same Figure being compared with a common Spherical Glass did far excel it, by magnifying the Letters to which it was applied up to the very Edges, and by shewing them distinctly from one Brim through the Center to the other, which the Spherical Glass came far short of.

Lastly, The two Burning Concaves of this new-invented Figure, were the one of 6 Inches Diameter, its Focus 3 Inches distant from the Center thereof ; the other of the same Diameter, but less Concave, and its Focus 10 Inches distant. These, when approached to a large Candle lighted, did somewhat warm the Faces of those that were 4 or 5 Foot distant at least ; and when held to the Fire, burned Gloves and Garments at the Distance of about 3 Foot from the Fire.

The Bishop of *Salisbury*, Dr. *Seth Ward*, was by at another time, when the deeper of the two Concaves turned a Piece of Wood into Flame in the Space of 10 Sec. of Time, and the shallower in 5 Sec. at most, in the Season of *Autumn*, about Nine of the Clock in the Morning, the Weather gloomy. The Inventor adds, That the deeper Concave, when held to a lucid Body, would cast a Light strong enough to read by at a considerable Distance ; and that exposing the same to a Northern Window, on which the Sun shined not at all, or very little, he had perceived that it would warm one's Hand sensibly, by collecting the warmed Air in the Day-time, which it would not do after Sun-set.

By an Artist at Paris. N. 40.
p. 795.
Oct. An. 1668.

6. We have an Artist at *Paris* that polishes Optick-Glasses on a Turn. I have seen a Glass of his Workmanship which is very good. He turns these Glasses as he does Wood, that is with the same Facility.

7. *M. Borelli* hath found out a sure and very easy Method to work all sorts of great Glasses. He hath already made one of them very good of 200 Foot, wrought on both sides on the same Rule. His Desire of advancing Astronomical Discoveries hath induced him to make Presents of them to several Persons capable to make use of them. He hath intrusted the Secret to one of the *Royal Academy of Sciences*. By *M. Borelli*.
N. 128. p. 691.
Aug An. 1676.

Campani and *Divini* have commonly sold their Glasses at a Pistole the Foot. Sometimes they have far exceeded that Price. One of *Divini's* of 12 Foot was sold for 400 Livres; and another of *Campani's*, of 34 Foot, for 2000 Livres. Notwithstanding which, *S. Borelli* is willing to part with the best of his own Glasses, of 50, 60, or 65 Foot, for 500 (*French*) Crowns; and the small Glasses, from 6 to 12 Foot, at a (*French*) Crown a Foot; from 12 to 18, at half a Pistole; and from 18 to 26, at a Pistole. N. 140. p. 1005.
July, An. 1678.

8. Though it be commonly believed, that *Rock Crystal* is not fit for Optick-Glasses, because there are many Veins in it; yet *Enstachio Divini* made one of it, which he said proved an excellent one, though full of Veins: But perhaps they were only superficial Strictures and slight Scratches, not Veins. Optick Lens of
Rock Crystal, by
Eust. Divini.
N. 20 p. 362.
Dec. An. 1666.

9. Drops of fair Water being let fall on a Piece of plain Glas, from themselves into Plano-convexes, having a Convexity proportionable to the Heights from which they descend; from a greater Height a less, from a less a greater Degree of Convexity. I applied some of these as Reading-Glasses for single Words of small Letters, as on the Globes and Maps, and found no other Inconveniency, than that the Fluidity of the Water obliges one to keep the Glas Horizontal, which I after devised a Way to remedy. I took a sufficient Quantity of Izing-Glas, and dissolved it in Water over the Fire, and whilst it was warm I dipt a Strick into the Solution, and let some Drops of it fall on the Glas as before; and in a quarter of an Hour they acquire a Consistency, that permits them to be held in any Position, and though they are not altogether so transparent, yet this is little or no Impediment to their Use. The Drops of this Solution are more exactly defined than those of common Water, having their Edges exactly circular, and one may make them of a much longer Focus than those. Of Water; by
Mr. Stephen
Gray.
N. 228. p. 539.
May, An. 1697.

A thin flat Ring of Brass, not exceeding 4 Tenths of an Inch Diameter in its interior Circle, being cemented to a plain Piece of Glas, and filled with Water, or the Solution now mentioned, then by pressing the Finger into it, till what is superfluous be taken off, there will be formed a Plano-concave, which may serve as an Eye-Glas to a Perspective, or to any other optical Use Concave-Glasses are applicable.

I have tried what would be the Success of combining Portions of Water by the help of Brass Rings, and plain Pieces of Glas, to give them their true Figure and requisite Apertures, and inserted them at the Ends of Tubes of several Lengths; and find, that though these *Natural Lenses* may serve as Eye-Glasses, yet when used as Object ones, either to Telescopes, or double Microscopes, the Effects will not compensate the Trouble there is in using them.

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*The Advantages
of Reflection in
Optick Instru-
ments; by
Mr. Newton.
N. 80. p. 3079.
Feb. An. 1672.*

XVII. 1. When I had found, that *Light consists of Rays differently refrangible*, I left off my Glas-works; for I saw, that the Perfection of Telescopes was hitherto limited, not so much for want of Glasses truly figured according to the Prescriptions of Optick Authors, (which all Men have hitherto imagined) as because that *Light itself is an heterogeneous Mixture of differently Refrangible Rays*: So that were a Glas so exactly figured as to collect any one sort of Rays into one Point, it could not collect those also into the same Point, which having the same Incidence upon the same Medium, are apt to suffer a different Refraction. Nay, I wondred, that seeing the Difference of Refrangibility was so great as I found it, Telescopes should arrive to that Perfection they are now at: For, measuring the Refractions in one of my Prisms, I found, that supposing the common Sine of Incidence upon one of its Plain was 44 Parts, the Sine of Refraction of the utmost Rays on the red End of the Colours, made out of the Glas into the Air, would be 68 Parts, and the Sine of Refraction of the utmost Rays on the other End 69 Parts; so that the Difference is about a 24th or 25th Part of the whole Refraction. And consequently, the Object-Glas of any Telescope cannot collect all the Rays, which come from one Point of an Object, so as to make them convene at its Focus in less room than in a circular Space, whose Diameter is the 50th Part of the Diameter of its Aperture; which is an Irregularity, some hundreds of times greater than a circularly figured *Lens*, of so small a Section as the Object-Glasses of long Telescopes are, would cause by the Unfitness of its Figure, were Light uniform.

This made me take Reflections into Consideration; and finding them regular, so that the Angle of Reflection of all sorts of Rays was equal to their Angle of Incidence, I understood that by their Mediation Optick Instruments might be brought to any Degree of Perfection imaginable, provided a reflecting Substance could be found, which would polish as finely as Glas, and reflect as much Light as Glas transmits, and the Art of communicating to it a parabolick Figure be also attained. But there seemed very great Difficulties, and I have almost thought them insuperable, when I farther consider'd that every Irregularity in a reflecting Superficies makes the Rays stray 5 or 6 times more out of their due Course, than the like Irregularities in a refracting one: So that a much greater Curiosity would be here requisite, than in figuring Glasses for Refraction.

Amidst these Thoughts I was forced from *Cambridge, Anno 1666.* by the intervening Plague, and it was more than two Years before I proceeded further. But then having thought on a tender Way of Polishing, proper for Metal, whereby, as I imagined, the Figure would be corrected to the last. I began to try what might be effected in this kind, and by degrees so far perfected an Instrument (in the essential Parts of it like that I sent to *London*) by which I could discern *Jupiter's* four Concomitants, and shew'd them divers times to two others of my Acquaintance. I could also discern the Moon-like *Phase* of *Venus*, but not very distinctly, nor without some Niceness in disposing the Instrument.

From

From that time I was interrupted till this last *Autumn*, when I made another. And as that was sensibly better than the first, (especially for Day-Objects) so I doubt not, but they will be still brought to a much greater Perfection by their Endeavours, who, as you inform me, are taking care about it at *London*.

2. This new Instrument is composed of two Metalline Speculums, the one Concave (instead of an Object-Glass) the other Plain: And also of a small Plano-convex Eye-Glass; as in the Figure, where A B is a Concave Speculum, of which the Radius or Semidiameter is $12\frac{1}{2}$ or 13 Inches.

*A new Catadi-
optical Telescope
invented by
Mr. Newton.
N. 81. p. 400d.
Mar. An. 1672.
Fig. 99.*

C D, another Metalline Speculum, whose Surface is flat, and the Circumference oval.

G D, an Iron Wire, holding a Ring of Brass, in which the Speculum C D is fixed.

F, a small Eye-Glass, flat above, and convex below, of the 12th Part of an Inch Radius, if not less.

GGG, the fore Part of the Tube (which is open) fastned to a Brass Ring H I, to keep it immoveable.

P Q K L, the hinder Part of the Tube, fastned to another Brass Ring P Q.

O, an Iron Hook fastned to the Ring P Q, and furnish'd with a Skrew N, thereby to advance or draw back the hinder Part of the Tube, and so by that means to put the Specula in their due Distance.

M Q G I, a crooked Iron sustaining the Tube, and fastned by the Nail R to the Ball and Socket S, whereby the Tube may be turned every Way.

The Center of the flat Speculum C D, must be placed in the same Point of the Tube's Axe, where falls the Perpendicular to this Axe, drawn to the same from the Center of the little Eye-Glass, which Point is here marked at T.

And to give the Reader some Satisfaction to understand, in what Degree it represents Things distinct, and free from Colours, and to know the Aperture by which it admits Light, he may compare the Distances of the Focus E from the Vertexes of the little Eye-Glass and the Concave Speculum; that is, E F, $\frac{1}{6}$ of an Inch, and E T V, $6\frac{1}{2}$ Inches, and the Ratio will be found as 1 to 38; whereby it appears, that the Objects will be magnified about 38 times; and be represented bigger by $2\frac{1}{2}$ times in Diameter, when seen thro' this, than thro' an ordinary Telescope of about 2 Foot long.

Thus far as to the Structure of this Telescope. Concerning the metalline Matter, fit for these Reflecting Speculums, the Inventor hath also consider'd the same, and gives this Caution, That whilst Men seek for a white, hard, and durable metalline Composition, they resolve not upon such an one as is full of small Pores, only discoverable by a Microscope: For tho' such an one may, to Appearance, take a good Polish, yet the Edges of those small Pores will wear away faster in the Polishing, than the other Parts of the Metal; and so, however the Metal seem polite, yet it shall not reflect with such an accurate Regularity as it ought to do. Thus Tin-Glass mixt with ordinary Bell-Metal makes it more white, and apt to reflect a greater Quantity of Light;

Light; but withal, its Fumes raised in the Fusion, like so many aerial Bubbles, fill the Metal full of the microscopical Pores. But white Arsenick both blanches the Metal, and leaves it solid, without any such Pores, especially if the Fusion hath not been too violent. What the Stellate-Regulus of *Mars*, (which I have sometimes used) or rather such like Substance will do, deserves particular Examination.

To this he adds this further Intimation, That Putty, or other such like Powder, with which 'tis polish'd, by the sharp Angles of its Particles, fretteth the Metal, if it be not very fine, and fills it full of such small Holes as he speaketh of. Wherefore Care must be taken of that before Judgment be given, whether the Metal be throughout the Body of it porous or not.

But not having tried, as he saith, many Proportions of the Arsenick and Metal, he does not affirm which is absolutely best, but thinks there may conveniently be used any Quantity of Arsenick equalling in Weight between a sixth and an eighth Part of the Copper, a greater Proportion making the Metal brittle.

The Way which he used was this. He first melted the Copper alone, then put in the Arsenick, which being melted, he stirred them a little together, bewaring, in the mean time, not to draw in Breath near the pernicious Fumes. After this he put in Tin; and again, so soon as that was melted, (which was very suddenly) he stirred them well together, and immediately poured them off.

He saith, He knows not, whether by letting them stand longer on the Fire after the Tin was melted, a higher Degree of Fusion would have made the Metal porous; but he thought that Way he proceeded to be the safest.

He adds, That in that Metal, which he sent to *London*, there was no Arsenick, but a small Proportion of Silver; as he remembers, one Shilling in three Ounces of Metal. But he thought withal, that the Silver did as much harm in making the Metal soft, and so less fit to be polish'd, as good in rendring it white and luminous.

At another time he mixed Arsenick one Ounce, Copper six Ounces, and Tin two Ounces; and this an Acquaintance of his hath, as he intimates, polish'd better than he did the other.

As to the Objection, That with this kind of Perspectives, Objects are difficultly found; he answers, That that is the Inconvenience of all Tubes that magnify much; and that after a little Use, the Inconvenience will grow less, seeing that himself could readily enough find any Day Objects by knowing which Way they were posited from other Objects that he accidentally saw in it. But in the Night to find Stars, he acknowledges it to be more troublesome; which yet may, in his Opinion, be easily remedied by two Sights affixed to the Iron Rod, by which the Tube is sustained, or by an ordinary Perspective-Glass fastned to the same Frame with the Tube, and directed towards the same Object, as *Des Cartes* in his *Dioptricks* hath described for remedying the same Inconvenience of his best Telescopes.

3. I see by the Description you have sent me of Mr. *Newton's* admirable Telescope, that he hath well considered the Advantage which a Concave Speculum hath above Convex-Glasses in collecting the parallel Rays; which certainly, according to the Calculation I have made thereof, is very great. Hence it is, that he can give a far greater Aperture to that Speculum, than to an Object-Glass of the same Distance of the Focus; and consequently, that he can much more magnify Objects this Way, than by an ordinary Telescope. Besides, by it he avoids an Inconvenience, which is inseparable from Convex Object-Glasses, which is the Obliquity of both their Surfaces, which vitiates the Refraction of the Rays that pass towards the Sides of the Glass, and does more hurt than Men are aware of. Again, by the meer Reflection of the metalline Speculum there are not so many Rays lost as in Glasses, which reflect a considerable Quantity by each of their Surfaces, and besides intercept many of them by the Obscurity of their Matter.

Approved by
M. Hugen de
Zulichem.
Ibid. p. 408.

Mean time the main Business will be, to find a Matter for this Speculum that will bear so good and even a Polish as Glasses, and a Way of giving this Polish without vitiating the Spherical Figure. Hitherto I have found no Specula that had near so good a Polish as Glass: And if Mr. *Newton* hath not already found a Way to make it better than ordinarily, I apprehend his Telescope will not so well distinguish Objects as those with Glasses. But 'tis worth while to search for a Remedy to this Inconveniency, and I despair not of finding one. I believe that Mr. *Newton* hath not been without considering the Advantage, which a Parabolical Speculum would have above a Spherical one in this Construction; but that he despairs, as well as I do, of working other Surfaces than Spherical ones with due Exactness; tho' else it be more easy to make a Parabolical, than Elliptical or Hyperbolical ones, by reason of a certain Propriety of the Parabolick Conoid, which is, that all the Sections parallel to the Axis make the same Parabola.

But though Mr. *Newton* (with M. *Hugen*) despairs of performing that work by Geometrical Rules, yet he doubts not but that the thing may in some measure be accomplish'd by mechanical Devices.

Ibid. p. 409.

4. In my last Letter I gave you occasion to suspect that the Instrument which I sent you is in some respect or other indisposed, or that the Metals are tarnished; and by yours I am fully confirm'd in that Opinion: For, whilst I had it, it represented the Moon in some Parts of it as distinctly as other Telescopes usually do which magnify as much as that. Yet I very well know, that that Instrument hath its Imperfections both in the Composition of the Metal, and in its being badly cast, as you may perceive by a scabrous Place near the Middle of the Metal of it on the polish'd Side, and also in the Figure of that Metal near that scabrous Place: And in all those Respects that Instrument is capable of further Improvement.

A further Account
of this Instru-
ment; by Mr.
Newton.
Ibid.

You seem to intimate, that the Proportion of 38 to 1, holds only for its magnifying Objects at small Distances. But if for such Distances, suppose 500 Feet, it magnify at that Rate, by the Rules of Opticks it must for the greater Distance imaginable magnify more than $37\frac{1}{2}$ to 1, which is so inconsiderable a Diminishing, that it may be even then as 38 to 1.

Here

Here is made another Instrument like the former, which does very well. Yesterday I compared it with a six Foot Telescope, and found it not only to magnify more, but also more distinctly: And to Day I found, that I could read in one of the Philosophical Transactions, placed in the Sun's Light, at an 100 Foot Distance, and that at an 120 Foot Distance I could discern some of the Words. When I made this Tryal, its Aperture (defined next the Eye) was equivalent to more than an Inch and a third Part of the Object-Metal. This may be of some Use to those that shall endeavour any thing in Reflections; for hereby they will in some measure be enabled to judge of the Goodness of their Instruments.

*The Apertures
and Charges of
these Instru-
ments, by*

Mr. Newton.

N. 82. p. 4032.

April, An. 1672.

I know that the Aperture was $1\frac{1}{3}$ of an Inch, by trying that an Obstacle of that Breadth was requisite to intercept all the Light which came from one Point of the Object.

I should tell you also, that the little plain Piece of Metal, next the Eye-Glass is not only truly figured: Whereby it happens, that Objects are not so distinct at the Middle as at the Edges. And I hope, that by correcting its Figure (in which I find more Difficulty than one would expect) they will appear all over distinct, and distincter in the Middle than at the Edges. And I doubt not but that the Performances will then be greater.

But yet I find, that there is more Light lost by Reflection of the Metal which I have hitherto used, than by Transmission through Glasses: For which Reason a shallower Charge would probably do better for obscure Objects; suppose such a one as would magnify 34 or 32 times. But for bright Objects at any Distance, it seems capable of magnifying 38 or 40 times, with sufficient Distinctness. And for all Objects, the same Charge, I believe, may with Advantage be allowed, if the steely Matter, employ'd at London, be more strongly reflective than this which I have used.

The Performances of one of these Instruments of any Length being known, it will appear by this following Table, what may be expected from those of other Lengths by this Way, if Art can accomplish what is promised by the Theory. In the first Column is expressed the Length of the Telescope in Feet; which doubled, gives the Semidiameter of the Sphere on which the Concave Metal is to be ground. In the second Column are the Proportions of the Apertures for those several Lengths. And in the third Column are the Proportions of the Charges, or Diameter of the Spheres, on which the Convex Superficies of the Eye-Glasses are to be ground.

| Lengths. | Apertures. | Charges. | Lengths. | Apertures. | Charges. |
|---------------|------------|----------|----------|------------|----------|
| $\frac{1}{2}$ | 100 | 100 | 8 | 800 | 200 |
| 1 | 168 | 119 | 10 | 946 | 211 |
| 2 | 283 | 141 | 12 | 1084 | 221 |
| 3 | 383 | 157 | 16 | 1345 | 238 |
| 4 | 476 | 168 | 20 | 1591 | 251 |
| 5 | 562 | 178 | 24 | 1824 | 263 |
| 6 | 645 | 186 | | | |

The Use of this Table will best appear by Example: Suppose therefore a half Foot Telescope may distinctly magnify 30 times with an Inch Aperture, and it being required to know, what ought to be the analogous Constitution and Performance of a four Foot Telescope: By the second Column, as 100 to 476; so are the Apertures, as also the Number of Times which they magnify. And consequently since the half Foot Tube hath an Inch Aperture and magnifieth 30 times, a four Foot Tube proportionally should have $4 \frac{76}{100}$ Inches Aperture, and magnify 143 times. And by the third Column, as 100 to 168; so have their Charges: And therefore if the Diameter of the Convexity of the Eye-Glass for a half Foot Telescope be $\frac{1}{2}$ of an Inch, that for a four Foot should be $\frac{68}{100}$, that is about $\frac{1}{2}$ of an Inch; and so of other Lengths. But what the Event will really be, we must wait to see determined by Experience. Only this I thought fit to insinuate, that they which intend to make Trial in other Lengths, may more readily know how to design their Instruments. Thus for a four Foot Tube, since the Aperture should be 5 or 6 Inches, there will be required a Piece of Metal 7 or 8 Inches broad at least, because the Figure will scarcely be true to the Edges. And the Thickness of the Metal must be proportional to the Breadth, lest it bend in the Grinding. The Metals being polished, there may be Trials made with several Eye-Glasses, to find what Charge may with best Advantage be made use of.

XVIII. 1. I doubt not but *M. A.* will allow the Advantage of Reflection in the Theory to be very great, when he shall have informed himself of the *Different Refrangibility* of the several Rays of Light. And for the practick Part, it is in some measure manifest by the Instruments already made, to what Degree of Vivacity and Brightness a metalline Substance may be polished. Nor is it improbable but that there may be new Ways of Polishing found out for Metal, which will far excel those that are yet in Use. And when a Metal is once well polished, it will be a long while preserved from tarnishing, if Diligence be used to keep it dry and close shut up from Air: For the principal Cause of Tarnishing seems to be the condensing of Moisture on its polish'd Surface, which, by an acid Spirit, wherewith the Atmosphere is impregnated, corrodes and rusts it; or at least at its exhaling leaves it cover'd over with a thin Skin, consisting partly of an earthly Sediment of that Moisture, and partly of the Dust, which, flying to and fro in the Air, had settled and adhered to it.

*Some Objections
of M.
Answered, by
Mr. Newton.
ib. p. 4234.*

When there is not occasion to make frequent Use of the Instrument, there may be other Ways to preserve the Metal for a long time; as perhaps by immersing it in Spirit of Wine, or some other convenient Liquor. And if they chance to tarnish, yet their Polish may be recover'd by rubbing them with a soft Piece of Leather, or other tender Substance, without the Assistance of any fretting Powders, unless they happen to be rusty; for then they must be new polished.

I am very sensible, that Metal reflects less Light than Glass transmits ; and for that Inconvenience, I gave you a Remedy in my last Letter, by assigning a shallower Charge in Proportion to the Aperture, than is used in other Telescopes. But as I have found some metalline Substances to be more strongly reflective, and to polish better, and be freer from tarnishing than others ; so I hope there may in time be found out some Substance much freer from these Inconveniencies than any yet known.

*The Considerations of
Answer'd; by
Mr. Newton.
N. 88. p. 5084.
Nov. An. 1672.*

2. The *Considerer* is pleas'd to reprehend me for laying aside the Thoughts of improving Opticks by Refractions. If he had oblig'd me by a private Letter on this Occasion, I would have acquainted him with my Success on the Trials I have made of that kind, which I shall now say have been less than I sometimes expected, and perhaps than he at present hopes for. But since he is pleas'd to take it for granted, that I have let this Subject pass without due Examination, I shall refer him to my former Letters, by which that Conjecture will appear to be ungrounded. For what I said there, was in respect of Telescopes of the ordinary Construction, signifying, that their Improvement is not to be expected from the well-figuring of Glasses, as Opticians have imagin'd ; but I despaired not of their Improvement by other Constructions, which made me cautious to insert nothing that might intimate the contrary. For although successive Refractions that are all made the same Way, do necessarily more and more augment the Errors of the first Refraction ; yet it seem'd not impossible for contrary Refractions so to correct each others Inequalities, as to make their Difference regular ; and if that could be conveniently effected, there would be no further Difficulty. Now to this end, I examin'd what may be done, not only by Glasses alone, but more especially by a Complication of divers successive Mediums, as by two or more Glasses or Crystals with Water or some other Fluid between them ; all which together may perform the Office of one Glass, especially of the Object-Glass, on whose Construction the Perfection of the Instrument chiefly depends.

To the Assertion, That Rays are less true reflect'd to a Point by a Concave, than refract'd by a Convex, I cannot assent ; nor do I understand, That the Focus of the latter is a less Line than that of the former. The Truth of the contrary you will rather perceive by the following Table, computed for such a Reflecting Concave, and the Refracting Convex, on Supposition that they have equal Apertures, and collect parallel Rays at an equal Distance from their Vertex ; which Distance being divided into 15000 Parts, the Diameter of the Concave Sphere will be 60000 of those Parts, and of the Convex 10000 ; supposing the Sines of Incidence and Refraction to be, in round Numbers, as 2 to 3. And this Table following shews, how much the exterior Rays, at several Apertures, fall short of their principal Focus.

| The Diameter of the Aperture. | The Parts of the Axis intercepted between the Vertex and the Rays. | | The Errors by | |
|-------------------------------|--|------------|-----------------|-------------|
| | Reflected. | Refracted. | Reflection. | Refraction. |
| 2000 | 14991 $\frac{1}{2}$ | 14865 | 8 $\frac{1}{2}$ | 135 |
| 4000 | 14966 | 14449 | 33 | 551 |
| 6000 | 14924 | 13699 | 76 | 1301 |
| 8000 | 14865 | 12475 | 135 | 2525 |
| 10000 | 14787 | 9472 | 213 | 5528 |

By this you may perceive, that the Errors of the Refracting Convex are so far from being less, that they are more than 16 times greater than the like Errors of the Reflecting Concave, especially in great Apertures; and that without respect to the heterogeneous Constitution of Light. So that, however the contrary Supposition might make the Author of these Animadversions reject Reflections as useless for the promoting of Opticks; yet I must for this, as well as other Considerations, prefer them in the Theory before Refractions.

Whether the *Parabola* be more difficult to describe than the *Hyperbola*, or *Ellipsis*, may be a Quære; but I see no absolute Necessity of endeavouring after any of their Descriptions. For if Metals can be ground truly Spherical, they will bear as great Apertures, as I believe Men will be well able to communicate an exact Polish to. And for Dioptrique Telescopes, I told you, that the Difficulty consisted not in the Figure of the Glass, but in the Deformity of Refractions; which if it did not, I could tell you a better and more easy Remedy than the Use of the *Conic Sections*.

3. We see that a Picture made by an Object-Glass of 12 Foot in a dark Room, is too distinct, and too well defined, to be produced by Rays, that should stray the 50th Part of the Aperture.

Objections, by
M.
N. 96. p. 6087.
July, An. 1673.

To take away this Difficulty, I must acquaint you, that though I put the greatest Lateral Error of the Rays from one another to be about $\frac{1}{70}$ of the Glass's Diameter; yet their greater Error from the Points on which they ought to fall, will be but $\frac{1}{100}$ of that Diameter: And then, that the Rays, whose Error is so great, are but very few in Comparison to those, which are refracted more justly; for the Rays which fall upon the middle Parts of the Glass, are refracted with sufficient Exactness, as also are those that fall near the Perimeter, and have a mean Degree of Refrangibility; so that there remain only the Rays, which fall near the Perimeter, and are most or least refrangible, to cause any sensible Confusion in the Picture. And these are yet so much further weakned by the greater Space, through which they are scattered, that the Light which falls on the due Point, is infinitely more dense than that which falls on any other Point round about it. And by this

Answer'd, by
Mr. Newton.
N. 97. p. 6110.
Oct. An. 1673.

Excess of Density, the Light, which falls in or invisibly near the just Point, may, I conceive, strike the *Sensorium* so vigorously, that the Impress of the weak Light, which errs round about it, shall, in Comparison, not be strong enough to be animadverted, or to cause any more sensible Confusion in the Picture than is found by Experience. But if this satisfy not, *N.* may try, if he please, how distinct the Picture will appear, when all the Lens is cover'd excepting a little Hole next its Edge on one side only: And if in this Case he please to measure the Breadth of the Colours thus made at the Edge of the Sun's Picture, he will perhaps find it to approach nearer to my Proportion than he expects.

A Reply by
M.
Ibid. p. 6112.

4. I am satisfied with the Manner whereby Mr. *Newton* reconciles the Effect of Convex-Glasses with his Theory; but then he is also to acknowledge, that this Aberration of the Rays is not so disadvantageous to Optick-Glasses, as he seems to have been willing to make us believe. His Invention is very good; but the Defect of the Metal seems to render it as impossible to execute, as the Difficulty of the Form obstructs the Use of the *Hyperbole* of M. *Des Cartes*.

Answer'd, by
Mr. Newton.
N. 96. p. 6091.
July, An. 1673.

If M. *N.* pleases to compare the Errors of a Glass and Speculum that collect Rays at equal Distances, he will find how much he is mistaken; and that I have not been extravagant, as he imagines, in preferring Reflections. And as for what he says of the Difficulty of the *Praxis*, I know it is very difficult, and by those Ways which he attempted it I believe it impracticable. But there is a Way insinuated above by which it is not improbable, but that as much may be done in large Telescopes as I have thereby done in short ones, but yet not without more than ordinary Diligence and Curiosity.

A Cata-Dioptri-
cal Telescope;
by M. Casse-
grain. N. 83.
p. 4056.
May, An. 1672.
Fig. 100.

XIX. 1. M. *Cassegrain* has communicated the Figure of a Telescope, almost like that of Mr. *Newton*.

A B C D. is a strong Tube, in the Bottom of which there is a great Concave Speculum C D, pierced in the middle E.

F is a Convex Speculum, so disposed, as to its Convexity, that it reflects the Species, which it receives from the great Speculum, towards the Hole E, where is an Eye-Glass, which one looketh through.

The Advantage which I find in this Instrument above that of Mr. *Newton*, is, 1. That the Mouth or Aperture A B of the Tube may be of what Bigness you please; and consequently you may have many more Rays upon the Concave Speculum, than upon that of which you have given us the Description. 2. The Reflection of the Rays will be very natural, since it will be made upon the Axis itself, and therefore more vivid. 3. The Vision of it will be so much the more pleasing, in that you shall not be incommoded by the great Light, by reason of the Bottom C D, which hideth the whole Face. Besides, you'll have less Difficulty in discovering the Objects, than in that of Mr. *Newton*.

Consider'd by
Mr. Newton.
Ibid. p. 4057.

2. When I first applied my self to try the Effects of Reflections, Mr. *Gregory's Optica Promota* (printed in the Year 1663.) being fallen into my Hands, where there is an Instrument (described p. 94.) like that of Mr. *Cassegrain's*, with

with a Hole in the midst of the Object Metal, to transmit the Light to an Eye-Glass placed behind it: I had thence an Occasion of considering that sort of Constructions, and found these Disadvantages in it; *viz.* 1. There will be more Light lost in the Metal by Reflection from the little Convex Speculum, than from the oval Plane. For it is an obvious Observation, That Light is most copiously reflected from any Substance when incident most obliquely. 2. The Convex Speculum will not reflect the Rays so truly as the oval Plane, unless it be of an hyperbolick Figure; which is incomparably more difficult to form than a Plane; and if truly formed, yet would only reflect those Rays truly which respect the Axis. 3. The Errors of the said Convex will be too much augmented by the too great Distance, through which the Rays reflected from it must pass, before their Arrival at the Eye-Glass. For which Reason, I find it convenient to make the Tube no wider than is necessary, that the Eye-Glass be placed as near to the oval Plane as is possible, without obstructing any useful Light in its Passage to the Object-Metal. 4. The Errors of the Object-Metal will be more augmented by Reflection from the Convex than from the Plane, because of the Inclination or Deflexion of the Convex on all sides, from the Points on which every Ray ought to be incident. 5. For these Reasons there is requisite an extraordinary Exactness in the Figure of the little Convex, whereas I find by Experience that it is much more difficult to communicate an exact Figure to such small Pieces of Metal, than to those that are greater. 6. Because the Errors of the Perimeter of the Concave Object-Metal, caused by the Sphericalness of its Figure, are much augmented by the Convex, it will not with Distinctness bear so large an Aperture as in the other Construction. 7. By reason that the little Convex conduces very much to the magnifying Virtue of the Instrument, which the oval Plane does not, it will magnify much more in Proportion to the Sphere, on which the great Concave is ground, than in the other Design; and so magnifying Objects much more than it ought to do in Proportion to its Aperture, it must represent them very obscure and dark; and not only so, but also confused, by reason of its being over-charged. Nor is there any convenient Remedy for this. For if the little Convex be made of a larger Sphere, that will cause a greater Inconvenience, by intercepting too many of the best Rays; or if the Charge of the Eye-Glass be made so much shallower as is necessary, the Angle of Vision will thereby become so little, that it will be very difficult and troublesome to find an Object; and of that Object, when found, there will be but a very small Part seen at once.

By this you may perceive, that the three Advantages, which Mr. Cassaign propounds to himself, are rather Disadvantages. For according to his Design, the Aperture of the Instrument will be but small, the Object dark and confused, and also difficult to be found. Nor do I see, why the Reflection is more upon the same Axis, and so more natural in one Case than in the other; since the Axis itself is reflected towards the Eye by the oval Plane, and the Eye may be defended from external Light, as well at the Side as at the Bottom of the Tube.

Mr. Gregory

Mr. Gregory speaking of these Instruments, in the aforefaid Book, p. 95. faith; *De Mechanica horum Speculorum & Lentium, ab aliis frustra tentata, ego in Mechanicis minus versatus nihil dico.* So that there have been Trials made of these Telescopes, but yet in vain. And I am informed, that about 7 or 8 Years since, Mr. Gregory himself, at London caused one of 6 Foot to be made by Mr. Reive, which I take to have been according to the aforefaid Design described in his Book; but, though made by a skilful Artist, yet it was without Success.

A Cata-Dioptrick
Telescope, by
S. Salvetti.
N. 87. p. 506.
Sept. An. 1672.

XX. S. Salvetti hath made a Prospective-Glass according to Mr. Newton's new Invention. It was not above half a Foot long, it had the same Effect of one of two. He is now making another after the Conceit of Mr. Cassegrain, though he agrees not with him in making Convex the little Speculum, which one looks into through the Eye-Glass, but believes the French Author only devised that to disguise as much as was possible his pretended new Invention, which he endeavours to make anterior to Mr. Newton's most noble one.

To make the Pi-
cture of any
thing appear in
a Light Room,
by Dr. Hook.
N. 38. p. 741.
Aug. An. 1688.

XXI. 1. Opposite to the Place or Wall, where the Apparition is to be, let a Hole be made of about a Foot in Diameter, or bigger: If there be a high Window, that hath a Casement in it, 'twill be so much the better. Without this Hole or Casement open'd, at a convenient Distance (that it may not be perceiv'd by the Company in the Room) place the Picture or Object, which you will represent, inverted, and by means of Looking-Glasses placed behind, if the Picture be transparent, reflect the Rays of the Sun so, as that they may pass through it towards the Place where it is to be represented; and to the End that no Rays may pass besides it, let the Picture be encompass'd on every side with a Board or Cloth. If the Object be a Statue, or some Living Creature, then it must be very much enlightned by casting the Sun-Beams on it by Refraction, Reflection, or both. Between this Object and the Place where 'tis to be represented, there is to be placed a broad Convex-Glass, ground of such a Convexity, as that it may represent the Object distinct on the said Place; which any one, that hath an Insight in the Opticks, may easily direct. The nearer it is placed to the Object, the more is the Object magnified on the Wall, and the further off the less; which Diversity is effected by Glasses of several Spheres. If the Object cannot be inverted (as 'tis pretty difficult to do with Living Animals, Candles, &c.) then there must be two large Glasses of convenient Spheres, and they placed at their appropriated Distances (which are very easily found by Trials) so as to make the Representations erect, as well as the Object.

These Objects, Reflecting and Refracting Glasses, and the whole Apparatus; as also the Persons employ'd to order, change, and make use of them, must be placed without the said high Window or Hole, so that they may not be perceiv'd by the Spectators in the Room; and the whole Operation will be easily perform'd.

What-

Whatsoever may be done by means of the Sun-Beams in the Day-time, the same may be done with much more ease in the Night, by the help of Torches, Lamps, or other bright Lights, placed about the Objects, according to the several sorts of them.

2. There are every where made of these Lanthorns to represent and magnify Figures upon a Wall; but then 'tis only in the Dark; wherefore to give Variety of Colours, take Oyl of Spike, and therein mix the several Colours, wherewith you will have your Glafs to be stained, paint them finely on, they dry presently, and penetrate any Glafs.

The Magick Lanthorn improved, by Sir Rob. Southwell.
N. 245. p. 364.
Oct. An. 1698.

XXII. Having found by many Trials, that some short-sighted Persons could find little or no Relief by the Use of Concave-Glasses for seeing Objects at any Distance distinct, and that any one may be made short-sighted, and to be able to distinguish nothing but what is placed very near his Eye, but within certain Limits of Distance, by putting on and looking through a very deep Pair of Spectacles, such as Ancient Men use: I concluded that what Glasses should make this Man, whilst looking thro' these Spectacles, to see Things at a greater Distance, would also help any other Person that should be short-sighted by Nature. I then consider'd, that by the help of a Convex-Glass, placed between the Object and the Eye, the Image of the Object may be made to appear at any Distance from the Eye; and consequently all Objects may thereby be made to appear in any convenient Distance from the Eye: So that the short-sighted Eye shall contemplate the Picture of the Object, in the same manner as if the Object itself were in that Place. But then, because the Pictures themselves are so inverted, and therefore will be uncouth to one, not used to see them in that Posture, I consider'd of these Expedients to help that Defect also.

A Way to help Short-sightedness, by Dr. Hook.
Pb. Call.
N. 3. p. 59.
Dec. An. 1681.

First, If it be only for Reading of a Book, or Writing, there needeth nothing but the Inversion of the Book, and then holding the Convex at a due Distance, for the Picture of the Letters will appear erected in the due Place, for the Eye to see and distinguish them very plainly.

Secondly, For seeing to Write, I thought this would be the best Expedient, That the Person short-sighted should first learn to Read with his naked Eye (both printed Letters and also written Hand) upside downwards, which is quickly attained to by one that can do both the right Way.

Thirdly, For distinguishing Objects at a Distance, I can assert by my own Experience, that with a little Use of contemplating Objects inverted, one shall have as good an Idea, and as true a Knowledge of all manner of Objects, as if they were seen erected in their natural Posture.

XXIII. 1. *Eustachio Divini* hath made a Microscope of a new Invention, wherein, instead of an Eye-Glass Convex on both sides, there are two Plano-Convex Glasses, which are so placed, as to touch one another in the middle of their Convex Surface. It hath this Peculiar, that it shews the Objects flat and not crooked; and although it takes in much, yet nevertheless magnifieth extraordinarily.

Microscopes by S. Divini.
N. 42. p. 842.
Dec. An. 1668.

It is almost 16 $\frac{1}{2}$ Inches high, and adjusted at four different Lengths. In the first, which is the least, it shews Lines 41 times bigger than they appear to the naked Eye; in the second 90 times; in the third 111 times; and in the fourth 143 times: Whence one may easily calculate how much it augments Surfaces and Solidities.

By S. Piet. Salvetti. N. 87.
p. 5065.
Oct. An. 1672.

2. *S. Salvetti* lately shew'd one of his Microscopes, made in Imitation of those of *Divini* and *Campani*, to the Great Duke of *Tuscany*, which was judged by all much better than any of the best his Highness hath. It was found, for Magnifying, Defining, and Clearness, to be very excellent.

By M. Leewen-
wenhoeck.
N. 94. p. 6037.
May, An. 1673.
By Mr. Butter-
field. N. 141.
p. 1026.
Sept. An. 1678.

3. *M. Leewenhoeck* hath lately contrived Microscopes, excelling those that have been hitherto made by *Eustachio Divini*, and others.

4. I have Microscopes of the manner lately brought out of *Holland* by *Mr. Huygens*, of several Fashions ready made. I have tried several Ways for the making of Glasses of the Bigness of a great Pin's Head and less; as in the Flame of a Tallow-Candle, and of one of Wax. But the best Way of all I have yet found, to make them clear and without Specks, is with the Flame of Spirit of Wine well rectified, and burned in a Lamp. Instead of Cotton I make use of very small Silver-Wire, doubled up and down like a Skein of Thread; which being wet with the Spirit of Wine, and made to burn in the Lamp, giveth through the Veril of the Lamp, a very ardent Flame. Then take your beaten Glass, being first washed very clean, upon the Point of a Silver Needle filled very small, and wet with Spittle. Hold it thus in the Flame till it be quite round, and no longer, for fear of burning it; and if the side of the Glass next the Needle be not melted, you may put it off, and take it up with the Needle on the round side, presenting the rough side to the Flame, till it be every where round and smooth, then wipe and rub one or several of them together with soft Leather, which makes them much the better. Then put them between two Pieces of thin Brass, the Apertures very round and without Bur, and that towards the Eye so big almost as the Diameter of the Glass, and so placed in a Frame with the Object conveniently for Observation.

By Mr. Stephen
Gray. N. 221.
p. 280.
June, An. 1626.

5. I took a small Particle of Glass, about the Bigness I designed my Globule, and laying it on the End of a Charcoal, I could, by the Help of a Blast-Pipe, with the Flame of a Candle, soon melt it into a Spherule; and by this means could make them indifferently clear, and the smallest very round, and I could make them much larger, than by the unassisted Heat of the Candle; but these latter were attended with an Inconvenience, they were, on that side that rested on the Coal, flattened, and received a rough Impression from it. To remedy this Inconvenience, I was wont to grind them and polish them on a Brass-Plane, and so reduce them to Hemispheres; but I found the clear small Globules, not to mention that they magnify more, shew Objects more distinctly.

XXIV. 1. A B, I call the Frame of the Microscope. It may be about $\frac{1}{4}$ of an Inch in Thickness. At A there is a small Hole, near $\frac{1}{4}$ of an Inch Diameter, this serves for the Aperture of the Water, being in the Center of a larger Spherical Cavity, about $\frac{1}{2}$ of an Inch Diameter, and in Depth somewhat more than half the Thickness of the Brass. Opposite to this, at the other side, there is another Concave, but half the Breadth of the former; which is so deep, as to reduce the Circumference of the small Hole in the Center, to almost a sharp Edge. In these Cavities the Water is to be placed, being taken upon a Pin, or large Needle, and convey'd into them till there be formed a double Convex Lens of Water; which, by the Concaves being of different Diameters, will be equivalent to a double Convex, of unequal Convexities. By this means, I find the Object is render'd more distinct than by a Planoconvex of Water, or by a double one, formed on the plain Surface of the Metal.

A Water Microscope by Mr. Stephen Gray. N. 221. p. 281. N. 223. p. 353.

Fig. 101;

C D E, is the Supporter, whereon to place the Object; if it be Water, in the Hole G; if a solid Object, on the Point F. This is fix'd to the Frame of the Microscope, by the Skrew E, where 'tis bent upwards, that its upper part may stand at a Distance from the Frame; 'tis moveable on the Skrew as a Center, to the end that either the Hole C, or the Point F, may be expos'd before the Microscope; and that the Object may be brought to, and fix'd in its Focus. There is another Skrew, about half an Inch in Length, which goes through the round Plate into the Frame of the Microscope A E, the Skrew and Plate taking hold of the Supporter about D, where there is a Slit somewhat larger than the Diameter of the Skrew, which is requisite for the Admission of the Hole C, or Point F, according to the Nature of the Object, into the Focus of the Glass; for by turning the Skrew G, the Supporter is carried to or from the same, which may be sooner done, if whilst one turn the Skrew with one Hand, the other hold the Microscope by the end B, and one continue looking through the Water till the Object be seen most distinctly.

The Supporter must be made of a thin Piece of Brass well hammer'd, that by its Spring it may the better follow the Motion of the Skrew, I chose rather to fix the Supporter by the Skrew E, than by a Rivet; because it may now, by help of a Knife, be unscrew'd, and by the other Skrew G, be brought close to the Frame of the Microscope without weakening its Spring, and so become more conveniently portable. If the Hole at G be fill'd with Water, but not so as to be Spherical; all Objects that will bear it, are seen therein more distinctly.

2. Having observ'd some irregular Particles in Globules of Glass, and finding them distinct, but prodigiously magnified, when held close to the Eye, I concluded that if I convey'd a small Globule of Water to my Eye, and that there were any opacous or less transparent Particles than the Water therein, I might see them distinctly. I therefore took on a Pin a small Portion of Water, which I knew to have in it some minute Animals, and laid it on the End of a small Piece of Brass Wire (there lay then by me) of about $\frac{1}{2}$ of

Another. Ibid. N. 221. p. 282.

N. 223. p. 355.

an Inch Diameter, till there was formed somewhat more than an Hemisphere of Water; then keeping the Wire erect, I applied it to my Eye, and standing at a proper Distance from the Light, I saw them and some other irregular Particles, as I had predicted, but most enormously magnified; for whereas they are scarce discernable by my Glass-Microscopes, or first Aqueous one, within the Globule they appeared not much different both in their Form, nor less in Magnitude than ordinary Pease. They cannot well be seen by Day-light, except the Room be darkned, after the manner of the famous Dioptrical Experiment, but most distinctly by Candle-light; they may be very well seen by the full Moon-light. If the Water be conveyed into the Hole B (which may be about $\frac{1}{10}$ of an Inch Diameter) till there remain near an Hemisphere of Water on each side of the Hole, the Objects are seen more distinctly; and the Spherical Form of the Water is this way better secured, than on the Point of a Pin-Wire.

N. 221. p. 285.

Fig. 102.

The Reason of this Appearance may be thus explained. Let the Circle DBBD represent a Sphere of Water, A an Object placed in its Focus, sending forth a Cone of Rays, two of which are AB, AB, which Opticians know coming into the Water at B and B, will be refracted from their direct Course, and become BD, BD; at D they will, at their passing into the Air, be again refracted into DE, DE, and so run parallel to one another, and to the Axis of the Sphere AFCG. Now 'tis a known and fundamental Principle in Opticks, that the Angle of Reflexion is equal to the Angle of Incidence; wherefore let the Rays BD, BD, be imagined to come from some Point of an Object placed within a Sphere of Water, by being reflected from the interior Surface of the Sphere at B B, CBD is the Angle of Reflexion, to which making CBF equal, F will be the place where an Object sending forth a Cone of Rays, two of which are FB, FB, which are reflected into the Rays BD, BD, and then coming to the other side the Sphere at D and D, they are refracted into DE, DE, as before; and consequently be as fit for distinct Vision, whether the Object be placed in F within, or in A without the Sphere, if its interior Surface be considered as a Concave Reflecting Speculum.

Microscopes improved; by
Mr. Newton.
N. 88. p. 5095.

XXV. From the Distinction I have elsewhere given between *Compounded* and *Uncompounded Colours*, I take occasion to communicate a Way for the Improvement of Microscopes by Refraction; viz. By illuminating the Object in a darkned Room with Light of any convenient Colour not too much *Compounded*; for by that means the Microscope will with Distinctness bear a deeper Charge and larger Aperture, especially if its Construction be such as I may hereafter describe; for the Advantage in ordinary Microscopes will not be so sensible.

A Reflecting
Microscope; by
Mr. Newton.
N. 89. p. 3080.

Fig. 103.

XXVI. 1. I have sometimes thought to make a Microscope, which should have, instead of an Object-Glass, a reflecting Piece of Metal. For these Instruments seem as capable of Improvement as Telescopes, and perhaps more; because but one reflective Piece of Metal is requisite in them, as you may

may perceive by the Diagram, where A B representeth the Object Metal, C D the Eye-Glass, F their common Focus, and O the other Focus of the Metal, in which the Object is placed.

2. A represents a small flat Ring of Brass, whose interior Circle must not much exceed $\frac{1}{4}$ of an Inch Diameter, and about $\frac{1}{5}$ of an Inch thick: This we may call the Frame or Cell of the Glass; it must be prepared for Use after the following manner. Take a small Globule of Quick-silver, and dissolve it in a few Drops of *Aqua Fortis*, to which you may add 10 Parts of common Water; dip the End of a Stick in this Liquor, and rub the inward Circle of the Ring with it; so as it will have acquired a mercurial Tincture, and being wiped dry, be fit for Use. Then let it be laid on the Table, and pour a Drop of Quick-silver within it, which press gently with the Ball of the Finger, and it will adhere to the Ring; then cleanse it with a Hare's Foot, and you will have a Convex Speculum. Take up the Ring and Speculum carrying it Horizontal, and lay it on the Brims of the hollow Cylinder B; so will the Mercury become a Concave Reflecting Speculum, which from the Smallness of the Sphere of which it seems to be a Section, may be used as a Microscope. The Cylindrick Vessel B has a Skrew-Hole at the Bottom, by which it is skrewed to the Top of the Pedestal C D; C E F G is the Supporter of the Object Plate, which, as you see, may be raised higher, or let lower, as there is Occasion, by the Skrew on the Pedestal: The Object Plate must be of Glass cemented to the Ring G.

By Mr. Stephen Gray. N. 228. p. 541.

Fig. 104.

This Instrument, with a little Variation, may be made a Microscope of Water, if, instead of the Ring G, there be only a small Arm with a Hole in it to receive a Drop of Water, and the Cylindrick Vessel B be either taken away, or skrewed on with its Bottom upwards, so as to make an Object Plate. This will be more convenient for viewing the Textures of opacous Objects, than that above described, which is more fit for fluid and transparent ones.

XXVII. 1. The Figure of it is round, being 30 Inches, and somewhat better in Diameter, On one side it hath a Frame of a Circle of Steel, to the end that it may keep its just Measure. 'Tis easy to remove it from Place to Place, tho' it be above an Hundred Weight, and 'tis easily put in all sorts of Postures. The Burning Point is distant from the Center of the Glass about 3 Foot. The Focus is about half a *Louis d'Or* large. One may pass one's Hand through it, if it be done nimbly; for if it stay there the time of a Second of a Minute, there is danger of receiving much hurt. Green Wood takes Fire in it in an instant, as do also many other Bodies.

A Burning Case at Lyons, made by M. de Vilette, N. 6. p. 95.

| | Seconds. |
|---|----------|
| A small Piece of Pot-Iron was melted, and ready to drop down in | 40 |
| A Silver Piece of Fifteen Pence was pierced in | 24 |
| A Gross Nail (called <i>le Clou de Paisan</i>) was melted in | 30 |
| The End of a Sword-Blade of <i>Olinde</i> was burned in | 43 |
| A Brass Counter was pierced in | 06 |
| A Piece of Red Copper was melted ready to drop down in | 42 |

| | Seconds. |
|---|----------|
| A Piece of a Chamber Quarry-Stone was vitrified, and put into } | |
| a Glas-Drop, in - - - - - } | 45 |
| Steel, whereof Watchmakers make their Springs, was found melt- | |
| ed, in - - - - - } | 09 |
| A Mineral-Stone, such as is used in Harquebusses à rouët, was } | |
| calcin'd and vitrified, in - - - - - } | 01 |
| A Piece of Mortar was vitrified, in - - - - - } | 52 |

In short, There is hardly any Body which is not destroy'd in this Fire. If one would melt by it any great Quantity of Metal, that would require much Time. the Action of Burning not being perform'd but within the Bigness of the Focus, so that ordinarily none but small Pieces are expos'd to it. One M. de Alibert buys it, paying for it 1500 Livres.

Ibid. p. 97.

You incline to believe, that the Glasses of *Maginus* and *Septalius* do approach to that of *Lyons*; but I can assure you they come very far short of it. You may consult *Maginus* his Book, where he describes his; and there are some Persons here who have seen one of his best, which had but about 20 Inches Diameter; so that this of *Lyons* must perform at least twice as much. As to *Septalius*, we expect the Relation of it from Intelligent and Impartial Men. It cannot well be compared with that of *Lyons*, but in Bigness; and in this Case, if it have five Palms (as you say) that would be but $3\frac{1}{2}$ Foot *French*, and so it were a Foot bigger, which would make it half as much greater in Surface: But as to the Effects, seeing it burns so far off, they cannot be very violent. And I have heard one say, that had seen it, that it did not set Wood on Fire but after the time of saying a *Miserere*. You may judge of the Difference of the Effects, since that of *Lyons* gathers its Beams together within the Space of 7 or 8 Lines; and that of *Septalius* must scatter them in the Compass of 3 Inches.

N. 49. p. 986.

Another by the same.

N. 49. p. 986.

It was disposed of to the King of *Denmark*.

The same M. de Kilette of *Lyons* hath made another Burning Concave. It is of 34 Inches Diameter, and melts all sorts of Metals, and Iron itself of the Thickness of a Silver Crown in less than a Minute of Time, and vitrifies Brick in the same time; and as for Wood, whether green or dry, it sets it on Fire in a Moment. The King hath seen it, and the Performances of it, with great Satisfaction; and his Majesty is likely to make it his, and then to bestow it on his *Royal Academy* of Philosophers, for making of farther Experiments with it.

By
Ibid. p. 987.

This kind of Concaves burning the most forcibly of any Fire we know of, would be of great Use, if they could be so contrived as to have a Focus of any considerable Largeness, to take in a good Quantity of combustible Matter at once.

By S. Serralla.
N. 40. p. 796.

3. S. Serralla at *Milan*, causeth to be made a Burning-Glass of 7 Foot in Diameter. He pretends to make it burn at the Distance of 50 Palms, which is about 33 Foot.

4. The

4. The outer Circle of the Concave Burning Speculum, which I lately caus'd to be made in *Lusace*, is near 3 *Leipsick* Ells in Diameter, exceeding that great one at *Paris* by $\frac{1}{2}$ of such an Ell. It is made of a Copper Plate scarce twice so thick as the Back of an ordinary Knife, and may therefore be easily removed from Place to Place, and order'd for Use: And the Workmanship of it may, by the Contrivances I have invented, be easily, and in little time perform'd by one Man. The Polish thereof is very good, and represents by distinct Reflections all those Appearances which arise from the Concave Figure thereof.

A Burning Concave in Germany, by N. 188. p. 42.

The Force of this Speculum is incredible. For, 1. A Piece of Wood put into the Focus (which is 2 Ells off) flames in a Moment, so as a fresh Wind can hardly put it out. 2. Water applied in an Earthen Vessel presently boils, so as to boil an Egg; and the Vessel being held there some time, the Water evaporates all away. 3. A Piece of Tin or Lead 3 Inches thick, as soon as it is put into the Focus, melts away in Drops; and held there a little time is in a perfect Fluor, so as in two or three Minutes to be quite pierced through. 4. A Plate of Iron or Steel placed in the Focus immediately is seen to be red-hot on the Backside, and soon after a Hole is burnt through: I have made three such Holes in a Plate in six Minutes time. 5. Copper, Silver, and the like, applied to the Focus, melt; which I have tried with several sorts of Coin; among the rest, with a *Rix-Dollar*, and the same happen'd to it as to the afore said Iron Plate in five or six Minutes. 6. Things not apt to melt, as Stones, Brick, and the like, soon become red-hot like Iron. 7. Slate at first is red-hot, but in a few Minutes turns into a fine sort of black Glas; of which, if any part be taken in the Tongs and drawn out, it runs into Glas Threads. 8. Tiles, which had suffer'd the most intense Heat of Fire, in a little time melt down in a yellow Glas: As do, 9. Pot-Shreds, not only well burnt at first, but much us'd in the Fire, into a blackish yellow Glas. 10. Pumice-Stone, said to be that of Burning Mountains, in this Solar Fire, melts into a white Transparent Glas. 11. A Piece of a very strong Crucible put into the Focus, in eight Minutes, was melted into a Glas. 12. I have seen Bones turn'd into a kind of opaque Glas; and a Clod of Earth into a yellow or greenish Glas.

These Experiments were made in *August* and *September*, when the Sun has not the same Force as when he is about the Summer Solstice. The Beams of the Full Moon, concentred by this Speculum, did not produce any Degree of Heat, tho' the Light was not a little encreas'd.

5. Some Years ago, Dr. *Hook* made a Proposal to the *Royal Society* concerning the same thing. He conceives one may be made of many Foot Diameter, for a small Price, being hammer'd out of a Copper Plate, and tinned over with a Mixture of Tin, Lead and Tin-Glas, which is found to bear a very good Polish. Such a Speculum might be of great Use in perfecting the Art of Pastes, or fictitious Jewels, which require the most intense Degree of Heat, to bring them to an exact Mixture.

By Dr. Hook. Ibid. p. 354.

*Concave Specula
nearly of a Para-
bolick Figure;
attempted by
Mr. Stephen
Gray.
N. 228. p. 542.
N. 235. p. 787.*

XXVIII. A Linnen-Cloth, first being wet in fair Water, and then laid on a Conclave Cylinder, as the Verge of a Sieve, Keeler, or the like, its central Parts will descend so as to form a very regular Concave Superficies. And a Thread, being first wet in common Water, and then suspended with its two Ends, or any two Points nearer than their utmost Extent, so as it might touch the Center of the suspended Cloth, and its two opposite Points on the Ring, was found to have the same Curvature. My Business was then to examine the Figure of the Thread thus suspended, which I did in manner following. On the side of a Wall I described *Parabola's* of several Species, whose Axes were perpendicular, and Perimeter horizontal, to which the Line being applied, so as it might touch the Vertex, past very nearly through all the intermediate Points of the *Parabola*, much nearer than the Portion of a *Circle*, which past through the Extremity of the Perimeter, and *Latus Rectum*, would do.

From hence I conclude, That a ponderous and pliable Substance, being suspended on a Ring or hollow Cylinder, so as that its central Parts may descend, will form itself into a Figure that is more commodious for Burning-Glasses than the Spherical, of which they are now made, being much nearer their most absolute Figure, the *Parabola*.

Now if there may be a Way found to give to Cloth or Leather a metalline Surface, or a Varnish that may bear a good Polish; or if this be found impracticable, perhaps Plates of Metal may be beat out so thin, as being suspended on a large Ring, will by their own Gravity receive their true Figure, one may make Speculums of what Largeness he pleaseth.

Upon this Consideration, I devised the following Experiment. There was taken a sufficient Quantity of Potter's Clay, of which there was formed a plain circular Plate, by help of an Iron Ring about 13 Inches Diameter. This was laid on a lesser Ring, which was supported by four Feet, and it immediately became a very regular Concave on its upper, and Convex on its under Superficies; but notwithstanding 'twas set to dry in the Shade, yet before it was dry enough, its central Parts extended so as to become almost plain, not without some Defects; if it had continued in its Regularity, I designed to have burned and glazed it in a Potter's Furnace.

*To make the
Globe Looking-
Glass, by Sir
R. Southwell.
N. 241. p. 363.*

XXIX. Take Quick-silver, Marchasite of Silver, each three Ounces; Tin and Lead, each half an Ounce; these two first throw on the Marchasite, and last of all the Quick-silver; stir them well together; but they must be taken from the Fire, and be towards Cooling before the Quick-silver be added; let your Glass be well warmed, and pour in the Mixture, and roll it from side to side.

Note, This will do also when Cold, but 'tis best when the Glass is heated and very dry.

Note also, That if at the Glass-House your Ball be of yellow Glass, then all will shine like Gold.

XXX. *Papers*

XXX. Papers (of less General Use) omitted.

1. **D**R. Hook having (in his *Micrographia*) described a new Engine for Grinding Optick-Glasses of very great Lengths, M. Auzout (in a small French Tract) objects several Difficulties to this Engine itself: But however he thinks it impracticable to make any Glasses of above 300 or 400 Foot at most (and fears that neither Matter nor Art will go even so far) which will be very far from showing us *Plants* or *Animals* in the *Moon*; and then proposes Remedies to some of the Inconveniences of the Turn. To all this, Dr. Hook here replies; He answers the Objections, and rejects the proposed Expedients.

2. Carlo Ant. Mancini having, in his *Occhiale all' Occhio*, described a particular Way for making Convex-Glasses upon a Plain, his Method is here translated from the *Italian* into *English*. But 'tis added, That though the Contrivance be ingenious, yet it is conceiv'd by skilful Artists, that it will be very difficult to put it into Practice.

XXXI. Accounts of Books omitted.

1. **P**hyfico-Mathesis de Lumine, Coloribus, & Iride, &c. Auth. Franc. Maria Grimaldo, S. J. Bononiæ 1665. in 4to. N. 99. p. 3068.
2. Cogitationes Physico-Mechanicae de Natura Visionis. Auth. Jo. Ott. Schafhusia Helvetio. Heidelbergæ 1670. in 4to. N. 71. p. 2163.
3. Synopsis Optica. Auth. Honorato Fabri. Soc. Jesu. Lugduni 1667. N. 32. p. 626. in 4to.
4. L' Occhiale all' Occhio, ovvero Dioptrica Pratica del Carlo Ant. Mancini. N. 42. p. 837. in Bologna 1660. in 4to.
4. Lectiones 18; Cantabrigia in Scholis publicis habita, in quibus Opticorum Phenomenon Genuina Rationes investigantur, & exponuntur, ab Isaaco Barrow. Lond. 1669. in 4to. N. 75. p. 2258.
6. La Dioptrique Occulaire, par le Pere Cherubin d' Orleans, Capucin. A N. 78. p. 3045. Paris, 1671. in Folio.
7. Christiani Hugenij Astrocopia Compendiaria, Tubi Optici molimine Liberrata: or, The Description of an Aerial Telescope. Hague, 1684. in 4to. N. 161. p. 668.
8. A Treatise of Dioptricks. By Will. Molineux, Esq; F. R. S. in 4to. N. 205. p. 967.
9. Catoptrica & Dioptrica Elementa. Auctore Davide Gregorio, D. M. Oxon, 1695. in 8vo. N. 219. p. 214.



C H A P. IV.

A S T R O N O M Y.

*The Observatory
of Tycho
Brahe; by
Mr. Gourdon.
N. 266. p. 692.*

I. **T**HE Island *Ween*, (vulgarly term'd the *Scarlet-Island*) famous for the Observations of *Tycho Brahe*, that renowned *Danish Astronomer*, (with all Submission to better Judgments) was none of the fittest for Astronomical Observations of all sorts, such as the taking the exact Time of the true Rising and Setting of Celestial Bodies, together with their respective Amplitudes; because the Island lies low, and is Land-lock'd on all the Points of the Compass, save three. Besides, the sensible Land-Horizon of the *Ween* is extremely uneven and rugged, the North and Eastern Parts thereof being some rising Hills in the Province of *Schonen*; and the Western Part is mostly overspread with Trees on the Island *Zealand*; from the remotest of which Coasts the *Ween* is not distant above three Leagues.

*A new Astronomical Instrument;
by M. Weigheli-
us.
N. 74. p. 2219.*

II. M. *Weigheli* hath invented an Instrument, which he calls *Astrodicticum*, by the means whereof very many Persons shall be able at one and the same time to behold one and the same Star. He hath also invented an exceeding great Globe of the World, capable of 10 Persons to sit in it all at once, and to behold the Motions of the celestial Bodies, &c.

*A Celestial Globe;
by M. Didier,
P. Alleman.
N. 136. p. 905.*

III. The Bigness of this Globe is only of four Inches Diameter. The Body of the Globe of burnish'd Steel, where all the Figures of the Constellations are designed in Silver-colour, but the Stars themselves of all Magnitudes are put on in emboss'd Gold.

This Globe moves from East to West in 24 Hours; and you may there see the Sun exactly rise and set as in the great World, together with the Moon, as also the Stars of the Constellations; likewise, how the Sun of this Globe comes to his Meridian, with an admirable Regularity, conform to the *Primum Mobile*. And you may also there perceive the mean Motions of the Sun and Moon from West to East, and all the Lunations; and by the Diurnal Motion of the Moon, it shews the Flux and Reflux of the Sea.

The Meridian serveth for a Needle to shew the Hours, which are marked upon the Zodiack, which the Sun marcheth regularly, which hath two main Rays, one whereof goeth directly Northward, the other Southward. That of the North marks the Way or Degree, which the Sun maketh from West to East upon the Signs of the Zodiack, and upon a Circle of Silver, where the 360 Degrees of the Circle are mark'd. The other Ray of the South, marks upon another Circle of Silver the Days of the Month, where the 365 Days are noted. The Circles of the Longitude of the Stars, which separate the Signs,

Signs, and which come from the Poles of the Zodiack, are marked by Gold-Wires; as also the Equator, the Tropicks, and the Polar Circles.

There is but one great Spring, the *Pinnam Mobile*, which puts all the rest in Motion: It is wound up by the *Antarctique* Pole, and you may wind it up to the Right or Left Hand, without wronging any contrary Motion: And by the *Arctique* Pole, you may advance and retard this Movement, if you should find any Inequality, without altering at all the great Spring.

IV. I applied my self the last Summer to the taking of the Diameters of the Sun, Moon, and the other Planets, by a Method which one M. *Picard* and my self have, esteemed by us the best of all those that have been practis'd hitherto; since we can take the Diameters to Second Minutes, being able to divide one Foot into 24000 or 30000 Parts, scarce failing as much as in one only part, so as we can in a manner be assured, not to deceive our selves in 3 or 4 Seconds. I shall not now tell you my Observations; but I may very well assure you, that the Diameter of the Sun has not been much less in his *Apogee*, than 31 min. 37 or 40 sec. and certainly not less than 31 min. 35 sec. and that at present in his *Perigee* it passes not 32 min. 45 sec. and may be less by a Second or two: That which is at the present troublesome is, that the vertical Diameter, which is the most easy to take, is diminish'd, even at Noon, by 8 or 9 sec. because of the Refractions, which are much greater in Winter than Summer at the same Height; and that the horizontal Diameter is difficult, because of the swift Motion of the Heavens.

A Way to measure the Diameters of the Planets, and the Parallax of the Moon; by M. Auzout. N. 21. p. 373. Jan. An. 1666.

As for the Moon, I never yet found her Diameter less than 29 min. 44 or 45 sec. and I have not seen it pass 33 min. or if it hath, it was only by a few Seconds. But I have not yet taken her in all the Kinds of Situations of the *Apogees* and *Perigees* which happen, with the Conjunctions and Quadratures. I do not mention all what can be deduced from thence; I shall only tell you, that I have found a Way to know the *Parallax* of the Moon, by the means of her Diameter: *viz.* If on a Day, when she is to be in her *Apogee* or *Perigee*, and in the most boreal Signs, you take her Diameter towards the Horizon, and then towards the South, with her Altitudes above the Horizon. For if the Observation of the Diameters be exact, as in these Situations the Moon changes not considerably her Distance from the Earth in 6 or 7 Hours, the Difference of the Diameters will shew the Proportion there is of her Distance with the Semidiameter of the Earth. I do not enlarge, because that as soon as one hath this Idea the rest is easy. The same would yet be practis'd better in the Places where the Moon passes through the Zenith, than here; for the greater the Difference is of the Heights, the greater is that of the Diameters. I do not note (for it easily appears) that if one were under the same Meridian, or the same *Azimuth* in two very distant Places, and took at the same time the Diameter of the Moon, one would do the same thing; tho' this Method goes not to Preciseness.

From what has been said may be collected the Reason of the Observation, which M. *Hevelius* made in the last Eclipse of the Sun (*July 2. St. N. 1666.*) touching the Encrease of the Moon's Diameter about the End. I am exte-

ing glad, that a Person, who probably knew not the Cause of it, has made the Experiment ; but it is strange, that until now no Astronomer has foreseen that That should happen, nor given any Precepts for the Change of the Moon's Diameter in the Eclipses of the Sun, according to the Places where they should happen, and according to the Hour and Height the Moon should have : For what happen'd in that Eclipse of Augmentation, would have fallen out contrarily, if it had been in the Evening ; for the Moon, which in that Eclipse that began in the Morning was higher about the End than at the Beginning, was nearer us, and consequently was to appear bigger : But if the Eclipse should happen in the Evening, she would be lower at the End, and therefore more distant from us, and consequently appear lesser. So also in two different Places, whereof one should have the Eclipse in the Morning, and the other at Noon, the Moon should appear bigger to him that hath it at Noon : And she must likewise appear to those who shall have a lesser Elevation of the Pole under the same Meridian, because the Moon will be nearer them.

An Account of
M. Gascoigne's
Micrometer ; by
Mr. Richard
Townley.
N. 25. p. 457.
May, An. 1667.

V. 1. I should be look'd upon as a great Wronger of our Nation, should I not let the World know, that I have, out of some scatter'd Papers and Letters that formerly came to my Hands of one Mr. *Gascoigne's*, found out, that before our late *Civil Wars* he had not only devised an Instrument of as great a Power as M. *Anzani's*, but had also for some Years made use of it ; not only for taking the Diameters of the Planets, and Distances upon Land ; but had farther endeavour'd, out of its Preciseness, to gather many Certainties in the Heavens ; amongst which I shall only mention one, viz. *The finding the Moon's Distance*, from two Observations of her Horizontal and Meridional Diameters ; which I the rather mention, because the *French Astronomer* esteems himself the first that took any such Notice, as thereby to settle the Moon's Parallax : For our Country-Man fully consider'd it before, and imparted it to an Acquaintance of his, who thereupon propos'd to him the Difficulties that would arise in the Calculation ; with Considerations upon the strange Niceties, necessary to give him a Certainty of what he desired. The very Instrument he first made I have now by me, and two others more perfected by him ; which doubtless he would have infinitely mended, had he not been slain unfortunately in His late Majesty's Service. He had a *Treatise of Opticks* ready for the Press ; but though I have used my utmost Endeavour to retrieve it, yet I have in that Point been totally unsuccessful : But some loose Papers and Letters I have, particularly about this Instrument for taking of Angles, which was far from perfect. Nevertheless, I find it so much to exceed all others, that I have used my Endeavours to make it exact, and easily tractable ; which above a Year since I effected to my own Desire, by the Help of an ingenious and exact Watchmaker : Since which time, I have not altogether neglected it, but employ'd it particularly in taking the *Distances*. (as Occasion served) of the *Circum-jovialists*, towards a perfect settling their Motion. I shall only say of it, That it is small, not exceeding in Weight, nor much in Bigness, an ordinary Pocket-Watch, exactly marking
above.

above 40000 Divisions in a Foot, by the help of two Indexes; the one shewing Hundreds of Divisions, the other Divisions of the Hundred; every list Division in my small one containing $\frac{1}{12}$ of an Inch, and that so precisely, that, as I use it, there goes about $2\frac{1}{2}$ Divisions to a Second. Yet I have taken Land-Angles several times to one Division, tho' (for the Reason mention'd by M. *Anzoni*) it be very hard to come to that Exactness in the Heavens, viz. the swift Motion of the Planets. Yet, to remedy that Fault, I have devised a *Rest*, in which I find no small Advantage, and not a little pleasing those Persons who have seen it, being so easy to be made, and by the Observer manag'd without the help of another; which second Convenience, my yet nameless Instrument hath in great Perfection, and is by reason of its Smallness and Shape, easily applicable to any Telescope.

2. *aaaa*, is a small oblong Brass Box, serving both to contain the Skrews and its Sockets, or Female Skrews, and also to make all the several moveable Parts of the Instrument to move very true, smooth, and in a simple direct Motion. To one End hereof is skrewed on a round Plate of Brass *bbbb*, about 3 Inches over; the extreame Limb of whose Outside is divided into 100 equal Parts, and numbred by 10, 20, and 30, &c. Through the Middle of this Plate, and the Middle of the Box *aaaa*, is placed a very curiously wrought Skrew, of about the Bigness of a Goose-Quill, and of the Length of the Box, the Head of which is, by a fixed Ring or Shoulder on the Inside, and a small springing Plate *dd*, on the Outside, so adapted to the Plate that it is not in the least subject to shake. The other End of this Skrew is by another little Skrew (whose small Points fill the Center or Hole made in the End of the longer Skrew for this purpose) rendred so fix'd and steady in the Box, that there appears not the least Danger of shaking. Upon the Head of this Skrew, without the springing Plate, is put on a small Index *ee*, and above that a Handle *mm*, to turn the Skrew round as often as there shall be Occasion, without at all endangering the displacing of the Index, it being put on very stiff upon a cylindrical Part of the Head, and the Handle upon a Square. The Skrew hath that Third of it, which is next the Plate, bigger than the other two Thirds of it, by at least as much as the Depth of the small Skrew made on it: The Thread of the Skrew of the bigger Third is as small again, as that of the Skrew of the other two Thirds. To the grosser Skrew is adapted a Socket *f*, fastned to a long Bar or Bolt *gg*, upon which is fastned the moveable Sight *h*, so that every Turn of the Skrew promotes the Sight *h*, either a Thread nearer, or a Thread farther off from the fix'd Sight *i*. The Bar *gg* is made exactly equal, and fitted into two small Staples *kk*, which will not admit of any shaking. There are 60 of these Threads, and answerable thereto are made 60 Divisions on the Edge of the Bolt or Ruler *gg*, and a small Index *l* fix'd to the Box *aaaa*, denotes how many Threads the Edges of the two Sights *h* and *i* are distant; and the Index *ee*, shews on the circular Plate what part of a Revolution there is more; every Revolution, as was said before, being divided into 100 Parts. At the same time that the moveable Sight *h* is moved forwards or backwards, one or more Threads of the courser Skrew, is the Plate *pp*, by the means of the Socket *q*,

A Description
of it, by
Dr. Hook.
N. 29. p. 542.
Nov. An. 1667.

Fig. 105.

Fig. 106.

to which it is skrewed, moved forward or backward, one or more Threads of the finer Skrew : So that this Plate being fix'd to the Telescope by the Skrews *r r*, so as the middle betwixt the Sights may lie in the Axis of the Glafs, however the Skrew be turned, the midst betwixt the Sights will always be in the Axis, and the Sights will equally either open from it, or shut towards it.

Fig. 107.

It is conceived by some ingenious Men, that it will be more convenient, instead of the Edges of the two Sights *b* and *i*, to employ two Sights *r* and *s*, fitted with the Hairs *r* and *v*, so that they may be conveniently used in the place of the solid Edges of the Sights *b* and *i*.

Fig. 108.

The Instrument is thus applied to the Telescope. The Tube *A D* is divided into three Lengths, of which (as in ordinary ones) *B C* is to lengthen or contract, as the Object requires: But *A B* is here added, that at *A* you may put such Eye-Glasses as shall be thought most convenient, and to set them still at the Distance most proper from the Indexes or Pointers, which here are supposed to be at *B*, which Length alters also in respect of divers Persons Eyes. *E* is a Skrew, by which the great Tube can be fix'd so, as by the help of the Figures any smaller part of it can immediately be found, measuring only, or knowing the Divisions on *B C*, the Distance of the Object-Glafs from the Pointers. *F* is the angular Piece of Wood, that lies on the upper Skrew of the Rest.

Fig. 109.

Ibid. p. 456.

This Rest (by Dr. Hook's Suggestion) may be rendred more convenient, if, instead of placing the Skrew Horizontal, it be so contrived, that it may be laid parallel to the Equinoctial, or to the Diurnal Motion of the Earth; for, by that Means, the same Thing may be performed by the single Motion of one Skrew, which in the other Way cannot be done but by the turning of both Skrews; as will easily appear to those that shall consider it.

More ways to
measure small
Distances inti-
mated, by
Dr. Hook.
N. 25. p. 459.
May, An. 1667

3. I have by me two or three several Ways of Measuring the Diameters of the Planets, whether Horizontal, Perpendicular, or Inclined, to the Exactness of a Second, by the help of a Telescope; as also of taking the Position and Distance of the small fix'd Stars one from another, or from any of the less bright Planets, if the Distance be not above two or three Degrees.

Excellence of the
Micrometer, by
Mr. Flamsteed.
N. 96. p. 6099.

4. Micrometro & Tubo Pedum 14, Planetarum frequenter Diametros & à Fixis Distantias, ad secundos fere Scrupulos, quod vix inexpertus credes, dimensus sum.

Plain Sights re-
jected; by
Mr. Flamsteed.
N. 89. p. 5119.
N. 96. p. 6102.

VI. 1. Pluribus Argumentis evinci potest, *Tychonem* sæpe cum in Locis, tum Latitudinibus, Fixis quibusdam assignatis, duos tresve & interdum quatuor aut quinque totos Scrupulos à vero aberrasse. Fixarum quidem Restitutionem suscepisse Celeberrimum *Joannem Hevelium* audivimus, attamen quandoquidem Pinacidiis Vitrorum Cassis fertur ipsum uti, dubium an multum ab ipso Emendatiores Locos habituri simus quam reliquit *Tycho*, nisi ubi valde hallucinatus est.

2. Per-

2. Percipio Vestrates non omnes mihi adstipulari in isto Dioptrarum Negotio, de quibus in Machinæ meæ Cœlestis *Organographia* tractavi. Verum etiam si *Cl. Hookins* & *Cl. Flamsteedius*, alique plane aliter sentiant, experientia tamen quotidiana me edocuit, atque etiamnum docet, rem longe aliter se habere in Magnis illis Organis, Quadrantibus scil. Sextantibus & Octantibus, imprimis Quadrantibus Azimuthalibus, aliisque Quadrantibus Regulis constructis, quæ nempe adeo procliviter commoveri & inverti (dum examinantur Dioptræ Telescopicæ) imo nullo modo possunt, ut quidem Instrumenta illa trium quatuorve Pedum Perpendiculo constructa. Res cum primis in eo consistit, quod nullam plane Observationem suscipere possint suis Dioptris Telescopicis, nisi prius denuo eas Examinent, & Rectificent; in quo tamen Examine varia via idque jugiter, utut studiosissime illud suscipias, hallucinari datur. Ad hæc, in Quadrantibus Azimuthalibus, Octantibus & Sextantibus, qua ratione Examen illud omni tempore, commode, & sine magno Temporis Dispendio institui possit, profecto nondum capio.

Plain Sights preferred to Telescopic; by M. Hevelius. N. 102. p. 27. Apr. An. 1674.

Video etiam aliquos (inter quos *Cl. Flamsteedius* invenitur) tulisse jam de nostris Observationibus, qualibus qualibus, judicium, priusquam alias viderunt, examinarunt, vel quicquam de iis cognoverunt. Nolo quidem vanus esse rerum mearum Jactator, nec unquam imaginatus mihi fui, me in omni isto Negotio, Restitutionis scil. Fixarum rem acu omnino tetigisse, aut tangere pro mea tenuitate posse: Sed hoc mihi penitus imaginor, si quidem totum illud Negotium suscepissem Dioptris Telescopicis, mihi non solum plurimos Annos Examinibus terendos, sed & spe, sine dubio, varia via (qua de re hic non est differendi locus) cadendum fuisse. Exinde gratulor mihi, me ad sententiam illam necdum transiisse, meaque me Methodo omnia perfecisse quicquid præstitum Dei Beneficio fuerit. Quando vero Observationes habebimus 20 & 30 Annorum spatium continuatas utrinque, nimirum tum quæ Dioptris Telescopicis, tum quæ solummodo nostris de Cœlo depromptæ fuerint, res omnino clarior futura est. Interea suo quilibet Ingenio fruatur, remque sua ratione pro Libitu, tentet.

VII. 1. It is well known that the mean apparent Magnitude of the Moon is 30 min. 30 sec. we will take it *Numero Rotundo* to be 30 min. at a Full Moon in the midst of Winter, and when she's in the Meridian, and at her greatest Northern Latitude, and consequently the utmost that she can be elevated in our Horizon: 'Tis as well known also, that when she is in this Posture, being looked upon by the naked Eye, she appears (that we may accommodate all to sensible Measures) to be *Magnitudinis Pedalis*, about a Foot broad. But the same Moon being looked upon just as she rises, she appears to be three or four Foot broad, and yet if with an Instrument we take her Diameter, both in one Posture and the other, we shall find that still she shall be but 30 min. That this Matter of Fact is true, besides the Authority of many Authors, I can assert that I have accurately tried it my self, and I have so found it. One of the ways I proceeded was thus: I took a very good Telescope of about 6 Foot long, in the inward Focus of whose Eye-Glass I apply'd a very fine Lattice made of the single Hairs of a Man's Head; then looking

Why Celestial Objects appear greater when nigh the Horizon than when higher elevated; Examined by Mr. Will. Mo-lineux. N. 187. p. 314. Apr. An. 1687.

looking with this at the Moon, when she was just risen, and looked extraordinary big, I observed what Number of the Squares of the Lattice were occupied by her Body ; and then observing her again, when more elevated and free from all extravagant Greatness, I still found the same Square of the Lattice possessed by her. This Way is equivalent to that now more used, of taking her Diameter by Mr. *Townley's* Micrometers ; but I have also tried and found the same thing by an accurate Sextant, taking the Distance of the Moon's opposite Limbs.

The celebrated *Des Cartes* attributes this Appearance rather to a deceived Judgment, than to any natural Affection of the Organ or Medium of Sense : For the Moon (says he) being nigh the Horizon, we have a better Opportunity and Advantage of making an Estimate of her, by comparing her with the various Objects that incur the Sight, in its Way towards her ; so that tho' we imagine she looks bigger, yet 'tis a meer Deceit ; for we only think so, because she seems nigher the Tops of Trees, or Chimneys, or Houses, or a Space of Ground, to which we can compare her, and estimate her thereby ; but when we bring her to the Test of an Instrument, that cannot be deluded or imposed upon by these Appearances, then we find our Estimate wrong, and our Senses deceived. These Thoughts, methinks, are much below the accustomed Accuracy of the noble *Des Cartes* ; for certainly if it be so, I may at any time increase the apparent Bigness of the Moon, tho' in the Meridian ; for it would be only by getting behind a Cluster of Chimneys, a Ridge of a Hill, or the Tops of Houses, and comparing her to them in that Posture, as well as in the Horizon ; besides, if the Moon be looked at just as she is rising from an Horizon determined by a smooth Sea, and which has no more Variety of Objects to compare her to, than the pure Air, yet she will seem big, as if look'd at over the rugged Top of an uneven Town, or rocky Country. Moreover, all Variety of adjoining Objects may be taken off, by looking through an empty Tube, and yet the deluded Imagination is not at all helped thereby.

Fig. 119.

The famous *Thomas Hobbs* gives this Solution. Let the Point G be the Center of the Earth, and F the Eye on the Surface of the Earth ; on the same Center G let there be struck the two Arches E H, determining the Atmosphere, and A D to represent that blue Surface in which we imagine the Fixed Stars, and let F D be the Horizon : Divide the Arch A D into three equal Parts by the Lines B F, C F ; it is manifest that the Angle A F B is greater than the Angle B F C, and this again is greater than the Angle C F D. Wherefore (says he) to make the Angle C F D equal to the Angle C F B, the Arch C D must be greater than the Arch C B ; and consequently, that the Moon may in the Horizon appear under the same Angle, as when elevated, she must cover a greater Arch, and therefore seem greater ; that is, the Moon in the Meridian appearing under the Angle B F C, that she may appear under an equal Angle in the Horizon, as suppose C F D, 'tis necessary that the Arch C D should be greater than C B ; and consequently, tho' she appear to subtend a greater Arch when in the Horizon than when elevated, yet she appears under the same Angle ; and all this without Refraction.

The

The Geometry of this Figure is most certainly true and demonstrable. At this I quarrel not; but it makes no more in our present Difficulty than if nothing had been said: For he has made the Circle GF , representing the Earth, very large in Proportion to the Circle AD ; and then indeed taking the Point F in the Earth's Surface, and by Lines from thence dividing the Angle AFD into whatever equal Parts, the intercepted Arches AB , BC , CD , shall be unequal. But if he had considered, that the Earth is, as it were, a Point in respect of the Sphere of the fixed Stars, nay the very annual Orbit of the Earth is almost imperceptible, he would have found that the Lines FB , FC , FD , must be all conceived as drawn from the Point G , and then equal Angles will intercept equal Arches, and equal Arches equal Angles: And so it happens (at least beyond the Possibility of the Discovery of Sense) to the Eye on the Surface of the Earth; so that his drawing his Lines so far from G as F is, and to another concentrick Circle so nigh as AD , deceived him in this Point.

The famous *Gassendus* has written four large *Epistles* on this Subject, the Substance of all which is, That the Moon being nigh the Horizon, and looked at through a more foggy Air, casts a weaker Light, and consequently forces not the Eye so much as when brighter; and therefore the Pupil does more enlarge itself, thereby transmitting a larger Projection on the Retina. In this Opinion I find he is not alone; for this Disquisition being lately revived by a *French Abbe*, he therein follows the Sentiment of *Gassendus*, with this Addition, That this contracting and enlarging of the Pupil causeth a different Shape in the Eye; an open Pupil making the Crystalline flatter, and the Eye longer, and the narrower Pupil shortening the Eye, and making the Crystalline more convex: The first attends our looking at Objects which are remote, or which we think so; the latter accompanies the viewing Objects nigh at hand. Likewise an open Pupil and flat Crystalline attends Objects of a more sedate Light, whilst Objects of more forcible Rays require a greater Convexity and narrower Pupil. From these Positions, the *Abbe* endeavoured to give an Account of our *Phænomenon*, as follows: When the Moon is nigh the Horizon, by Comparison with interposed Objects, we are apt to imagine her much farther from us than when more elevated; and therefore (says he) we order our Eyes as for viewing an Object farther from us; that is, we something enlarge the Pupil, and thereby make the Crystalline more flat: moreover, the Duskiness of the Moon in that Posture does not so much strain the Sight; and consequently the Pupil will be more large, and the Crystalline more flat. Hence a larger Image shall be projected on the Fund of the Eye, and therefore the Moon shall appear larger. And this Disposition of the Eye that magnifies her, magnifies also the Divisions of our foremention'd Lattice, and consequently she by her Body shall possess no more of the Divisions than when she seems less. These two foremention'd Accidents, *viz.* The Moon's imaginary Distance and Duskiness, gradually vanishing as she rises, a different Species is hereby introduced in the Eye, and consequently she seems gradually less and less, till again she approaches nigh the Horizon. These two Opinions of *Gassendus* and the *Abbe* being so near a-kin, I shall consider them both.

Fig. 111.

both together; and first I assert, That a wider or narrower Aperture of the Pupil increases not, neither diminishes the Projection on the Retina. I know, *Honoratus Faber* in his *Synopsis Optica* endeavours to prove the clear contrary to this my Assertion, and that after this manner. A B is an Object, E F the greater Aperture of the Pupil, admitting the Projection K I on the Retina, whereas the lesser Aperture C D, admits only the Projection G H; but G H is less than K I, wherefore a lesser Aperture diminishes the Projection. I admire that any Man that undertook (as *Honoratus Faber*) to write of *Opticks* more accurately than all that went before him, should be guilty of so very gross an Error; and I do more admire, that the celebrated *Gassendus*, and with him the noble *Hevelius*, should be of the same Opinion: For tho' the aforesaid Demonstration hold most certainly true in direct Projections, as in a dark Room with a plain Hole; yet it will not hold in Projections made by Refraction, as it is in those on the Retina in the Eye, by means of the Crystalline, and other Coats and Humours of the Eye. For let A B be a remote Object, and E F the Crystalline at its large Aperture, projecting the Image I M on the Retina. Let then C D be the lesser Aperture of the Pupil before the Crystalline: I say, the Image I M shall be projected as large as before; for the Cone of Rays E A F consists partly of the Cone of Rays C A D, therefore where the former E A E is projected, the latter C A D, as being a part of the former, shall be projected also. So that no more is effected by this narrow Aperture, but that the Sides of the radiating Cones are intercepted, and consequently the Point I shall be affected with less Light, but it shall still be in the same Place: What is said of that Cone and that Point, may be said of all other Cones and other Points of the Object. From hence appears, *First*, The Invalidity of the Account given of the Moon's Appearance by *Gassendus* from this Reason. *Secondly*, The Reason appears why a Telescope's lesser or greater Aperture, makes no Difference in the Angle it receives: For imagine E F to be an Object-Glass of a Telescope, and 'tis plain. *Thirdly*, 'Tis evident why a greater or less Aperture on a Telescope should make the Objects appear lighter or darker, for thereby more or less Rays are admitted to determine on the Projection of each Point. But all this by the By: And this is sufficient for a Confutation of *Gassendus* and *Faber*. But our forementioned *Abbe* superadds to a greater or lesser Aperture of the Pupil, as a necessary Consequent, a greater and lesser Convexity of the Crystalline, as also a Lengthening and Shortening the Tube of the Eye. And this I must confess would do something, if we find it true in our Case; and this let us try. *First*, (says he) The Duskyhness of the Moon nigh the Horizon admits the Pupil to enlarge itself, the Crystalline to flatten, and the Eye to lengthen: But what if we change our Object, and instead of the Moon take the Distance between some of the Fix'd Stars (as suppose those of *Orion's Girdle*) we shall find the same *Phenomenon* in them, and yet I hope neither he nor *Gassendus* will assert, that they at one time strain the Eye more than at another, or that at any time their Fulgur strains the Eye at all; if he do, let him take Stars of the lesser Magnitudes, nay even those that can but just be perceiv'd, and then he will be convinc'd: Or let him consider whether this

will

will hold in looking at the Sun through very dark Glasses, which render the Sight thereof as inoffensive to the Eye, as that of a Green Field; but perhaps he will then say, that this other Reason holds, which is, *Secondly*, That the greater imaginary Distance at which we think the Moon near the Horizon, than when more elevated, makes us contemplate her as if really she was so, viz. with ample Pupils, &c. But this I have sufficiently overthrown in my Remarks against *Des Cartes*: Therefore I pass it over, only subjoyning, that if there were any thing in this Surmise, methinks the horizontal Moon should be fancied nigher to us than farther from us; for if we are for trying natural Thoughts, let us take Children to determine the Matter, who are apt to think, that could they go to the Edge of that Space that bounds their Sight, they should be able (as they call it) to touch the Sky; and consequently the Moon seems then rather nigher to us than farther from us.

After I had writ thus far, I accidentally cast my Eye upon *Riccioli's* Treatise of *Refraction*, at the End of his Second Volume of *Almagest*, Lib. 10. Sect. 6. Cap. 1. *Quest.* 13. wherein he speaks of our present Difficulty; but to my Wonder I find him assert, That he and Father *Grimaldi* had often taken the horizontal Sun and Moon's Diameters by a Sextant, when to the naked Eye they appeared very large (*Grimaldus* directing his Sight to the left Edge, and *Ricciolus* to the right) and that even by the Instrument they always found the Diameters greater than when more elevated, the Sun often subtending an Angle of almost a Degree, and frequently 45 Minutes, the Moon also 38 or 40 Minutes. This is downright contrary to the Matter of Fact which I have before alledged, and directly repugnant to the Matter of Fact asserted by the forementioned French *Abbe*: Whether of us be in the right, I leave to accurate Experiment to determine, and submit the Whole to the Decision of the illustrious *Royal Society*. Only give me leave to add one Word against *Riccioli*, for had his Experiments been accurately prosecuted, he should have tried them when the horizontal Moon had look'd ten times more large in Diameter than ordinary; and then if it be true, that even by an Instrument she will be found proportionally broader than really she should, subtend an Angle of 300 Min. or 5 Deg. for very often I have seen the Moon when she appeared ten times broader than ordinary, which the small Addition of 8 or 10 Min. to her usual Diameter will never cause.

2. I discoursed of this Appearance near 40 Years ago with Mr. *Foster*, then Professor of *Astronomy* in *Gresham-College*, who did then assure me (from his own Observation I suppose) that the apparent Magnitude taken by Instruments (however the Fancy may apprehend it) is not greater at the Horizon than when higher. Mr. *Caswell* affirms the same thing; and I do not doubt but the thing is so: For though Refraction near the Horizon alters the Altitude of the thing seen; yet it cannot alter the Azimuth at all. For since this equally respects all Points of the Horizon; let the Refraction be what it will, the whole Horizon can be but a Circle: So that there is no room for the Breadth of a thing (as to the Angle at the Eye) to be made greater, whatever its Tallness may (the Refraction not equally affecting all Parts in the Circles of Altitude). Nor is there any Reason, why this should

*This Phenomenon
consider'd by
Dr. Wallis.
Ibid. p. 323.*

rather thrust the other, than that the other thrust this, out of Place. Whereas, in the Altitude, it is otherwise: For while what is near the Horizon is enlarged, that which is further off is thereby contracted; which as to the Azimuth, or Horizontal Position, cannot be.

Supposing then that the Sun's apparent Horizontal Diameter, taken by Instrument, is the same near the Horizon, as in a higher Position, I take its imaginary Greatness, which is fancied near the Horizon, to be only a Deception of the Eye, or rather the Imagination from the Eye.

For sure it is, that the Imagination doth not estimate the Greatness of the Object seen, only by the Angle which it makes at the Eye; but, by this compared with the supposed Distance. True it is, that *Ceteris paribus*, we judge that to be the greater Object, which makes at the Eye the greater Angle; but not so, if apprehended at different Distances.

For if through a Casement (or lesser Aperture) we see a House at 100 Yards Distance; this House (though seen under a less Angle) doth not to us seem less than the Casement through which we see it, (or this greater than that, because it makes at the Eye the greater Angle :) But the Imagination makes a comparative Estimate from the Angle and Distance jointly consider'd.

So that of two things seen under the same or equal Angles, if to one of them there be ought which gives the Apprehension of a greater Distance, that to the Imagination will appear greater. Now, sure it is, that one great Advantage for estimating the Distance of a thing seen, is from the Variety of intermediate Objects between the Eye and the thing seen. For then the Imagination must allow room for all these things.

Now when the Sun or Moon is near the Horizon, there is a Prospect of Hills, and Valleys, and Plains, and Woods, and Rivers, and Variety of Fields and Inclosures, between it and us; which present to our Imagination a great Distance capable of receiving all these: Or if it so chance that (in some Position) these Intermediates are not actually seen; yet having been accustomed to see them, the Memory suggests to us a View as large as is the visible Horizon.

But when the Sun or Moon is in a higher Position; we see nothing between us and them, (unless perhaps some Clouds) and therefore nothing to present to our Imagination so great a Distance as the other is.

And therefore, though both be seen under the same Angle, they do not appear (to the Imagination) of the same Bigness, because not both fancied at the same Distances: But that near the Horizon is judged bigger (because supposed farther off) than the same when at a greater Altitude.

'Tis true, that as to small and middling Distances (besides this Estimate from Intermediates) the Eye hath a Means within itself to make some Estimate of the Distance. As, when we already know the Bigness of a thing seen, to which we have been accustomed; as, a Man, a Tree, a House, or the like: If such thing appear to us under a small Angle, and indistinct, and faintly coloured, the Imagination doth allow such Distance as to make such a thing so to appear. And if this, through a Prospective-Glass, be repre-

represented to us under a bigger Angle, and more distinct, it is accordingly apprehended as so much nearer. But the Case is otherwise, when we do not, by the known Bigness, judge the Distance; but, by the supposed Distance, judge of the Bigness, as in the Case before us. And accordingly, different Persons, according to different fancied Distances, judge very differently.

Again; In our two Eyes (when the Object is seen by both) there is yet another Means of estimating how far off it is. (And it is this by which we judge of Distances.) Namely, there are, from the same Object, two different visual Cones, terminated at the two Eyes; whose two Axes contain, at the Object, different Angles, according to different Distances; an acuter Angle at a great Distance, and more obtuse when nearer.

Now, that such Object may be seen by both Eyes clearly; it is requisite that the Eyes be put in such a Position, as that the Sight of each Eye receive the respective Axis at Right Angles; which requires a different Position of the two Eyes, according to the different Distance of the Object; as will manifestly appear, if we look, with Attention, on a Finger (or other small Object) at 2 or 3 Inches Distance from the Eye; and then upon another like Object at 3 or 4 Yards beyond it, (and this alternately several times). For 'twill be manifest, that while we look intently on the one, we do not see the other, (or but confusedly) though both be just before us. And, as we change our View, from the one to the other, we manifestly feel a Motion of the Eyes (by their Muscles) from one Posture to another.

And according to the different Posture in the Eyes, requisite to a clear Vision by both, we estimate the Distance of the Object from us.

And hence it is, that they who have lost the Sight of one Eye, are at a great Disadvantage, as to estimating Distances, from what they could do while they had the Use of both.

But now when the Distance grows so great, as that the Position of these visual Axes become parallel, or so near to parallel as not to be distinguishable from it: This Advantage is lost, and we can thenceforth only conclude, that it is far off; but not how far. Hence it is, that our View can make no Distinction of the Moon's Distance from that of the other Planets, or even of the fixed Stars: But they seem to us as equally remote from us; though we otherwise know their Distances from us to be vastly different. Because the Parallax (as I may so call it) from the different Position of the two Eyes, is quite lost, and undiscernable in Distances much less than the least of these.

So that, though as to small Distances, we may make some Estimate from the known Magnitude of the Object; and as to middling Distances, from the Parallax (as I may call it) arising from the Interval of the two Eyes: Yet even this latter will hardly reach beyond, if so far as the visible Horizon; and all beyond it is lost. And therefore there being nothing left to assist the Fancy in estimating so great a Distance, but only the intermediate Objects: Where these Intermediates appear to the Eye, (as when the Sun or Moon are near the Horizon) the Distance is fancied greater, than where they

appear not (as when farther from it), and consequently (tho' both under the same or equal Angles) that near the Horizon is fancied the greater: And this I judge to be the true Reason of that Appearance.

*An Experiment
of the Refraction
of the Air; by
Mr. Lowthorp.
N. 257. p. 329.
Mar. An. 1699.*

Fig. 113.

VIII. We took a Cylinder of Cast-Brass, A B C D, and cut one End of it C D, perpendicular to the Axis $a c x$, the other End A B, enclined to it at an Angle of about 27 deg. 30 min. and therefore the Perpendicular to this enclining Plain $p c$, and the Axis of the Cylinder $a c x$, comprehended an Angle $p c a$, of about 62 deg. 30 min. These Ends were ground very true upon a Glass-Grinder's Brass Tool, and each of them was compass'd about with a narrow Feril of thin Brass $b b b b$. Into the upper Side of the Cylinder, at E, was soldered the Brass-Pipe E F, and into the under Side, at G, the other Brass-Pipe G H; the former of these Pipes being about 3 Inches long, and the latter 6 Inches. Upon the Plate $d d d$, were fix'd two other Plates, L L, perpendicular to it and parallel to each other. Each of these two Plates had an Arch of a Circle (whose Diameter was equal to that of the Cylinder) cut out of its upper Edge; so that when the Pipe G H was let thro' a Hole near the Middle of the Plate $d d d$, the Cylinder fell into the Arches; and being fastned there with Solder, the Axis $a c x$ laid parallel to the Plate $d d d$, and about an Inch and a half above it. The perpendicular End of the Cylinder, D C, was closed with an Object-Glass of a 7 Foot Telescope $o o$, and the enclining End A B, with a well-polish'd flat Glass, $f f$; which was carefully chosen to transmit the Object distinct enough, notwithstanding its Obliquity to the visual Rays. The Ferils were filled with Cement round about the Edges of the Glasses, which laid flat, and every where touch'd the smooth Ends of the Cylinder, that they might firmly support the Weight and Pressure of the excluded Air.

Fig. 114.

Instead of a Cistern (as in the *Torricellian* Experiment) we made use of the inverted Syphon of Brass M N O, solder'd to the Plate $g g g$. One of the Sides M N, stood perpendicular to the Plate $g g g$, and the other Side N O, enclined to it, and was supported near the upper End O, with a little Piece of Brass $k k$.

Fig. 115.

We then placed the Cylinder upon a Table, which was well fastned to a firm Floor: The Pipe G H, was let through a Hole in the Top of the Table; and the Plate $d d d$, was nailed down to it: The Tube of the Telescope $s s s$, with the Eye-Glass in it, was applied to the Object-Glass, and a Hair fix'd at x , the common Focus of both Glasses, in the Axis of the Cylinder continued to it. Upon the Floor (under the Cylinder) we nailed the Plate $g g g$, with the inverted Syphon upon it, and joyned M to H, by the Insertion of the Glass-Tube T. The Joints were very carefully closed with Cement, and then covered over with Pieces of a Bladder wrap'd hard with strong Thread. There was also a Bladder tied below each Joint at m , and when it was filled with Water it was tied about it at n ; so that no Air could come to the Cement, to insinuate itself through its Pores or Fissures, if any happen'd to be left unclosed.

It

It will not (I hope) be thought more than necessary, that in this Account of the *Apparatus*, I have mention'd so many minute Circumstances; for we found it difficult enough to exclude the Air, and almost impossible to discover the very little Holes through which so subtile a Fluid would freely enter, and possess the Spaces, deserted by the *Subsiding Mercury*. But with all this Precaution, the Experiment succeeded at last, as I wish'd; after this manner:

We placed the Object *a* (which was a black Thread fasten'd in a little Frame over a Piece of white Paper) in the Axis of the Cylinder *x c a*: We fill'd the Pipes and Cylinder with *Mercury*; and having stopp'd the upper End of the Pipe at F, with the little Iron-Stopple K, and closed it, as the upper Part of the Tube and other Joints, we let the *Mercury* run out gently at O (into the Bladder *u*) till it remain'd suspended at the usual Height (as in the *Barometer*) leaving the upper Part of the Tube and the Cavity of the Cylinder between the Glasses *o o*, and *f f*, void of Air. We then saw the Object, which before appear'd in the Axis at *x*, rais'd considerably above it; and we reduced it to appear again at *x*, by removing it from *a* to *a'*. The Axis therefore of the visual Ray (which was also the Axis of the Cylinder) *x c a*, falling perpendicularly on the void Space, pass'd through it without any Refraction: But emerging obliquely into the Air, it was refracted towards the Perpendicular *p c*, and received a new Direction to *a*. And therefore the Distance *a a'*, subtended the Angle of Refraction *a c a'*; all which we measur'd, and found as follows; *viz.*

| | Inches | Dec. Parts |
|---|--------|------------|
| The Height of the Object above the Axis, or the unrefracted visual Ray <i>a a'</i> - - - - - | 000 | 425 |
| The Distance of the Object from the refracting Plain <i>a c</i> , about 51 Feet, or - - - - - | 612 | 000 |

| | Deg. | Min. | Sec. |
|---|------|------|------|
| Therefore the Angle of Refraction <i>a c a'</i> , was - - - | 00 | 02 | 23 |
| The Angle of Emerision <i>p c a</i> , (by the Construction of the Cylinder) was - - - - - | 62 | 30 | 00 |
| Therefore the Angle of Incidence <i>p c a = p c a' + a c a'</i> , was - - - - - | 62 | 27 | 37 |

And therefore *universally*, (according to the known Laws of Refraction)

| | |
|--|--------|
| The Sines of the Angles of Incidence being - - - | 100000 |
| The Sines of the Angles of Emerision are - - - | 100036 |
| And the refractive Power of the dense Air - - - | 36 |

By the *refractive Power* of a pellucid Body, I mean that Property in it whereby the oblique Rays of Light are diverted from their direct Course, and which is measured by the proportional Differences (always observed) between the Sines of the Angles of Incidence and Emerision.

This

This Property is not always proportional to the Density (at least not to the Gravity) of the refracting Medium: For the refractive Power of Glass to that of Water is as 55 to 34, whereas its Gravity is as 87 to 34; that is, the *Squares* of their refractive Powers are (very near) as their respective Gravities. And there are some Fluids, which, tho' lighter than Water, yet have a greater Power of Refraction: Thus the refractive Power of Spirit of Wine (according to Dr. Hook's Experiment, *Microgr. Obs.* lviii. p. 220.) is to that of Water, as 36 to 23; and its Gravity *reciprocally* as 33 to 36, or $36\frac{1}{3}$. But the refractive Powers of Air and Water seem to observe the *simple* Proportion of their Gravities *directly*, as I have compared them in the following Table. The Numbers there expressing the Refraction of Water are taken from the Mean of Nine * Experiments, made at so many several Angles of Incidence, *Jan.* 25. 1642. by Mr. Gascoigne, (the ingenious first Inventor of the Micrometer, and the Ways of measuring Angles by Telescopes) and those of Air are produced by the preceding Experiment.

| | Water. | Air. |
|--|--------|--------|
| The (assum'd) Sines of the Angles of Incidence }
on the Void from - - - - - } | 100000 | 100000 |
| The Sines of the correspondent Angles of Emer- }
sion out of - - - - - } | 134400 | 100036 |
| The refractive Power of - - - - - | 34400 | 36 |
| The specifick Gravity (if as 900 to 1 at the }
Time of the Experiment) of - - - - - } | 34400 | 38 |
| Or (if as 850 to 1) of - - - - - | 34400 | 40 |

From hence it seems very probable, that their respective Densities and refractive Powers are in a just *simple* Proportion. And if this should be confirmed by succeeding Experiments, made at different Angles of Incidence, and with Cylinders continuing exhausted through several Changes of the Air, it would be more than probable that the refractive Powers of the Atmosphere are every where, and at all Heights above the Earth, proportional to its Densities and Expansions: And then it would be no difficult Matter to trace the Light through it, so as to terminate the Shadow of the Earth, and (together with proper Expedients for measuring the Quantity of Light illuminating an opaque Body) to examine at what Distances the Moon must be from the Earth to suffer Eclipses of the observed Duration.

* I am indebted for these Experiments to the Reverend and very Accurate Astronomer Mr. Flamsteed; who copied them, together with many other Observations and several Passages relating to them, from Mr. Gascoigne's Letter to Mr. Crabtree: They were happily preserved, in the Time of our Civil War, by the late Sir Jonas Moor, and Mr. Chr. Townley, and they are now in the Hands of Mr. Rich. Townley of Townley in Lancashire, by whom they were imparted some Time ago to Mr. Flamsteed.

IX. Give me leave to suggest a Speculation, which hath been in my Thoughts these Forty Years or more; but I have not had the Opportunity of reducing it to Practice: It is concerning the Parallax of the fixed Stars, as to the Earth's annual Orb.

To find the Parallax of the Fixed Stars; by Dr. Wallis, to Mr. William Molyneux. July, An. 1693. N. 202. p. 844.

Galileo complains of it a great While since (in his *Systema Cosmicum*) as a thing not attempted to be observed with such Diligence as he could wish, and I doubt we have the same Cause of complaining still. I know that Dr. Hook and Mr. Flamsteed have attempted somewhat that Way, but have desisted before they came to any thing of Certainty. What hath been done to that purpose Abroad I know not.

Galileo hath suggested divers Things considerable in order to it; as, the Times of Observation, the Stars to be observed, and the Manner of observing them, which yet I doubt is not practicable. That which occurred to my Thoughts upon these Considerations, was to this purpose: That some circumpolar Stars (nearer to the Pole of the Equator than is our Zenith, and not far from the Pole of the Zodiack) should be made choice of for this purpose. And in case the meridional Altitude be discernably different at different Times, so will also be their utmost East and West Azimuth, which may be better observed than their Rising or Setting: And this will not be obnoxious to the Refraction, as is the meridional Altitude (for though the Refraction do affect the Altitude, yet not the Azimuth at all); and we may here have choice of Stars for the purpose; which, in Observations from the Bottom of a Well, we cannot have; being there confined to those only which pass very near our Zenith, tho' very small Stars.

I would then take for granted, as a thing at least very probable, that the fixed Stars are not all (as was wont to be supposed) at the same Distance from us, but the Distance of some vastly greater than of others; and consequently, though as to the more remote, the Parallax may be undiscernable, it may, perhaps, be discernable in those that are nearer to us.

And those we may reasonably guess (though we are not sure of it) to be nearest to us, which to us do appear biggest and brightest, as are those of the *First* and *Second* Magnitude; and there are at least of the *Second* Magnitude pretty many not far from the Pole of the Ecliptick, (as that in particular in the *Shoulder* of the *Lesser Bear*:) And in case we fail in one, we may try again and again on some other; which may chance to be nearer to us than what we try first. And Stars of this Bigness may be discerned by a moderate Telescope even in the Day-time; especially when we know just where to look for them.

The Manner of Observation, I conceive, may be thus. Having first pitched upon the Star we mean to observe, and having then considered (which is not hard to do) where such Star is to be seen in its greatest East

or

or West Azimuth ; it may be then convenient to fix (very firm and steadily on some Tower, Steeple, or other high Edifice, in a convenient Situation) a good Telescopic Object-Glass, in such Position as may be proper for viewing that Star. And at a due Distance from it near the Ground, build on purpose (if already there be not any) some little Stone-Wall, or like Place, on which to fix the Eye-Glass, so as to answer that Object-Glass : And having so adjusted it, as through both to see that Star in its desired Station (which may best be done while the Star is to be seen by Night in such Situation, near the Time of one of the Solstices) let it be there fixed so firmly, as not to be disturbed (and the Place so secured, as that none come to disorder it) and care be taken so to defend both the Glasses, as not to endamaged by Wind and Weather. In which Contrivance, I am beholden to Mr. *John Caswell*, M. A. of *Hart-Hall* in *Oxford*, for his Advice and Assistance, with whom I have many Years since communicated the whole Matter.

This Glass being once fixed (and a Micrometer fitted to it, so as to have its Threads perpendicular to the Horizon, to avoid any Inconvenience which might arise from Diversity of Refraction, if any be) the Star may then be viewed from Time to Time (for the following Year or longer) to see if any Change of Azimuth can be observed.

This I thought fit to recommend to your Consideration, who do so well understand Telescopes, and the Managery of them : But when I suggest (as a convenient Star for this purpose) the *Shoulder* of the *Lesser Bear* (as being the nearest to the Pole of the Zodiack of any Star that is of the First or Second Magnitude) I do not confine you to that Star ; but (without retracting that) suggest another ; namely, the *Middle Star*, in the *Tail* of the *Great Bear*, which (though somewhat further from the Pole of the Zodiack) is a brighter Star than the other, and may be nearer to us.

But I do it principally upon this Consideration ; namely, That there is adhering to it a very small Star, (which the *Arabs* call *Alcor*, of which they have a proverbial Saying, when they would describe a sharp-sighted Man, That he can discern the *Rider* on the *Middle Horse* of the *Wayn* : And of one who pretends to see small Things, but overlooks much greater, *Vidit Alcor at non Lunam Plenam* :) Which *Hevelius* in his Observations finds to be distant from it about nine Minutes, and five or ten Seconds : So that besides the Advantage of discovering the Parallax of the greater Star, if discernable ; the Difference of the Parallax of that and of the lesser Star (being both within the Reach of a Micrometer) may do our Work as well. For if that of the greater Star be discernable, but that of the lesser be either not discernable or less discernable, their different Distances from each other at different Times of the Year may perhaps (without farther Apparatus) be discerned by a good Telescope of a competent Length, furnished with a Micrometer, if carefully preserved from

from being disordered in the Intervals of the Observations ; and discover at once, both that there is a Parallax, and that the Fixed Stars are at different Distances from us ; wherein, that I may not be mistaken, my Meaning is not that the Instrument or Micrometer should be removed for the observing of the Lesser Star, but that (when the Azimuth of the greater Star is taken) by a Micrometer (consisting of divers fine Threads parallel and transverse) may (at the same time) be observed the Distance of the two Stars, each from other, in that Position (both being at once within the reach of the Micrometer;) which Distance (the Instrument remaining unmoved) if it be found (at different Times of the Year) not to be the same, this will prove that there is a different Parallax of these two Stars.

This latter Part of the Observation (of their different Distances at different Times) I suggest, as more easily practicable, though not so nice as the former. For it may be done, I think, without any further Apparatus there than a good Telescope of ordinary Form, furnished with a Micrometer, (this being carefully kept unvaried during the Interval of these Observations.) And if this Part only of the Observation (without the other) be pursued ; it matters not though the two Observations (near the two Solstices) be, one at the Eastern, the other at the Western, Azimuth (whereby both may be taken in the Night-time) for the Distance must (at both Azimuths) be the same. If, after observing the Azimuth of the greater Star, it be necessary to move the Micrometer for measuring its Distance from *Alcor*, that may be done another Night, (and it is not necessary to be done at one Observation) for that Distance cannot be discernably varied in a Night or two.

X. Since the *Pythagorean System* of the World has been revived by *Copernicus*, (and now by all Mathematicians accepted for the true one) there seemed Ground to imagine, that the Diameter of the Earth's annual Course (which, according to our best Astronomers, is at least 40000 times bigger than the Semidiameter of the Earth) might give a sensible Parallax to the Fixed Stars, and thereby determine their Distance. But there are some Considerations which make us suspect that even this Basis is not large enough for that purpose.

Concerning the Distance of the Fixed Stars; by Mr. Fr. Roberts. N. 209. p. 101.

M. *Hugens* (who is very exact in his Astronomical Observations) tells us, He could never discover any visible Magnitude in the Fixed Stars, though he used Glasses which magnified the apparent Diameter above 100 times.

Now, since in all likelihood the Fixed Stars are Suns, (perhaps of a different Magnitude) we may as a reasonable Medium presume they are generally about the Bigness of our Sun.

Let us then (for Example) suppose the *Dog-Star* to be so. The Distance from us to the Sun being about 100 times the Sun's Diameter, it is evident, that the Angle under which the *Dog-Star* is seen in Mr. *Hugen's* Telescope, must be near the same with the Angle of its Parallax to the Sun's Distance, or Semidiameter of the Earth's annual Course; so that the Parallax to the whole Diameter, can be but double such a Quantity, as even to Mr. *Hugen's* nice Observation is altogether insensible.

The Distance therefore of the Fixed Stars seems hardly within the reach of any of our Methods to determine: But from what has been laid down, we may draw some Conclusions that will much illustrate the prodigious Vastness of it.

1. That the Diameter of the Earth's annual Orb (which contains at least 160 Millions of Miles) is but as a Point in Comparison of it; at least it must be above 6000 times the Distance of the Sun: For if a Star should appear through the aforesaid Telescope half a Minute broad, (which is a pretty sensible Magnitude) the true apparent Diameter would not exceed $18''$, which is less than the 6000th Part of the apparent Diameter of the Sun, and consequently the Sun's Distance not the 6000th Part of the Distance of the Star.

2. That could we advance towards the Stars 99 Parts of the whole Distance, and have only 100th Part remaining, the Stars would appear little bigger to us than they do here: For they would shew no otherwise than they do through a Telescope, which magnifies an hundred-fold.

3. That at least 9 Parts in 10, of the Space between us and the Fixed Stars can receive no greater Light from the Sun, or any of the Stars, than what we have from the Stars in a clear Night.

4. That Light takes up more Time travelling from the Stars to us, than we in making a *West-India* Voyage, (which is ordinarily performed in six Weeks:) That a Sound would not arrive to us from thence in 50000 Years, nor a Cannon-Bullet in a much longer Time. This is easily computed, by allowing (according to Mr. *Newton*) 10 Minutes for the Journey of Light from the Sun hither, and that a Sound moves above 1300 Foot in a Second.

The Places of
the chiefest Fixed
Stars, according
to the best Anti-
ent Observers;
by Dr. Edward
Bernard, to
D. Robert
Huntington.
N. 158. p. 567.

XI. Inter Codices tuos Arabicos in *Musæo Mertonensi* (numeras autem plus quadraginta Doctrinæ & Observationis Sideralis refertis) in *Tabulis Ilchanicis* studio *Persæ* præclari *Choagæ Nasirodini Tusii* factum offendi, ut præcipui Honoris Stellas Fixas, secundum variorum Astronomorum varia Observata, qua Latæ (ut loquuntur) quæve illæ Longæ in Cælo deprehendebantur, brevis Pagina representaret. Hoc ergo Canonion, partim ex tuo Peculio, ut vides, partim ab aliis, adauxi. Non equidem quasi Rem Magnam mihi viderer adeo fecisse; verum ut nostri Homines sensum

ac Opinionem aliquam concipiant Astronomiæ Orientis, ubi Ars ea primum nata constat. Multa sane commendant Astronomiam Orientalium; Felicitas quidem & Claritas Regionum, ubi Observatum; Machinarum Granditas & Accuratio, Quantas plerique nostri credere nolunt Cælo ipso obvertisse; Contemplantium insuper Numerus & Scribentium, decuplo major quam apud Græcos Latinosque celebratur; Adde decuplo Plures Munificentiores, ac Potentiores Principes, qui viris boni Ingenii sumtus & Arma Cœlestia dederunt. Quid vero Astronomi Arabum in *Cl. Ptolæo*, magno Constructore Artis Cœlestis, injuria nulla reprehenderint; quam illi solícite Temporis Minutias, per Aquarum Guttulas, Immanibus Sciotheris, imo (mirabere) *Fili Penduli Vibrationibus*, jampridem distinxerint & mensurarent; quam etiam perite & accurate versaverint in magno molimine Ingenii Humani, de Ambitu Intervalloque binorum Luminarium & nostri Orbis, una Epistola narrare non debet.

| na Epitola narrare non debet. | | | | | | | | | |
|-------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 |
| 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 |
| 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 |
| 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 |
| 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 |
| 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 | 22 |
| 23 | 23 | 23 | 23 | 23 | 23 | 23 | 23 | 23 | 23 |
| 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| 26 | 26 | 26 | 26 | 26 | 26 | 26 | 26 | 26 | 26 |
| 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 28 | 28 | 28 | 28 | 28 | 28 | 28 | 28 | 28 | 28 |
| 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 |
| 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |
| 31 | 31 | 31 | 31 | 31 | 31 | 31 | 31 | 31 | 31 |
| 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 |
| 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 |
| 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 |
| 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 |
| 37 | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 37 |
| 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 |
| 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 |
| 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 |
| 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 | 41 |
| 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 |
| 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 |
| 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 |
| 45 | 45 | 45 | 45 | 45 | 45 | 45 | 45 | 45 | 45 |
| 46 | 46 | 46 | 46 | 46 | 46 | 46 | 46 | 46 | 46 |
| 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 | 47 |
| 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 |
| 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 | 49 |
| 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 |
| 52 | 52 | 52 | 52 | 52 | 52 | 52 | 52 | 52 | 52 |
| 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | 53 |
| 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |
| 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 |
| 56 | 56 | 56 | 56 | 56 | 56 | 56 | 56 | 56 | 56 |
| 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 | 57 |
| 58 | 58 | 58 | 58 | 58 | 58 | 58 | 58 | 58 | 58 |
| 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 |
| 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 61 | 61 | 61 | 61 | 61 | 61 | 61 | 61 | 61 | 61 |
| 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 | 62 |
| 63 | 63 | 63 | 63 | 63 | 63 | 63 | 63 | 63 | 63 |
| 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 |
| 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 |
| 67 | 67 | 67 | 67 | 67 | 67 | 67 | 67 | 67 | 67 |
| 68 | 68 | 68 | 68 | 68 | 68 | 68 | 68 | 68 | 68 |
| 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 |
| 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 |
| 71 | 71 | 71 | 71 | 71 | 71 | 71 | 71 | 71 | 71 |
| 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| 73 | 73 | 73 | 73 | 73 | 73 | 73 | 73 | 73 | 73 |
| 74 | 74 | 74 | 74 | 74 | 74 | 74 | 74 | 74 | 74 |
| 75 | 75 | 75 | 75 | 75 | 75 | 75 | 75 | 75 | 75 |
| 76 | 76 | 76 | 76 | 76 | 76 | 76 | 76 | 76 | 76 |
| 77 | 77 | 77 | 77 | 77 | 77 | 77 | 77 | 77 | 77 |
| 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 |
| 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 | 79 |
| 80 | 80 | 80 | 80 | 80 | 80 | 80 | 80 | 80 | 80 |
| 81 | 81 | 81 | 81 | 81 | 81 | 81 | 81 | 81 | 81 |
| 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 |
| 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 | 84 |
| 85 | 85 | 85 | 85 | 85 | 85 | 85 | 85 | 85 | 85 |
| 86 | 86 | 86 | 86 | 86 | 86 | 86 | 86 | 86 | 86 |
| 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 |
| 88 | 88 | 88 | 88 | 88 | 88 | 88 | 88 | 88 | 88 |
| 89 | 89 | 89 | 89 | 89 | 89 | 89 | 89 | 89 | 89 |
| 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 |
| 91 | 91 | 91 | 91 | 91 | 91 | 91 | 91 | 91 | 91 |
| 92 | 92 | 92 | 92 | 92 | 92 | 92 | 92 | 92 | 92 |
| 93 | 93 | 93 | 93 | 93 | 93 | 93 | 93 | 93 | 93 |
| 94 | 94 | 94 | 94 | 94 | 94 | 94 | 94 | 94 | 94 |
| 95 | 95 | 95 | 95 | 95 | 95 | 95 | 95 | 95 | 95 |
| 96 | 96 | 96 | 96 | 96 | 96 | 96 | 96 | 96 | 96 |
| 97 | 97 | 97 | 97 | 97 | 97 | 97 | 97 | 97 | 97 |
| 98 | 98 | 98 | 98 | 98 | 98 | 98 | 98 | 98 | 98 |
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Canon Præcipuarum è Stellis Fixis secundum Observata Majorum.

| | | | Cl. Ptolemaeo in Magna
Syntaxi.
A. C. 137. | |
|----|----|--|--|----------|
| | | | Long. | Lat. |
| | | | S. ° ' " | ° ' " |
| 1 | 34 | <i>Ultima Fluvii</i> — — — | 1 0 10 30 al. 50 | 53 30 m. |
| 2 | 12 | <i>Lucida Cathedræ Cassiopeiæ</i> — | 3 0 7 50 | 51 40 b. |
| 3 | 12 | <i>Perfei Caput Medusæum. Algol.</i> | 2 0 29 40 al. 20 | 23 0 b. |
| 4 | 14 | <i>Oculus Tauri Austrinus.</i> — | 1 1 12 40 al. 20 | 5 10 m. |
| 5 | 35 | <i>Orionis Pes Sinister. Rigel.</i> — | 1 1 20 50 | 31 30 m. |
| 6 | 3 | <i>Capella Aurigæ.</i> — — — | 1 1 25 0 | 22 30 b. |
| 7 | 2 | <i>Orionis Humerus Dexter.</i> — | 1 2 2 0 | 17 0 m. |
| 8 | 1 | <i>Sirius. Alhabor.</i> — — — | 1 2 17 40 al. 20 | 39 10 m. |
| 9 | 2 | <i>Procyon. Algomeisæ.</i> — — — | 1 2 29 30 al. 10 | 16 10 m. |
| 10 | 8 | <i>Cor Leonis. Regulus</i> — — — | 1 4 2 30 al. 10 | 0 10 b. |
| 11 | 14 | <i>Spica Virginis.</i> — — — | 1 5 26 40 | 2 0 m. |
| 12 | 23 | <i>Arcturus.</i> — — — | 1 5 27 0 | 31 30 b. |
| 13 | 8 | <i>Antares. Cor Scorp.</i> — — — | 2 7 12 40 al. 20 | 4 0 m. |
| 14 | 1 | <i>Caput Ophiuchi.</i> — — — | 3 7 24 50 | 36 0 b. |
| 15 | 1 | <i>Lucida Lyreæ.</i> — — — | 1 8 17 20 | 62 0 b. |
| 16 | 3 | <i>Aquilæ Lucida.</i> — — — | 2 9 3 50 | 29 10 b. |
| 17 | 5 | <i>Lucida in Cauda Cygni.</i> — — | 2 10 9 10 | 60 0 b. |
| 18 | 3 | <i>Lucid. in Crure Dext. Pegasi. Seat.</i> | 2 11 2 10 | 35 0 b. |
| 19 | 2 | <i>Ala Pegasi Ultima. Algenib.</i> — | 2 11 12 10 | 12 30 b. |
| 20 | 7 | <i>Perfei Lucidum Latius.</i> — — | 2 1 4 50 | 30 0 b. |
| 21 | 27 | <i>Cauda Leonis.</i> — — — | 1 4 24 30 | 11 50 b. |
| 22 | 1 | <i>Os Piscis Aust. in Aqua extrema.</i> | 1 10 7 0 | 23 0 m. |
| 23 | 44 | <i>Canopus.</i> — — — | 1 2 17 10 | 75 0 m. |

Canon Præcipuarum è Stellis Fixis secundum Observata Majorum.

| <i>Ali Abolcasimo. A.C. 938.</i> | | <i>Abdoralmano
Sophio.
A.C. 964.*</i> | | <i>Ebnolalano. A.C. 980.</i> | |
|----------------------------------|---------|---|----------|------------------------------|-------|
| Long. | | Latit. | | Long. | |
| S. | ° / | ° / | S. | ° / | ° / |
| 1 | 0 11 50 | 53 30 | 0 12 53 | 0 16 42 | 53 30 |
| 2 | 0 20 20 | 51 45 | 0 20 32 | 0 24 9 | 51 45 |
| 3 | 1 12 17 | 22 45 | 1 12 22 | 1 15 1 | 22 45 |
| 4 | 1 24 35 | 5 15 | 1 25 22 | 1 28 40 | 5 15 |
| 5 | 2 2 1 | 31 4 | 2 2 32 | 2 5 55 | 31 20 |
| 6 | 2 6 49 | 22 50 | 2 7 42 | 2 10 50 | 22 50 |
| 7 | 2 13 16 | 16 45 | 2 14 42 | 2 17 21 | 16 45 |
| 8 | 2 29 30 | 39 20 | 3 0 22 | 3 3 40 | 39 20 |
| 9 | 3 10 40 | 16 0 | 3 11 12 | 3 15 45 | 16 0 |
| 10 | 4 14 40 | 0 15 | 4 15 12 | 4 18 45 | 0 15 |
| 11 | 6 8 23 | 2 6 | 6 9 22 | 6 12 33 | 2 6 |
| 12 | 6 8 50 | 31 12 | 6 9 42 | 6 12 55 | 31 12 |
| 13 | 7 24 35 | 4 24 | 7 25 22 | 7 28 40 | 4 24 |
| 14 | 8 6 53 | 36 0 | 8 7 32 | 8 11 2 | 36 0 |
| 15 | 9 0 40 | 61 45 | 9 0 2 | 9 4 45 | 61 45 |
| 16 | 9 15 53 | 29 12 | 9 16 32 | 9 20 58 | 29 14 |
| 17 | 10 20 4 | 59 36 | 10 21 52 | 10 24 9 | 59 36 |
| 18 | 11 14 0 | 31 10 | 11 14 52 | 11 18 54 | 31 10 |
| 19 | | | 11 24 52 | 11 24 52 | 12 30 |
| 20 | 1 16 15 | 30 8 | 1 17 32 | | |
| 21 | 5 6 40 | 11 50 | 5 7 12 | | |
| 22 | | | 10 18 12 | | |
| 23 | | | 2 29 52 | | |

*. Is cum *Ptolemaeo* convenit κατὰ πλάτθ.

Canon Præcipuarum è Stellis Fixis secundum Observata Majorum.

| <i>Ex Canonibus Hacimicis
Fohanide Egypti.
A. C. 996.</i> | | | | | <i>Choaga Nasirodino Tusio
in Tabulis Ilchanicis.
A. C. 1233.</i> | | | | | <i>Ex Sultanicis Ologbeci.
A. C. 1437.</i> | | | | | |
|---|----|----|--------|----|---|----|----|--------|----|--|----|----|--------|----|----|
| Long. | | | Latit. | | Long. | | | Latit. | | Long. | | | Latit. | | |
| S. | ° | ' | ° | ' | S. | ° | ' | ° | ' | S. | ° | ' | ° | ' | |
| 1 | 0 | 16 | 20 | 53 | 28 | 0 | 24 | 55 | 51 | 45 | 0 | 15 | 40 | 53 | 45 |
| 2 | 0 | 24 | 34 | 51 | 50 | 0 | 24 | 35 | 51 | 40 | 0 | 28 | 1 | 50 | 48 |
| 3 | 1 | 15 | 24 | 22 | 39 | 1 | 16 | 25 | 23 | 0 | 1 | 18 | 54 | 22 | 0 |
| 4 | 1 | 29 | 7 | 5 | 15 | 1 | 29 | 22 | 5 | 13 | 2 | 2 | 31 | 5 | 15 |
| 5 | 2 | 5 | 30 | 31 | 35 | 2 | 6 | 35 | 31 | 30 | 2 | 9 | 25 | 31 | 18 |
| 6 | 2 | 11 | 32 | 22 | 3 | 2 | 11 | 10 | 22 | 40 | 2 | 14 | 43 | 22 | 42 |
| 7 | 2 | 17 | 46 | 16 | 50 | 2 | 18 | 0 | 16 | 50 | 2 | 21 | 13 | 16 | 43 |
| 8 | 3 | 4 | 2 | 39 | 30 | 3 | 3 | 50 | 39 | 10 | 3 | 6 | 19 | 39 | 30 |
| 9 | 3 | 15 | 52 | 16 | 2 | 3 | 15 | 45 | 16 | 5 | 3 | 18 | 22 | 16 | 0 |
| 10 | 4 | 19 | 10 | 0 | 10 | 4 | 19 | 14 | 0 | 17 | 4 | 22 | 13 | 0 | 9 |
| 11 | 6 | 12 | 58 | 2 | 10 | 6 | 13 | 25 | 1 | 52 | 6 | 16 | 10 | 2 | 9 |
| 12 | 6 | 13 | 49 | 31 | 33 | 6 | 13 | 0 | 31 | 25 | 6 | 16 | 31 | 31 | 18 |
| 13 | 7 | 29 | 9 | 4 | 25 | 7 | 29 | 0 | 4 | 10 | 8 | 2 | 16 | 4 | 30 |
| 14 | 8 | 11 | 34 | 35 | 59 | 8 | 11 | 15 | 35 | 55 | 8 | 15 | 13 | 35 | 51 |
| 15 | 9 | 5 | 0 | 61 | 55 | 9 | 4 | 40 | 61 | 50 | 9 | 8 | 19 | 62 | 0 |
| 16 | 9 | 20 | 34 | 29 | 10 | 9 | 20 | 40 | 29 | 15 | 9 | 24 | 10 | 29 | 15 |
| 17 | 10 | 23 | 31 | 59 | 38 | 10 | 24 | 30 | 59 | 50 | 10 | 28 | 46 | 59 | 42 |
| 18 | 11 | 18 | 44 | 31 | 12 | 11 | 18 | 44 | 31 | 12 | 11 | 21 | 37 | 30 | 51 |
| 19 | | | | | | 11 | 18 | 55 | 31 | 1 | 0 | 1 | 22 | 12 | 34 |
| 20 | | | | | | 1 | 21 | 35 | 30 | 0 | 1 | 25 | 7 | 29 | 21 |
| 21 | | | | | | 5 | 11 | 55 | 11 | 50 | 5 | 13 | 49 | 12 | 0 |
| 22 | | | | | | 10 | 23 | 45 | 23 | 0 | 10 | 20 | 40 | 21 | 30 |
| 23 | | | | | | 3 | 3 | 55 | 7 | 0 | | | * | | |

* Non vidit unquam Nepos *Timuri* Clarum *Canopum*; non cæteri hujus Canonis exceptis *Alexandrinis*.

Canon

Canon Præcipuarum è Stellis Fixis secundum Observata Majorum.

| | <i>Abdalgatio Segazio
in Genethliacis.
A. C. 1261.</i> | | | <i>Ex Persicis Chry-
soccæ.
A. C. 1115.</i> | | | <i>Ex Canonib. Persæ.
Pemb. A. C. 1346
Melixæ 338.</i> | | | <i>Ex Cod. Arnoldino
apud Gassendum
pro A. C. 1364.</i> | | |
|----|--|----|----|---|----|----|--|----|----|---|----|----|
| | Long. | | | Long. | | | Long. | | | Long. | | |
| | S. | ° | ' | S. | ° | ' | S. | ° | ' | S. | ° | ' |
| 1 | 0 | 6 | 45 | 0 | 15 | 10 | | | | 0 | 18 | 20 |
| 2 | 0 | 25 | 2 | 0 | 22 | 50 | | | | | | |
| 3 | 1 | 16 | 52 | 1 | 14 | 40 | 18 | 10 | | 1 | 17 | 45 |
| 4 | 1 | 29 | 45 | 1 | 27 | 0 | 2 | 1 | 30 | 2 | 1 | 0 |
| 5 | 2 | 6 | 52 | 2 | 4 | 50 | 8 | 20 | | 2 | 8 | 0 |
| 6 | 2 | 11 | 43 | 2 | 10 | 50 | 13 | 3 | | 2 | 13 | 10 |
| 7 | 2 | 19 | 12 | 2 | 17 | 0 | 20 | 30 | | 2 | 18 | 10 |
| 8 | 3 | 4 | 28 | 3 | 2 | 40 | | | | 3 | 6 | 0 |
| 9 | 3 | 16 | 11 | 3 | 14 | 10 | 18 | 0 | | 3 | 17 | 20 |
| 10 | 4 | 19 | 42 | 4 | 17 | 30 | 9 | 31 | | 4 | 20 | 40 |
| 11 | 6 | 13 | 26 | 6 | 11 | 40 | 15 | 10 | | 6 | 15 | 10 |
| 12 | 6 | 13 | 47 | 6 | 12 | 0 | 15 | 30 | | 6 | 14 | 10 |
| 13 | 7 | 29 | 45 | 7 | 27 | 40 | 8 | 1 | 10 | 8 | 27 | 40 |
| 14 | 8 | 11 | 49 | | | | | | | | | |
| 15 | 9 | 5 | 38 | 9 | 2 | 20 | 5 | 0 | | 9 | 5 | 30 |
| 16 | 9 | 20 | 56 | 9 | 18 | 50 | 22 | 20 | | 9 | 22 | 0 |
| 17 | 10 | 26 | 15 | 10 | 24 | 10 | 10 | 20 | | 9 | 27 | 18 |
| 18 | 11 | 18 | 58 | 11 | 17 | 10 | 20 | 40 | | 11 | 20 | 20 |
| 19 | | | | 11 | 27 | 10 | | | | | | |
| 20 | 1 | 22 | 2 | 1 | 19 | 50 | | | | | | |
| 21 | 5 | 11 | 38 | | | | 5 | 13 | 2 | 5 | 12 | 40 |
| 22 | 10 | 23 | 53 | 10 | 22 | 00 | | | | | | |
| 23 | 3 | 4 | 3 | 2 | 2 | 10 | | | | | | |

Præcipuarum Fixarum Canon juxta Ob- servationes Selectas.

| | <i>Ex Codicibus Savi-
lii & Bodlei. quasi
pro A. C. 750.</i> | | | | <i>Mohammedi Tizanio.
A. C. 1533.</i> | | | | <i>Sabebodino Alpenfi ad Il-
chanicas, pro
A. C. 1436.</i> | | | |
|----|--|----|----|----|---|----|---------|----|--|----|---------|----|
| | Long. | | | | Ascens. Rect. | | Declin. | | Ascens. Rect. | | Declin. | |
| | S. | ° | ' | '' | ° | ' | ° | ' | ° | ' | ° | ' |
| 1 | 0 | 12 | 20 | | 130 | 17 | 41 | 0 | 132 | 10 | 41 | 30 |
| 2 | | | | | 86 | 55 | 56 | 41 | 84 | 2 | 55 | 45 |
| 3 | 1 | 2 | 50 | | 130 | 14 | 39 | 43 | 127 | 44 | 39 | 4 |
| 4 | 1 | 19 | 50 | | 153 | 0 | 15 | 43 | 150 | 40 | 15 | 20 |
| 5 | | | | | 163 | 43 | 9 | 13 | 162 | 20 | 9 | 20 |
| 6 | 2 | 2 | 10 | | 161 | 39 | 45 | 0 | 158 | 30 | 44 | 43 |
| 7 | 2 | 9 | 10 | | 173 | 15 | 6 | 28 | 171 | 55 | 6 | 15 |
| 8 | 2 | 24 | 50 | | 187 | 0 | 15 | 50 | 185 | 45 | 15 | 43 |
| 9 | 3 | 6 | 40 | | 199 | 0 | 6 | 5 | 197 | 25 | 6 | 7 |
| 10 | 4 | 9 | 39 | | 236 | 0 | 14 | 8 | 234 | 0 | 15 | 10 |
| 11 | 6 | 4 | 50 | | 285 | 36 | 8 | 29 | 283 | 40 | 7 | 30 |
| 12 | 6 | 4 | 10 | | 299 | 0 | 22 | 14 | 297 | 39 | 33 | 8 |
| 13 | 7 | 19 | 20 | | 330 | 57 | 22 | 42 | 331 | 0 | 24 | 12 |
| 14 | | | | | 348 | 57 | 13 | 9 | 347 | 5 | 13 | 20 |
| 15 | 8 | 24 | | | 38 | 37 | 5 | 29 | 38 | 32 | 4 | 20 |
| 16 | 9 | 15 | 0 | | 22 | 5 | 7 | 24 | 20 | 5 | 7 | 3 |
| 17 | | | | | 36 | 40 | 43 | 50 | 36 | 20 | 43 | 15 |
| 18 | | | | | 71 | 0 | 25 | 16 | 68 | 50 | 24 | 30 |
| 19 | | | | | 88 | 10 | 12 | 23 | 85 | 54 | 11 | 30 |
| 20 | | | | | 133 | 46 | 47 | 52 | 130 | 20 | 47 | 11 |
| 21 | 5 | 1 | 45 | | 261 | 20 | 17 | 55 | 259 | 30 | 17 | 55 |
| 22 | | | | | 68 | 45 | 33 | 51 | 67 | 25 | 35 | 2 |
| 23 | | | | | 183 | 40 | 51 | 35 | 183 | 30 | 5 | 25 |

Præcipuarum Fixarum Canon juxta Observationes Selectas.

| <i>Olavino Sæteride in Tabulis Damascenis.</i>
A. C. 1480. | | | | | | <i>Ex Tabulis Rodolphinis, A. C.
1600. ad Contemplationes
Tythonis Brahe.</i> | | | | | | | | |
|---|----|----|---------|-----|----|---|----|--|-------|----|-----|------|-----|--|
| Long. | | | Ascens. | | | Declin. | | | Long. | | | Lat. | | |
| S. | ° | ' | ° | ' | | ° | ' | | S. | ° | ' | ° | ' | |
| 1 | 0 | 20 | 34 | 129 | 30 | 17 | 0 | | 0 | 21 | 10 | 53 | 30 | |
| 2 | 0 | 18 | 16 | 83 | 50 | 55 | 50 | | 0 | 29 | 35½ | 51 | 14½ | |
| 3 | 1 | 20 | 4 | 127 | 55 | 38 | 50 | | 1 | 20 | 37 | 22 | 22 | |
| 4 | 1 | 3 | 4 | 149 | 50 | 15 | 0 | | 2 | 4 | 12½ | 5 | 31 | |
| 5 | | | | 167 | 50 | 5 | 10 | | 2 | 11 | 37 | 31 | 11½ | |
| 6 | 2 | 15 | 24 | 157 | 40 | 44 | 40 | | 2 | 16 | 16 | 22 | 50½ | |
| 7 | 2 | 22 | 24 | 170 | 10 | 6 | 10 | | 2 | 15 | 23 | 16 | 36 | |
| 8 | 4 | 4 | 4 | 184 | 20 | 15 | 45 | | 3 | 8 | 35½ | 39 | 30 | |
| 9 | 4 | 19 | 44 | 196 | 20 | 6 | 30 | | 3 | 20 | 18½ | 15 | 57 | |
| 10 | 4 | 24 | 44 | 233 | 30 | 15 | 5 | | 4 | 24 | 17 | 0 | 26½ | |
| 11 | 6 | 16 | 30 | 283 | 20 | 7 | 30 | | 6 | 18 | 16 | 1 | 59 | |
| 12 | 6 | 17 | 24 | 297 | 0 | 23 | 30 | | 6 | 18 | 39½ | 31 | 2½ | |
| 13 | 8 | 3 | 4 | 322 | 0 | 24 | 15 | | 8 | 4 | 13 | 4 | 27 | |
| 14 | | | | | | | | | 8 | 16 | 50 | 35 | 57 | |
| 15 | 9 | 7 | 44 | 4 | 0 | 38 | 30 | | 9 | 9 | 43 | 61 | 47½ | |
| 16 | 9 | 24 | 14 | 20 | 0 | 7 | 5 | | 9 | 26 | 9 | 29 | 21½ | |
| 17 | | | | 35 | 10 | 4 | 30 | | 10 | 29 | 53½ | 59 | 56½ | |
| 18 | 11 | 22 | 34 | 68 | 20 | 24 | 0 | | 11 | 23 | 49½ | 31 | 7½ | |
| 19 | 0 | 2 | 34 | 85 | 5 | 11 | 30 | | 0 | 3 | 28½ | 12 | 35 | |
| 20 | | | | 127 | 20 | 1 | 40 | | 1 | 26 | 17 | 30 | 5 | |
| 21 | 5 | 14 | 54 | 259 | 5 | 18 | 10 | | 5 | 16 | 3 | 12 | 18 | |
| 22 | 10 | 24 | 24 | | | | | | 10 | 28 | 11½ | 21 | 0 | |
| 23 | | | | 181 | 15 | 51 | 50 | | | | | | | |

Canon Præcipuarum è Stellis Fixis secundum Observata Majorum.

Ricciola Bononiensis in Astronomia Reformata. A. C. 1660.

| | Long. | | | | Lat. | | | Ascens. | | | Declin. | | |
|----|-------|----|----|----|------|----|----|---------|----|----|---------|----|----|
| | S. | ° | ' | " | ° | ' | " | ° | ' | " | ° | ' | " |
| 1 | 0 | 22 | 0 | 14 | 33 | 30 | 0 | | | | | | |
| 2 | 1 | 0 | 22 | 12 | 51 | 17 | 10 | 337 | 44 | 33 | 57 | 18 | 40 |
| 3 | 1 | 11 | 32 | 10 | 22 | 22 | 40 | 41 | 33 | 34 | 39 | 36 | 30 |
| 4 | 2 | 5 | 1 | 43 | 6 | 30 | 30 | 64 | 56 | 38 | 15 | 46 | 10 |
| 5 | 2 | 12 | 3 | 10 | 31 | 10 | 40 | 74 | 32 | 38 | 8 | 37 | 30 |
| 6 | 2 | 17 | 6 | 15 | 22 | 51 | 45 | 72 | 52 | 20 | 45 | 36 | 0 |
| 7 | 2 | 16 | 13 | 0 | 16 | 52 | 30 | 76 | 45 | 14 | 35 | 59 | 50 |
| 8 | 3 | 9 | 3 | 0 | 39 | 32 | 5 | 97 | 30 | 18 | 16 | 16 | 30 |
| 9 | 3 | 21 | 6 | 30 | 15 | 57 | 10 | 110 | 22 | 32 | 16 | 14 | 0 |
| 10 | 4 | 26 | 4 | 45 | 0 | 26 | 20 | 147 | 31 | 14 | 13 | 36 | 30 |
| 11 | 6 | 19 | 6 | 0 | 1 | 59 | 30 | 196 | 51 | 0 | 9 | 20 | 30 |
| 12 | 6 | 19 | 30 | 40 | 31 | 0 | 40 | 210 | 4 | 38 | 20 | 59 | 50 |
| 13 | 8 | 5 | 1 | 40 | 4 | 26 | 30 | 242 | 10 | 40 | 25 | 33 | 30 |
| 14 | 8 | 17 | 43 | 20 | 35 | 56 | 15 | 259 | 47 | 14 | 12 | 52 | 30 |
| 15 | 9 | 10 | 32 | 40 | 41 | 47 | 0 | 276 | 19 | 32 | 30 | 30 | 40 |
| 16 | 9 | 26 | 58 | 30 | 29 | 20 | 40 | 293 | 31 | 59 | 8 | 1 | 20 |
| 17 | 11 | 0 | 41 | 25 | 59 | 57 | 20 | 307 | 26 | 35 | 44 | 26 | 40 |
| 18 | 11 | 24 | 40 | 20 | 31 | 8 | 20 | 341 | 51 | 48 | 29 | 15 | 50 |
| 19 | 0 | 4 | 40 | 20 | 12 | 37 | 0 | 358 | 57 | 10 | 13 | 19 | 20 |
| 20 | 1 | 27 | 0 | 8 | 30 | 6 | 40 | 44 | 52 | 30 | 48 | 34 | 30 |
| 21 | 5 | 16 | 53 | 0 | 12 | 16 | 20 | 172 | 53 | 58 | 16 | 27 | 40 |
| 22 | 10 | 29 | 2 | 56 | 20 | 59 | 40 | 339 | 37 | 10 | 13 | 20 | 10 |
| 23 | | | | | | | | | | | | | |

Præcipuarum Fixarum Distantiæ juxta Observationes Selectas.

| 1 | Ultima Fluvii | à Seat | 33 54 | à Cauda Cygni | 33 32 | à Capella | 23 57 | à Algomeisa | 46 22 | à Capella | 54 15 |
|----|----------------------|-----------------|-------|-----------------|-------|-------------|-------|-----------------|-------|-----------------|-------|
| 2 | Luc. Carb. Cassiop. | à Latere Persei | 9 23 | à Capella | 23 57 | à Algomeisa | 46 22 | à Capella | 54 15 | à Persei Latere | 19 11 |
| 3 | Persei caput Med. | à Sirio | 24 58 | à Algomeisa | 46 22 | à Capella | 54 15 | à Persei Latere | 19 11 | à Capella | 54 15 |
| 4 | Ocul. Taur. Austr. | ab Oculo Tauri | 28 33 | à Persei Latere | 19 11 | à Capella | 54 15 | à Algomeisa | 46 22 | à Capella | 54 15 |
| 5 | Orion. Pes Sinist. | ab Oculo Tauri | 38 43 | à Persei Latere | 19 11 | à Capella | 54 15 | à Algomeisa | 46 22 | à Capella | 54 15 |
| 6 | Capella Aurigæ | ab Oculo Tauri | 15 47 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 7 | Orion. Humer. dex. | ab Algomeisa | 25 42 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 8 | Sirius. | à Regulo | 37 20 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 9 | Procyon. | à Spica | 34 2 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 10 | Cor Leonis. | à Spica | 34 2 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 11 | Spica Virginis. | ab Antare | 33 02 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 12 | Anturus. | à Regulo | 39 42 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 13 | Antares. | à Lucida Aquilæ | 60 29 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 14 | Caput Ophiuchi. | à Spica | 36 14 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 15 | Lucida Lyra. | à Seat | 55 30 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 16 | Aquilæ Lucida. | à Lucida Lyra | 34 09 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 17 | Luc. in Cauda Cyg. | à Lucida Lyra | 23 52 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 18 | Luc. in Crure dext. | ab Algenib. | 20 37 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 19 | Ala Peg. ult. (Peg.) | à Latere Persei | 31 45 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 20 | Persei Luc. Lat. | ab Oculo Tauri | 36 20 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 21 | Cauda Leonis. | à Spica | 33 2 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 22 | Os Piscis Australis. | à Spica | 33 2 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |
| 23 | Canopus. | à Spica | 33 2 | à Rigel | 23 40 | à Rigel | 38 37 | à Cauda Cygni | 24 39 | à Spica | 33 02 |

Octo Fixarum Nobilium Declinationes ex Curis Antiquis,
 Consulto etiam Illustri *Brabeo*.

| | | <i>Aristyllo.</i>
A. a. C. 300. | <i>Timocharidi.</i>
A. a. C. 295. | <i>Hipparcho.</i>
A. a. C. 128. | <i>Menelao.</i>
A. C. 97. | <i>Ptolemaeo.</i>
A. C. 137. | <i>Tychoni.</i>
A. C. 1600. |
|----|---------------------|------------------------------------|--------------------------------------|------------------------------------|------------------------------|---------------------------------|--------------------------------|
| 4 | <i>Ocul. Tauri.</i> | | 8 45 b. | 9 45 b. | | 11 0 b. | 15 28 b. |
| 6 | <i>Capella. —</i> | 20 0 b. | | 40 24 b. | | 41 10 b. | 25 28½ b. |
| 8 | <i>Sirius. —</i> | | 15 20 m. | 16 0 m. | | 16 45 m. | 16 11 m. |
| 10 | <i>Regulus. —</i> | | 21 20 b. | 20 40 b. | | 19 50 b. | 13 53½ b. |
| 11 | <i>Spica. —</i> | | 1 24 b. | 0 36 b. | 0 40 m. | 0 36 m. | 9 1 m. |
| 12 | <i>Arcturus. —</i> | | 31 30 b. | 31 0 b. | | 29 50 b. | 21 18½ b. |
| 13 | <i>Antares. —</i> | | 18 20 m. | 19 0 m. | | 20 25 m. | 25 26 m. |
| 16 | <i>Aqu. Lucida</i> | | 5 48 b. | 5 48 b. | | 5 50 b. | 7 54 b. |

| <i>Oculi Tauri Longitudo.</i> | | | | <i>Spica Longitudo.</i> | | | |
|--|---------------------------|---------|--|--|-------------------------|---------|--|
| | <i>Hermeti. — —</i> | κ 25 17 | | A. a. C. 295. | } <i>Timocharidi</i> } | α 22 20 | |
| | <i>Hipparcho. — —</i> | γ 10 0 | | A. a. C. 283. | | α 22 30 | |
| A. C. 140. | <i>Ptolemaeo. — —</i> | γ 12 40 | | A. a. C. 128. | <i>Hipparcho. — —</i> | α 24 0 | |
| A. C. 851. | <i>Abomasaro. — —</i> | γ 19 15 | | A. C. 97. | <i>Menelao. — —</i> | α 25 45 | |
| A. C. 831. | <i>Thabero. — — —</i> | γ 21 37 | | | | | |
| | <i>Arzachel. — — —</i> | γ 23 20 | | | | | |
| | <i>Albaranio. — — —</i> | γ 24 30 | | | | | |
| | <i>R. Grotetio. — — —</i> | γ 28 40 | | | | | |
| A. C. 1316. | <i>W. Eveshamio. — —</i> | γ 29 0 | | | | | |
| <i>Reguli Longitudo.</i> | | | | <i>Addit. Fixarum Longitudinibus</i> | | | |
| A. a. C. 128. | <i>Hipparcho. — —</i> | ε 29 50 | | | <i>Ptolemaicis.</i> | | |
| A. C. 137. | <i>Ptolemaeo. — —</i> | ε 2 30 | | A. C. 1303. | <i>Alphonfus. — —</i> | ε 17 8 | |
| A. C. 879. | <i>Albaranio. — —</i> | ε 14 0 | | A. C. 1320. | <i>Wymundus. — —</i> | ε 15 52 | |
| | | | | A. C. 1440. | <i>Walterus Vigorn.</i> | ε 19 5 | |
| Spatio itaque annorum 742. grad. 11. 50'.
aut potius gr. 11. 30'. propter <i>Reguli</i>
Long. <i>Ptolemaicam</i> ε 2°. 30'. non 2°. 10'.
sicut olim legerat <i>Astronomus Raccensis</i> . | | | | Prima <i>Arietis</i> distat ab <i>Æquinoctio</i> verno.
25°. 40'.
Hec omnia <i>Walterus Vigorniensis, Cod. 13.</i>
inter <i>Digbianos</i> . | | | |

Progrediuntur autem Stellæ Fixæ Gradum unum Annis Solaribus.

1. **H** Ipparcho, Ptolemaeo, Theoni, Proclo & Alfergano ——— 100
2. Timocharidi Alexandrino, qui observarat Spicam Caelestem Annis Nabon, 454 & 466. Abdorahmano Salchio, & de D. Petavio, 72 ;
sive $\frac{60 \times 12}{10}$: & 50". quovis Anno.
3. Johanne de Egyptio, Canonum Hamicorum Conditori ——— 70 $\frac{1}{2}$
4. Fabie f. Abomansori, aliisque Probatæ, quam vocarunt, Astronomiæ Auctoribus : nec non Nasirodino Tusio, Cothodino Sirasio, Ologbeco Mogolorum Domino, Xacholgio, Abolphetacho, Abensfara, Maimonide, & plerisque juniorum ——— 70. & 51". 26'''.
5. Chrysococce in Persicis, & Astron. Anglicis. An. Chr. 1300. — 68. & 52". 23'''.
6. Astronomis plerisque Arabum sub Mamone Principe ——— 66 $\frac{1}{2}$
7. Abdorahmano Sophio, Bahodino Chorcio, Alphonsi Regi, Albatanio ex Racca, (quæ est Callinicos Mesopotamia) Abdolgalilo Segazio, Levi & Zacuto Judæis, & Observatorum Maragensium nonnullis ——— 66. & 53". 33'''.
8. Copernico, Mastlino, aliisque fide illorum ——— fere 71. & 50". 12'''.
9. Nonnullis apud Chorcium Arabem ——— ——— ——— 54".
10. Tychoni Braheo, Keplero, Bullialdo, ex obliquitate Zod. — 23 $\frac{1}{2}$ °. 70 $\frac{1}{2}$ '. & 51'.
11. Langomontano ——— ——— ——— 72 $\frac{1}{2}$ '. & 49". 54'''.
12. Gassendo ——— ——— ——— 70 $\frac{1}{2}$ '. & 51". 19'''.
13. Ricciolo in Afr. Reform. ex obliq. Zod. 23°. 30'. 20". 71. 19 $\frac{1}{2}$ d. 50". 40'''.
14. Nobis, & Ægyptiorum Hierophantis — 71. 9 $\frac{1}{2}$ mens. & 50". 94'''.

XH. Tubum habeo Pedum 13 $\frac{1}{2}$, Lentibus Convexis, & Micrometore exactissimo Townleniano instructum, quocum Noctibus serenioribus Mensium Octobris & Novembris, nuper elapsum, Minutas frequenter Pleiadum Stellularum Intercapedines dimensus sum, idque adeo auspicio, ut nunquam 20". imo per-raro 10". inter se dissiderere repetitæ Observationes ; à prægressis etiam Defuncti D. Gascognii, & nuperis Generosissimi Townleii (quantorum Hominum!) eadem ratione peractis Observationibus, confirmatæ ; quæ quidem Distantiæ limatissimæ sic se habent.

The Pleiades observed, in 1671. by Mr. Flamsteed. N. 79. p. 3051, 3062.

Stellæ.

| Stella. | Distantia. | | Stella. | Distantia. | |
|---------|------------|------|---------|------------|------|
| | Mihi | Muto | | Mihi | Muto |
| ab | 35 | 40 | ad | 18 | 30 |
| ae | 27 | 40 | ed | 26 | 10 |
| eb | 20 | 00 | di | 16 | 25 |
| cb | 21 | 45 | ai | 18 | 18 |
| ec | 10 | 00 | fi | 29 | 04 |
| eg | 14 | 40 | fa | 23 | 00 |
| cg | 11 | 55 | ha | 23 | 20 |
| bd | 22 | 04 | fa | 04 | 45 |

Addit Vinc. Minus in Epistola ad Doctrinum Ricciolum, (cujus metacrit in Appendice ad Ann. No. Tom. 1. Pag. 747.) Occidentalem Lucidiorem transisse Meridianum in eadem omnino Altitudine ac Lucidum Pleiadum. Qua fre-
tus Adversione & Observatis Distantiis, Loca Stellis assignavi infra scripta;
Media prius Lucida, eodem omnino Loco & Latitudine donata, quæ Auliori
Carolino arrideat; cæteris etiam abinde dispositis, quas tamen omnes, propri-
am si hac in re sententiam sequi licuisset, tres vel saltem duos Scrupulos Pri-
mos Promotiores, nec non & Latiores ab Ecliptica proponerem. Ineunte
Ann. 1672. constituta,

| Stella. | Long. 8 | Lat. Bor. | mag. |
|---------------------------------------|------------|-----------|------|
| Pleiadum. | 19 | 48 | 11 |
| Occidentalis Lucidior | b 24 45 15 | 4 08 51 | 5 |
| Intra hanc & Borealiorem Telescopicam | g 24 46 47 | 4 19 31 | 8 |
| Occidentalis Borealior | c 24 54 48 | 4 28 19 | 6 |
| Suprema in Quadrilatero | e 25 01 24 | 4 20 39 | 5 |
| Infima Australis opposita | d 25 02 18 | 3 53 19 | 6 |
| Media & Lucida | a 25 19 48 | 4 00 00 | 3 |
| Quæ in Cuspide ad Ortum | f 25 41 29 | 3 52 19 | 5 |
| Orientalium superior Telescopica | h 25 42 55 | 3 56 51 | 7 |
| Telescopica alia | i 25 14 04 | 3 42 37 | 9 |

XIII. 1. Inter *Canem Majorem & Navem* nuper deprehendi *Nebulosam* visu Pulcherrimam, si magnis Telescopiis inspicatur, ex Stellis Confertissimis compositam, quæ Cælum mediat cum *Cane Minori*. *A Nebulous Star, by M. Cassini. N. 123. p. 565.*

2. Cælus infra Procyonem petustrans, *Nebulosam* offendi, Latam, & Stellulis Confertissimam. Hanc eandem credo quam *G. Cassinus* observavit. *B. Mr. Flamsteed. Ibid. p. 567.*

XIV. *Ann. 1664.* I discovered the first Star in the Head of *Aries* to be a double Star, made of two considerable Stars, so near as not to be discovered two, but by a Glas of 6 or 8 Foot long. *The first of Aries a Double Star; by Dr. Hook. Phil. Coll. N. 2. p. 108. Marg.*

XV. 1. Desunt in Cælo duæ Stellæ Secundæ Magnitudinis in Puppi *Navis* ejusque Transtris, *Bayero β & γ* , prope *Canem Majorem*, à me & aliis occasione præsertim *Cometae. Ann. 1664.* Observatæ & recognitæ. Earum Disparitionem cui Anno debeam, non novi, hoc indubium, quod à die 10 April. 1668. ne Vestigium quidem illarum adesse amplius observo; cæteris circa eas, etiam Quartæ & Quintæ Magnitudinis, immotis. Plura de aliarum Stellarum Mutationibus, plusquam Centenis, at non tanti ponderis, annotavi. *Changes amongst the Fix'd Stars; by S. Montanari. N. 73. p. 2202. Marg.*

2. *M. Cassini* hath discovered many New Stars; viz. One of the Fourth Magnitude, and two of the Fifth, in *Cassiopeia*. He hath discovered two others towards the beginning of *Eridanus*, where we were sure they were not yet about the end of the Year 1664. considering that this Place of the Heavens, where passed the then appearing *Comet*, was diligently beheld by many, who perceived divers other small Stars, without observing those two. The same hath also observed, towards the *Arctick* Pole, four of the Fifth or Sixth Magnitude. *By M. Cassini. N. 73. p. 2202.*

He hath also observed, That the Star which *Bayerus* puts near that which he marketh in the Figure of *Ursa Minor*, appears no more; That that which is marked A, in the Figure of *Andromeda*, is also disappear'd; That in lieu of that which is marked v, at the Knee of the same Figure, there are two others more Northward; and that, that which is ξ , is very much diminish'd; the Star, which *Tycho* placeth at the Extremity of *Andromeda's Chain*, and calls it of the Fourth Magnitude, is now so small that one can scarce see it; and that which is in his *Catalogue* the 20th of the Constellation of *Pisces*, is now no more seen.

3. On the 24th of *Sept. (St. N.) 1666.* I have observ'd that New Star in *Pectore Cygni*, (which from the Year 1662. until this Time, hath been almost altogether hid) not only with my naked Eye, like a Star of the Sixth or Seventh Magnitude, but also with a very great Sextant. It is still in the very same Place of the Heavens, where it was from *Ann. 1661.* to almost 1662. For its Distance from *Scheat Pegasi* hath been by me found $35^{\circ} 51' 20''$. and from *Marcab*, $43^{\circ} 10' 50''$. which Distances are altogether equal to those which I observed *Ann. 1658.* the First of *November*. For the Distance from *Scheat* at that Time was $35^{\circ} 51' 20''$. and from *Marcab*, $43^{\circ} 10' 25''$. *To: New Star in Pectore Cygni; by M. Hevelius. N. 19. p. 349. N. 21. p. 372.*

where

where that former from *Scheat* exactly answers to the Recent ; and that from *Marcab*, 'tis true, differs in a very few Seconds, but that Disparity is of no moment, since it only proceeded from thence, that this New Star is not yet so distinctly to be seen as at that Time, when it was of the Third Magnitude. It is therefore certain, that it is the self-same Star, which *Kepler* did first see *Ann.* 1601. and continued till *Ann.* 1662. He that will observe this Star, must take care lest he mistake the Three more Southern ones of the Sixth Magnitude ; the highest of which is distant from *Scheat Pegasi*, $36^{\circ}. 25' 45''$. the Middlemost from the same $37^{\circ}. 25'. 20''$. and the Lowest $38^{\circ}. 4'. 30''$.

N. 134. p. 855.

Ann. 1662. *Novemb.* 28. Stella illa nova in *Pectore Cygni* quæ aliquamdiu ab *Ann.* 1662. plane dilituit, Cælo sereno quasi Reviviscere videbatur.

Ann. 1666. *Septemb.* 21. Nudis Oculis (etiam *Luna* splendente) apparuit. *Sept.* 24. Minor erat illis tribus præcedentibus in *Collo* & vix 6 Magn. videbatur.

N. 65. p. 2089.

Ann. 1670. *Aug.* 26. Sensim Crescere videtur quanquam necdum Major est Stellis 6 Magn. *Sept.* 3. Adhuc Crescere videbatur ; 8. Paulo adhuc Crescere deprehendimus ; & *Octob.* 13. Satis clare apparuerit.

N. 134. p. 855.

Ann. 1671. *April.* 29. Vix major adhuc apparuit quam Anno Præterito ; si quidem Stellis 6 Magn. æquabatur.

Ann. 1671. *Jun.* 26. Major fere videbatur.

Ann. 1672. *Mart.* 29. Adhuc Crescere videbatur.

Ann. 1675. *Jul.* 22. Apparuit instar 6 Mag.

N. 154. p. 864.

Ann. 1677. Nondum ad priorem Magnitudinem (Tertii videlicet Honoris) atque Claritatem & Splendorem (qua Magnitudine *Ann.* 1657, 1658, & 1659. apparuit) pervenit : siquidem non nisi instar 6 Magn. adhuc fulget.

Pb. Col. N. 5.
p. 162.

Ann. 1681. *Aug.* 18. Nova in *Collo Cygni*, Nudis Oculis ob ejus tenuitatem parvitatemque haud quidem conspecta, sed Telescopio tamen deprehensa est.

The New Star,
sub Capite
Cygni.

N. 65. p. 2092.
N. 73. p. 2198.

4. I. Don *Anhelme*, a *Carthusian* at *Dyon*, on the 20th of *June*, *Ann.* 1670. discover'd a Star of the Third Magnitude beneath the Head of *Cygnus*, situated in the Section of the two streight Lines, one of which goeth from *Lyra* to the nearest of the *Quadrangle* in the *Dolphin*, and the other from the *Eagle* to the Star, which is on the Top of the *Upper Wing* of *Cygnus*. He sent the News of this Discovery to M. *L' Abbe Mariotte*, one of the *Royal Academy*, who communicated it to the rest. They all agree, 'tis a New Star, though M. *B.* opposed it at first, affirming it to be in *Bayernus's* Tables ; but they prove that Star in *Bayernus* to be another ; giving for Distinguishment these Measures.

| | | | | |
|---------------------------------------|----------------------------|-----|----|----|
| The Bright Star ad Rostrum Cygni, its | <i>Ascensio Recta</i> | 289 | 22 | 00 |
| | <i>Declinatio Borealis</i> | 27 | 19 | 20 |
| But this New Star's | <i>Ascensio Recta</i> , is | 293 | 33 | 00 |
| | <i>Declinatio Borealis</i> | 26 | 33 | 20 |

Longi

| | ° | ' | " |
|---|----|----|----|
| <i>Longitudo</i> $\approx$ | 1 | 55 | |
| <i>Latitudo</i> | 47 | 28 | 10 |
| Its Distance from that <i>ad Rostrum Cygni</i> towards <i>Jaculum</i> | 3 | 47 | 30 |
| From the Tail of <i>Cygnus</i> ————— | 20 | 54 | 30 |
| And from the <i>Lucida Lyra</i> ————— | 18 | 39 | 40 |
| It came to the Meridian after the Star in <i>Rostrum Cygni</i> ——— | 16 | 44 | |
| And before the <i>Lucida Aquila</i> ————— | 0 | 27 | |

In the Beginning of *July*, this Star was observed to decrease: *July* 11. it scarce appear'd of the Fourth Magnitude.

Aug. 10. it was of the Fifth, and continued to decrease till it wholly disappear'd.

Anno 1671. *March* 17. D. *Anthelme* spied it again of the Fourth Magnitude.

April 4. M. *Cassini* found it greater than the two Stars of the Third Magnitude that are below in the Constellation of *Lyra*, and a little smaller than that in the Book of *Cygnus*, but more radiant.

April 9. he found it a little diminish'd, and almost equal to the greatest of the two Stars that are below in *Lyra*.

The 12th, it was equal to the least of these two Stars.

The 15th, he perceived that it increased, and found it equal the second time to the greatest of these two Stars.

From the 16th unto the 27th, it appeared of different Magnitudes, being sometimes equal to the biggest of these two Stars, sometimes equal to the least, and now and then between both.

But the 27th and 28th, it was become as big as the Star in the *Swan's Beak*.

The 30th, it appeared a little clearer; and the first Six Days in *May* it was greater.

The 15th of *May*, it was seen smaller than the same Star.

The 16th, it was in Bigness between the two Stars that are below in *Lyra*: And ever since she hath still diminished.

Thus this Star hath been twice in her greatest Splendour; first on the 4th of *April*, and the second time in the Beginning of *May*.

Hicse te invisere volui, quo vos de Observatione quadam notatu digna certiores facerem, mentemque simul meam ea de re vobis exponerem: De N. 65. p. 2087.

Nova; viz. illa Fixa 3 Mag. fere, Circa & Infra Caput *Cygni*, inter informes conspicua, cujus Longitudo 1°. 52'. 26''. $\approx$. & Latitudo 47°. 25'. 22''. Bor. modo existit; ut Observationes, Die 25 *Julii*, Anno 1670. à me habitæ, luculenter ostendunt. *Novam* autem Stellam hanc ipsam omnino esse, & in Cœlo ad Ann. 1660. penitus inconspicuam fuisse, non est quod quiscquam dubitet. Accidit enim ut Annis 1659, 1660, & 1661. pleraque Stellas illa omnes, in Asterismo *Cygni* apparentes summa diligentia, debitis Organis Dimensus fuerim, atque ita omnes illas, etiam circa Collum, & Caput

Deprehensibiles notaverim, earumque Distantias à diversis Fixis ceperim; nullam autem Stellam 3 Mag. eo loco ubi jam dicta *Nova* notatur, tum deprehendi; quam tamen optime, si adfuisset, conspexissem. Sic ut primo hinc Certus sim, Ann. 1660, & 1661. hanc Stellam nondum extitisse visibilem. Deinde clare etiam patet ex *Bayeri Uranometria*, hanc modo dictam *Novam* Stellam neque Ann. 1603. Apparuisse, & per consequens neque *Tychoni*, multo minus *Hipparcho*. Si quidem & *Bayerus* Fixam tantæ Magnitudinis deprehendisset, cum haud procul ab illa aliam 6 Mag. depinxerit: prout in Asterismo ejus *Cygni* videre est. At, inquires, forte ea ipsa est, quam tu *Novam* dicis, quippe cum *Bayerus* Congruis Organis Stellas illas haud Observaverit, fieri facile potuit, ut à vero ejus loco ad Gradum, vel paulo plus, aberravit. Sed, crede, id factum non est, quandoquidem Stella illa Parvula adhuc eodem loco, ubi *Bayerus* fere eam deposuit, commoratur, nec Major est Stella 6 Mag. ut eodem tempore ei videbatur. Distat enim, ut ipsemet nuper deprehendi, ab Ore *Pegasi* $32^{\circ} 39' 00''$. Dextro Genu *Pegasi* $39^{\circ} 23' 45''$. hinc provenit ejus Long. $00^{\circ} 06' 28''$. & Lat. $46^{\circ} 11' 14''$. Bor. ad Annum scilicet Currentem 1670. Compleat. *Julio*. At *Nova* Elongatur ab Ore *Pegasi* $32^{\circ} 31' 35''$. & à Dextro Genu *Pegasi* $38^{\circ} 18' 50''$. ex quibus Distantiis Long. $1^{\circ} 52' 26''$. & Lat. $47^{\circ} 25' 22''$. Bor. Elicitur. Adeo ut hæc *Nova* plane sit diversa ab illa 6 Mag. à *Bayero* notata; (quanquam hæc duæ nondum ad duos gradus ab invicem remouentur.) Atque ex Dictis manifestum fit, hanc *Novam* nec Ann. 1603. nec Ann. 1670. inter cæteras emicuisse Stellas.

Cum primitus à me observabatur, quoad Magnitudinem & Splendorem, Stellæ in Pectore *Aquila* æquabatur, nisi quod aliquanto Obtusioris fuerit Luminis; quo ad Situm, respectu reliquarum Stellarum, in Linea Recta cum illa in Ancona Alæ Superioris *Cygni*, & illa in Humero *Aquila*, nec non cum *Luci da Lyra*, & illa in Rhombo *Delphini*, Mediarum Borealiori consistebat. Triangulum vero *Æquilaterum* cum illa in Capite & Rostro *Cygni* constituebat.

N. 66. p. 2028. Mirum vero in modum M. *Sept.* decrevit, adeo ut 14 *Oct.* nulla ratione amplius Sectante observari à me potuerit; licet omnem adhibuerim diligentiam.

N. 134. p. 855. Ann. 1671. *Apr.* 29. Denuo Observavi. Excedit illam in Rostro *Cygni*, nec non eam quæ est in Ancone Inferioris Alæ *Cygni*, fereque illi quæ est in Pectore *Cygni* æquatur, nisi quod Lumine paulo Obtusiori & Rubicundiori modo Luceat. Qua vero Die primum rursus illuxerit, affirmare adeo certo non possum. Certus interim sum, ad Mens. *Dec. Jan.* imo *Feb.* haud Conspicuum fuisse. Etenim post 14 *Oct.* quo videri desiit, memini me eam sæpius quævisisse eo in Loco, sed nusquam Apparuisse. Id circo quantum colligere datur, vix ante initium *Mar.* quin sine dubio adhuc tardius, iterum Prodiit. *Apr.* 30. eam à Reliquis quibusdam Fixis sum dimensus. Distat à Cauda *Cygni* $26^{\circ} 55' 20''$. ab Ancone Alæ Superioris *Cygni*, $17^{\circ} 47' 50''$; à Capite vero *Serpentarii* $34^{\circ} 19' 40''$; sic ut eodem plane Loco adhuc persistat, ubi antea fuerat.

N. 144. p. 856. *Maii* 17. Aliquanto Minor videbatur Rostro *Cygni*, & illa in Humero *Aquila*, tum etiam Lumine Obrusior; Major tamen illa in Cuspide *Sagittæ*, & æqualis fere illi Seq; in Jugo *Lyra*.

Maii 25.

Maii 25. Minor videbatur quam Die 29 *Apr.* qua primum visa fuit; sic ut decrefcere videretur. Minor jam erat Rostro *Cygni*, nec non illa in *Ancone Alæ Austr.* etiam Minor illis in Jugo *Lyra*, & Humero *Aquila*; vix Major apparuit Minori duarum in Pede *Cygni*, & illa in Pectore *Aquila*.

Jun. 26. Minor apparuit illa in Collo *Cygni*; sicut notabiliter decreverit.

Jul. 3. Minor fere illa in Collo *Cygni*; & 18, vix Steillis; Mag. æquiparari videbatur.

Aug. 2. Vix 6 Magn. apparuit imo Minor quam reliquæ omnes circa Caput & Collum *Cygni* existentes; per intervalla tantummodo micabat.

Sept. 11. Haud amplius confpecta.

Ann. 1672. Mar. 6. I observ'd it again; but it can hardly be seen with N. 81. p. 4018. the naked Eye.

Mar. 29. Vix 6 Magn. apparuit; à quo tempore neutiquam amplius N. 134. p. 857, (viz. ad *Ann. 1677.*) in Conspectum venit, utut sæpius illam diligenter Quæfiverim.

5. *Ann. 1667. in Jan.* The *Nebulosa* in *Andromeda's* Girdle (which may well enough be seen by the bare Eye) appeared much obscurer than the Year before. In the Months of *February* and *March* I did not see it.

6. 1. *Ann. 1667. Jan. 20.* The *New Star* in the Neck of the *Whale*, did approach to the Bigness of a Star of the Sixth Magnitude, and grew bigger afterwards.

The *Nebulosa* in the Girdle of *Andromeda*, by M. Bullialdus. N. 25. p. 459. The *New Star* in Collo *Ceti*, by M. Bullialdus. *Ibid.*

Feb. 12. I saw it at last of the Fourth Magnitude.

Feb. 24. It was equal to the Stars of the Third Magnitude, shining very bright.

Feb. 26 and *27.* It appeared yet to encrease.

2. *Ann. 1667.* In the Beginning of *January* this Star did not appear.

Jan. 23. I found a little Star of the Sixth or Seventh Magnitude about the same Place where the said *New Star* uses to appear. But it then seemed to me not the genuine *New Star*, but another; to wit, preceding the *New*, whose Longitude in *Ann. 1660.* was defined by me, $\gamma. 25^{\circ}. 43'. 3''$. and the Latitude $14^{\circ}. 41'. 32''$.

By M. Hevelius. N. 25. p. 460. N. 134. p. 855.

Feb. 2. It appeared very bright, and that, when the Moon shone, of the Bigness of that in the Mouth of the *Whale*, or *Nodo Lini*; from which time I always observ'd it to grow bigger.

Mar. 13. I did still find it extremely bright, but could not by my naked Eye, because of the vivid Crepuscle, and the low Sight of the Star, accurately determine its Magnitude.

Ann. 1668. Octob. 26. *Nova* in Collo *Ceti* primum visa; sed instar minutissimæ Stellulæ.

N. 132. p. 855.

Nov. 7. *Nova* in Collo *Ceti* Medium fere in ore æquabat.

Ann. 1669. Jan. 28. Minor erat illa in Ore.

Sept. 26. Instar 6 Magn. apparuit.

Oct. 16. Illa in Ore Major erat, & Clarior.

Oct. 27. Lucidam *Mandib.* æquabat.

Nov. 19. Major illa in Ore & Minor. *Mandib.*

- Anno 1670. *Aug.* 27. Maximo gaudebat Lumine. æquales fere Stellis
 2 Mag. *Mandib. Ceti.*
Sept. 3. Admodum Fulgida extitit; & 8 æqualis *Mandib. Ceti.*
 N. 66. p. 1028. Ad medium usque Mensis *Oct. Mandib. Ceti* æqualis fere existit Magnitu-
 dine, & Claritate eam propemodum superavit; adeo ut hoc Anno secundæ
 fuerit Magn. ac Major quam præcedentibus Annis, excepto Anno 1660. quo
 Major etiam *Mandibula Ceti* à me fuit deprehensa. Aliis temporibus non me-
 mini eam tertiz Magnitudinis Stellas superasse, Certum igitur est ipsam non
 eandem semper præ se ferre Magnitudinem nec Claritatem, utut in maximo
 suo existat Incremento.
- N. 134. p. 856. *Dec.* 5. Adeo decreverat, ut vix Stellæ 6 Magn. æquaretur.
 Anno 1671. *Aug.* 14. Æquabatur Stellæ ad *Genam*, imo fere Major paulo
 videbatur.
Sept. 12. Æquabatur illi in *Ore* 4 Magn.
Oct. 30. Vix 6 Magn. apparuit.
Nov. 3. Non amplius apparuit.
 Anno 1672. *Aug.* 9. Clarissimis fulgebat Radiis, Major erat illa in *Ore*,
 & Minor *Mandibula*.
Sept. 17. Minor illa ad *Genam*, vix 4 imo 5 Magn. & 25. vix 6 Magn.
 Ibid. p. 854. A Mense circiter *Octob.* ad *Decemb.* 23. Anno 1676. ne semel quidem
 Prodiisse, utut semper omni studio Vigiles Oculos ad eam, quoties Observa-
 tionibus operam serenis noctibus dedi, direxerim.
- Ibid. p. 858. Anno 1676. *Dec.* 10. Bene memini me *Novam* hanc in *Collo Ceti* haud vi-
 disse, licet ea in *Cœli* parte plurimas Stellulas observaverim.
Dec. 23. *Novam* hanc in *Collo Ceti* *Cœlo* admodum sereno clarissime vidi-
 mus; & quidem tanta Claritate & Magnitudine fulgentem, ut *Mandibulam*
Ceti non solum æquaret, sed Magnitudine & Claritate vinceret.
Dec. 31. Fere Major *Mandib. h. e.* 2 Magn.
 Anno 1677. *Jan.* 1. Clarissime rursus Affulgebat, Major fere *Mandib. Ceti*,
 Major quoque quam extrema *Ala* & *Marcab. Pegasi*, Colore & Lumine fere
 æqualis *Mandib.* Memini tamen me olim observasse, quando Secundæ existe-
 bat Magnitud. eam paulo Albicantiorem & Splendidiorem.
- Phil. Col.
 N. 5. p. 162. Anno 1681. *Aug.* 18. *Nova* Stella in *Collo Ceti*, hac nocte Luna etfi
 Plena & Splendente, major erat ea in *Ore Ceti*, sed nondum æquabat Luci-
 dam *Mandibula*.
- By M. Cassini.
 N. 123. p. 565. 3. Anno 1676. *Mar.* Inspecta mihi est Stella *Nova* in *Ore Ceti*, quæ An-
 nos aliquot latuit, Solaribus Radiis tempore Maximæ Fullionis immerfa;
 nunc vero Stellas 3 Mag. facile superat.
- By Mr. Flam-
 steed.
 Ibid. p. 567. 4. *Novam* in *Pectore Ceti* sæpius ante *Octo* Menses vidi, nec Minorem
 quam innuit Cl. Cassinus.
- A New Star in
 Eridanus, by
 M. Cassini.
 N. 35. p. 683. 7. *March* 10. 1668. Not far from the Star in *Eridanus*, which is called
 the 14th by *Bayerus*, there appeared a Star equal to the brightest of the Fourth
 Magnitude, almost in the same Place, where was observed the Comet of *Ann.*
 1664. *Dec.* 31. which Star was not then seen, nor at other times elsewhere,
 nor is described in any Catalogue, on any Globe or Map, that I can learn;
 which therefore I deem to be a *New* one, that is, of *New Appearance*.

8. The Comet, Anno 1672. had (on the 1st of April, N. S.) pass'd 45'. beyond the most Northern Star of the Head of Taurus, and was distant 1°. 43'. from the Star that was nearest to that towards the South. M. Cassini having consider'd these two Stars, observ'd that the Second is not less bright than the First, and yet that *Bayerus* hath not marked it; and that at first Sight it seems that *Tycho* hath left it out in his Catalogue: For he puts four Stars in the Place he calls in *Quadrilatero Cervicis*, and he speaks not of this which is the fifth, and maketh, with the other four, an irregular Pentagone. This Omission of *Bayerus*, and the Denomination which *Tycho* useth to denote these Stars, which suits not with the Number nor the Configuration that now appears, do administer Cause to doubt, whether the Star in Question be not one of those that appear from Time to Time.

A New Star in Taurus, by M. Cassini. N. 82. p. 4046.

XVI, 1. Supponit Cl. *Cassinus*, ad Planetam in Ellipsi moventem extendi ab utroque Foco duas Rectas, quarum altera sit Medii, altera autem Veri Motus Linea. Constructio porro talis est:

To find the Aphelia of the Planets directly; by M. Cassini. Considered by Mr. Nic. Mercator. N. 57. p. 1168. Mar. An. 1670.

Fig. 117.

| | |
|---|--|
| L est Centrum Concentrici ABCDE. | B I est Perpendicularis ad R H G. |
| BLD est Diameter. | I est Centrum Ellipseos. |
| BA, BC, BP, sunt Intervalla Apparentia. | LI est Excentricitatis. |
| DE, DF, DQ, sunt Intervalla Mediorum Motuum. | IO = LI. |
| BE, BF, BQ; item DA, DC, DP, sunt Lineæ Rectæ. | O est Focus, circa quem ordinatur Medius motus; L, circa quem Verus. |
| BE secat DA in H; BF secat DC in G; BQ secat DP in R. | IM = IN = LB. |
| R H G est Linea Recta. | M, est Apogeon; N, Perigeon; B L M, Anomalia Vera. |

Demonstratio. 1. Illustrissimus ac Reverendiss. *Sethus Wardus*, Episcopus *Sarisburyensis*, in Examine Astronomiæ Philolaicæ, docuit Methodum, ex data Anomalia Media Planetarum, investigandi Verum; quæ est hujusmodi.

Fig. 118.

C, est Centrum Ellipseos AEP: F, focus circa quem ordinatur Medius Motus. S, focus circa quem ordinatur Verus Motus. A, Apogeon. P, Perigeon. E, Erro sive Planeta. AFE, Anomalia Media. ASE, Anomalia Vera. FET, Linea Recta; ET = SE. ST, est Linea Recta.

In Triangulo SFT, dantur, 1. SF, distantia focorum: 2. FT = FE + ES = AP. 3. AFT, Angulus Externus, sive Anomalia Media, æqualis Summæ Angulorum, FST & T. Ergo inveniri potest FSE, sive Anomalia Vera, æqualis Differentiæ Angulorum, FST & T. Nimirum.

Ut Semi-summa Laterum FT & FS, ad Semi-differentiam eorundem; Ita Tangens Semi-summæ Angulorum FST & T, ad Tangentem Semi-differentiæ eorundem.

Sed Semi-summa laterum FT & FS invenitur, substituendo pro FT æqualem AP, cujus Semis est AC, qui additus CS, semilli ipsius FS, facit Semi-summam AS, Distantiam Planetæ Maximam.

Tum :

Tum si ex Semi-summa A S, auferatur Latus Minus F S, restant Semi-differentia Laterum F A, æqualis P S, distantia Planetæ minimæ: ut si Regula ex Anomalia Media data inveniendi veram.

Ut A S, Distantia Planetæ Maxima, ad P S, Distantiam Minimam; Ita Tangens dimidia Anomalia Media, ad Tangentem dimidia Anomalia Vera.

Corollar. 1. Si continuetur S E usque ad V, ita E V sit = ipsi F E, & tota S V = Axi A P, erit Trianguli, F S V, Angulus V, Semis Prosthæreseos F E S, ideoque æqualis Semi-differentiæ Angulorum Anomaliæ Mediæ & Veræ, h. e. ipsorum A F E, & A S E; & Externus A F V = Semi-summa eorundem, A F E & A S E, Angulorum, ablata scil. Semi-differentia, V F E, ex Majori A F E; unde oriuntur duæ Analogiæ,

1. Ut Sinus Semi-summa Anomaliæ Mediæ & Veræ, A F V, ad Sinum Semi-differentiæ eorundem V; ita S V, (= Axi transverso A P) ad S F, Distantiam Focorum.

2. Ut Sinus Semi-summa Anomaliæ Mediæ & Veræ A F V, ad Sinum Anomaliæ Veræ, F S V; ita S V, (vex Axis A P) ad F V, Subtensam Anomaliæ Veræ: Ita quoque Semiaxis, A C, ad Semi-subtensam V X, vel F X.

Corollar. Si in eodem Triangulo, F S V, ex Subtensæ F V, Puncto medio X, erigatur Perpendicularis X E; secabit illa S V in duas partes, quarum altera V E = est lineæ Medii Motus F E, altera vero S E, est ipsa linea Veri Motus.

Fig. 191.

2. Sit a Centrum Concentrici c h f i. c d h, est Angulus dimidiæ Anomaliæ Veræ, & c a d, Diameter, eademque Linea Ap-sidum. d c i, Angulus dimidiæ Anomaliæ Mediæ. c b, Arcus Anomaliæ Veræ, cui re-spondet. c i, & d h, sunt Lineæ Rectæ, secan-tes se mutuo in g. d i, Arcus Anomaliæ Mediæ. Itaque

Ab Intersectionis Puncto g, demittatur ad c d, Perpendicularis g b. Erit igitur,

$$d b : b g :: \text{Radius} : \text{Tang. } b d g, \text{ vel } c d h.$$

$$\text{Et } c b : b g :: \text{Radius} : \text{Tang. } b c g, \text{ vel } d c i.$$

$$\text{Ergo } d b \times \text{Tang. } c d h = b g \times \text{Rad.} = c b \times \text{Tang. } d c i.$$

Quare d b : c b :: Tang. d c i : Tang. c d h; hoc est, d b erit ad c b, ut Tangens dimidiæ Anomaliæ Mediæ & Tangentem dimidiæ Anomaliæ Veræ; adeoque (per Regulam supra expositam) ut distantia Planetæ Maxima, ad distantiam Minimam. Quamobrem d b = erit distantia Planetæ Maximæ; & c b, Minimæ; & a b, Excentricitati.

Cumque idem eodem modo Demonstretur de cæteris omnibus Interfectionum Punctis, nimirum Perpendiculares ab ipsis ad c d Lineam incidere in Punctum b; oportet, ut Recta, jungens ipsas Interfectiones, congruat Perpendiculari, b g f.

3. Ducta

3. Ducta Diametro hak , fiat Arcus $kl = id$, & ducantur kc , & bt , Secantes se mutuo in p . Ab h , in bgf , demittatur Perpendicularis, hr , eademque Parallela Apfidum Lineæ cd ; erit Angulus rbs , Semi-differentia Arcuum Anomalix Veræ ch , & mediæ di . Tum ab eodem h Puncto ducatur Recta hp , faciens cum kb , Angulum = Angulo rbs , & occurrens Lineæ Apfidum in β , erit Trianguli apb , Angulus pab , Mensura Arcus ch , sive Anomalix Veræ, & βha , semi-differentia Anomalix Veræ & Mediæ (ex Constructione); & Externus $c\beta b$, (æqualis duobus Internis & Oppositis βab , & βha , adeoque compositus ex Anomalia Vera & Semi-differentia ejus à Media) erit Semi-summa Anomalix Veræ & Mediæ. Ergo per Corollarii primi Analogiam priorem; Ut sinus $c\beta b$, ad sinum βha ; Ita Radius ab , ad Excentricitatem $a\beta$. Sed supra Demonstravimus quoque ab æqualem Excentricitati. Ergo Punctum β congruit Puncto b .

Tum ex b excitetur ipsi hb , Perpendicularis bt ; Aio, hanc continuatam Incidere in Punctum Interfectionis p . Nam Triangula, rbs , & bht , sunt Similia, ex Constructione; quemadmodum & Triangulum bpk , simile est Triangulo hgi , cum eidem Peripheriæ ch , insistentes Anguli pkl , & gih , sint æquales, nec non æqualibus Peripheriis kl , & id , insistentes Anguli phk , & ghi , æquales; quare & Tertius bpk , æqualis est Tertio hgi . Et ex æqualibus phk , & ghi , ablatis æqualibus bht , & rbs , restant æquales phb , & ghr . Unde sic Arguo; $srh = tbb$, & $rbs = bht$, Ergo $hbr = htb$; ergo & Complementa horum ad Semicirculum sunt æqualia, nimirum $rsi = btk$, & $sig = tkp$, Ergo & $igs = kpt$, quibus ablatis ex æqualibus igh , & kpb , restat $hgs = hpt$; & $ghr = phb$, Ergo & $hrg = hbp$. Sed hrg , est Rectus, Ergo & hbp , Rectus est, Cum vero, & bht Rectus sit, ex Constructione, erit tb inde Rectum ipsi bp . Cumque idem eodem modo Demonstraretur de quavis alia Interfectione linearum ab h , & k , ad Congruentia Anomalix Veræ & Mediæ Functæ ductarum; patet, non modo Rectam, jungentem Interfectiones, transituram per b punctum; sed & hb , lineam perpendicularem fore ad eandem Jungentem. Q. E. D.

Corollarium. Si à quovis puncto Anomalix Veræ, puta h ad respondens punctum Anomalix Mediæ i , ducatur Recta hi ; excitata è Centro Excentrici b , ipsi cbd , Perpendicularis bf , secabit ipsam hi in s , ea ratione quam linea Medii Motus obtinet ad lineam Veri Motus.

Nam per Corollarii Primi Analogiam Posteriolem, hb , est Semi-subtensa; ergo per Coroll. 2. Perpendicularis erecta ex b , nimirum bt , secat Diametrum bk , in t , ea ratione quam linea Medii Motus obtinet ad lineam Veri Motus. Ergo & rs , (sive bf), secat hi , lineam eadem ratione in s ; propter Demonstratam modo Figurarum $tbbkphb$, & $srhigbr$, Similitudinem.

Cæterum ex laudata superius Reverendiss. Wardi Methodo inveniendi primam inequalitatem, non est difficile, alium adhuc modum investigandi Apogea & Excentricitates, non minus Directum & Geometricum, & Observationes quotvis admittentem, producere; quem & paucis exponam. Plures modos

modos invenient Astrophili in Reverendiss. Viri *Astronomia Geometrica*, ad quam eos remitto. Interim

Fig. 120.

Sint l & d , duco Foci Ellipseos; t & u , duo Puncta Veri Motus Planetæ; Arcus Ellipseos $t u$, ex l spectatur sub Angulo $t l u$, & ex d , sub Angulo $t d u$; item Distantia Focorum $l d$, ex t spectatur sub Angulo $d t l$, & ex u , sub Angulo $d u l$: Aio differentiam Angulorum $t l u$, $t d u$, æqualem esse differentiæ Angulorum $d t l$, & $d u l$.

Cum enim Trianguli $l u x$, tres Anguli simul sumpti æquales sint Trianguli $d t x$, tribus Angulis simul sumptis; si auferantur utrinque æquales $l x u$, & $d x t$, reliquorum duorum summa, $u l x + l u x$, erit = summæ reliquorum $t d x + d t x$, & ab his æqualibus Summis si auferantur inæquales, *V. g.* $u l x$, ex priori, & $t d x$, ex posteriori; reliquorum $l u x$, & $d t x$, differentia = est differentiæ ablatorum $u l x$, & $t d x$; quod erat propositum.

Centro l . Intervallo Axis Transversi $m n$, describatur Circulus $a b c$, cujus Arcus $a b$, rursus ex l spectatur sub Angulo $a l b$, & ex d , sub Angulo $a d b$, item distantia focorum $l d$, ex a spectatur sub Angulo $l a d$, & ex b , sub Angulo $l b d$. Ergo rursus differentia Angulorum $a l b$, & $a d b$, = est differentiæ Angulorum $l a d$, & $l b d$. Sed per *Coroll. 1.* Angulus $l a d$, Semis est Anguli $l u d$, & Angulus $l b d$, Semis Anguli $l t d$. Ergo horum Angulorum $l a d$, & $l b d$, differentia = est Semi-differentiæ Angulorum $l u d$, & $l t d$; ergo & Angulorum $a l b$, & $a d b$, differentia = est Semi-differentiæ Angulorum $u l t$, & $u d t$, quorum prior est Intervallum Apparens duarum Observationum, posterior autem, Intervallum Motus Medii. Data igitur horum intervallorum differentia, datur quoque hujus (differentiæ) Semis, nimirum differentia Angulorum $a l b$, & $a d b$. Sed $a l b$, idem est cum $u l t$, dato; ergo datur quoque $a b d$, Angulus, sub quo Peripheria $a b$, spectatur ex d .

Simili modo ostendetur, differentiam Angulorum $t l y$, & $t d y$, æqualem esse Summæ Angulorum $l t d$, & $l y d$; nec non differentiam Angulorum $b l c$, & $b d c$ = esse Summæ Angulorum $l b d$, & $l c d$. Cumque $l b d$, semis sit ipsius $l t d$, & $l c d$, semis ipsius $l y d$; erit sane Summa ipsorum $l b d$, & $l c d$, = semi-summæ Angulorum $l t d$, & $l y d$; hoc est, differentia Angulorum $b l c$, & $b d c$, = erit semi-differentiæ Angulorum $t l y$, & $t d y$, quorum prior est Intervallum Apparens duarum Observationum, posterior autem, Intervallum Motus Medii. Quare, data horum Intervallorum differentia, datur quoque hujus Semis nimirum differentia Angulorum $b l c$, & $b d c$. Sed $b l c$, idem est cum $t l y$, dato; ergo datur quoque $b d c$, Angulus, sub quo Peripheria $b c$, spectatur ex d .

Unde liquet, & datis Intervallis Observationum Mediis & Apparentibus dari Angulos, sub quibus ex d spectantur Circuli $a b c$, Peripheriæ quotvis, interceptæ lineis Veri Motus. Ergo per *H. rigoni Theor. Plan. lib. 1. cap. 3. Prop. 12. Schol. 1.* totidem Circuli Segmenta describi possunt, capacia Angulorum sub quibus isti Arcus conspiciuntur ex d , quæ segmenta omnia se mutuo interfecabunt in d . Possunt igitur & hac Methodo inventi Apogea &

& Excentricitates Planetarum, delineatione Geometrica, adhibitis Observationibus quotvis ; nec difficilius est, Circulos ducere, quam Lineas Rectas.

Sed ut demus id, quod verum est, Clarissimi *Cassini* Delineationem Geometricam non nihil expeditiorem esse ; verendum est interim ne, si ἀκριβῶς Astronomis expetitam sectemur, Diagrammata requirat Enormis Magnitudinis, adeoque operosior evadat, quam ipse Calculus. Ad hunc autem accedentes, utramque Methodum æquipollere deprehendemus. Ne quis vero Apogei & Excentricitatis utraque Methodo inventæ à vero discrepantiam censeat Errori Calculi imputandam : restat ut Hypothesin Excutiamus.

Et Ellipticæ quidem Orbitæ inventio, sine controversia *Keplero* debetur ; sed quibus Accelerationis & Retardationis gradibus incedant Planetæ, definire, non minus pertinet ad integrandam Hypothesin, quam ipsius Orbitæ determinatio. Quamquam autem ex Cl. *Cassini* (vel Interpretis ejus) sermone id nusquam apparet ; attamen ex Constructione Problematis, & ejus Analyti, manifestum est, eum supponere, Planetem ex Foco superiori videri prorsus æquabili Motu incedere. Fuit sane, cum id Observationibus nequaquam Congruere animadverteret, mutavit sententiam, & Lineam Veri Motus Planetæ æqualibus temporibus æquales Areas Ellipticas verrere professus est : Punctum autem, ex quo Planeta exacte æquabili Motu procedere videtur, nullum omnino extare in hoc Universo, nisi id libratile statuere libeat. Nulli interim puncto propius æquabilem videri incesum Planetæ, quam ipsi Foco superiori Ellipseos. Neque inventus fuit hactenus, qui Areas *Kepleri* Phænomenis satisfacere posse negaret ; sed cum eas Calculo directo exhibere nec ipse, nec post eum quisquam potuerit, causati sunt nonnulli *Keplerum*, nimis indulgentem causis, Physicis à Geometria diversum abiisse ; quasi causæ Physicæ repugnent Geometria, aut minus Geometricum sit Problema ; quod, nulla injecta Physicarum Causarum mentione sic proponitur : *Data Area Trilinei, inter Lineas Apsum, & veri Motus, nec non Peripheriam Ellipticam intercepti, invenire Angulum ad Solem.* Habent igitur à *Keplero* responsum, qui illi ἀποκρίσεις objiciunt ; nim. *Eant ipsi & Schema solvant.*

Quamvis autem Religio fuerit *Keplero*, ab Hypothesi, quam naturalem esse plane persuasum habeat, recedere ; quidni liberum foret allis periculum facere, num via quævis alia detur, inequalitatem Planetarum Primam directo Calculo investigandi ? Ideoque Vir Cl. *Isma. Bullialdus* aggressus est Ratiocinio Geometrico indagare, qua semita, & quibus intentionis ac remissionis gradibus conveniret Planetas ferri, ut ab æquabili incesus Norma, Astronomis ante *Keplerum* assumpta, ad eam quam spectamus Inæqualitatem perduceremur. Perennant Illust. Viri monumenta, unde omnem hujus Inventi rationem hauriri licet Astrophilis. Amplexus eandem Reverendiss. *Seth. Wardus*, primum ostendit, paria facere cum linea æquabilis Motus circa alterum Ellipseos Umbilicum gyrata ; deinde & Calculi Directi Methodo ornavit ea, quam paulo ante recitavimus : Ita ut nihil amplius desiderari posset, quam ut Urania felicibus Coeptis annueret. Cujus quidem Nomine suscipere ausus fuit Illustriss. *Comes Paganus*, edito biennio post ejusdem fere tenoris scripto

adeo veram esse Hypothesin, ut deprehensam circa Octantes discrepantiam, Astronomorum Inficitæ tributam mallet. At Cl. *Bullialdus*, audiendum potius ipsam Astronomiam ratus, Observatorum ore loquentem, secundis Curis, adhibita prioribus Inventis Limitatione quadam, discrepantiam illam exterminavit. Unde porro intelligitur, Hypothesin illam, cui Cl. *Cassinus* Investigationem Apogeorum & Excentricitatum superstruit, tantundem fere deficere à vero quantum Cl. *Bullialdi* Limitatio pollet, atque ab illo defectu pullulare Calculi à Cælo dissensum.

By Mr. Edm.
Halley.
Aug. An. 1676.
N. 128. p. 683.

2. Motus Terræ Annuus per Eclipticam, Opticam Inæqualitatem inducit Motibus cæterorum planetarum; Astronomis *Copernicanis* nomine Parallaxeos Orbis notissimam; quam quidem inæqualitatem, ex Observationibus non multa opera datam, Methodi sequentis basin firmissimam constituo; ubi præter Observata nihil aliud supponitur, quam quod Orbes Planetarum sint Ellipses, quodque Sol in Foco, omnium Orbibus communi, sit constitutus & denique quod Tempora Periodica singulorum ita innotescant, ut non sentiantur error aliquis, saltem in duabus vel tribus Revolutionibus: His concessis, Motus Terræ, pro cæteris Planetis necessario requisitus, primo aggrediendus est.

Fig. 121.

Sit S, Sol; A B C D E, orbis Terræ; P. Planetà Mars, (qui in hanc rem plurimis de causis longe præferendus est; & primo observetur Verum Tempus & Locus, quo Mars opponitur Soli; Tunc enim Sol & Terra coincidunt in Lineam Rectam cum Marte; vel (quod fere semper accidit si habuerit Latitudinem, cum puncto, ubi Perpendicularis à Marte demissa in planum Eclipticæ incidit. Sic in Schemate, S, A, & P, sunt in Linea Recta; deinde post 687 dies, Mars revertitur ad idem punctum P, ubi in priori Observatione Soli opponebatur; Terra vero cum non revertatur ad A, nisi post 730 $\frac{1}{2}$ dies, in B, Solem respicit in Linea S B, Martem vero in Linea B P, & Observatis Longitudinibus Solis & Martis, omnes Anguli Trianguli, P B S, dantur, & supposita PS 100000, in iisdem partibus invenitur Longitudo Lineæ S B; pari ratione post alteram Martis Periodum, Terra existente in C, invenitur Linea S C, nec absimiliter Linea S D, S E, S F, differentiarque Observatorum Locorum Solis, sunt Anguli ad Solem A S B, B S C, C S D, D S E: Sic tandem ventum est ad hoc Problema Geometricum; *Datus tribus lineis, in uno Ellipseos Foco coeuntibus, tam Longitudine quàm positione, invenire Longitudinem Transversæ Diametri, cum Distantia Focorum*: Cujus Resolutio extenditur etiam ad Reliquos Planetas, si post Theoriam Motus Terræ cognitam, scrutemur (secundum Methodum propositam à Reverendiss. Episcopo *Sarisburyensi* in *Astronomia* ejus *Geometrica*, Lib. 2. Part. 2. Cap. 5.) tres Distantias Planetæ alicujus à Sole in positionibus suis. Quoniam vero Rev. *Episcopus* supponit Planetam ita ferri in Orbe suo, ut æqualibus temporibus æquales Angulos ad Focum alterum Ellipseos absolvat, & ei Calculum suum superstruit, non incongruum videtur, ostendere, quomodo id ipsum fieri posset absque ista suppositione, quam Observatio nos rejiciendam monet.

Sit S.

Sit S, Sol; A L B K, Orbis Terræ; P. Planeta, vel punctum in Plano Fig. 122.
 Eclipticæ, ubi Perpendicularis, à Planeta demissa, incidit; A B, Linea Ap-
 sidum Orbis Terræ: Observentur primo Planetæ, in P, Longitudo & Lati-
 tudo, simulque Solis Longitudo à Terra in K; & post Periodum ejusdem
 Planetæ. Terra existente in L, Observentur denuo positiones Planetæ, Solis-
 que, ut prius: jam ex Observatis Longitudinibus Solis & Aphelii Terræ,
 Anguli A S K, A S L, dantur, & consequenter Latera, S K, S L. Jam
 in Triangulo K S L, dantur Latera K S, L S, & Angulos K S L, quærun-
 tur Latus K L, & Anguli S K L, S L K: Deinde Triangulo K L P, dan-
 tur K L, K P L differentia Observatarum Longitudinum Planetæ, & P K L
 Differentia Angulorum S K L, ultimo inventi, & S K P, Elongationis Plan-
 etæ à Sole in prima Observatione, quæritur L P: Tum in Triangulo L S P,
 Latera L S, L P, & Angulus $\angle$ L S, Elongatio Planetæ à sole in secunda
 Observatione, dantur; Latus S P, & Angulus L S P, requiruntur quibus
 Inventis, ut S P ad L P ita Tangens Latitudinis Observatæ ex L, ad Tan-
 gentem Inclinationis sive Latitudinis ad Solem; & ut Co-sinus Inclina-
 tionis ad Radium, ita S P Curtata Distantia, ad Veram Distantiam Planetæ
 à Sole: Sic tandem invenimus Positionem & Longitudinem desideratam.
 Jam restat ut ostendam, quomodo ex Datis tribus Distantiis à Sole cum
 Angulis interceptis invenienda sit Media Distantia cum Excentricitate Elip-
 seos.

Sit S, Sol, & S A, S B, S C, tres Distantia in debita Positione, ductis- Fig. 123.
 que A B, B C, sit A B, Distantia Focorum Hyperbolæ, & S A - S B
 = E H, Transversa Diameter; quibus positis, describatur Linea ista Hyper-
 bolica, cujus Focus Interior est punctum A, extremitas Lineæ Longioris
 S A: pari modo sint B C, Foci Alterius Hyperbolæ, cujus Diameter S B
 - S C = K L; ex quibus describatur Linea Hyperbolica Focum habens In-
 teriorem in puncto B: Dico has duas Hyperbolas sic descriptas sese interfecare
 in puncto F, qui est alter Ellipseos quæsitæ Focus, ductaque Linea F A, F B,
 vel F C, S A, + F A, S B + F B, vel S C + F C æquabitur Transversæ
 Diametro, & S F est Distantia Focorum: quibus positis descriptio Ellipseos
 facillima est. Cum vero hujus Constructionis ratio non omnibus ita facile
 percipiatur, non abs re erit, illustrationem ejus aliquam afferre; ideo dico quod
 ex notissima Ellipseos proprietate $S B + F B = S A + F A$, & transpositis
 Equationis partibus $F B - F A = S A - S B$, ita ut etiam si F B & F A
 nos lateant, earum tamen differentia æqualis sit S A - S B, hoc est, E H,
 cumque sit ex natura Hyperbolæ, ut habeat quasvis duas lineas à suis Focis
 ad quodvis punctum in sua Curva constanter Differentes quantitate Transversæ
 Diametri; constat punctum F esse alicubi in Curva Hyperbolæ, cujus Di-
 ameter Transversæ æquatur S A - S B, & Foci, A B; Pari modo Demon-
 strari potest punctum F esse in Hyperbola cujus Diameter est S B - S C, &
 Foci B, C. Ergo necesse est, ut sit in Interfectione duarum istarum Hyper-
 bolarum, quæ, cum sese interfecent in unico solum puncto, clare ostendunt
 ubi sit Focus alter Ellipseos quæsitæ.

Jam ut id ipsum Analytice expediatur, puta Factum, fitque $FB = a$,
 $SA - SB = FB - FA = b$, $AB = c$, $SB - SC = FC - FB = d$,
 $BC = f$, fitque Sinus Anguli $ABC = S$, Co-sinus ejusdem $= s$.

Tum ut c ad b , ita $2a - b$ ad $\frac{2ab - bb}{c}$; & $\frac{2ab - bb + cc}{2c}$

$= BD$, per 36. 3. *Eucl.* & ut f ad d , ita $2a + d$ ad $\frac{2ad + dd}{f}$

& $\frac{ff - 2ad - dd}{2f} = BG$, per eandem; & ut minuat labor calculi,

fit $\frac{cc - bb}{2c} = g$, & $\frac{b}{c} = h$, similiter fit $\frac{ff - dd}{2f} = k$, & $\frac{d}{f} = t$,

tunc $BD = g + ha$, & $BG = k - la$; & quoniam in omni Triangulo

| | | | |
|---|---|--|---|
| $\left\{ \begin{array}{l} \text{Obtusangulo} \\ \text{Acutangulo} \end{array} \right\}$ | $\left\{ \begin{array}{l} \\ \\ \end{array} \right\}$ | $\left\{ \begin{array}{l} \text{Summæ} \\ \text{Differentiæ} \end{array} \right\}$ | $\left\{ \begin{array}{l} \\ \\ \end{array} \right\}$ |
| | | | |

Quadratorum Laterum, & Dupli Rectanguli Laterum in Co-sinum Anguli
 comprehensi ducti, erit $gg + 2gha + hbha + kk - 2kla + llaa$
 $+ 2gks - 2glsa + 2khsa - 2hlsaa$ æqualis quadrato DG : Sed
 DG æqualis est Sinui Anguli DFG , vel DBG , in a , id est FB , ducto,
 (est enim Quadrilaterum $FBDG$, Circulo, cujus Diameter est FB , In-
 scriptum;) ideo $SSaa = gg + 2gha + hbha + kk - 2kla + llaa$
 $+ 2gks - 2glsa + 2khsa - 2hlsaa$; quæ æquatio facile resol-
 vitur, cum non excedat Quadraticam Affectam, semperque componitur ex istis
 Quadratis & Rectangulis; signa tamen $+$ & $-$ ob diversam trium linearum
 constitutionem multa cautione sunt Rectangulis adhibenda.

The Obliquity of
 the Ecliptick from
 the Observations
 of the Ancients;
 by Dr. Ed. Ber-
 nard.
 Sept. An. 1684.
 N. 163. p. 721

XVII. 1. Obliquitatem Zodiaci repetit *Eratosthenes*, ante Natum *Christum*,
 Ann. 230. Grad. 23. atque insuper $51'. 19''. 31'''$. $5''$.

Distantia enim Tropicorum ipsi fuit $\frac{1}{2}$. Circuli Meridiani sive $47^\circ 19'$.
Ptol. Meg. Σωτ. p. 18. 21. Quare *Λόγων* *Eratosthenica* minor erat *Ptole-*
maico tantum $\frac{1}{4}$ unius Minuti Secundi, re sane contemnenda.

Eratosthenes apud *Cleomedem*, *Ricciolo* eruente, (supra Grad. 23.) $46'. 00''$.
Eratosthenes a *Ricciolo* quasi correctus, $31'. 5''$.

Hipparchus, (ante *Christ.* 10.) *Eratosthenicam* retinuit. *Ptol. Σωτ. Meg.*
 p. 18. & p. 60.

Theonis, *ὁ ἀνέβη τὸ ἀντικείμενον*. $51'. 19''. 31'''$. $5'''$.

Tabulæ tamen *Chouaresmica*, conditæ post *Christ.* 830. exhibent Canoni-
 cam *Λόγων Alexandrinorum*, juxta MS. Lat. D. *Hatoni*, $51'$.

Pytheas Massiliensis, ante *Christ.* 324. *Ricciolo* $52'. 41''$.

Aristarchus, ante *Christ.* 280. *Illustri Savilio* supputante, $51'. 20''$.

Aristarchus,

Aristarchus, ex ratiocinio *Riccioli*, $30', 00''$.

Strabo Geographus, p. 93. post *Christi*. 30. $\frac{4}{5}$. Circuli, sive præter Gradus 23, adhuc unius, sive $60'$.

Nec aliter *Geminus* (tempore *Christi*) cap. IV. Element. Astron. Et *Tavius* c. 26. atque *Proclus* de Sphæra. Indeque sive Astrologi, apud *Noddamum* Arabem, *Abrahamum Abenesdram*, &c.

Noddamus Astronomus, qui floruit circa Ann. Dom. 1200. notat $\Delta\delta\epsilon\omega\sigma\iota\nu$ neque observatam unquam Majorem Gr. 23, neque Minorem $23^{\circ} 33'$. continuo tamen decrevisse.

Cl. Ptolemaus, post *Christi*. 140. sæpius expertus, & Crico suo & Plinthide, semper reperit proxime eandem cum *Eratosthenica*, $51'. 20''$.

Distantia enim Tropicorum versabatur inter 47° . & 47° . Sed elegit pro *Selidio* suo $47. 42. 40''$. *Σωτ.* *Μαγ.* p. 18, 20, 21. & p. 27. capit pene Medium, $\mu\omicron\sigma\iota\varsigma. \kappa\gamma. \nu\delta. \kappa''$. *ἑγχεσθαι*. Nec aliter in Hypothesibus Planetarum. Theo vero in Canonibus *ἑγχεσθαι* facilitatis causa præterit Minuta Secunda. Fallitur autem *Ricciolus*, dum ex Climate *Rhodi* colligit $\Delta\delta\epsilon\omega\sigma\iota\nu$ modum pro *Ptolemaeo* $23^{\circ} 30'$.

Pappus Alexandrinus (post *Christi*. 390.) l. 6. Theor. $35'$. *Ricciolo* $30'$.

Pappus, Fr. *Commandino* colligente $50'. 00''. 00'''$.

Theo, (post *Christi*. 370.) p. 88. accuratius $51'. 20''. 00'''$.

Alibi numero rotundo, ut p. 57. & passim in Canonibus suis *ἑγχεσθαι* nondum vulgatis $51', 00''. 00'''$.

Almamoni Princeps Ann. *Christi* 825. *Hegira* 210. $23^{\circ} 35'$. *Grav.* p. 44. ex *Ebn-Shatir* Damasceno MS. *Seld.* adsistentibus ei plurimis Astronomis. Ita enim refert *Abnesdras* MS. Lat. in Archivis *Digbeanis*. Insuper *Astronomus Incertus* in Arch. *Seld.* affirmat *Jahia Ebn Albimansur* cum multis aliis Philosophis, tempore *Almamoni* τὸ $\Delta\delta\epsilon\omega\sigma\iota\nu$ Experimento

deprehendisse, $23^{\circ} 35'$.

Idem tradit de observatis *Almamoni* Doctissimus *Al Noddam* in Commentariis suis ad Astronomica *Hosein Nisaburiensis*. Imo addit ille eodem Ævo sæpius observasse *Beni Musa* modum eundem $23^{\circ} 35'$ *Bagdadi* in Campis. MS. Arab. Coll. S. *Joan.* *Oxon.* Hunc etiam placuisse plerisque sequentium Astronomorum. Sane in eo quiescit *Alferganus* Astron. suæ c. 3.

Mohammed Ebn. Gaber Al. Batanius, (*Al. Bategnius*), *Racca*; *Ricciolo*, A. D. 880 Illi. *Savilio*, 890. *Gravio* p. 44. 882. *Hegira*, 269. Obiit ille *Hegira*, 317. A. D. 929. *Abolfaragi* Hist. p. 191. $35'. 00''$.

Al Batanius hac in re suas τῆς *ἑγχεσθαι* præferre non dubitat *Ptolemai* dictis, c. 4. atque se adjutum longissima *Alhidada*, seu Regula Parallactica ad formam *Ptolemaicæ* cum cura & assiduitate reperisse apud *Racoam* Tropico- rum Distantiam, $47^{\circ} 10'$. (hoc est $59^{\circ} 36'$. minus $12^{\circ} 26'$) atque adeo Latitudinem *Pacca* 35° . quam tamen *Viabegus* statuit $36^{\circ} 10'$. *Schickardus* apud *Curtium*, (p. 33.) & *Ricciolus*, 36° .

Thabet Ebn Corra (Ricciolo, A. D. 1210. rectius 901, *Hegira* 289.) reperit $\Delta\delta\xi\omega\sigma\iota\nu$ 33' 30''.

Abul Hossein Ebn Suphi, 35'. 00''.

Abul Waffi Albuziani, & *Abn Hamed Saganienfis*, Vir Ingeniosissimus, (A.D. 987. *Heg.* 377.) Bagdadi repererunt $\Delta\delta\xi\omega\sigma\iota\nu$ tantum non 35'.

Ita & auctor *يحيى بن جابر* Perfa in Arch. *Seld.* 35'.

Tabulæ iridem Perficæ *Chrysococca*, 35'.

Al Batrunius Riban, (A. A. 995. *Hegira* 385. *Abolfaragius* hunc ponit ad *Hegira* 463. seu A. D. 1070.) usus Quadrante, cui Radius xv. cubitorum Grav. p. 44, ex Cod. Arab. *Birunii*. 35'.

Verum *Abu Joaser Alchazan*, cum Socio suo *Abufaldo Harwanensi* apud *Edeffam*, & istius *Ævi* alii (A.D. 970.) observarunt $\tau\lambda\omega$ $\Delta\delta\xi\omega\sigma\iota\nu$ ad 23°. 35'. plane non accessisse, sed paulo fuisse Minorem.

Almaon F. Almanforis (A. D. 1140 *Ricc.*) 30'. 30''. at ille *Clavio* & *Mestlino* 33'.

Ismael Abulfeda Princeps *Hama*, (A.D. 1311. *Hegira* 711. in Tabulis suis MS. Arab. Coll. S. *Joan.* retinet forte ab *Almamonis* auctoritatem 35. 00'.

Propharius Judeus (A.D. 1300. *Ricc.* 1303. *Mestlino* apud *Curtium*, p. 40. 230 annis post *Arzahelem*, inquit *Copernicus*) & *Ricciolo*, & MS. Coll. *Merton.* 32'. 00''.

Abu Mahmud Al Chogandi (A.D. 992. *Hegira* 382. tempore *Fecroddanle*, Sextante cujus Radius erat Cubitorum XL. limbusque in minuta secunda distinctus, invenerat $\Delta\delta\xi\omega\sigma\iota\nu$ Minorem quam unquam captaverat aliquis Majorum suorum, nimirum 32'. 21''.

Hinc *Noddamus* Astronomus adfirmat MS. Coll. *Joan.* Solis Declinationem Maximam vix unquam Minorem fuisse repertam 23°. 33'.

Arzachel Hispanus, (Gravio p. 44. A.D. 1689. *Hegira* 482. *Ricciolo* 1070. *Mestlino* apud *Curtium*, p. 35. 1075. *Copernico*, l. 3. c. 6. Annis 190. post *Al Batanium*) proposuit $\Delta\delta\xi\omega\sigma\iota\nu$ 23°. 33' 30''. Ita MS. Coll. *Merr. Oxon.* ubi dicitur differentia 17'. 30''. intercedere inter $\Delta\delta\xi\omega\sigma\iota\nu$ *Ptolemai* & *Arzachelis*.

Apud *Maragam* Nobilissimus Perfa *Chojah Nasiroddinus Tusensis*, A.D. 1269. *Hegira* 668. (at *Gravio*, p. 44. 1261. *Hegira* 660.) accuratissime observavit $\tau\lambda\omega$ $\Delta\delta\xi\omega\sigma\iota\nu$ 23° 30'. 00''.

Hæc est minima ex Maximis Solis Declinationibus, quæ ad hunc usque diem reperta fuit, ait Doctiss. Commentator ad *Astronomica Hossein Nisaburiensis*

Ebn Shatir Damascenus, MS. *Seld.* A. D. 1363. ait se emendasse $\Delta\delta\xi\omega\sigma\iota\nu$, non neglecta Solis Parallaxi, quæ Horizontalis capta est 2'. 59''. Huic Solis Max. Declin 23°. 31'. 00''.

Olochebus Princeps, A. D. 1437. *Hegira* 841. cum *Aly Cusbgio* aliisque Astronomis, usus summa cura, & maximis Instrumentis (vide *Gravium*, p. 44.) reperit $\Delta\delta\xi\omega\sigma\iota\nu$ 23°. 30' 17''. Ita MSS. Coll. D. *Joan.* & *Bibliotheca Savilliana*, nam MS. *Seld.* exhibet 23°. 30'. 27'.

Rabbi Moyses Ben Maimon Judæorum Doctissimus ait in *Jad. de Consecratione Calendarum*, c. ult. sect. 4. Maximam Zodiaci Obliquitatem fuisse, A. D. 1174. $23^{\circ} 30' 45''$.

Scias vix dimidiam partem Astronomorum Orientalium quorum Scripta Academiæ Oxon. Bibliothecis servantur à me consultam fuisse. Ex hisce autem Observatis aliisque quæ mecum adhuc cis vulgus servo, unam eandemque suspicor fuisse à primordio Mundi $\Lambda\delta\epsilon\tau\epsilon\sigma\nu$ Zodiacam. Æva enim recentia, quod vides, melioribus Organis errorum excessumque veteris Astronomiæ probe correxerunt.

2. Whether the Poles and Axis of the Earth be really fix'd in the Globe, or subject to be transferr'd from place to place, is an old Enquiry, though now lately revived by Mr. *Hook* in his ingenious Essays upon the great Mutations and Catastrophes which in all appearance have happened to the Earth's Surface. A necessary Consequence of such a Translation of the Poles would be the Change of the Latitudes of Places, which would Encrease in those Regions towards which the Poles approach, and Decrease in those from which they recede: and under the Meridian 90 Degrees removed from that in which the Poles shift, the Latitudes continuing the same, the Meridian Line would only alter; but not two Places considerably differing in Longitude can be supposed, wherein, if there be any sensible Motion of the Poles, it shall not be perceived by the Alteration of the Latitude of one or both of them.

The Obliquity of the Ecliptick, and Elevation of the Pole continue unaltered; by Nov. An. 1687. N. 190. p. 403.

The accurate M. *Wurtzelbauer*, has lately furnished us with the Means of examining this Hypothesis by Observation, having sent us the Meridian Altitude of the Sun taken at *Nurenburch* about the two Solstices in the Year 1686. *Jun.* 10. He found the Meridian Altitude of the Sun $64^{\circ} 2' 20''$. and the next day, $64^{\circ} 2' 25''$. and on *Decemb.* 14. (three days after the Solstice, wherein the Sun was got two Minutes higher) he found the Meridian Altitude $17^{\circ} 9' 10''$. wherefore the Solstitial Altitude was $17^{\circ} 7' 10''$. These Heights were taken by an Instrument of 6 Foot Radius of Brass; and the Skill and Diligence of the Observer is not to be doubted.

To compare with these, I find among *Bernard Walther's* Observations made in the same City of *Nurenburch* two hundred Years before, viz. In the Year 1487. that the Meridian Altitude of the Sun in the Summer Solstice was observed by the parallactick Instrument of *Ptolemy*, whereby the Chord of the Sun's Distance from the Zenith was observed 44890 parts of 100000 Radius; the same being observed by the Concurrence of the Observations of several Years both before and after. The Arch answering to this Chord gives the Sun's Distance from the Zenith $25^{\circ} 56' 30''$ and consequently the Meridian Altitude, its Complement to a Quadrant $64^{\circ} 3' 30''$. Again, The same Year 1487. the Chord of the Meridian Distance of the Sun from the Zenith, on the day of the Winter Solstice, was found 118790, confirmed likewise by many subsequent Observations; the Arch answering to this Chord is $72^{\circ} 52' 40''$. and its Complement $17^{\circ} 7' 20''$. the Meridian Height of the Sun in the Winter Solstice.

Hence

Hence it appears, That the Solstitial Heights were very nearly the same at *Nuremburg* 200 Years ago as now they are, that of the Summer Solstice being but one Minute differing, the other only 10'. both which may possibly arise from the Defects of the Instruments of these Observers, being made with Plain Sights : But what I shall necessarily conclude from hence is, That if there be such a Motion of the Poles, it is either very slow, or else nearly at Right Angles to the Meridian of *Nuremburg* ; in which latter case, the Latitudes of Places about *Tunking, Siam, Malacca, and Java*, on the one side, and in our *American Plantations of New-England, Virginia, Jamaica, &c.* on the other, ought to change fastest ; but I have never yet heard of any such thing observed by any of our Navigators ; whence, if there be such a Change of the Earth's Poles, it must necessarily require a long time to become sensible.

Besides, from these Observations, it appears, That the Obliquity of the Ecliptick has continued unaltered for these 200 Years last past ; that is to say, that the Angle which the Earth's Axis makes with the Plain of the Ecliptick or Orb wherein she moves annually round the Sun, has been without sensible Change in all that time ; which will be very hard to conceive, if we allow a Translation of the Earth's Poles ; for the Direction of the Axis being perfectly at Liberty, it must be purely casual, if it so hit, that after such Change, it make the same Angle with the Ecliptick as before.

A farther Argument of this Slowness of the Change of the Poles, is the Latitude of *Alexandria*, the Habitation of those famous Astronomers of Antiquity, *Erasthenes, Timocharis, Hipparchus, and Ptolemy*, and for that Reason it may be concluded, that this, of all the Latitudes the Ancients have left us, ought to be one of the most Correct. This by *Ptolemy* is said to be $30^{\circ}, 58'$ North, (which he uses in all his Computations in his *Almagest*, and seems derived from the proportion of the Gnomon to its Equinoctial Shadow, as 5 to 3. but in his *Geography*, 31° . just. In the Year 1638. the Curious and Ingenious Mr. *Graves*, when he went to visit the *Egyptian Pyramids*, of which he has given so good an Account, did, with a sufficient Instrument, observe the Latitude of *Alexandria*, and found it $31^{\circ}, 4'$. or 6 Minutes more than it is reputed by *Ptolemy*, and before him by *Erasthenes* ; so that in about 2000 Years the Latitude of *Alexandria* has altered only a few Minutes, and so few, that the Accuracy of the Observations of the Ancients may well be questioned : but both being granted, this Motion will amount to no more than a Degree in 20000 Years.

This is said not with intent to invalidate what Mr. *Hook* hath from so good Grounds advanced, viz. That the Ball of the Earth, at least the Fluids thereof, being necessarily of the Figure of a *Spheroides Prolatus*, or flat Oval, whose shortest Diameter is the Axis, and greatest Circle the Equinoctial ; if the Poles be supposed changed, the Equinoctial will be so too ; and consequently the Water must rise and cover those Parts from which the Poles recede, and fall off, and leave bare those Places towards which the Poles approach. By this Means it may be accounted for, how such strange Marine things are found on the Tops of Hills, and so deep under Ground ; and scarce
any

any other way. But from these, and the like Observations, it will follow, That if these Inundations are produced by any regular Motion of the Poles, it would require a prodigious number of Ages to effect the Changes we may be certain have been. Besides, if the Access and Recess of the Sea were after such a gradual manner, as when produced by such an easie Translation of the Poles, as can by Observation be admitted, those Inundations could never be fatal to the Inhabitants, for that they would always give notice of their coming, so that the People might provide for their Safety. But the *Holy Scriptures*, and Pagan Tradition, do unanimously agree, That the last great *Deluge* was brought to pass in a few Days, with no previous Notice; so that the Account we have thereof could not, by this *Hypothesis* be made out, without the Supposition of a great and sudden Alteration in the Poles of the Earth's Diurnal Revolution; for which, whether we should have recourse to the intelligent Powers that first impress'd this whirling Motion on the Ball, or leave it to be performed naturally, by the casual Shock of some transient Body, such as a Comet, or the like, whereby the former Axis might be lost, and a New Revolution produced, differing both in Time and Position from the Old, I shall not undertake to dispute; such a Supposition would include likewise a Change of the Length of the Year, and Eccentricity of the Earth's Orb; for which yet we have no sort of Authority.

1. As I was wondering how an ordinary Mathematician could miss so easie a thing as the drawing a true Meridian, so far as in the instance of the old Meridian in the Church of St. *Petronio* in *Bononia*, which is found by M. *Cassini* to vary 8 or 9 Degrees from the true Meridian of the Place; and in that of the Meridian of *Uraniburge*, which is found by M. *Picart*, and others, to vary 18'; I hit upon this Thought, that Meridians must needs vary. For you know, that (taking it for granted that the Earth moves, &c.) besides the Diurnal and Annual Revolutions, there must be also a third to account for that slow Motion of the Fixed Stars, upon the Poles of the Ecliptick, in about 25000 Years; which is solved by the Direction of the Earth's Axis from one Point to another of the polar Circle. And that Direction being nothing but a certain Wobble in the Earth's Motion, must needs make the Noon-shade of a Perpendicular not lie always in the same Line.

A supposed Alteration of the Meridian Line.
n. 255. p. 285.
June An. 1699.

2. This being a new Suggestion, deserves to be consider'd: For it is not probable that so careful a Man as *Tycho*, and those concerned in the Church of St. *Petronio*, should be so much mistaken in the Meridian Line. But if there be ought of this Nature, it must arise from a Change of the Terrestrial Poles (here on Earth of the Earth's Diurnal Motion; (not of their pointing to this or that of the Fixed Stars:)) For if the Poles of this Diurnal Motion remain fixed to the same Place on the Earth, the Meridians, which pass thro' these Poles) must remain the same.

Consider'd by Dr. Wallis.
Ibid p. 286.

XVIII. I have had the good hap to measure the Distances of *Mars* from two Stars the same Night; whereby I find, that his Parallax was very small, certainly not 30": So that I believe the Sun's Parallax is not more than 10".

The Parallax of the Sun, by Mr. Flamsteed.
n. 89 p. 1419.
Nov. An. 1672.
n. 96. p. 6100.

To find the Sun's
Ingress into the
Tropical Signs;
by Mr. Edmund
Halley
N. 215, p. 12.
Feb. An. 1695.

XIX. It may perhaps pass for a Paradox, if I should assert, That it is an easier Matter to be assured of the Moments of the Tropicks, or of the Times of the Sun's entrance into *Cancer* and *Capricorn*, than it is to observe the true Times of the Equinoctials or Ingress into *Aries* and *Libra*. But I here design to shew a Method to find the Moment of the Tropicks capable of all the exactness the most Accurate can desire; and that without any consideration of the Parallax of the Sun, of the Refractions of the Air, of the greatest Obliquity of the Ecliptick, or Latitude of the Place: All which are required to ascertain the Times of the Equinoctial, from Observation; and which being faultily assumed, have occasioned an Error of near three Hours in the Times of the Equinoctials deduced from the Tables of the Noble *Tycho Brahe* and *Kepler*, the Vernal being so much later, and the Autumnal so much earlier than by the *Calculus* of these famous Authors.

Now before we proceed, it will be necessary to premise the following *Lemmata*, serving to demonstrate this Method; viz.

1. That the Motion of the Sun in the Ecliptick, about the Time of the Tropicks, is so nearly Equable, that the Difference from Equality is not sensible, from 5 days before the Tropick to 5 Days after, by reason of the nearness of the *Apogeon* of the Sun to the Tropick of *Cancer*.

2. That for 5 Deg. before and after the Tropicks, the Differences whereby the Sun falls short of the Tropicks, are as the *Versed Sines* of the Sun's Distance in Longitude from the Tropicks, which *Versed Sines* in Arches under 5 Degrees, are beyond the utmost Nicety of Sense, as the Squares of those Arches. From these two follow a third,

3. That for 5 Days before and after the Tropicks, the Declination of the Sun falls short of the utmost Tropical Declination, by Spaces which are in *Duplicate* Proportion, or as the Squares of the Times by which the Sun is wanting of, or past, the Moment of the Tropick.

Hence it is evident, That if the Shadows of the Sun, either in the Meridian, or any other Azimuth, be carefully observed about the time of the Tropicks, the Spaces whereby the tropical Shade falls short of, or exceeds those at other times, are always proportionable to the Squares of the Intervals of Time between those Observations and the true Time of the Tropick, and consequently if the Line, on which the Limits of the Shade is taken, be made the Axis, and the correspondent Times from the Tropick *Expounded* by Lines, be erected on their respective Points in the Axis as Ordinates, the extremities of those Lines shall touch the Curve of a *Parabola*. Thus *a, b, c, e*, being supposed Points observed, the Lines *a B, b C, c A, e F*, are respectively proportional to the Times of each Observation before or after the tropical Moment in *Cancer*.

This premised, we shall be able to bring the Problem of finding the true Time of the Tropick by three Observations, to this Geometrical one. *Having three Points in a Parabola A, B, C, or A, F, C, given, together with the Direction of the Axis, to find the Distance of those Points from the Axis.*

Of

Of this there are two Cases ; the one, when the time of the second Observation B is precisely in the middle between A and C : In this Case, putting t for the whole Time between A and C, we shall have Ac , the Interval of the remotest Observation A, from the Tropick, by the following Analogy.

As $2ac - bc$ to $2ac - \frac{1}{2}bc$; so is t , or A E to Ac , the Time of the remotest Observation A, from the Tropick.

But the other Case, when the middle Observation is not exactly in the middle between the other two Times, as at F, is something more operose, and the whole Time from A to C, being put $= t$, and from A to F, $= s$, $ce = c$, and $bc = b$, the Theorem will stand thus, $\frac{tcc - bss}{2tc - 2bs} = Ac$, the Time sought.

To illustrate this Method of Calculation, it may perhaps be requisite to give you one or two Examples.

Ann. 1500. Bernard Walser in the Month of June at Nuremberg, observ'd the Chord of the Distance of the Sun from the Zenith, by a large *Parallactic* Instrument of Ptolemy, as follows ;

| | | | | | | |
|----------|-------|---|-----|---|----------|--------|
| June 2. | 45467 | } | and | { | June 8. | 44975. |
| June 9. | 44934 | | | | June 12. | 44883. |
| June 16. | 44990 | | | | June 16. | 44990. |

In both these Cases the Middle Term is exactly in the middle between the Extrems, and therefore in the former three, $ac = 533$, $bc = 477$, and t , the Time between, being 14 Days, by the first Rule, the Time of the Tropick will be found by this Proportion ; as 589 to 827 $\frac{1}{2}$, So t , or 7 Days to 9 Days, 20 h . 2'. Whence the Tropick, *Ann.* 1500, is concluded to have fallen June 11 20 h . 2'. In the latter three ac is $= 107$, and $bc = 15$, and the whole interval of Time is 8 Days $=$ to t ; whence, as 199 to 206 $\frac{1}{2}$; so is 4 Days to 4 h . 3 h . 37'; which taken from the 16th Day at Noon, leaves 11 d . 20 h . 23'. for the Time of the Tropick, agreeing with the former to the third part of an Hour.

Again, *Ann.* 1636. Gassendus at Marfeilles, observed the Summer Solstice by a Gnomon of 55 Foot high, in order to determine the Proportion of the Gnomon to the Solstitial Shade, and he hath left us these Observations, which may serve for an Example for the Second Rule.

| | | | | | | |
|---------|---|------------------|---|-------|---|-------------------------------------|
| June 19 | } | S r N. shadow | { | 31766 | } | Parts, whereof the Gnomon was 89428 |
| 20 | | | | 31753 | | |
| 21 | | | | 31751 | | |
| 22 | | | | 31759 | | |

These being divided into two Setts of three Observations, each, *viz.* The 19th, 20th, and 22d, and the 19th, 21st, and 22d, we shall have in the first three, $c = 13$, and $b = 7$, $t = 3$ Days, $s = 1$; and in the second, $c = 15$, and $b = 7$, $t = 3$, and $s = 2$. Whence, according to the Rule, the 19th Day at Noon, the Sun wanted of the Tropick a Time proportionate to one Day, as $ttc - ssb$ to $2tc - 2bs$, that is, as 100 to 64 in the first Sett, or 105 to 62 in the second Sett; that is, $1^d. 17^h. 15'$ in the first, or $1^d. 17^h. 25'$ in the second Sett: So that we may conclude the Moment of the Tropick to have been *June* $10^d. 17^h. 20'$ in the Meridian of *Mar-seilles*.

Now that these two Tropical Times thus obtained, will be found to confirm each others Exactness from their near Agreement, appears by the Interval of Time between them, *viz.* $1^d. 2^h. 30'$ less than 136 *Julian* Years, whereof $1^d. 1^h. 8'$ arises from the defect of the length of the Tropical Year from the *Julian*, and the rest from the Progression of the Sun's *Apogeon* in that Time; so that no two Observations made by the same Observer in the same Place can better answer each other, and that without any the least Artifice or Force in the managment of them.

What were the Methods used by the Ancients to conclude the Hour of the Tropicks, *Ptolemy* has nowhere delivered; but it were to have been wished that they had been aware of this, that so we might have been more certain of the Moments of the Tropicks we have received from them, which would have been of singular Use to determine the Question, Whether the Sun's *Apogeon* be fix'd in the Starry Heaven; or if it move, What is the true Motion thereof? It is certain, that if we take the Account of *Ptolemy*, the Tropick said to be observed by *Euctemon* and *Meton*, *Junii* 27 *Maie*, *Ann.* 432 *Ante Christum*, can no ways be reconciled, without supposing the Observation made the next Day, or *June* 28. in the Morning. And *Ptolemy*'s own Tropick, observed in the third Year of *Antoninus*, *An. Christ.* 140. was certainly on the 23d, and not on the 24th Day of *June*, as will appear to those that shall duly consider and compare them with the length of the Year deduced from the diligent and concordant Observations of these two great Astronomical Genii, *Hipparchus* and *Albatani*; established and confirmed by the Concurrence of all the Modern Accuracy. For these Observations give the Length of the Tropical Year, such as to anticipate the *Julian* Account only one Day in 300 Years; but we are now secure, that the said Period of the Sun's Revolution does anticipate very nearly 3 Days in 400 Years; so that the Tables of *Ptolemy*, founded on that Supposition, do Err about a whole Day in the Sun's Place, for every 240 Years. Which principal Error in so Fundamental a Point, does vitiate the whole Superstructure of the *Almagest*, and serves to convict its Author of want of Diligence, or Fidelity, or both.

But to return to our Method, the great Advantage we have hereby, is, That any very high Building serves for an Instrument, or the top of any high Tower or Steeple, or even any high Wall whatsoever, that may be sufficient

ficient to intercept the Sun, and cast a true Shade, and make the Spaces large and fair, though the Height and Distance of the Building, and Position of the Plain, upon which you receive the Shade, and of the Line on which you measure the Spaces, be not exactly known. But it is convenient that the Plain on which you take the Shade be not far from Perpendicular to the Sun, at least not very Oblique, and that the Wall which casts the Shade, be straight and smooth at the top, and its Direction nearly East and West. The principal Objection is, That the *Penumbra*, or Partile Shade of the Sun is in its Extrems very difficult to distinguish from the true Shade, which will render this Observation hard to determine nicely. But if the Sun be transmitted through a Telescope, after the manner used to take his *Species* in a *Solar Eclipse*, and the upper half of the Object-Glass be cut off by a Paper pasted thereon, and the exact upper Limb of the Sun be seen just emerging out of, or rather continging the Species of the Wall, (the Position of the Telescope being regulated by a fine Hair extended in the Focus of the Eye-Glass) I am assured, that the limit of the Shade may be obtained to the utmost Exactness. I shall only further advertise, That the Winter Tropick by this Method may be more certainly obtained than the Summer's, by reason that the same Gnomon does afford a much larger Radius for this manner of Observation.

XX. I have found it necessary to make new Solar Numbers, because in my old, I have neglected to apply Refractions in all the Altitudes above 30 Degrees; wherein yet Reason and some little Experience hath shewed me, they are not insensible. I found S. Cassini's Observations, which I took from Ricciolus his *Astronomia Reformata*, much more Accurate than Tycho's, and therefore sought out Numbers that might answer them. The *Apogoeum* I found it necessary to promote 44 Minutes; so that *Anno Incunte* 1655. it might be in $\odot$ $7^{\circ} 30' 00''$. and to make the greatest Equation only $1^{\circ} 54' 13''$. whereby I found the *Phenomena* would be answered much more accurately than I expected, and as near, all things considered, as I could desire.

The Solar Numbers corrected by Mr. J. Flamsteed. n. 110. p. 220. Jan. Ann. 1674.

But still I was uncertain, whether the Refractions in the said Cassini's Tables were just Measures or not, and I had no Conveniencies for making Trial. At last I thought on this Expedient, which fully satisfied me; viz.

I considered, That if some of those Observations of the Distances of $\odot$ from the $\odot$ by Day, and from the Stars in the Night preceding or following, were skilfully Examined, they might shew me the true Quantity of the Equations of the Sun's Orb, or rather the Difference of his Mean and Equal Motion. I turned over his *Progymnasmatia*, and pitched on two. The first made *Ann. 1585, March 5. 4^h. 42'* and *7^h. 12' post Meridiem*: whereby I found the $\odot$ at $4^h 42'$ was $94^{\circ} 47'$ in Antecedence of the *Lucida Calcis* II. The second made *Ann. 1585, Septemb. 15. 5^h. 15'* and *6^h. 55' Mane*. Where-from, (applying and considering the Refractions

fractions in both) I found the Sun at $6^{\circ}.55'$. to be $7^{\circ}.30'$. in Consequence of the *Lower Head* of Π . The Difference of Longitude betwixt these two Stars is $17^{\circ}.59'$: And therefore now the Sun in Consequence of the *Lucida Calcis* $11^{\circ}.92'.29'$. So that the Sun's apparent Motion betwixt the Year 1582. *Mar.* 5. $4^{\circ}.42'$. and the Year 1585. *Septemb.* 15. $6^{\circ}.55'$. *Manè*, (besides the whole Revolutions) was $187^{\circ}.16'$. but the Mean Motion is $191^{\circ}.2'$. greater than the Apparent by $3^{\circ}.46'$. which parted in proportion to the Equation of the Earth's Motion, collected for those Times from my New Table, gives the greatest Equation of the Orb, $1^{\circ}.54'.15''$. consenting to my wonder, without any wresting of the Observations with that, which I deduced from *Cassini's* Correct Meridional Altitudes.

The Sun's Motion, by the Tables which I now use, grounded on this Equation, is less than *Tycho's* by no less than 9. That great Equation made him commit no small Errors, and put him upon strange Shifts to hide and solve them. So that all his Observations of the Planets in their Oppositions to the Sun, are to be Corrected, before we attempt to represent them by Numbers: For his Errors in the Sun's Place made him Err sometimes 5 or 6 Hours in the time of the Opposition; which must be Reformed.

The Equality of Natural Days;
by
Professor of Mathematicks at Seville.
N. 118. p. 426
Oft. An. 1676.

XXI. Non parvum adhuc est Dissidium inter Astronomos, quanta sit in a quando Tempore Prosthaphæresis, ita ut *Longomontanus* fateatur, nullam in Astronomico Pulvere majorem Difficultatem se invenisse; quod cum notarem, animadverti in quibusdam Cœli Observationibus à me factis, quid ex illis inveniret; & cum mihi esset Horologium Rotatile Pendulum, admodum exactum, linea Meridiana artificialiter Constituta, examinabam Solis in Meridianum Ingressum, singulis diebus, cum quo ad amissim Horologium meum congruebat, & si discrepabat aliquando, rarissime, duobus minutis discriminabat, quod, cum opus erat emendabam. Quare per triennium continuando & quotidie Solem in Meridiano observando, cum licebat (quod in hac Regionis parte sæpe sæpius fit) inveni tandem nullam Diem Naturalem Longiorem Revolutionem, in Uno vel Alio Anni Tempore, alia Die habuisse, unde intrepide dico omnes Dies Naturales æquales esse, & si adhuc aliqua Differentiolarum intercedit, non esse Sensibilem. Hæc volui notum facere, ut Astronomos hoc Scrupulo liberarem, quod tam multos torset & indies torquet, quanquam *Tychonica* Aequatio propter Eclipticæ Obliquitatem non sit rejicienda.

Refuted; by Mr. Flamsteed.
Ibid. p. 430.

2. Dies quomodo Aequales esse possint, & tamen Aequatio *Tychonica* admitti, vix me capere fateor. Ob inæquales etenim æqualium Eclipticæ partium Rectas Ascensiones, dies unus Aequinoctialis Tropico uno Brevior erit Scrupulis Horæ secundis 40; & Dies 14 Tropici, totidem Aequinoctialibus. Longiores sunt sexta Horæ parte, seu Scrupulis primis 10. Hanc autem Differentiam Majorem credo, quam ut eam in Observationibus suis non perciperet *Professor Hispanensis*, proindeque ipsum in examinandis iis *Tychonicam* Temporis Aequationem adhibuisse autumem.

Sed & ponamus, *Æquales* esse *Primi Mobilis Revolutiones* (quod nulli, qui *Ptolemaicam Hypothesin* admittant, unquam iverunt inficias) necessario consequitur, nec *Æquationem Temporis*, ad inæquali *Solis in Orbita sua Incessu* enatam, rejiciendam esse; Eteniam cum *Apogæus* quotidie promoveatur tantum $57'. 10''$. *Perigæus* vero $61'. 15''$; *Apogæus* equidem citus $16''$, (seu Tempore absumpto dum primum Mobile revolvit $4'. 5''$.) a Meridie in Meridianum Diei sequentis recurrit, quam *Perigæus*: Attamen quandoquidem progenita ex hac causa *Æquatio* tardius admittit *Diurnum Incrementum*, Scil. $8''$. quotidie, ad summum cum velocissima, & vix Diebus 15 ad duorum *Scrupulorum* *Quantitatem* excrefcit, eâ, cujus ille meminit, duorum *Scrupulorum* *Emendatione*, in *Horologio suo* ablatam credo: de qua videat propterea *Vir Doctissimus*.

Demum vero, si in *Copernicanam Hypothesin* fit pronior, quam in *Ptolemaicam*, in ea etiam, suppositis *Terræ Isochronis Revolutionibus*, eadem consequuntur *Æquationes*. Fateor equidem, amoveri posse, & in contrarium trahi, ab *Inæquali incessu Solis in Orbita sua* proveniente *Temporis Æquationem*, si inæquales *Terræ vel primi Mobilis* perinde enim est utrum horum statuerimus: *Revoluciones* supponamus: Sed si *Temporis Naturam* bene perpendat, facile intelliget, impossibile esse, omnem ejus *Inæqualitatem* removeri.

| | | | | | | | | | | | | |
|-----|----|-----|---|-----|---|-----|---|-----|---|-----|---|-----|
| 12 | 11 | 24 | 1 | 48 | 1 | 14 | 1 | 42 | 1 | 42 | 1 | 51 |
| 04 | 1 | 03 | 1 | 24 | 1 | 12 | 1 | 08 | 1 | 22 | 1 | 13 |
| 07 | 1 | 23 | 1 | 47 | 1 | 14 | 1 | 40 | 1 | 41 | 1 | 50 |
| 09 | 1 | 21 | 1 | 45 | 1 | 11 | 1 | 38 | 1 | 39 | 1 | 48 |
| 01 | 1 | 20 | 1 | 44 | 1 | 10 | 1 | 37 | 1 | 38 | 1 | 47 |
| 17 | 1 | 04 | 1 | 02 | 1 | 04 | 1 | 22 | 1 | 03 | 1 | 23 |
| 12 | 1 | 05 | 1 | 03 | 1 | 05 | 1 | 23 | 1 | 04 | 1 | 24 |
| 14 | 1 | 06 | 1 | 04 | 1 | 06 | 1 | 24 | 1 | 05 | 1 | 25 |
| 08 | 1 | 05 | 1 | 03 | 1 | 05 | 1 | 23 | 1 | 04 | 1 | 24 |
| 06 | 1 | 04 | 1 | 02 | 1 | 04 | 1 | 22 | 1 | 03 | 1 | 23 |
| 19 | 1 | 07 | 1 | 05 | 1 | 07 | 1 | 25 | 1 | 06 | 1 | 26 |
| 20 | 1 | 08 | 1 | 06 | 1 | 08 | 1 | 26 | 1 | 07 | 1 | 27 |
| 21 | 1 | 09 | 1 | 07 | 1 | 09 | 1 | 27 | 1 | 08 | 1 | 28 |
| 22 | 1 | 10 | 1 | 08 | 1 | 10 | 1 | 28 | 1 | 09 | 1 | 29 |
| 23 | 1 | 11 | 1 | 09 | 1 | 11 | 1 | 29 | 1 | 10 | 1 | 30 |
| 24 | 1 | 12 | 1 | 10 | 1 | 12 | 1 | 30 | 1 | 11 | 1 | 31 |
| 25 | 1 | 13 | 1 | 11 | 1 | 13 | 1 | 31 | 1 | 12 | 1 | 32 |
| 26 | 1 | 14 | 1 | 12 | 1 | 14 | 1 | 32 | 1 | 13 | 1 | 33 |
| 27 | 1 | 15 | 1 | 13 | 1 | 15 | 1 | 33 | 1 | 14 | 1 | 34 |
| 28 | 1 | 16 | 1 | 14 | 1 | 16 | 1 | 34 | 1 | 15 | 1 | 35 |
| 29 | 1 | 17 | 1 | 15 | 1 | 17 | 1 | 35 | 1 | 16 | 1 | 36 |
| 30 | 1 | 18 | 1 | 16 | 1 | 18 | 1 | 36 | 1 | 17 | 1 | 37 |
| 31 | 1 | 19 | 1 | 17 | 1 | 19 | 1 | 37 | 1 | 18 | 1 | 38 |
| 32 | 1 | 20 | 1 | 18 | 1 | 20 | 1 | 38 | 1 | 19 | 1 | 39 |
| 33 | 1 | 21 | 1 | 19 | 1 | 21 | 1 | 39 | 1 | 20 | 1 | 40 |
| 34 | 1 | 22 | 1 | 20 | 1 | 22 | 1 | 40 | 1 | 21 | 1 | 41 |
| 35 | 1 | 23 | 1 | 21 | 1 | 23 | 1 | 41 | 1 | 22 | 1 | 42 |
| 36 | 1 | 24 | 1 | 22 | 1 | 24 | 1 | 42 | 1 | 23 | 1 | 43 |
| 37 | 1 | 25 | 1 | 23 | 1 | 25 | 1 | 43 | 1 | 24 | 1 | 44 |
| 38 | 1 | 26 | 1 | 24 | 1 | 26 | 1 | 44 | 1 | 25 | 1 | 45 |
| 39 | 1 | 27 | 1 | 25 | 1 | 27 | 1 | 45 | 1 | 26 | 1 | 46 |
| 40 | 1 | 28 | 1 | 26 | 1 | 28 | 1 | 46 | 1 | 27 | 1 | 47 |
| 41 | 1 | 29 | 1 | 27 | 1 | 29 | 1 | 47 | 1 | 28 | 1 | 48 |
| 42 | 1 | 30 | 1 | 28 | 1 | 30 | 1 | 48 | 1 | 29 | 1 | 49 |
| 43 | 1 | 31 | 1 | 29 | 1 | 31 | 1 | 49 | 1 | 30 | 1 | 50 |
| 44 | 1 | 32 | 1 | 30 | 1 | 32 | 1 | 50 | 1 | 31 | 1 | 51 |
| 45 | 1 | 33 | 1 | 31 | 1 | 33 | 1 | 51 | 1 | 32 | 1 | 52 |
| 46 | 1 | 34 | 1 | 32 | 1 | 34 | 1 | 52 | 1 | 33 | 1 | 53 |
| 47 | 1 | 35 | 1 | 33 | 1 | 35 | 1 | 53 | 1 | 34 | 1 | 54 |
| 48 | 1 | 36 | 1 | 34 | 1 | 36 | 1 | 54 | 1 | 35 | 1 | 55 |
| 49 | 1 | 37 | 1 | 35 | 1 | 37 | 1 | 55 | 1 | 36 | 1 | 56 |
| 50 | 1 | 38 | 1 | 36 | 1 | 38 | 1 | 56 | 1 | 37 | 1 | 57 |
| 51 | 1 | 39 | 1 | 37 | 1 | 39 | 1 | 57 | 1 | 38 | 1 | 58 |
| 52 | 1 | 40 | 1 | 38 | 1 | 40 | 1 | 58 | 1 | 39 | 1 | 59 |
| 53 | 1 | 41 | 1 | 39 | 1 | 41 | 1 | 59 | 1 | 40 | 1 | 60 |
| 54 | 1 | 42 | 1 | 40 | 1 | 42 | 1 | 60 | 1 | 41 | 1 | 61 |
| 55 | 1 | 43 | 1 | 41 | 1 | 43 | 1 | 61 | 1 | 42 | 1 | 62 |
| 56 | 1 | 44 | 1 | 42 | 1 | 44 | 1 | 62 | 1 | 43 | 1 | 63 |
| 57 | 1 | 45 | 1 | 43 | 1 | 45 | 1 | 63 | 1 | 44 | 1 | 64 |
| 58 | 1 | 46 | 1 | 44 | 1 | 46 | 1 | 64 | 1 | 45 | 1 | 65 |
| 59 | 1 | 47 | 1 | 45 | 1 | 47 | 1 | 65 | 1 | 46 | 1 | 66 |
| 60 | 1 | 48 | 1 | 46 | 1 | 48 | 1 | 66 | 1 | 47 | 1 | 67 |
| 61 | 1 | 49 | 1 | 47 | 1 | 49 | 1 | 67 | 1 | 48 | 1 | 68 |
| 62 | 1 | 50 | 1 | 48 | 1 | 50 | 1 | 68 | 1 | 49 | 1 | 69 |
| 63 | 1 | 51 | 1 | 49 | 1 | 51 | 1 | 69 | 1 | 50 | 1 | 70 |
| 64 | 1 | 52 | 1 | 50 | 1 | 52 | 1 | 70 | 1 | 51 | 1 | 71 |
| 65 | 1 | 53 | 1 | 51 | 1 | 53 | 1 | 71 | 1 | 52 | 1 | 72 |
| 66 | 1 | 54 | 1 | 52 | 1 | 54 | 1 | 72 | 1 | 53 | 1 | 73 |
| 67 | 1 | 55 | 1 | 53 | 1 | 55 | 1 | 73 | 1 | 54 | 1 | 74 |
| 68 | 1 | 56 | 1 | 54 | 1 | 56 | 1 | 74 | 1 | 55 | 1 | 75 |
| 69 | 1 | 57 | 1 | 55 | 1 | 57 | 1 | 75 | 1 | 56 | 1 | 76 |
| 70 | 1 | 58 | 1 | 56 | 1 | 58 | 1 | 76 | 1 | 57 | 1 | 77 |
| 71 | 1 | 59 | 1 | 57 | 1 | 59 | 1 | 77 | 1 | 58 | 1 | 78 |
| 72 | 1 | 60 | 1 | 58 | 1 | 60 | 1 | 78 | 1 | 59 | 1 | 79 |
| 73 | 1 | 61 | 1 | 59 | 1 | 61 | 1 | 79 | 1 | 60 | 1 | 80 |
| 74 | 1 | 62 | 1 | 60 | 1 | 62 | 1 | 80 | 1 | 61 | 1 | 81 |
| 75 | 1 | 63 | 1 | 61 | 1 | 63 | 1 | 81 | 1 | 62 | 1 | 82 |
| 76 | 1 | 64 | 1 | 62 | 1 | 64 | 1 | 82 | 1 | 63 | 1 | 83 |
| 77 | 1 | 65 | 1 | 63 | 1 | 65 | 1 | 83 | 1 | 64 | 1 | 84 |
| 78 | 1 | 66 | 1 | 64 | 1 | 66 | 1 | 84 | 1 | 65 | 1 | 85 |
| 79 | 1 | 67 | 1 | 65 | 1 | 67 | 1 | 85 | 1 | 66 | 1 | 86 |
| 80 | 1 | 68 | 1 | 66 | 1 | 68 | 1 | 86 | 1 | 67 | 1 | 87 |
| 81 | 1 | 69 | 1 | 67 | 1 | 69 | 1 | 87 | 1 | 68 | 1 | 88 |
| 82 | 1 | 70 | 1 | 68 | 1 | 70 | 1 | 88 | 1 | 69 | 1 | 89 |
| 83 | 1 | 71 | 1 | 69 | 1 | 71 | 1 | 89 | 1 | 70 | 1 | 90 |
| 84 | 1 | 72 | 1 | 70 | 1 | 72 | 1 | 90 | 1 | 71 | 1 | 91 |
| 85 | 1 | 73 | 1 | 71 | 1 | 73 | 1 | 91 | 1 | 72 | 1 | 92 |
| 86 | 1 | 74 | 1 | 72 | 1 | 74 | 1 | 92 | 1 | 73 | 1 | 93 |
| 87 | 1 | 75 | 1 | 73 | 1 | 75 | 1 | 93 | 1 | 74 | 1 | 94 |
| 88 | 1 | 76 | 1 | 74 | 1 | 76 | 1 | 94 | 1 | 75 | 1 | 95 |
| 89 | 1 | 77 | 1 | 75 | 1 | 77 | 1 | 95 | 1 | 76 | 1 | 96 |
| 90 | 1 | 78 | 1 | 76 | 1 | 78 | 1 | 96 | 1 | 77 | 1 | 97 |
| 91 | 1 | 79 | 1 | 77 | 1 | 79 | 1 | 97 | 1 | 78 | 1 | 98 |
| 92 | 1 | 80 | 1 | 78 | 1 | 80 | 1 | 98 | 1 | 79 | 1 | 99 |
| 93 | 1 | 81 | 1 | 79 | 1 | 81 | 1 | 99 | 1 | 80 | 1 | 100 |
| 94 | 1 | 82 | 1 | 80 | 1 | 82 | 1 | 100 | 1 | 81 | 1 | 101 |
| 95 | 1 | 83 | 1 | 81 | 1 | 83 | 1 | 101 | 1 | 82 | 1 | 102 |
| 96 | 1 | 84 | 1 | 82 | 1 | 84 | 1 | 102 | 1 | 83 | 1 | 103 |
| 97 | 1 | 85 | 1 | 83 | 1 | 85 | 1 | 103 | 1 | 84 | 1 | 104 |
| 98 | 1 | 86 | 1 | 84 | 1 | 86 | 1 | 104 | 1 | 85 | 1 | 105 |
| 99 | 1 | 87 | 1 | 85 | 1 | 87 | 1 | 105 | 1 | 86 | 1 | 106 |
| 100 | 1 | 88 | 1 | 86 | 1 | 88 | 1 | 106 | 1 | 87 | 1 | 107 |
| 101 | 1 | 89 | 1 | 87 | 1 | 89 | 1 | 107 | 1 | 88 | 1 | 108 |
| 102 | 1 | 90 | 1 | 88 | 1 | 90 | 1 | 108 | 1 | 89 | 1 | 109 |
| 103 | 1 | 91 | 1 | 89 | 1 | 91 | 1 | 109 | 1 | 90 | 1 | 110 |
| 104 | 1 | 92 | 1 | 90 | 1 | 92 | 1 | 110 | 1 | 91 | 1 | 111 |
| 105 | 1 | 93 | 1 | 91 | 1 | 93 | 1 | 111 | 1 | 92 | 1 | 112 |
| 106 | 1 | 94 | 1 | 92 | 1 | 94 | 1 | 112 | 1 | 93 | 1 | 113 |
| 107 | 1 | 95 | 1 | 93 | 1 | 95 | 1 | 113 | 1 | 94 | 1 | 114 |
| 108 | 1 | 96 | 1 | 94 | 1 | 96 | 1 | 114 | 1 | 95 | 1 | 115 |
| 109 | 1 | 97 | 1 | 95 | 1 | 97 | 1 | 115 | 1 | 96 | 1 | 116 |
| 110 | 1 | 98 | 1 | 96 | 1 | 98 | 1 | 116 | 1 | 97 | 1 | 117 |
| 111 | 1 | 99 | 1 | 97 | 1 | 99 | 1 | 117 | 1 | 98 | 1 | 118 |
| 112 | 1 | 100 | 1 | 98 | 1 | 100 | 1 | 118 | 1 | 99 | 1 | 119 |
| 113 | 1 | 101 | 1 | 99 | 1 | 101 | 1 | 119 | 1 | 100 | 1 | 120 |
| 114 | 1 | 102 | 1 | 100 | 1 | 102 | 1 | 120 | 1 | 101 | 1 | 121 |
| 115 | 1 | 103 | 1 | 101 | 1 | 103 | 1 | 121 | 1 | 102 | 1 | 122 |
| 116 | 1 | 104 | 1 | 102 | 1 | 104 | 1 | 122 | 1 | 103 | 1 | 123 |
| 117 | 1 | 105 | 1 | 103 | 1 | 105 | 1 | 123 | 1 | 104 | 1 | 124 |
| 118 | 1 | 106 | 1 | 104 | 1 | 106 | 1 | 124 | 1 | 105 | 1 | 125 |
| 119 | 1 | 107 | 1 | 105 | 1 | 107 | 1 | 125 | 1 | 106 | 1 | 126 |
| 120 | 1 | 108 | 1 | 106 | 1 | 108 | 1 | 126 | 1 | 107 | 1 | 127 |
| 121 | 1 | 109 | 1 | 107 | 1 | 109 | 1 | 127 | 1 | 108 | 1 | 128 |
| 122 | 1 | 110 | 1 | 108 | 1 | 110 | 1 | 128 | 1 | 109 | 1 | 129 |
| 123 | 1 | 111 | 1 | 109 | 1 | 111 | 1 | 129 | 1 | 110 | 1 | 130 |
| 124 | 1 | 112 | 1 | 110 | 1 | 112 | 1 | 130 | 1 | 111 | 1 | 131 |
| 125 | 1 | 113 | 1 | 111 | 1 | 113 | 1 | 131 | 1 | 112 | 1 | 132 |
| 126 | 1 | 114 | 1 | 112 | 1 | 114 | 1 | 132 | 1 | 113 | 1 | 133 |
| 127 | 1 | 115 | 1 | 113 | 1 | 115 | 1 | 133 | 1 | 114 | 1 | 134 |
| 128 | 1 | 116 | 1 | 114 | 1 | 116 | 1 | 134 | 1 | 115 | 1 | 135 |
| 129 | 1 | 117 | 1 | 115 | 1 | 117 | 1 | 135 | 1 | 116 | 1 | 136 |
| 130 | 1 | 118 | 1 | 116 | 1 | 118 | 1 | 136 | 1 | 117 | 1 | 137 |
| 131 | 1 | 119 | 1 | 117 | 1 | 119 | 1 | 137 | 1 | 118 | 1 | 138 |
| 132 | 1 | 120 | 1 | 118 | 1 | 120 | 1 | 138 | 1 | 119 | 1 | 139 |
| 133 | 1 | 121 | 1 | 119 | 1 | 121 | 1 | 139 | 1 | 120 | 1 | 140 |
| 134 | 1 | 122 | 1 | 120 | 1 | 122 | 1 | 140 | 1 | 121 | 1 | 141 |
| 135 | 1 | 123 | 1 | 121 | 1 | 123 | 1 | 141 | 1 | 122 | 1 | 142 |

XXII.
An Equation
Table, by M.
Cassini. N. 214.
p. 248.

Tabula *Aequationis* Dierum cum Solis Loco adeunda.

| G. | ♈ | | ♉ | | ♊ | | ♋ | | ♌ | | ♍ | |
|----|----------|---------|----------|----|----------|--------|--------|----|----------|----|----------|--------|
| | ' Sub. " | | ' Add. " | | ' Add. " | | Sub. " | | ' Sub. " | | ' Sub. " | |
| 0 | 7 | 45 | 1 | 11 | 4 | 3 | 0 | 59 | 5 | 43 | 2 | 8 |
| 1 | 7 | 26 | 1 | 24 | 4 | 0 | 1 | 15 | 5 | 45 | 1 | 53 |
| 2 | 7 | 7 | 1 | 37 | 3 | 56 | 1 | 29 | 5 | 46 | 1 | 37 |
| 3 | 6 | 48 | 1 | 49 | 3 | 51 | 1 | 42 | 5 | 47 | 1 | 21 |
| 4 | 6 | 29 | 2 | 1 | 3 | 45 | 1 | 54 | 5 | 48 | 1 | 5 |
| 5 | 6 | 10 | 2 | 12 | 3 | 39 | 2 | 6 | 5 | 48 | 0 | 48 |
| 6 | 5 | 51 | 2 | 23 | 3 | 32 | 2 | 19 | 5 | 48 | 0 | 30 |
| 7 | 5 | 31 | 2 | 33 | 3 | 25 | 2 | 32 | 5 | 46 | 0 | 12 |
| 8 | 5 | 11 | 2 | 43 | 3 | 17 | 2 | 44 | 5 | 44 | 0 | Add. 7 |
| 9 | 4 | 51 | 2 | 53 | 3 | 9 | 2 | 56 | 5 | 40 | 0 | 26 |
| 10 | 4 | 31 | 3 | 3 | 3 | 0 | 3 | 8 | 5 | 36 | 0 | 35 |
| 11 | 4 | 11 | 3 | 13 | 2 | 51 | 3 | 20 | 5 | 31 | 1 | 3 |
| 12 | 3 | 52 | 3 | 22 | 2 | 41 | 3 | 32 | 5 | 25 | 1 | 21 |
| 13 | 3 | 33 | 3 | 30 | 2 | 31 | 3 | 43 | 5 | 19 | 1 | 40 |
| 14 | 3 | 14 | 3 | 37 | 2 | 21 | 3 | 54 | 5 | 13 | 1 | 59 |
| 15 | 2 | 55 | 3 | 43 | 2 | 10 | 4 | 4 | 5 | 6 | 2 | 19 |
| 16 | 2 | 37 | 3 | 48 | 2 | 0 | 4 | 14 | 4 | 58 | 2 | 40 |
| 17 | 2 | 19 | 3 | 53 | 1 | 49 | 4 | 26 | 4 | 49 | 3 | 1 |
| 18 | 2 | 1 | 3 | 57 | 1 | 37 | 4 | 34 | 4 | 39 | 3 | 22 |
| 19 | 1 | 43 | 4 | 1 | 1 | 25 | 4 | 43 | 4 | 30 | 3 | 44 |
| 20 | 1 | 26 | 3 | 5 | 1 | 13 | 4 | 51 | 4 | 20 | 4 | 6 |
| 21 | 1 | 9 | 4 | 8 | 1 | 1 | 4 | 59 | 4 | 9 | 4 | 29 |
| 22 | 0 | 52 | 4 | 10 | 0 | 49 | 5 | 6 | 3 | 57 | 4 | 51 |
| 23 | 0 | 35 | 4 | 12 | 0 | 37 | 5 | 13 | 3 | 45 | 5 | 13 |
| 24 | 0 | 19 | 4 | 13 | 0 | 24 | 5 | 19 | 3 | 32 | 5 | 35 |
| 25 | 0 | 3 | 4 | 11 | 0 | 10 | 5 | 24 | 3 | 19 | 5 | 57 |
| 26 | 0 | Add. 12 | 4 | 9 | 0 | Sub. 3 | 5 | 29 | 3 | 5 | 6 | 19 |
| 27 | 0 | 27 | 4 | 8 | 0 | 16 | 5 | 33 | 2 | 51 | 6 | 41 |
| 28 | 0 | 42 | 4 | 6 | 0 | 29 | 5 | 37 | 2 | 37 | 7 | 2 |
| 29 | 0 | 57 | 4 | 5 | 0 | 44 | 5 | 40 | 2 | 23 | 7 | 23 |
| 30 | 1 | 11 | 4 | 3 | 0 | 59 | 5 | 43 | 2 | 8 | 7 | 44 |

Tabula

Tabula *Aequationis* Dierum cum Solis
Loco adeunda.

| G. | ♈
'Add." | | ♉
'Add." | | ♊
'Add." | | ♋
'Add." | | ♌
'Sub." | | ♍
'Sub." | |
|----|-------------|----|-------------|----|-------------|----|-------------|--------|-------------|----|-------------|----|
| 0 | 7 | 44 | 15 | 34 | 13 | 25 | 0 | 59 | 11 | 48 | 14 | 36 |
| 1 | 8 | 5 | 15 | 42 | 13 | 7 | 0 | 27 | 12 | 4 | 14 | 29 |
| 2 | 8 | 25 | 15 | 48 | 12 | 48 | 0 | Sub. 5 | 12 | 19 | 14 | 21 |
| 3 | 8 | 45 | 15 | 53 | 12 | 29 | 0 | 35 | 12 | 35 | 14 | 13 |
| 4 | 9 | 5 | 15 | 57 | 12 | 10 | 1 | 4 | 12 | 50 | 14 | 4 |
| 5 | 9 | 25 | 16 | 1 | 11 | 50 | 1 | 33 | 13 | 5 | 13 | 55 |
| 6 | 9 | 44 | 16 | 5 | 11 | 30 | 2 | 3 | 13 | 19 | 13 | 46 |
| 7 | 10 | 3 | 16 | 7 | 11 | 10 | 2 | 32 | 13 | 32 | 13 | 37 |
| 8 | 10 | 22 | 16 | 8 | 10 | 49 | 3 | 1 | 13 | 44 | 13 | 27 |
| 9 | 10 | 41 | 16 | 9 | 10 | 28 | 3 | 29 | 13 | 55 | 13 | 17 |
| 10 | 11 | 00 | 16 | 9 | 10 | 6 | 3 | 57 | 14 | 5 | 13 | 7 |
| 11 | 11 | 19 | 16 | 9 | 9 | 42 | 4 | 25 | 14 | 14 | 12 | 56 |
| 12 | 11 | 38 | 16 | 8 | 9 | 17 | 4 | 53 | 14 | 22 | 12 | 44 |
| 13 | 11 | 57 | 16 | 7 | 8 | 51 | 5 | 20 | 14 | 29 | 12 | 32 |
| 14 | 12 | 15 | 16 | 5 | 8 | 25 | 5 | 48 | 14 | 35 | 12 | 19 |
| 15 | 12 | 33 | 16 | 1 | 7 | 58 | 6 | 15 | 14 | 40 | 12 | 6 |
| 16 | 12 | 50 | 15 | 56 | 7 | 31 | 6 | 42 | 14 | 45 | 11 | 52 |
| 17 | 13 | 7 | 15 | 50 | 7 | 5 | 7 | 9 | 14 | 50 | 11 | 37 |
| 18 | 13 | 22 | 15 | 44 | 6 | 38 | 7 | 34 | 14 | 54 | 11 | 21 |
| 19 | 13 | 36 | 15 | 37 | 6 | 12 | 7 | 58 | 14 | 56 | 11 | 4 |
| 20 | 13 | 49 | 15 | 30 | 5 | 45 | 8 | 21 | 14 | 58 | 10 | 46 |
| 21 | 14 | 2 | 15 | 22 | 5 | 19 | 8 | 45 | 14 | 59 | 10 | 28 |
| 22 | 14 | 14 | 15 | 13 | 4 | 52 | 9 | 8 | 15 | 00 | 10 | 10 |
| 23 | 14 | 26 | 15 | 3 | 4 | 26 | 9 | 31 | 15 | 00 | 9 | 52 |
| 24 | 14 | 37 | 14 | 52 | 3 | 58 | 9 | 53 | 15 | 00 | 9 | 34 |
| 25 | 14 | 47 | 14 | 40 | 3 | 30 | 10 | 13 | 14 | 58 | 9 | 16 |
| 26 | 14 | 57 | 14 | 27 | 3 | 1 | 10 | 32 | 14 | 55 | 8 | 58 |
| 27 | 15 | 7 | 14 | 13 | 2 | 31 | 10 | 51 | 14 | 51 | 8 | 40 |
| 28 | 15 | 16 | 13 | 58 | 2 | 1 | 11 | 10 | 14 | 47 | 8 | 22 |
| 29 | 15 | 25 | 13 | 42 | 1 | 30 | 11 | 29 | 14 | 42 | 8 | 4 |
| 30 | 15 | 34 | 13 | 25 | 0 | 59 | 11 | 48 | 14 | 26 | 7 | 45 |

Spots Observ'd
in the Sun;
by Mr. Boyle.
N. 74. p. 2216.

XXIII. *Ann.* 1660, *April* 27. About 8 of the Clock in the Morning there appear'd a Spot in the lower Limb of the Sun, a little towards the South of its *Æquator*, which was entred about $\frac{1}{4}$ of the Diameter of the Sun itself, being about $\frac{1}{10}$ in its shortest Diameter, of that of the Sun, its longest, about $\frac{1}{10}$ of the same. It disappeared upon Wednesday Morning, *May* 9. though we saw it the day before about 10 in the Morning, to be near about the same Distance from the Westward Limb, a little South also of its *Æquator*, that it first appeared to be from the Eastward Limb, a little South also of its *Æquator*. It seem'd to move faster in the middle of the Sun than towards the Limb. It was a very dark Spot, almost of a Quadrangular Form, and was enclosed round with a kind of dusky Cloud.

We first observed this very same Spot both for Figure, Colour, and Bulk, to be Re-entred the Sun *May* 25, when it appear'd to be in a part of the same Line it had formerly traced; and was Entred about $\frac{1}{4}$ of its Diameter about 7 a Clock in the Afternoon. At the same time there appear'd another Spot, which was just Entered, and appeared to be Entered not above $\frac{1}{11}$ part of the Sun's Diameter. It appeared to be longest towards the North and South, and shortest towards the East and West. There seem'd to be dispers'd about it divers small Clouds here and there.

Spots Observ'd
in the Sun;
by M. Picard.
N. 74. p. 2238.
N. 75. p. 2253.
By M. Cassini.
N. 74. 2238.
N. 75. 2250.

XXIV. 1. *Ann.* 1671. M. Picard, at Sea near the *Texel*, observ'd a Spot in the Sun from *Aug.* 3. *St. N.* to the 19th. It appear'd at first, like the Tail of a *Scorpion*, but on the 19th day resembling a Melon-Seed.

2. *Aug.* 11. (*St. N.*) 1671. About 6 a Clock a Night, M. Cassini, with a three-Foot Glas, remark'd in the Sun's Disk, two Spots very dark, distant from its apparent Center about the third part of his Semi-Diameter. They were in the Southern part of the Sun, and their Elongation from the *Parallel of the Æquator* passing through the Center of the Sun, was about $\frac{1}{6}$ part of his Diameter. The Time which lapsed between the Transite of the Sun's Center and that of the first of these Spots was 22". or 23". the Semi-diameter of the Sun then passing in 66". The first of these Spots, being looked upon with a Telescope of 17 Foot long, appear'd with a somewhat Oval Figure; the other was Oblong and a little curved, like the *Hebrew* Letter *Jod*; and both together were surrounded by a *Corolla*, or Coronet, made up of little dark Points, which conformed itself to the Figure of the Spots, considered as they were joyned together.

Aug. 12. He perceived that they were nearer his Center. The Time between the passage of the Sun's Center, and that of the interior edge of the *Coronet* which encompass'd them both, was then of 16". At 7 a Clock it was but of 15". and the Southern Limb of the *Coronet* touched the *Parallel* passing through the Sun's Center. The first Spot was composed of two others almost round, and conjoined. The second represented the shape of a *Scorpion*. The third was round. And they were all three environed with a *Coronet*, which was composed, as we said above, of abundance of little obscure Pricks. This *Coronet* appear'd to be clearer than the rest of the Sun, when looked

look'd upon with the short Glas, and darker when seen with the long. Without it there were other Points, but very black ones; viz. Five near the round Spot on the South-side and another near the *Scorpion's* Tail on the North-side.

At 8 a Clock 48'. the Figure of the *Scorpion* was seen divided into several pieces, as if the Tail and Arms had been cut off. The Northern Point appear'd no more, there remaining none but those on the South-side; and the length of the enclosure of all the *Spots*, comprehended between the Extremities, was of 1'. 15". and the Breadth of 30".

The same 12th day at 6 in the Evening, he found no great Change in the first *Spot*. The other two were severed into 5 distinct ones, compass'd about with a *Coronet*, together with 5 black Points, which stood in a streight row, and after another manner than they did in the Morning. From 6 at Night unto 7, the time when the passage of the *Sun's* Center, and that of the *Coronet's* Limb, was found to be at one time of 8". and another time of 7". The Distance of the *Spots* unto the Parallel passing through the *Sun's* Center, was near the same on the North-side with what it had been observed to be in the Morning on the South-side.

Aug. 13. Between the Rising of the *Sun*, and half an Hour past 6 in the Morning, the edge of the *Coronet* was turned to a Point on the South-side, and was distant from the *Aequator* on the North-side, half a Minute; and there was but a Second of Time from the passage of the *Sun's* Center unto the passage of the same anterior edge of the *Coronet*.

At 8 a Clock, 30'. the fore-edge was in the same Horary Circle with the Center of the *Sun*; so that in one day and an half, these *Spots* have run thro' very near the third part of the *Sun's* apparent Semidiameter, which giveth an Arch of 19 Degrees and an half of the Circumference of the *Sun's* Body; and consequently their *Diurnal* Motion about the *Sun's* Axe hath been of 13 Degrees; and the time of their *Periodical* Revolution, as far as we could conjecture in so little a time, must be about 27 days and an half.

Aug. 14. At 6 in the Morning, there pass'd 15'. of Time between the passage of the Anterior Limb of the *Crown*, and the passage of the *Sun's* Center through the same Horary Circle: And then the Southern Limb of the *Crown* was a Minute and an half distant toward the North, from the Parallel of the *Aequator*, passing through the same Center of the *Sun*. The Figure of the first *Spot* was almost the same with that of the day before. The second had taken the Form of an Heart, the Point of which was turned to the North-side, and its base between the South and the East. Three other small *Spots*, dispos'd Triangle-wise, stood over the said Base, and were accompanied with two others upon a Line turned Southward. And they were all encompass'd by a *Crown* running out into a Point on the South-side; and on the North-side, Eastward, it had an *Appendix*.

Aug. 15. At 6 in the Morning, there pass'd 27" between the passage of the Anterior Limb of the *Crown*, and that of the *Sun's* Center through the same Horary Circle. The Southern Limb of the same *Crown* was two Minutes and an half distant from the Parallel of the *Aequator*, passing through the

the Center of the *Sun*, whose Diameter pass'd in $2'. 9''$. through the same Horary Circle. The first *Spot* had a little changed the Figure; the second was Quadrangular, longer from East to West, than from North to South: It appeared bigger than ordinary, and had withal on its sides, within the compass of the *Crown*, three other small *Spots*. There were also seen four more without the said *Crown* on the South-side.

Aug. 16. At 6 in the Morning, there was $27''$. between the passage of the *Sun's* Anterior Limb, and the passage of the Anterior Limb of the *Crown* through the same Horary Circle; and $38'$. between the passage of the Anterior Limb of the *Crown*, unto the passage of the *Sun's* Center. The Southern Limb of the *Crown* was $3\frac{1}{2}'$ off from the Parallel of the *Equator*, passing thro' the Center of the *Sun*, towards the North. And the Observation having been made yet more exactly at half an hour past 7 of the same Morning, this Distance was found of $3'. 33''$. The Figure of the first *Spot* in the beginning of the Observation, differ'd not much from that of the precedent day; but afterwards it was seen divided into two. The second, which likewise seem'd to be the same in the beginning, was afterwards divided into three, accompanied with black and dark Points without the *Crown* on the South-side. The same day at 6 a Clock, and 15. at night, the Figures of these *Spots* were much changed. There were 5 *Spots* enclosed in the *Crown*; The two foremost were part of that which had been seen in the Morning as one; the two others following those two first, were part of the second in the Morning; and without there were 5 Points on the South-side, and two more a little further to the North, which Points were ranged as in another Area made up of other Points so small that they could scarce be perceived.

Aug. 17. in the Morning, immediately after the Rising of the *Sun*, there appeared three very dark *Spots*, which form'd in a manner these Letters *F n 7*, posited from East to West, and included in their wonted *Crown*, which stretch'd out, as 'twere two Arms, or two Handles, one to the South, and the other to the North. There pass'd $18''$. between the passage of the foremost Limb of the *Sun*, and that of the foremost Limb of the *Crown*, and $47\frac{1}{2}''$. between the passage of the Anterior Limb of the *Crown*, unto the passage of the *Sun's* Center. The Southern Limb of the same *Crown* was distant $11'. 17''$. from the Parallel that touch'd the *Sun* on the North-side, and $4'. 38''$. from the Parallel that pass'd through the Center.

Aug. 18. at 7 in the Morning, the *Spots* which appear'd through some Clouds, had almost the same shape with those of the day before, only with this difference, that they were a little closer together, drawing from East to West. There lapsed $13''$. between the passage of the Anterior Limb of the *Sun*, and that of the Anterior Limb of the *Spot*, through the same Horary Circle, and $52''$. of the foremost Limb of the *Spot* unto the passage of the Center. The Southern Limb of the *Spot* was $9'. 13''$. distant from the Parallel that touch'd the Northern Limb of the *Sun*, and $6'. 41''$. from the Parallel that pass'd through his Center. At 5 a Clock and 55. at night of the same day, there lapsed $11''$. between the passage of the Anterior Limb of the *Sun* through the same Horary Circle, and the passage of the Anterior Limb

Limb of the *Crown*, and from thence unto the Passage of the *Sun's* Center 54". The Limb of the *Crown* next the *Parallel* passing through the Center of the *Sun*, was distant from the same *Parallel* 7'. 40".

Aug. 19. from 4 to 5 in the Evening, the *Spot* appear'd Oblong near the *Sun's* Circumference, from which it was distant about the breadth of the same *Spot*.

Aug. 20. in the Morning, which was not the full seventh from the day that they were arriv'd to the middle of the Disk, they disappeared.

The apparent Velocity nigh the Center was such, that if it had continued the same, the *Spots* would have arrived almost in 4 days to the Limb of the Disk; but in this Hypothesis, that the *Spots* were adherent to the *Sun's* Surface, or at least very nigh to it, this apparent Velocity was to lessen according as they should remove from the Center, as hath come to pass in effect. The Diminution of the length of the Misty *Crown* was in a manner proportionable to the Diminution of the apparent Velocity; since that, when this *Crown* was in the middle, and in a Situation, wherein its true Figure could be best seen, it appear'd Oblong, and of the Form of an human Ear, its greatest Diameter respecting East and West; but being nigh the Limb, this same Diameter seem'd to shorten; and having appeared greatest in its first Situation, it appear'd least in this, because it was almost in a Circle that pass'd through the Center of the *Sun*, whose equal Arches are by so much the more Oblique, by how much they approach more to the Limb of his Disk, and consequently appear less, according to the Rules of *Opticks*; mean time, the Diameter, that was turned from South to North, apparently kept the same bigness it had near the Center, because it was in a Circle almost Parallel to the Horizon of the *Sun*, which formed the Representation of its Limb, and whose equal Arches, (by the same *Optical* Reasons) do not appear contracted.

3. Several curious Observers at *London*, have seen one of these *Spots* recurr'd to the *Sun's* Eastern Limb, about *Aug.* 25. *St. N.* as *M. Cassini* predicted they should return. By several at
London.
N. 75. p. 5293.

Aug. 30. 1671. I saw a large *Spot* in the Center of the *Sun's* Face about Noon. By Dr. Hook :
N. 77. p. 2294.

Sept. 1. At 3 a Clock, I saw the same *Spot* moved about a quarter of the Diameter of the *Sun* Westward: it consisted of one greater and two lesser black *Spots*, with a *Dusky Cloud* incompassing them: The Diameter of the whole Phænomenon was about $\frac{1}{2}$ of the Diameter of the *Sun*, and it was distant from the next adjoining Limb $\frac{1}{2}$ (that is exactly one quarter) of the Diameter of the *Sun*.

5. *Maculæ Solares* Observatæ fuere nobis *Hamburgi* à 26. *Aug. St. K.* (pri-
mo fere die, quo iterum Apparere cœperunt) ad usque 5. *Sept.* quo ad Lim-
bum quam proxime accessere. By Mr. Hen.
Siterus.
N. 78. p. 3033.

XXV. *Ann.* 1676. *Jun.* 28. *St. N.* Habemus in sole satis Ingentem *Ma-*
culam, quæ *Solem* ipsum mediavit h. 4. post Meridiem, cum Latitudine Au-
strali 4'; ejus Distantiam a Polo Australi *Solis* ex pluribus Observationibus
supputavi gr. 78'. Si satis habuerit Consistentiæ ad absolvendum Circulum
expectanda Restitutio ejus ad Medium, Diei 25. *Jul.* vespere, cum majore La-
titudine Australi. Spots Observ'd
in the Sun by
M. Cassini.
N. 127. p. 6642.

XXVI. 1.

Spots Observed in the Sun, by Mr. Flamsteed, and Mr. Halley. N. 128. p. 687.

| Grenovici,
A. D. 1676. | | | | Oxonii,
A.D. 1676. | | |
|---------------------------|------------------------------------|--|-------------------------|------------------------------------|-------------|-----------------------|
| <i>Jul.
St. V.</i> | <i>Tempus Observa-
tionum.</i> | <i>Longit.
à Solis
Centro.</i> | <i>Latit.
Aust.</i> | <i>Tempus Observa-
tionum.</i> | <i>Lon.</i> | <i>Lat.
Aust.</i> |
| | <i>h. ' "</i> | <i>' "</i> | <i>' "</i> | <i>h. ' "</i> | <i>' "</i> | <i>" "</i> |
| 25 | | | | 6 46 p.m.Con. | 13 40 | 2 8 |
| 27 | 10 3 a. m. Con. | 9 34 | 3 25 | | | |
| 28 | 4 51 | 5 40 | 2 50 | | | |
| 29 | 10 31 a. m. — | 3 05 | 3 27 | 6 21 a. m. — | 3 55 | 3 22 |
| | 3 54 p. m. — | 2 25 | 3 10 | | | |
| 30 | 9 15 a.m Ant. | 0 37 | 3 33 | 7 20 a. m. — | 0 00 | 3 30 |
| 31 | | | | 7 40 a. m. — | 3 36 | 3 28 |
| Aug. 1. | 9 25½ a. m. — | 6 48 | 4 09 | 7 3 a. m. — | 6 54 | 3 50 |
| | | | | 5 6 p. m. — | 8 7 | 3 53 |
| 2 | 8 8 a. m. — | 9 49 | 3 55 | 7 16 a. m. — | 9 57 | 3 40 |
| 3 | 9 36 a. m. — | 82 28 | 3 27 | 5 9 p. m. — | 13 15 | 3 56 |
| | 4 16 p.m.— | 12 55 | 3 58 | 6 2 p. m. — | 13 25 | 3 26 |
| 4 | 7 38 a.m. — | 14 02 | 4 4 | 7 33 a. m. — | 14 7 | 3 14 |
| | | | | 4 54 p. m. — | 14 43 | 3 23 |

Mr. Halley faith, That he saw a Spot again on the fifth day, 8^h. 30'. *mane*, very near the Limb of the Sun, so that it appeared only as a fine Line; but by reason of its fineness and the too great Height of the Sun, he could not take any measures to determine its Place and Latitude by; and that while the Spot continued one, as it was July 25. he measured to the middle of it; as also when the Pieces were divided, but not far disjoined: Afterwards, when they were separated considerably, he observed the middle of the bigger Spot, which was to the South, apparently, I suppose; but really North: for so only his Observations will agree with those of Mr. Flamsteed exactly.

Hence it seems very evident, (saith Mr. Flamsteed) that the Spot's way was not inclined to the *Ecliptick* six or seven Degrees, as *Scheiner* and some others make it, but much less by the joint Consent of the Observations of both our Observers. Mr. Halley adds, That considering the Motion of the Spot cross the Sun's Disk, as both their Observations give it, it appears that the Latitude was not so great at its Entrance into the Sun as in the middle of him. And by Mr. Flamsteed's Observation it was greatest on the first of *August*, and then again inclining towards the *Ecliptick*. If you grant this, it will follow, (saith Mr. Flamsteed) that the Sun's Axis was inclined to the Plane of the *Orbis magnus*; but the quantity of this Inclination must not be very great

great. The *Nodes* of the Sun's Equinox and Ecliptick he guesses to be not far from the beginning of *Cancer* and *Capricorn*; and that from *Cancer* to *Capricorn* the Earth is North of the Sun's *Æquator*; from *Capricorn* to *Cancer*, South of the same: And the Period of the Sun's Revolution in respect of the fixed Stars 25 Days, 9^h hours sufficiently exact. Of which things these two Observators say, they might have been more certain, had not the *Spot* in its passage broken into so many parts, and those often varied their Positions to each other.

2. *Solarem Maculam* hic observavimus à die 6 *Aug.* ad 14 *St. N.* Collationeque Observationum didicimus, eam mediam Itineris sui in solis Disco Apparente tenuisse circa Mediam Noctem post 8. diem *Aug.* in distantia Apparenti trium minutorum à Centro Austrum versus. In plures distracta partes est, quæ invicem Boream & Austrum versus indies satis manifesto intervallo disjungebantur, adeo ut, præter Motum Communem circa Solis Axem, singulæ partes Proprium inter se directum habuerit. Hanc Porro *Maculam* diversam esse sentio ab ea quam præcedenti Mense *Junio* observaveramus. Illa quippe cum medium Itineris sui in Disco Solis Apparente tenuerit die 28 ejusdem Mensis ad eundem proxime situm reversa esset si fuisset superstes die 25 *Julii* Nocte sequente, ut deducitur tum ex ejus Velocitate, Tempore suæ Apparitionis observata, tum etiam ex Cursu aliarum *macularum*, quæ periodum suam circa Solem à nobis videntur absolvere Spatio dierum 27 cum tridente, vel 27 cum semisse. Ejus præterea semita diversa est à præcedenti; prior quippe paulo remotior fuit ab *Æquatore macularum* quam posterior. Hæc porro, si satis habuerit Consistentiæ, ad Medium Solem rediit die 5 *Septembris* mane.

XXXVII. *Ann.* 1684. *Apr.* 25. About an Hour before Noon I discover'd a large Spot entered within the Sun's Disk a little distant from his following Limb. These Appearances however frequent in the days of *Scheiner* and *Galileo*, have been so rare of late, that this is the only one I have seen in his Face since *December* 1676. By the observed Meridional Distances of it, and the Sun's Southern Limb from the *Vertex* at Noon, I found it to have 3' 40". more North Declination than the Sun's Center, and at 3^h. 35'. after Noon, I measured its Distance from his next Limb 40".

Spots Observed in the Sun; by Mr. Flamsteed. N. 157. p. 934.

Next Morning, *April* 26. I saw it more Remote from his Limb, and by the Observations then made (at 8 h. *mane*) determined its Longitude from the Sun's Axis 66^h Deg. and its Declination from the Solar *Æquator* 9^h Deg. South. Whence supposing the Revolution of any point of the Sun to the same fixed Star to be performed in 25 days 6 hours, the Angle of his *Æquator* and our *Ecliptick* 7 Deg. and the Longitude of his Northern Pole 16 Deg. I designed the Line of its Way or Trace over the Sun, and the Points in it where the Spot would appear every Morning after at the same hour, till its Egress on the 8th of *May*, which I found altogether confirmed by such Observations as I made till then; so that I had no Reason to doubt of the Theory.

When

When the *Spot* was near the middle of the Sun, it appeared very broad, and almost square, the *Nucleus* of the same Figure about 40". Diameter; but when it was near the Limb much narrower, and almost Oval: It seemed to have Consistence enough to endure a second Return; if it shall, it will enter the Visible Disk of the Sun on the 21st of May in the Evening, and in its passage over him describe a Line nearly streight, with greater Latitude from the *Ecliptick*,

To find in what
Proportion the
Planets are En-
lightened by the
Sun; by
M. Auzout.
N. 4. p. 68.

XXVIII. One of the Means used by M. Auzout to Enlighten an Object in what Proportion one pleaseth, is by some great Object-Glass, by him called a *Planetary* one, because that by it he shews the difference of Light, which all the Planets receive from the Sun, by making use of several Apertures, proportionate to their Distances from the Sun, provided that for every 9 Foot Draught, or thereabout, an Inch of Aperture be given for the Earth. Doing this, one sees (saith he) that the Light which *Mercury* receives, is far enough from being able to burn Bodies, and yet that the same Light is great enough in *Saturn* to see clear there, seeing that (to him) it appears greater in *Saturn* than it doth upon the Earth, when it is over-cast with Clouds, which (he adds) would scarce be believed if by means of the Glass it did not sensibly appear so.

The Equinoxes;
by M. Wortzel-
baur.
N. 264. p. 623.

XXIX. The *Equinoxes* of this Year 1699, according to the Observations of M. Wortzelbaur at Nuremberg, happened March 9. 20^h. 35'. 27". and Sept. 12. 10^h. 22'. 42". which by his Tables ought to have been March 9. 20^h. 40' 30". and Sept. 12. 10^h. 32'. 52".

To Observe Solar
Eclipses; by
Mr. Flamsteed.
N. 55. p. 1104.

XXX. For the well observing of Solar Eclipses, cast the Species of the Sun through a good Telescope of a competent Length, on an extended Paper, placed behind the Eye-Glass so far, as that the said Species may appear at least 6 Inches over; then divide both his *Periphery* into 360 Degrees for the better Observing the Inclination of the *Cusps* of each Phasis, and his Diameter into Digits and their Parts, by Concentrick Circles, for measuring the Quantities of the obscured Parts.

An Eclipse of the
Sun, Ann. 1666.
June 22. at
London;
by Mr. Wil-
loughby,
Dr. Pope,
Dr. Hook, and
Mr. Phillips.
N. 14. p. 295.

XXXI. 1. The Eclipse began at 5^h. 43'.

| | | h. | ' |
|-----------------|---------------------|----|---------------|
| It was darkened | 1/2 Diam.—at 6 00 | 5 | Dig.— at 7 6 |
| | 4 Digits. at 6 7 | 4 | Dig.— at 7 13 |
| | 5 Dig.— at 6 13 | 3 | Dig.— at 7 20 |
| | 6 Dig.— at 6 21 | 2 | Dig.— at 7 26 |
| | 7 Dig.— at 6 39 1/2 | 1 | Dig.— at 7 32 |
| | 6 Dig.— at 6 57 | 0 | Dig.— at 7 37 |

Its Duration hence appears to have been one hour and 54'. Its greatest Obscurity somewhat more than 7 Digits. About the middle, between the Perpendicular and Westward Horizontal Radius of the Sun, viewing it thro' Mr. Boyle's

Mr. Boyle's 60 Foot Telescope, there was perceived a little of the Limb of the Moon without the Disk of the Sun ; which seemed to some of the Observers to come from some shining Atmosphere about the Body either of the Sun or Moon. They affirm to have observed the Figure of this Eclipse, and measured the Digits, by casting the Figure through a 5 Foot Telescope on an extended Paper, fix'd at a certain distance from the Eye-Glass, and having a round Figure ; all whose Diameters were divided by 6 concentrick Circles, into 12 Digits.

2. The *Eclipse* began at 5^h. 44'. 52'' *mane*. It ended at 7^h. 43'. 6''. So that its whole duration was 1^h. 58'. 14''. The greatest Obscuration was 7 Dig. 50'. but it seemed to have been greater by 3'. which Mr. *Payen* imputes to a particular Motion of Libration of the Sun's Globe, which entertained that Luminary in the same Phasis for the space of 8'. and some Seconds, as if it had been stopped in the midst of its Course ; rather than to a Tremulous Motion of the Atmosphere, as *Scheiner* would have it. The apparent Diameters were almost Equal : for in the Phasis of 6 Digits, the Circumference of the Moon's Disk passed thro' the Center of that of the Sun, so as that two Lines drawn through the two Horns of the Sun, made with the common Semidiameter two Equilateral Triangles.

At Paris ; by M. Payen. Ibid. p. 296.

The Beginning and Middle of the Eclipse happened to be in the North-Eastern Hemisphere, and the End in the South-Eastern. The first Contact (as 'twere) of the two Disks, was observed in the superior Limb of the Sun's Disk in respect to the Vertical Line, and in the Inferior in respect to the Ecliptick. But the Middle and the End were seen in the Superior Limb, in respect both to the Vertical and the Ecliptick : And what to M. *Payen* seems extraordinary, both the Beginning and the End of this Eclipse happen'd to be in the Oriental Part of the Sun's Disk.

3. The *Eclipse* began about 5 a Clock in the Morning ; at 5^h. 15'. The Sun's Altitude was 6°. 55'.

At Madrid, by the Earl of Sandwich. Ibid. p. 296.

The Middle of it was at 6^h. 2'. The Sun's Altitude 15°. 5'.

The End was exactly at 7^h. 5'. The Sun's Altitude 25°. 24'.

The Duration 2^h. 4'.

Thirty seven parts of the Sun's Diameter remained Light, and 63 were Darkened.

4. In this *Eclipse* it is chiefly observable, That the Semidiameter of the Moon, from the very beginning to about 5 or 6 Digits of the Increasing Phasis, was almost equal to the Semidiameter of the Sun : But, after the greatest Obscuration, when I again contemplated the Moon's Semidiameter, I found it 8'' or 9'' bigger than that of the Sun ; so that the Semidiameter of the Moon was not always, during the *Eclipse*, constant to itself. Of this Variation, the Excellent *Ismael Bullialdus* hath also observ'd something at *Paris*. For he hath written to me, That in this same *Eclipse*, the Semidiameter of the Sun to the Semidiameter of the Moon was, as 16. 9'' to 16'. 22''. but that in another Phasis of 6 Digits, the Semidiameters appeared equal.

At Dantzick ; by M. Hevelius, N. 19. p. 1347.

Ibid.
p. 349.
N. 21.
p. 370.

| Ordo Phasium. | Quantitas Phasium. | Tem. Est. Sec. Horol. Ambulat. | Temp. ex Sciather. | Altitud. C. | Tempus Correct. | Animadvertenda. |
|---------------|----------------------------|--------------------------------|--------------------|-------------|-----------------|---|
| | | h. / " | h. / " | p. / ' | h. / " | |
| | | 5 51 11 | 5 51 00 | 17 45 | 5 53 12 | Quod Sciatericum cum Correcto Tempore non omnino convenit, non nisi Lineæ Meridianæ imputandum. |
| | | 5 57 05 | 5 57 00 | 18 37 | 5 59 28 | |
| | | 6 00 00 | 6 00 00 | 18 55 | 6 1 28 | |
| | Initium. | 6 55 30 | | | 6 57 30 | Initium circa 79 gr. à puncto Zenith Occasum versus contigit. |
| 2 | 0 $\frac{1}{2}$ Dig. | 8 57 30 | | | 6 59 30 | |
| 2 | 0 $\frac{3}{4}$ | 7 0 23 | 7 0 0 | | 7 2 23 | |
| 3 | 1 $\frac{1}{2}$ | 7 2 30 | 7 2 0 | | 7 4 30 | |
| 4 | 1 $\frac{1}{2}$ dig. | 7 4 50 | 7 5 fere. | | 7 6 50 | Hucusque Semidiameter Lunæ æqualis extitit Solari. |
| 5 | 1 $\frac{3}{4}$ fere. | 7 10 57 | 7 10 | | 7 12 57 | |
| 6 | 3 $\frac{1}{2}$ | 7 14 59 | 7 15 | | 7 16 59 | |
| 7 | 3 $\frac{3}{4}$ | 7 17 50 | 7 18 fere. | | 7 19 50 | |
| 8 | 4 $\frac{1}{2}$ dig. | 7 21 35 | 7 21 | | 7 23 35 | |
| 9 | 4 $\frac{3}{4}$ | 7 23 43 | 7 23 fere. | | 7 25 43 | |
| 10 | 5 $\frac{1}{4}$ | 7 27 53 | 7 28 | | 7 29 53 | |
| 11 | 6 | 7 31 50 | 7 32 | | 7 33 50 | |
| 12 | 6 $\frac{1}{2}$ | 7 36 55 | 7 37 | | 7 38 55 | |
| 13 | 6 $\frac{3}{4}$ paul. plus | 7 38 5 | 7 38 | | 7 40 0 | |
| 14 | 7 $\frac{1}{4}$ | 5 39 45 | 7 39 | | 7 41 45 | |
| 15 | 7 $\frac{3}{4}$ paul. plus | 7 42 30 | 7 42 | | 7 44 30 | |
| 16 | 7 $\frac{1}{2}$ | 7 44 6 | 7 44 | | 7 46 6 | Mox. Obscuratio extitit. Dig. 8. 25'. hora 8. 2'. |
| 17 | 7 $\frac{3}{4}$ | 7 46 0 | 7 46 | | 7 48 0 | |
| 18 | 8 fere. | 7 48 25 | 7 48 fere. | | 7 50 25 | |
| 19 | 1 $\frac{1}{2}$ | 7 51 15 | 7 51 | | 7 53 15 | |
| 20 | 8 paul. plus | 7 53 37 | 7 52 | | 7 55 37 | |
| 21 | 8 $\frac{1}{4}$ | 7 55 45 | 7 56 fere. | | 7 57 45 | |
| 22 | 8 $\frac{3}{4}$ pau. min. | 7 59 5 | 7 59 | | 8 1 5 | |
| 23 | 8 $\frac{1}{2}$ | 8 6 30 | 8 6 | | 8 8 30 | |

| Ordo Phasium. | Quantitas Phasium. | Tem. Ast. Sec. Horol. Ambulat. | Temp. ex Sciother. | Altit. ☉. | Tempus Correct. | Animadvertenda. |
|---------------|-----------------------|--------------------------------|--------------------|-----------|-----------------|---|
| | | h. ' " | h. ' " | o. ' " | h. ' " | |
| 24 | 7 $\frac{1}{4}$ | 8 11 25 | 8 12 | | 8 13 25 | Hic Semidiameter Lunæ ad 8". vel 9". major apparuit |
| 25 | 7 $\frac{1}{2}$ fere. | 8 17 30 | 8 18 | | 8 19 30 | |
| 26 | 7 fere. | 8 19 41 | 8 19 | | 8 21 40 | |
| 27 | 5 $\frac{1}{2}$ | 8 28 8 | 8 28 | | 8 30 8 | |
| 28 | 5 $\frac{1}{2}$ fere. | 8 30 14 | 8 30 | | 8 22 14 | |
| 29 | 4 $\frac{3}{4}$ | 8 36 25 | 8 36 | | 8 38 25 | |
| 30 | 3 $\frac{3}{4}$ | 8 43 19 | 8 43 | | 8 45 19 | |
| 31 | 3 $\frac{1}{4}$ | 8 46 12 | 8 46 fere. | | 8 48 12 | |
| 32 | 3 | 8 47 32 | 8 47 | | 8 49 32 | |
| 33 | 2 $\frac{3}{4}$ | 8 50 57 | 8 50 | | 8 52 57 | |
| 34 | 2 $\frac{1}{2}$ fere. | 8 54 15 | 8 54 | | 8 56 15 | |
| 35 | 1 $\frac{3}{4}$ | 8 58 24 | 8 58 | | 9 0 24 | |
| 36 | 1 $\frac{1}{2}$ | 8 59 35 | 8 59 | | 9 1 35 | |
| 37 | 1 $\frac{1}{4}$ | 9 1 38 | 9 1 | | 9 3 38 | |
| 38 | 1 $\frac{1}{4}$ | 9 3 20 | 9 3 | | 9 5 20 | |
| 39 | Finis. | 9 6 53 | 9 6 | | 9 8 53 | |
| | | 9 23 6 | | 47 33 | 9 25 28 | Punctum Finis distat à Verticali ad Ortuum 143 gr. |
| | | 9 24 16 | | 47 42 | 9 26 45 | |
| | | 9 28 29 | | 48 10 | 9 30 42 | |
| | | 9 30 36 | | 48 28 | 9 33 12 | |

XXXII.

An Eclipse of
the Sun, Jun. 23.
(St. N.) 1674.
at Dantzick;
by M. Hevelius.
N. 127. p. 660.

| Ordo Phasium. | Animadvertenda. | Alt. Solis. | | Temp. Cor. | |
|---------------|--|-------------|----|------------|-------|
| | | h. | " | h. | " |
| | Solis Centrum in Horizonte. | | | 3 | 21 30 |
| | Nihil in Sole. | | | 4 | 42 0 |
| | Initium. | | | 4 | 44 0 |
| 1 | 6 $\frac{1}{2}$ Digni fere Obscurati erant. | | | 5 | 32 0 |
| 2 | 6 $\frac{1}{2}$ Dign. | | | 5 | 34 0 |
| 3 | Maxima fere obscurat. 6 Dign. 42'. | | | 5 | 38 50 |
| 4 | 6 $\frac{1}{2}$ Dign. | | | 5 | 43 30 |
| 5 | 6 $\frac{1}{2}$ Dign. | | | 5 | 47 30 |
| 6 | 6 Dign. | | | 5 | 49 30 |
| 7 | 5 $\frac{1}{2}$ Dign. Diamet. Lunæ, 14'. 37". Data nempe | | | 5 | 55 0 |
| 8 | 5 $\frac{1}{2}$ Dign. Semidiametro Sol. 15'. | | | 5 | 57 0 |
| 9 | 5 Fere Dign. | | | 5 | 59 30 |
| 10 | 4 $\frac{1}{2}$ Dign. | | | 6 | 2 0 |
| 11 | 4 $\frac{1}{2}$ Dign. | | | 6 | 4 15 |
| 12 | 4 Dign. | | | 6 | 6 30 |
| 13 | 3 $\frac{1}{2}$ Dign. | | | 6 | 9 15 |
| 14 | 3 $\frac{1}{2}$ Dign. | | | 6 | 14 35 |
| 15 | 2 $\frac{1}{2}$ Dign. | | | 6 | 18 10 |
| 16 | 2 $\frac{1}{2}$ Dign. | | | 6 | 20 0 |
| 17 | 2 Dign. | | | 6 | 22 0 |
| 18 | 1 $\frac{1}{2}$ Dign. | | | 6 | 24 0 |
| 19 | 1 $\frac{1}{2}$ Dign. | | | 6 | 25 25 |
| 20 | 1 Dign. | | | 6 | 27 10 |
| 21 | $\frac{1}{2}$ Dign. | | | 6 | 30 30 |
| | Finis. | 0. | / | 6 | 33 30 |
| | Altitudo ☉. | 25 | 20 | 6 | 41 22 |
| | Altitudo ☉. | 28 | 5 | 7 | 0 14 |

An Eclipse of the
Sun, June 1.
1676, at West-
minster; by
Mr. Francis
Smethwicke.
N. 126. p. 637.

XXXIII. 1. Initium Defectionis. ————— 7 $\frac{1}{2}$. 50'.

Finis. ————— 9 54 $\frac{1}{2}$.

Totius Eclipsis Duratio. ————— 2 4 $\frac{1}{2}$.

Tempus Observatum fuit cum Horologio Oscillatorio, Correcto per Observa-
tionē. Tubus adhibitus fuit bonæ notæ, pedum 7 $\frac{1}{2}$.

| <i>Temp. juxta
Horol. Oscil.</i> | <i>Phases.</i> | <i>Solis Alt.</i> | <i>Temp. Correct.
ex Altit.</i> | |
|--------------------------------------|------------------------|-------------------|-------------------------------------|---------------------|
| <i>h. ' "</i> | | <i>o ' "</i> | <i>h. ' "</i> | |
| 7 34 50 | | 22 46 | 7 36 0 | |
| 7 37 14 | | 33 10 | 7 38 40 | |
| 7 39 10 | | 33 30 | 7 40 48 | |
| 7 50 40 | 0 $\frac{1}{2}$ Digit. | | 7 51 51 | Tubo Optico æstim. |
| 8 8 34 | 1 $\frac{1}{2}$ Digit. | | 8 9 45 | Tub. Optico mensur. |
| 8 17 25 | 2 $\frac{1}{2}$ Digit. | | 8 18 36 | |
| 8 27 10 | 3 $\frac{1}{2}$ Digit. | | 8 28 21 | |
| 9 39 | 1 $\frac{1}{2}$ Digit. | | 9 40 | Tubo æstim. |
| 9 43 | 1 $\frac{1}{4}$ Digit. | | 9 44 | |
| 9 48 | 0 $\frac{1}{2}$ Digit. | | 9 49 | |
| 9 54 25 | Non Finita. | | 9 55 36 | |
| 9 55 55 | Finita. | | 9 57 6 | |
| 4 26 5 | | 32 10 | 4 26 56 | |
| 4 28 58 | | 31 53 | 4 29 52 | |
| 4 31 2 | | 31 31 | 4 32 16 | |

2.
At Wapping
by Mr. Colson.
Ibid.

Hiscæ Observationibus peragendis Socium acciveram Amicum meum *Ed. Halleium*. Tubos præparaveram duos, alterum Digitos 196 $\frac{1}{2}$. longum, quocum & *Micrometro Townleiano*, Ego ipse octo Phasium priorum cepi *N. 127. p. 662.* Mensuras, alterum Digitorum duntaxat 103 $\frac{1}{2}$, quocum & *Micrometro* meo, iis adscriptas Mensuras *Halleius* cepit: in duabus tamen ultimis Animadversionibus, Ego Minori Tubo & *Micrometro* meo (in hunc usum altero accommodatiore) Distantiam cepi *Azimutharum*, per Solis Limbum Lucidum, & Cupidem proximam *Eclipsis* decidentium; *Halleio* interea Partes Lucidas & Cupidem Distantiam majori Tubo dimetiente. Paulo ante Initium advenerat Nobilissimus Præses Regiæ Societatis Dom. Vice-Comes *Brouncker*, qui Mensuram Diametri Solaris, Tubo Longiori captam, suo Judicio probavit. Hora 7. 45'. Sol primum per Nubes apparuit. Observata deinde sic se habuerunt.

Phasium.

Fig.
125.

| Phasium
Ordo. | Hor. Horol.
Oscillatorii. | Correcta. | Longiori Tubo. | Breviori. |
|------------------|------------------------------|-----------|--|-------------------|
| | h ' " | ' " | | |
| | 7 46 00 | 7 45 00 | Nulla Eclipsis. | |
| 1 | 7 54 50 | 7 53 50 | { Solis Eluctati & Nubibus
Magno Dexter Eclipsatus
apparuit. " | |
| 2 | 7 58 24 | 7 57 24 | | |
| 3 | 8 04 12 | 8 03 12 | I C ————— 2040 = 10 10 | |
| 4 | 8 13 40 | 8 12 40 | I C ————— 2773 = 13 56 | PL - 3198 = 26 18 |
| 5 | 8 18 37 | 8 17 37 | I C ————— 3580 = 17 52 | IC - 2334 = 29 13 |
| 6 | 8 21 06 | 8 21 06 | PL ————— 4975 = 24 50 | PL - 2989 = 24 35 |
| | | | Sol. Diamet. 6360 = 31 43 | 3850 = 31 40 |
| 7 | 8 28 01 | 8 27 01 | | PL - 2888 = 23 57 |
| 8 | 8 29 01 | 8 28 01 | PL ————— 4565 = 21 46 | |
| 9 | 8 35 12 | 8 34 12 | PL ————— 4478 = 22 18 | AZ - 2310 = 19 00 |
| 10 | 8 40 20 | 8 39 20 | I C ————— 4417 = 22 00 | AZ - 2070 = 17 02 |
| | 10 02 00 | 10 01 00 | Sol deinceps sub Nubibus receptus latuit usque ad
Emergentis Limbus per Nubes Defectu Liber apparuit;
Prodibat Clarius, & nihil in ejus Limbo deficere com-
pertus. | |
| | 10 04 00 | 10 03 00 | | |

Pro Correctione Horologii, acceperam pridie *Eclipsis Maii 31. Mane.*

| Hor. Horologii. | | Alt. ☉. | Temp. inde sup. | Error. |
|-----------------------------|-------------------------------------|---------|-----------------|--------|
| h. ' " | | ° ' " | h. ' " | ' " |
| 7 07 12 | Altitudinem Limbi Solis Inferioris. | 27 47 | 7 06 09 | 1 03 |
| 7 10 16 | ejusdem Limbi. | 28 16 | 7 09 19 | 0 57 |
| Iterum <i>Jun. 1. p. m.</i> | | | | |
| 5 32 00 | Altitudinem Limbi Solis Inferioris. | 22 06 | 5 31 06 | 3 56 |
| 5 35 23 | Limbi Superioris | 22 06 | 5 34 34 | 0 49 |
| 5 45 17 | Inferioris. | 20 6 | 5 44 18 | 0 59 |
| Denique <i>Jun. 2. m</i> | | | | |
| 8 9 44 | Altitudinem Limbi Solis Inferioris. | 37 34 | 8 08 45 | 0 59 |
| 8 13 36 | | 38 09 | 8 12 34 | 1 02 |
| 8 15 44 | | 38 28 | | |
| 8 17 51 | | 38 47 | | |
| 8 20 1 | | 39 07 | 8 18 49 | 1 11 |

Unde liquet & Motus constantiam servasse Horologium, & in *Eclipsi* debite fuisse
correctum. Hora

| <i>Hora Horol.</i>
<i>Oscillatorii.</i> | <i>Correct. per</i>
<i>Lin. Merid.</i> | <i>Mensura.</i> | <i>Phasum.</i> |
|--|---|---|---------------------------|
| h. ' " | h. ' " | | ' " |
| 8 06 45 | 8 8 27 | I C 1190 | 10 09 forsan 1109 = 14 50 |
| 8 11 00 | 8 12 42 | P L 1935 | 26 15 |
| 8 18 00 | 8 19 42 | I C 1405 | 19 04 |
| 8 21 00 | 8 22 42 | P L 1805 | 24 30 |
| 8 26 14 | 8 27 56 | I C 1504 | 20 47 |
| 8 34 00 | 8 35 42 | P L 1711 | 23 13 |
| 8 42 15 | 8 43 57 | I C 1551 | 21 03 accurate. |
| 8 46 30 | 8 48 12 | P L 1702 | 23 20 vel -- 1720 = 23 15 |
| 8 51 45 | 8 53 27 | I C 1553 | 20 04 accurate. |
| 9 00 00 | 9 01 42 | P L 1801 | 24 33 |
| 9 12 34 | 9 14 16 | I C 1357 | 18 25 |
| 9 30 55 | 9 32 37 | I C 872 | 11 50 |
| 9 41 15 | 9 42 57 | Præcise. Desiit Eclipsis, quantum per Aeris | |

Vibrationem potui discernere. Exitus Locus adeo Vertici vicinus erat, ut inquam ab ea partem inclinaret, bene non potuerim definire; etiamsi hora 9 29'. per Horologium Cuspides Horizonti apparerent Parallela.

Solis Diameter hora 9. 10'. erat 2334; satis ut putavi præcise.

Deinde, accedente sole ad Meridiem per Lineam longam Meridianam, Horologium iusto tardius inventum fuit scrupulis 1'. 42". Magno autem Æquinoctiali Sciaterico, quo medias minoresve Scrupuli horarii partes possum distinguere, Horologium toto hoc mane tardius duntaxat 45". Correctioni tamen per Lineam Meridianam quam Sciaterico fidendum puto.

7h. 50. Nihil sub Sole.

7 50. Initium accurate.

7 52 Notabilis defectus.

9 00 Digiti 3.

9 11 Digiti 3.

9 21 Digiti 2.

9 47. Non Finita; Imminente Fine.

6. Cum Sol e nubibus emergeret, Altitudinem graduum 48 accedens, ad eum direxi Quadrantem, quem ad hanc Altitudinem immotum tenui. Ex quo, Solis Margo superior *a*, tetigit Filum Horizontale *c d*, in Foco Telescopii, ad adventum Centri *b*, fluxere secundæ horariæ 104 = *ab* vel *br*; A transitu Centri *b* ad transitum Marginis Lunæ Superioris *s*, Secundæ 11 = *bs*; A transitu Centri *b*, ad cornu Superioris occidentalis *e*, fluxere Secundæ 25 = *eb*; A transitu Centri *b*, ad transitum Cornu Inferioris & Orientalis *i*, Secundæ 93 = *ik*; Hinc determinatur Linea Cornutum *ie* (seclusa variatione) ejusque Inclinatio ad Horizontem *lk*; & Punctum *p* concursus Tangentis Lunam cum Secante *ie p*, & Tangens ipsa *po*, Media proportionalis inter *pi*, *pe*: & Anguli *noe*, *toi*, hinc Angulus *ioe*; & Triangulum *ioe*, Lunari Circumferentia inscriptum.

Ex iis, aliisque ex *Astronomia* datis, deduxi

Initium esse debuisse *Parisis*—7h. 55'.

Finem vero ——— 10 12', vel. circiter.

4.
At Townley;
by Mr. Rich.
Townley.
Ibid. 633.

Fig. 125.

5.
At Wingfield;
near Derby;
by Mr. Imman.
Halton.
Ibid. p. 664.

At Paris;
by M. Cassini.

Fig. 126.

Temp.

7.
At
Dant-
zick ;
by M.
He-
veli-
us. Ib.
p. 666.

| Temp. juxta
Schæरिकum
& Hor. Oscil. | | | Altitudi-
nes ☉. | | | Tempus ex
Altitud. Sol.
correct. | | | Ordo
Phasium. | Magnitudo
Phasium. | Animadvertenda. |
|---|----|----|---------------------|----|---|--|----|----|------------------|---------------------------|--|
| h | ' | " | h | ' | " | h | ' | " | | | |
| 7 | 58 | 10 | 36 | 17 | 0 | 7 | 58 | 18 | | | Sol omnino Purus apparuit.
Nihil adhuc in Sole.
Initium Eclipseos. |
| 8 | 1 | 30 | 36 | 41 | 0 | 8 | 1 | 6 | | | |
| 8 | 3 | 30 | 37 | 3 | 0 | 8 | 3 | 39 | | | |
| 8 | 50 | 30 | | | | 8 | 50 | 0 | | | |
| 9 | 21 | 30 | | | | 9 | 21 | 0 | 1 | Digit. | |
| 9 | 22 | 30 | | | | 9 | 22 | 0 | 2 | $\frac{1}{2}$ fere. | |
| 9 | 24 | 10 | | | | 9 | 23 | 40 | 3 | $\frac{1}{2}$ Digit. | |
| 9 | 24 | 55 | | | | 9 | 24 | 25 | 4 | $\frac{1}{2}$ Digit. | |
| 9 | 27 | 28 | | | | 9 | 27 | 0 | 5 | $\frac{1}{2}$ Digit. | |
| 9 | 29 | 40 | | | | 9 | 29 | 10 | 6 | $\frac{1}{2}$ Digit. | |
| 9 | 33 | 25 | | | | 9 | 33 | 0 | 7 | $\frac{1}{2}$ Digit. | |
| 9 | 36 | 30 | | | | 9 | 36 | 5 | 8 | $\frac{1}{2}$ fere. | |
| 9 | 39 | 35 | | | | 9 | 39 | 10 | 9 | 2 Digit. | |
| 9 | 45 | 49 | | | | 9 | 45 | 25 | 10 | $\frac{1}{2}$ Digit. | |
| 9 | 54 | 22 | | | | 9 | 54 | 0 | 11 | $\frac{1}{2}$ Digit. | |
| 10 | 3 | 44 | | | | 10 | 3 | 22 | 12 | $\frac{1}{2}$ Digit. | |
| 10 | 8 | 30 | | | | 10 | 8 | 10 | 13 | $\frac{1}{2}$ Digit. | |
| 10 | 18 | 17 | | | | 10 | 18 | 0 | 14 | $\frac{1}{2}$ fere. | |
| 10 | 22 | 42 | | | | 10 | 22 | 22 | 15 | $\frac{1}{2}$ pau. plus | |
| 10 | 26 | 19 | | | | 10 | 26 | 0 | 16 | $\frac{1}{2}$ fere. | |
| 10 | 35 | 24 | | | | 10 | 35 | 6 | 17 | 4 $\frac{1}{2}$ fere. | Hora 10. 31'. 0". fere, Max.
incidit Obscuratio. |
| 10 | 38 | 53 | | | | 10 | 38 | 38 | 18 | 4 Dig. fere. | |
| 10 | 47 | 34 | | | | 10 | 47 | 20 | 19 | $\frac{3}{4}$ Digit. | |
| 10 | 53 | 49 | | | | 10 | 53 | 30 | 20 | $\frac{3}{4}$ Digit. | |
| 10 | 58 | 17 | | | | 10 | 58 | 8 | 21 | $\frac{2}{3}$ Digit. | |
| 11 | 5 | 27 | | | | 11 | 5 | 20 | 22 | $\frac{2}{3}$ Digit. | |
| 11 | 8 | 50 | | | | 11 | 8 | 44 | 23 | $\frac{1}{2}$ fere. | |
| 11 | 22 | 13 | | | | 11 | 22 | 8 | 24 | $\frac{1}{2}$ fere. | |
| 11 | 29 | 14 | | | | 11 | 29 | 10 | 25 | $\frac{1}{2}$ paul. plus. | |
| 11 | 35 | 25 | | | | 11 | 35 | 20 | 26 | $\frac{1}{4}$ paul. plus. | |
| 11 | 36 | 59 | | | | 11 | 36 | 55 | | | Nond. Sol omnino Purus exstitit
Finis Eclipseos. |
| 11 | 39 | 15 | | | | 11 | 39 | 15 | | | |
| 11 | 39 | 40 | | | | 11 | 39 | 40 | | | |
| 4 | 18 | 10 | 33 | 13 | 0 | 11 | 18 | 19 | | | |
| 4 | 20 | 0 | 32 | 25 | 0 | 11 | 20 | 36 | | | |

| | Ex Calcu-
lo Rudolph. | Ex Obser-
vat. | Diff. | Temp. |
|----------|--------------------------|-------------------|-------|---------|
| | ° / " | ° / " | ' " | h. / " |
| Semid. ☉ | 0 15 0 | | | |
| Semid. ☾ | 0 15 3 | 0 13 53 | 1 10 | 10 0 0 |
| | | 0 14 0 | 1 3 | 10 24 0 |
| | | 0 14 50 | 0 13 | 11 0 0 |
| | | 0 15 0 | 0 0 | Ultimo |
| Duratio. | h. 1 " | h. 1 " | | |
| | 1 50 58 | 2 17 40 | | |

8. Diebus præcedentibus, locum aptissimum elegimus in quo Aëre Puro *At Avignon;* fruermur, videlicet *Conventum RR. PP. Carmelitarum Discalceatorum,* qui re- *by M. Gallet.*
spectu Civitatis *Aven.* ad Ortum vergit & moenia stringens Aëre, Fumo & *n. 141. p. 1020.*
Vaporibus-urbanis libero gaudet; in medio Horti Cameram Obscuram Tape-
tibus construximus, & in eâ Instrumenta ad observationem necessaria ritè collo-
cavimus.

Tubospicillum aptavimus Lente Oculari Concavâ, & Objectivâ Convexâ instructum, duplicem habens motum firmo Sustentaculo, Verticalem scil. & Horizontalem, affixam Tabellam immobilem firmatis cochleis secum circumducens, Oculari Vitro semper parallelam chartâ candidissima indutam, in qua Solarem Speciem, distantia Tubospicilli determinatam descripsimus, hujus Diameter Circulis Concentricis in duodecim Dignos divisimus, & quemlibet Dignorum in partes Sexagesimas.

Loco Quadrantis, qui pluribus indiget cautionibus, & nimium obnoxius est vacillationibus, Gnomonem ad captandas umbras Solis in partes 400, optimè divisum disposuimus, ita ut liberè moveretur Situm Verticalem ope perpendiculi conservans. Tandem Horologium Rotatile, minuta prima & Secunda indicans, motu Penduli cum Cycloide præparavimus.

Ipsa die Eclipsis undecimâ *Junii*, horâ unâ circiter post ortum Solis, usque ad Initium & Finem Eclipsis, Speciem ejus Lucidam in Charta, sine intermissione recepimus, & quilibet ex nobis Instrumento sibi destinato semper invigilavit; Dominus de *Beauchamps* Musarum *Avenionensium* *Mænas* Amplissimus. Ego quoque cum illo, Tubospicillo; Dominus de *S. Florent*, Visus perspicatissimi, Gnomoni; Dominus *Montonier*, Horologio, cum Domino *Marii* Presbytero, in Mathematicis & præsertim in Horologiis versatissimo.

Statim ac sensibilibus coepit Umbra Discum inire, quantitatem partium Obscuratarum, Umbram in partibus Gnomonis, & Horam horologii notavi è directo primæ Phasis, & ita collegi Phases 39, contentas in sequenti Tabella.

| <i>Num. Phas.</i> | <i>Digiti Obscur.</i> | <i>Umbra Gnomon. in partibus qualium Gnomon. continet.</i>
400 | <i>Altitudo Solis Apparentis.</i> | <i>Altitudo Solis Vera.</i> | <i>Hora Horologii Penduli.</i> | <i>Hora correcta per Altitudinem Solis.</i> | |
|-------------------|-----------------------|---|-----------------------------------|-----------------------------|--------------------------------|---|---|
| | <i>D. /</i> | | <i>o / ' "</i> | <i>o / ' "</i> | <i>h. / ' "</i> | <i>h. / ' "</i> | |
| 1 | 0 27 | 561 | 35 29 23 | 35 28 48 | 7 50 31 | 7 50 34 | |
| 2 | 1 0 | 536 | 36 44 0 | 36 43 28 | 7 57 25 | 7 57 28 | |
| 3 | 1 30 | 520 | 37 34 7 | 37 33 37 | 8 2 3 | 8 2 7 | |
| 4 | 3 0 | 478 | 39 55 23 | 39 54 57 | 8 15 14 | 8 15 13 | |
| 5 | 3 25 | 466 | 40 38 30 | 40 38 6 | 8 19 0 | 8 19 14 | |
| 6 | 4 30 | 438 | 42 24 14 | 42 23 53 | 8 29 19 | 8 29 6 | |
| 7 | 4 40 | 434 | 42 39 58 | 42 39 37 | 8 30 59 | 8 30 34 | |
| 8 | 5 0 | 424 | 43 19 53 | 43 19 32 | 8 34 34 | 8 34 18 | |
| 9 | 5 30 | 412 | 44 9 12 | 44 8 52 | 8 39 19 | 8 38 56 | |
| 10 | 6 0 | 394 | 45 10 57 | 45 10 39 | 8 44 54 | 8 44 44 | Cornu Verticalia. |
| 11 | 6 40 | 375 | 46 35 50 | 46 35 33 | 8 53 19 | 8 52 45 | |
| 12 | 6 50 | 371 | 46 54 15 | 46 54 0 | 8 54 54 | 8 54 31 | |
| 13 | 7 0 | 366 | 47 17 30 | 47 17 14 | 8 56 44 | 8 56 44 | |
| 14 | 7 20 | 350 | 48 30 37 | 48 30 23 | 9 3 44 | 9 3 44 | Maxima Obscuratio. |
| 15 | 7 8 | 339 | 49 29 10 | 49 28 58 | 9 9 14 | 9 9 15 | |
| 16 | 7 0 | | | | 9 11 0 | | |
| 17 | 6 35 | 325 | 50 39 22 | 50 39 10 | 9 15 54 | 9 16 12 | |
| 18 | 6 25 | 321 | 51 0 12 | 51 0 0 | 9 18 14 | 9 18 11 | |
| 19 | 5 25 | 296 | 53 10 29 | 53 10 19 | 9 31 5 | 9 31 1 | |
| 20 | 5 0 | 286 | 53 55 14 | 53 55 5 | 9 35 44 | 9 35 30 | |
| 21 | 4 40 | 283 | 54 27 6 | 54 26 57 | 9 38 39 | 9 38 43 | |
| 22 | 4 35 | | | | 9 42 3 | | Cornua Parallela Horizonti. |
| 23 | 4 0 | | | | 9 47 19 | | |
| 24 | 3 53 | 266 | 56 3 35 | 56 03 27 | 9 48 45 | 9 48 36 | |
| 25 | 3 35 | 262 | 56 28 29 | 56 28 22 | 9 51 29 | 9 51 11 | |
| 26 | 3 30 | 262 | 56 37 32 | 56 37 25 | 9 52 11 | 9 51 59 | |
| 27 | 3 26 | 260 | 56 43 34 | 56 43 27 | 9 52 34 | 9 52 45 | |
| 28 | 3 6 | 254 | 57 15 59 | 57 15 53 | 9 56 5 | 9 56 10 | |
| 29 | 3 0 | | | | 9 57 40 | | |
| 30 | 2 48 | 249 | 57 50 48 | 57 50 42 | 9 59 34 | 9 59 53 | |
| 31 | 2 35 | 246 | 58 9 32 | 58 9 26 | 10 1 34 | 10 1 53 | |
| 32 | 2 25 | 243 | 58 26 11 | 58 26 5 | 10 3 0 | 10 3 41 | |
| 33 | 2 0 | | | | 10 6 46 | | |
| 34 | 1 50 | 236 | 59 12 33 | 59 12 28 | 10 8 56 | 10 8 47 | |
| 35 | 1 0 | 226 | 60 16 59 | 60 16 55 | 10 15 51 | 10 16 0 | |
| 36 | 0 40 | 220 | 60 56 21 | 60 56 16 | 10 20 57 | 10 20 31 | |
| 37 | 0 30 | 217 | 61 16 11 | 61 16 6 | 10 22 54 | 10 22 50 | |
| 38 | 0 20 | 214 | 61 36 12 | 61 36 8 | 10 25 0 | 10 25 12 | Cornu occidentale verticale cum Centro Solis. |
| 39 | Finis. | 209 | 62 6 23 | 62 6 19 | 10 28 41 | 10 28 50 | |

Proportio Diametrorum apparuit æqualis in Eclipsi 6 Digitorum, tunc enim Cornua Solis Verticalia distabant à Verticali Solis hinc inde gradibus circiter 30. Unde patet Centrum Lunæ tunc reperiri Peripheria Solis, & Lineam Diacentron esse æqualem Semidiametro Solis. Verum post Medium Eclipsis, mutationem aliquam in Diametro Umbrae deprehendimus; apparuit enim Umbra paululum magis Convexa, & ideo Semidiameter brevior, sed ferè insensibiliter.

| Temp. per
Horolog.
Oscillator. | Temp. inde
ab Observ.
correcta. | Observationes. | Partes Ob-
scur. qua-
rum 32 æ-
quales erant
Diam. Solis. | Partes li-
bera Mi-
crometro. |
|--------------------------------------|---------------------------------------|----------------------------------|---|-------------------------------------|
| h. / ' / " | h. / ' / " | | | |
| 2 12 30 | 2 03 45 | Sol integer; tunc nubes | | |
| 2 21 2 | 2 12 17 | Sol def. perexiguum ima Ora. | | |
| 2 21 25 | 2 12 40 | Partes obscuratæ supra Scenam. | 0 $\frac{1}{2}$ | |
| 2 32 20 | 2 23 35 | | 4 | |
| 2 46 40 | 2 37 55 | | | 22 19 |
| 2 51 37 | 2 42 52 | | 11 | |
| 3 5 23 | 2 56 38 | | 15 | |
| 3 9 5 | 3 00 20 | Centrum | 16 | |
| 3 10 4 | 3 1 19 | | | 15 07 |
| 3 11 47 | 3 3 2 | | | 14 36 |
| 3 12 15 | 3 3 30 | | 17 | |
| 3 13 23 | 3 4 38 | Cornua Horizontalia | | |
| 3 17 43 | 3 8 58 | | 18 | |
| 3 21 48 | 3 13 3 | | | 12 30 |
| 3 23 0 | 3 14 15 | | 19 | |
| 3 25 40 | 3 16 55 | | | 12 11 |
| 3 29 51 | 3 20 6 | | 19 $\frac{1}{2}$ | |
| 3 33 54 | 3 25 9 | | 19 $\frac{1}{2}$ | |
| 3 40 8 | 3 31 23 | | 19 | |
| 3 43 16 | 3 34 31 | | 18 $\frac{1}{2}$ | |
| 3 45 35 | 3 36 50 | | 18 | |
| 3 49 15 | 3 40 30 | Cuspis inferior ad Nadir Solis | | |
| 3 50 2 | 3 41 17 | | 17 | |
| 3 54 20 | 3 45 35 | Centrum | 16 | |
| 3 56 45 | 3 48 00 | | 15 | |
| 4 2 58 | 3 54 13 | | 13 | |
| 4 6 0 | 3 57 15 | | 12 | |
| 4 8 45 | 4 00 00 | | 11 | |
| 4 16 32 | 4 7 47 | | 8 | |
| 4 17 44 | 4 9 00 | Cuspides Verticales dist. 84 gr. | | |
| 4 24 8 | 4 15 23 | Inter Cuspides | | 17 32 |
| 4 24 20 | 4 15 35 | | 5 | |
| 4 27 14 | 4 18 29 | Inter Cuspides | | 15 35 |
| 4 29 36 | 4 20 51 | | 2 $\frac{1}{2}$ | |
| 4 30 32 | 4 21 47 | Inter Cuspides | | 12 22 |
| 4 33 06 | 4 24 21 | | 1 | |
| 4 36 22 | 4 27 37 | Finis accuratè | | |

XXXIV.

An Eclipse of
the Sun, July
2d. 1684. at
Greenwich; by
Mr. Flamsteed.
n. 162, p. 691.

Deinde ad Errorem Horologii investigandum.

| Hor. Horol. | Distantia à Vertice. | Hor. inde sup. | Error. Hor. |
|-------------|----------------------|----------------|-------------|
| h. / " | o. / " | h. / " | h. / " |
| 4 43 38 | Limbi Solis 60 11 00 | 4 34 56 | 8 42 |
| 4 45 48 | Inferior. 60 31 20 | 4 37 7 | 8 41 |
| 4 46 56 | 60 41 50 | 4 38 15 | 8 43 |
| 4 49 3 | 61 1 40 | 4 40 22 | 8 41 |

At Paris, by
M. Bullialdus.
n. 162. p. 693.

2. Alt. $\odot$. 50° . Initium Elapsum erat, Sole Nubibus tecto, & Digitus circiter deficiebat.

Alt. $\odot$. 41° . $15'$. Paulo amplius quam Digitum 7. Attingit Digitos, 8.

Alt. $\odot$. 29° . $30'$. Finis.

At the Obser-
vatory; by M.
Cassini. n. 162.
p. 693. n. 163.
p. 715.

3. The Beginning of the Eclipse could not be seen, but was deduced from the following Phases. The Apparent Diameter of the Moon appeared less than that of the Sun. It was judged that the Dilatation of the Sun's Light, might make the Moon's Diameter seem less.

| Phases. | Time. | Phases. | Time. |
|----------|---------|----------|---------|
| | h. / " | | h. / " |
| Begin.. | 2 25 30 | 7 Digits | 3 35 00 |
| or | 2 25 55 | 7 Digits | 3 55 50 |
| 1 Digit | 2 32 50 | 6 Digits | 4 4 10 |
| 2 Digits | 2 40 00 | 5 Digits | 4 12 25 |
| 3 Digits | 2 47 40 | 4 Digits | 4 19 15 |
| 4 Digits | 2 54 10 | 3 Digits | 4 25 50 |
| 5 Digits | 3 2 00 | 2 Digits | 4 32 15 |
| 6 Digits | 3 10 5 | 1 Digit | 4 37 40 |
| 7 Digits | 3 20 10 | End. | 4 43 23 |

4.
By M. de la
Hire and Po-
thenot. *Ibid.*
p. 716.

| Phases. | Time. | Phases. | Time. |
|-----------------|---------|----------|---------|
| | h. / " | | h. / " |
| Begin-
ning. | 2 25 24 | 7 Digits | 3 53 34 |
| 1 Digit | 2 33 2 | 6 Digits | 4 3 53 |
| 2 Digits | 2 40 30 | 5 Digits | 4 11 3 |
| 3 Digits | 2 47 47 | 4 Digits | 4 17 42 |
| 4 Digits | 2 54 41 | 3 Digits | 4 25 14 |
| 5 Digits | 3 2 41 | 2 Digits | 4 31 56 |
| 6 Digits | 3 12 6 | 1 Digit | 4 38 11 |
| 7 Digits | 3 20 54 | End. | 4 43 27 |
| 7 Dig. 5' | 3 36 27 | | |

The Beginning was deduced from many Observations made soon after it. The Moon's Diameter appear'd then not to be more than about $30''$; though by the Observations of her Diameter some Days before and after, it was judg'd to be $31'. 30''$. But the Extremities of the Horns, on which depended the Exactness of the Determination, appear'd a little Blunted.

| Phases. | Time. | Phases. | Time. |
|-----------------------|---------|---------------------|---------|
| Less than | h. ' " | | h. ' " |
| $\frac{1}{2}$ Digit | 2 29 30 | 7 Digits | 3 51 20 |
| $1\frac{1}{2}$ Digit | 2 37 40 | 6 Digits | 4 2 25 |
| 2 Digits | 2 40 25 | 5 Digits | 4 10 50 |
| 3 Digits | 2 48 34 | 3 Digits | 4 24 31 |
| 4 Digits | 2 54 30 | 2 Digits | 4 29 54 |
| 5 Digits | 3 3 00 | $0\frac{1}{2}$ fere | 4 41 00 |
| 6 Digits | 3 12 40 | | |
| 7 Digits | 3 22 18 | | |
| $7\frac{1}{2}$ Digits | 3 38 + | | |

5.
At the College
of Lewis the
Great; by R.
P. Fontenay.
Ibid. p. 717.

6. The Beginning was at $2^h. 54'. 30''$. The End at $5^h. 9'. 9''$. The Greatness of the Eclipse $8\frac{1}{2}$ Digits.

At Aix; by M.
Gautier. *Ib.*
p. 718.

| Phases. | Time. | | | |
|-----------------------|----------------|-------|-------------|-------|
| | By Fixt Stars. | | By the Sun. | |
| | h. | ' " | h. | ' " |
| 1 Digit | 2 | 45 3 | 2 | 50 3 |
| $8\frac{1}{2}$ Digits | 3 | 53 52 | 3 | 58 52 |
| 1 Digit | 4 | 59 20 | 5 | 4 20 |

7.
At Lyons; by
R. P. Paul
Hoste. *Ib.*

At $3^h. 26'. 14''$. (by the Stars) The Diameter of the Sun and Moon $30'$. $58''$. but at $4^h. 20'. 34''$. The Diameter of the Sun $30'. 58''$. of the Moon, $30'. 5''$.

| Phases. | Times. |
|--|---------|
| | h. ' " |
| The Beginning of the Eclipse | 2 40 00 |
| The Edge of the Moon at the Sun's Center | 3 25 00 |
| The Horns Horizontal | 3 49 00 |
| The Horns Vertical | 4 15 00 |
| The End of the Eclipse | 5 1 30 |

8.
At the Bay de
Roses; by M.
Chasselles. *Ib.*
p. 719.

The Greatness of the Eclipse about $\frac{1}{4}$ of the Sun's Diameter; at which time Venus might be seen without Pain.

9. The

At Honſieur;
by M. de Glos.
Ibid.

At Pau; by R.
P. Richaud. *Ibid.*

9. The Beginning was at 2h. 15'. 2". The End, at 4h. 34'. 35". The Greatness more than 8 Digits, but less than 9.

10. At 1 $\frac{1}{4}$ h. The Eclipse was not Begun. At 3 $\frac{1}{4}$ h. at 10 Digits. The End, at 4 $\frac{1}{4}$ h.

II.
At Avignon;
by R. P. Bonfa.
ibid.

| Phases. | Time. | Phases. | Time. |
|----------------|---------|----------------------|---------|
| The Beginning. | h. ' " | | h. ' " |
| 1 Digit | 2 43 27 | Horns Vert. | 4 24 32 |
| 9 Digits | 2 51 58 | 1 st Dig. | 5 1 16 |
| | 4 2 00 | The End. | 5 4 37 |

The Sun's Diameter 31'. 38". The Moon's 30'. 6".

At Oxford; by
Dr. Ed. Bern-
ard. n. 164.
p. 747.

12. Phases Quadraginta hujus Defectus captavit definiuitq; bona manus Wal-
lisi. Tempus etiam justum æquumque ejusdem Deliquii Altitudinibus ali-
quot Solaribus comprobarunt Clarissimi Viri D. Casuallus & D. Rookius, ubi
Horologiis & Oscillis nostris fuerat peccatum.

| Digiti
Obscuri,
cum suis
decimis. | Tempora Os-
cillatoria à
Cælo Cor-
recta. | |
|--|--|--|
| | h. ' " | |
| | 2 03 00 | Initium Eclipsis. |
| 0 6 | 2 07 44 | |
| 1 0 | 2 10 44 | |
| 1 3 | 2 13 19 | |
| 1 6 | 2 15 44 | |
| 2 2 | 2 21 34 | |
| 2 6 | 2 23 44 | |
| 2 9 | 2 25 39 | |
| 3 5 | 2 29 54 | |
| 4 0 | 2 33 04 | |
| 4 4 | 2 36 34 | |
| 4 6 | 2 40 04 | |
| 5 0 | 2 43 14 | |
| 5 3 | 2 48 19 | |
| 5 6 | 2 50 09 | Medium quasi Solem fecit π' Αελαργίον. |
| 6 5 | 2 57 30 | |
| 6 7 | 2 59 14 | |
| 7 4 | 3 08 24 | Obscuritas Maxima. |
| 7 1 | 3 11 09 | |
| 6 8 | 3 29 19 | |
| 6 7 | 3 31 39 | αὐθιχὲς ἐπὶ κέντρον π' σκότης. |
| 6 0 | 3 37 24 | |
| 5 7 | 3 39 29 | |

| H. / " | | H. / " | |
|--------|---|--------|-------|
| 5 | 5 | 3 | 44 29 |
| 5 | 0 | 3 | 47 59 |
| 4 | 4 | 3 | 50 34 |
| 3 | 9 | 3 | 53 54 |
| 2 | 9 | 4 | 2 49 |
| 2 | 5 | 4 | 4 39 |
| 2 | 2 | 4 | 6 19 |
| 2 | 0 | 4 | 7 54 |
| 1 | 4 | 4 | 13 9 |
| 1 | 0 | 4 | 15 4 |
| 0 | 7 | 4 | 16 49 |
| 0 | 3 | 4 | 19 39 |
| | | 4 | 21 14 |

Finis Eclipsis.

13. Mr. *Jacobs* at *Lisbon* Noted.The Beginning of the Eclipse at 1^h. 30'. exactly.

The Ending at ————— 4 12.

*At Lisbon. 16.
p. 749.*

14. Mr. *Ash* and Mr. *Molynaux*, toward the Middle of the Eclipse, having ^{In Ireland. 16.}
a short View of the Sun; they Judged that about 8 Digits were covered:
at the Ending also having a faint View thereof; they assigned its End, at 3^h.
56'. p. m.

The same Eclipse was observed by one Mr. *Osburn*, nigh *Tredagh*. *Initium*
1^h. 37'. 30". *Finis* 3^h. 56'. 20".

15. *Defumebant Solis à Vertice Distantias D. D. Jo. Ludovicus Donellus, & At Bononia;*
Nicolanus Ignatius Fomettus; Phasibus determinandis tres aderamus, D. D. Jo. by S. Domin.
Galeatus Manzins, Hercules Vanotus, & Ego. Observationes scribebat D. Gre- Gulielmini.
gorius Malisardus; Horas vero in Horologio notabat D. Bartolomeus Ferrarius. n. 203. p. 858.

| Hora Ho-
rologii. | Hora crui-
ta ex Di-
stantiis ob-
servatis. | Distan-
tie Cen-
tri Solis
à Vertice. | Phases. |
|----------------------|--|--|-------------------------|
| h. / " | h. / " | o / " | / |
| 3 34 | | | Dig. 2 30 |
| 3 37 30 | | | Dig. 3 |
| 3 48 14 | 3 47 47 | 51 31 | Dig. 4 20 dubia ubique. |
| 3 52 00 | | | Dig. 5 optima. |
| 3 57 00 | 3 56 2 | 52 58 | Dig. 5 30 |
| 4 2 00 | 4 2 29 | 54 10 | Dig. 6 |
| 4 4 45 | 4 5 59 | 54 44 | Dig. 6 30 optima. |
| 4 9 20 | 4 9 1 | 55 17 | Dig. 7 exacta. |
| 4 14 40 | 4 15 43 | 56 28 | Dig. 7 |
| 4 19 00 | | | Dig 7 20 dubia. |

| h. 1 // h. 1 // 0 1 // | | | | | |
|------------------------|---------|----|----|---------|------------------|
| 4 27 44 | 4 28 42 | 58 | 39 | Dig. 7 | 20 |
| 4 32 35 | | | | Dig. 7 | fatis exacta. |
| 4 34 20 | | | | Dig. 6 | 45 |
| 4 44 30 | | | | Dig. 6 | 30 |
| 4 47 15 | | | | Dig. 6 | 30 |
| 4 51 30 | | | | Dig. 6 | optima. |
| 4 54 38 | 4 53 53 | 63 | 16 | Dig. 5 | 30 exacta. |
| 4 58 2 | 4 57 38 | 63 | 56 | Dig. 5 | exacta. |
| 5 1 55 | 5 1 2 | 64 | 32 | Dig. 4 | 30 fatis exacta. |
| 5 4 40 | 5 5 28 | 65 | 19 | Dig. 4 | Diligens ubique. |
| 5 7 0 | 5 6 23 | 65 | 28 | Dig. 3 | 30 exacta. |
| 5 10 30 | 5 9 9 | 65 | 58 | Dig. 3 | exacta. |
| 5 12 35 | 5 13 28 | 66 | 43 | Dig. 2 | 30 fatis exacta. |
| 5 16 15 | 5 16 14 | 67 | 13 | Dig. 2. | exacta. |
| 5 19 50 | 5 20 24 | 67 | 57 | Dig. 1. | 30 exacta. |
| 5 22 30 | 5 23 21 | 68 | 28 | Dig. 1. | exacta. |
| 5 25 0 | 5 25 14 | 68 | 48 | Dig. 0 | 30 diligens. |
| 5 27 40 | 5 28 7 | 69 | 18 | Dig. 0 | accuratissima. |

Notabile fuit ; cum Quantitas Eclipsis fuerit Dig. 7. 20'. quod non modicam Aëris Offuscationem debebat inducere, (ut aliàs multoties in consimilibus Defectibus observatum est,) nihilominus tamen vix sensibilibus consuetum in Sole Libero Aëris statum mutatum fuisse ; unde plurimis Solem non respicientibus orta suspicio, aut Solem non defecisse, aut minimum quidem ; cujus quidem rei non alia mihi videtur assignanda causa, quam ingens vis Nubium à Sole maximè illuminatarum, quæ non multum ab eo distabant ; ab his enim Solis Radius per Reflexionem & Refractionem multiplicatus certe Intensior redditus deficientem aliundè Splendorem potuit compensare.

An Eclipse of the
Sun, May 1.
1687. at Ox-
ford. n. 187.
p. 329.
In divers other
Places. n. 189.
p. 370.

XXXV. 1. Dr. Wallis writes from Oxford that this Eclipse of the Sun was observed there about $\frac{1}{2}$ a Digit ; between one and two a Clock Afternoon.

2. Hæc Eclipsis, etiamsi contemnendæ Quantitatis fuerit, ac nudis oculis non omnino percipi potuerit, tamen ad accuratam determinationem Parallaxis & Latitudinis Lunæ maximè idonea videtur.

Londini, seorsim observationibus Hockio & Halleio ; Initii Momentum, Cælo licet purissimo, ob Obliquam Incidentiam Lunæ, debite Definire non licuit. Sed 1h. 16'. jam cœpta erat Eclipsis satis notabiliter : circa 1h. 40'. prope Medium Eclipsis, Chorda partis Eclipsatæ, sive inter Cornua, inventa est 9'. 30''. cui respondet Arcus 36°, in Diametro vero non nisi 1'. 30''. Finis, consensu utriusque Observatoris, contigit accurate 2h. 3'. 0''.

Grenovici in Observatorio Regio, Flamsteedius eadem de causa, Initium non vidit : Finem vero determinavit 2h. 4'. 15''. Medio Eclipsis sive Maxima Obscuratione, Chorda Partis Eclipsatæ erat 9'. 54''.

Apud Totteridge prope Londinum versus Corum, Finem vidit D. Haines, R. S. S. ad 2h. 2'. Quantitatem vero Maximam dimidii Digiti ab Austro.

In

In Insula *Barbada*, ad Oppidum *Bridge-Town*, Finem habuit D. *Frank* 1'. 30". ante quam Solis Altitudo fuit 31°. 47'. ad ortum, hoc est 7h. 56'. 45". Quantitatem Maximam æstimatione definivit duorum Digitorum ab Austro.

Norimbergæ eandem Eclipsin observavit J. P. *Wurtzelbauer*. Initium quidem accurate 1h. 58½'. circa Medium, sc. ad 2h. 36½'. Quantitatem Maximam duorum Dig. præcise; Finem vero ad 3h. 18'. 33".

Ulma Sueviæ, Observavit *Honoldus* Initium ad 1h. 48'. Quantitatem Maximam 2½ Dig. Finem vero ad 3h. 16'.

Lipsiæ, observatore *Kirchio*, Eclipsis jam satis notabilis ad 2h. 20'. 10". ad 2h. 47½'. Digiti 1½ circiter. Finis vero Incidit præcise in 3h. 15'.

Uratiflavie Silesiæ denique observavit D. G. *Schulzcius* Maximam Obscuracionem paulo citius quam 3h. 12'. fuisse 1½ Dig. Finem vero 3h. 37'.

XXXVI. 1. I did not see the Beginning of the late Eclipse, but the End happen'd here, precisely 24'. 9". after 10 a-Clock in the Morning, apparent Time: The greatest Obscuration, which was 10 Digits and a Quarter, was about 7 Min. after 9.

An Eclipse of the Sun, Sept. 3. 1699. at Oxford, by Dr. D. Gregory. N. 246. p. 336.

| Phases. | Quantities eclipsed. | | Times by a Pendulum. | | Phases. | Quantities eclipsed. | | Times by a Pendulum. | |
|----------|----------------------|----|----------------------|-------|---------|----------------------|----|----------------------|-------|
| | Dig. | ' | h. | " | | Dig. | ' | h. | " |
| Initium. | | | 8 | 57 14 | 20 | 9 | 49 | 10 | 30 10 |
| 1 | 0 | 52 | 9 | 3 26 | 21 | 9 | 21 | 10 | 33 11 |
| 2 | 1 | 32 | 9 | 8 23 | 22 | 8 | 52 | 10 | 35 53 |
| 3 | 2 | 28 | 9 | 14 14 | 23 | 8 | 30 | 10 | 38 46 |
| 4 | 3 | 19 | 9 | 19 40 | 24 | 7 | 38 | 10 | 42 42 |
| 5 | 4 | 8 | 9 | 24 57 | 25 | 7 | 14 | 10 | 40 7 |
| 6 | 5 | 15 | 9 | 31 57 | 26 | 6 | 33 | 10 | 49 42 |
| 7 | 5 | 50 | 9 | 35 2 | 27 | 6 | 6 | 10 | 53 22 |
| 8 | 6 | 26 | 9 | 38 43 | 28 | 5 | 27 | 10 | 56 37 |
| 9 | 6 | 53 | 9 | 40 36 | 29 | 5 | 9 | 11 | 0 00 |
| 10 | 7 | 20 | 9 | 43 47 | 30 | 4 | 33 | 11 | 4 24 |
| 11 | 7 | 56 | 9 | 50 39 | 31 | 3 | 57 | 11 | 8 16 |
| 12 | 8 | 30 | 9 | 55 9 | 32 | 3 | 13 | 11 | 13 3 |
| 13 | 9 | 23 | 10 | 1 44 | 33 | 2 | 41 | 11 | 18 3 |
| 14 | 9 | 53 | 10 | 5 46 | 34 | 2 | 11 | 11 | 21 37 |
| 15 | 10 | 24 | 10 | 10 34 | 35 | 1 | 32 | 11 | 25 38 |
| 16 | 10 | 38 | 10 | 14 37 | 36 | 1 | 2 | 11 | 28 27 |
| 17 | 10 | 45 | 10 | 17 54 | Finis. | 0 | 00 | 11 | 33 56 |
| 18 | 10 | 45 | 10 | 22 29 | | | | | |
| 19 | 10 | 12 | 10 | 27 31 | | | | | |

2.
An Nuremberg; by M. Wurtzelbauer. N. 265. p. 619.

From the 8th to the 12th Phasis, the opake Limb of the Moon on the South-side was a little rough; but about the Northern Horn, to near a 4th part of the Segment, it was more smooth: But when the Horns of the Eclipse were almost parallel to the Horizon, before and after the 15th Phasis, the Extremity of the gibbous Limb of the Moon looking downward, was somewhat enlightened, and of a kind of a Saffron Colour; but though the Sky was free from Clouds, yet no Stars were visible. Nor was even *Venus* itself visible in the open Air, unless by some more sharp-sighted than ordinary.

Amongst many round Plates cut out of thick Paper of divers Magnitudes, differing from one another 5'', about the first Phasis, and after, none agreed to the Limb of the Moon but that which was cut to a Radius or Semidiameter of 15'. 30". (taking the Radius or Semidiameter of that of the Sun to be 16'. 4'') and that gradually so swelled or augmented, that larger Plates were necessary to be made use of; and about the 36th Phasis, none less than one described of a Radius of 16'. 5''. would agree with, or equal the Appearance; and consequently, that the Diameter of the Moon, about the End of the Eclipse, did equalize, if not exceed, that of the Sun. Besides, in the 27th Phasis (when the obscure Part was 6 Dig. 6') the Body of the Moon did obscure more than two 3ds of the Sun's Limb; which is an Argument that its Semidiameter at that Time was equal to that of the Sun.

By Others, ib.
p. 623.

3. This Eclipse by the Observations of Mr. *Godfred Taber* at *Ciza*, began at 9^h, and ended at 11^h. 35'. and increas'd to 11 Dig. by the Observations of Mr. *Jacob Honold* at *Harvelsing* near *Ulm* of *Suevia*, it began at 8^h. 55'. and ended at 11^h. 31'. and its greatest Defect was 10 Dig. And by Observations at *Leipsick*, it began at 9^h. 11'. and ended at 12^h. 38'. 30''. The greatest Obscurity was 11 Dig. 20'. which lasted from 10^h. 16'. 45''. for 6'. Ten Digits being obscured, the Sky (being otherwise very clear) began to appear of a more livid or wan Complexion, and more sad than it usually looks with a clear Sky when the Sun is set, or below the Horizon. The Cocks also, which had hitherto crowed very frequently, as if silenc'd, going to Roost, left off Crowing, and did not renew it, till, by the Recovery of the Sun's Light, they had recover'd their former Gaiety and Mirth: However, we cannot learn that any Star, besides that of *Venus*, was discover'd, by those which were Spectators of it, in the open Air.

Changes likely to
be discover'd in
the Moon; by
M. Auzout.
N. 7. p. 120.
Dec. An. 1664.

XXXVII. I sometimes think that the Earth must appear, to the supposed Inhabitants of the Moon, to have a different Face in the several Seasons of the Year; and to have another Appearance in Winter, when there is almost nothing Green in a very great Part of the Earth; when there are Countries all cover'd with Snow, others all cover'd with Water, others all obscur'd with Clouds, and that for many Weeks together; another in Spring, when the Forests and Fields are Green; another in Summer, when the whole Fields are Yellow, &c. Methinks, I say, that these Changes are considerable enough in the Force of the Reflections of Light to be observ'd, since we see so many Differences of Lights in the Moon. We have Rivers considerable enough to be seen, and they enter far enough into the Land, and have Breadth capable to be observed. There are Fluxes in certain Places, that reach into large Countries, enough to make there some apparent Change, and in some of our Seas there float sometimes such

such bulky Masses of Ice, that are far greater than the Objects, which we are assured we can see in the Moon. Again, we cut down whole Forests, and drain Marshes, of an Extent large enough to cause a notable Alteration: And Men have made such Works, as have produced Changes great enough to be perceived. In many places also are *Vulcans* that seem big enough to be distinguish'd, especially in the Shadow. And when Fire lights upon Forests of great Extent, or upon Towns, it can hardly be doubted, but these luminous Objects would appear either in an Eclipse of the Earth, or when such parts of the Earth are not illuminated by Sun: But yet, I know no Man, who hath yet observed such things in the Moon; and one may be rationally assured, that no *Vulcans* are there, or that none of them burn at this time. This it is, which all curious Men, that have good Telescopes, ought well to attend; and I doubt not, but if we had a very particular Map of the Moon, as I had design'd to make one, with a *Topography*, as it were, of all the considerable Places therein, that we or our Posterity would find some Change in her. And if the Maps of the Moon of *Hevelius*, *Divini*, and *Riccioli*, are exact, I can say that I have seen there some Places considerable enough, where they put Parts that are clear, whereas I there see dark ones. 'Tis true, that if there be Seas in the Moon, it can hardly fall out otherwise, than it doth upon our Earth, where Alluviums are made in some Places, and the Sea gains upon the Land in others; I say, if those Spots we see in the Moon are Seas, as I must believe them to be, whereas I have many Reasons, that make me doubt, whether they be so. And I have sometimes thought, whether it might not be, that all the Seas of the Moon, if there must be Seas, were on the side of the other Hemisphere, and that for this Cause it might be, that the Moon turns not upon its Axis, as our Earth, wherein the Lands and Seas are as it were ballanced: That thence also may proceed the Non-appearance of any Clouds rais'd there, or of any Vapours considerable enough to be seen as there are rais'd upon this Earth; and that this Absence of Vapours is, perhaps, the Cause that no Crepuscle is there, as it seems there is none, my self at least not having been hitherto able to discern any Mark thereof: For, methinks, it is not to be doubted, but that the reputed Citizens of the Moon might see our Corpuscle, since we see that the same is without Comparison stronger than the Light afforded us by the Moon, even when she is Full; for a little after Sun-set, when we receive no more the first Light of the Sun, the Sky is far clearer than it is in the fairest Night of the Full Moon. Mean while, since we see in the Moon, when she is Encreasing or Decreasing, the Light she receives from the Earth; we cannot doubt, but that the People of the Moon should likewise see in the Earth that Light wherewith the Moon illuminates it, with, perhaps, the Difference there is betwixt their Bigness. Much rather therefore should they see the Light of the Crepuscle, being, as we have said, incomparably greater. In the mean time, we see not any faint Light beyond the Section of the Light, which is every where almost equally strong, and we there distinguish nothing at all, not so much as that clearest part, which is call'd *Aristarchus* or *Porphyrites*, as I have often try'd; although one may there see the Light, which the Earth sends thither, which is sometimes so strong, that in the Moon's Decrease, I have often distinctly seen all the Parts of the Moon that were not enlightned by the Sun, together with the Difference of the

clear Parts and the Spots, so far as to be able to discern them all. The Shadows also of all the Cavities of the Moon seem to be stronger, than they would be, if there were a second Light. For although afar off the Shadows of our Bodies, environed with Light, seem to us almost dark, yet they do not so appear, so much as the Shadows of the Moon do; and those that are upon the Edge of the Section, should not appear in the like Manner. But I will determine nothing of any of these Things.

*To find the Pa-
rallax of the
Moon, by
N. 9. p. 151.
Feb. An. 1666.*

XXXVIII. At certain times agreed on by two Observators, making use of Telescopes large, good, and well-fitted for this Purpose, by a Measuring-Rod placed within the Eye-Glass at a convenient Distance, that it may be distinctly seen, and serve for measuring small Distances by Minutes and Seconds, (which is easy enough in large Telescopes) Let each of such Observators thus furnished, observe the visible Way of the Moon among the fix'd Stars (by taking her exact Distance from any fix'd Star, that lies in or very near her Way, together with the exact Time of her so appearing) and the then apparent Diameter of her Disk; continuing these Observations every time for two or three Hours; that so, if possible, two exact Observations of her apparent place among the fix'd Stars being made at two places thus distant in Latitude, and as near as may be under the same Meridian, by these Observations concurring at the same time, her true and exact Distance may be hence collected, not only for that time, but at all other times, by any single Observer's viewing her with a Telescope, and measuring exactly her apparent Diameter. It were likewise desirable, that as often as there happens any considerable Eclipse of the Sun, that this also might be observ'd by them, noting therein the exact Measure of the greatest Obscuration compared with the then apparent Diameter of his Disk. For by this means, after the Distance of the Moon hath been exactly found, the Distance of the Sun will easily be deduced.

As for the Time fittest for making Observations of the Moon, that will be when she is about a Quarter, or somewhat less illuminated, because then her Light is not so bright, but that with a good Telescope she may be observ'd to pass close by, and sometimes over several fix'd Stars, which is about four or five Days before or after the Change: Or else at any other time, when the Moon passes near or over some of the bigger sort of fix'd Stars, such as of the First and Second Magnitude; which may be easily calculated and foreseen: Or best of all, when there is any total Eclipse of the Moon; for then the smallest Telescopic Stars may be seen close adjoyning to the very Body of the Moon.

*A Method for
observing Lunar
Eclipses; by
Mr. Rook.
N. 22. p. 388.
Feb. An. 1666.*

XXXIX. 1. Eclipses of the Moon are observed for two principal Ends: One Astronomical, that by comparing Observations with Calculations, the Theory of the Moon's Motion may be perfected, and the Tables thereof reformed; the other Geographical, that by comparing among themselves the Observations of the same Ecliptick Phases, made in divers Places, the Difference of Meridians or Longitudes of those Places may be discerned.

The Knowledge of the Eclipse's Quantity and Duration, the Shadow's Curvity and Inclination, &c. conduce only to the former of these Ends. The exact Time of the Beginning, Middle, and End of the Eclipses, as also in total Ones, the Beginning and End of total Darkness, is useful for both of them.

But

But because in Observations made by the bare Eye, these times considerably differ from those with a Telescope; and because the Beginning of Eclipses, and the End of total Darkness, are scarce to be observed exactly, even with Glasses (none being able clearly to distinguish between the true Shadow and Penumbra, unless he hath seen for some time before, the Line separating them, pass along upon the Surface of the Moon:) And lastly, because in small partial Eclipses, the Beginning and End, and in total Ones of small Continuance in the Shadow, the Beginning and End of total Darkness are unfit for nice Observations, by reason of the slow Change of Appearances, which the oblique Motion of the Shadow then causeth. For these Reasons I shall propound a Method peculiarly design'd for the Accomplishment of the Geographical End in observing Lunar Eclipses, free (as far as is possible) from all the mention'd Inconveniencies.

For, *First*, It shall not be practicable without a Telescope. *Secondly*, The Observer shall always have Opportunity, before his principal Observation, to note the Distinction between the true Shadow and the Penumbra. And, *Thirdly*, It shall be applicable to those Seasons of the Eclipse, when there is the suddenest Alteration in the Appearances. To satisfy all which Intents,

Let there be of the eminentest Spots, dispers'd over all Quarters of the Moon's Surface, a select Number generally agreed on, to be constantly made use of, to this purpose, in all Parts of the World. As for Example, Those which M. Hevelius calleth, *M. Sinai*, *M. Aetna*, *M. Porphyrites*, *M. Sexorum*, *Inf. Besbicus*, *Inf. Creta*, *Palus Maeotis*, *Palus Maracotis*, *Lacus Niger Major*.

Let in each Eclipse, not all, but (for Instance) three of these Spots, which then lie nearest to the Ecliptick, be exactly observ'd, when they are first touch'd by the true Shadow; and again, when they are just compleatly entred into it; and (if you please) also in the Decrease of the Eclipse, when they are first fully clear from the true Shadow. For the accurate Determinations of which Moments of Time (that being in this Business of main Importance) let there be taken Altitudes of remarkable fix'd Stars; on this side of the Line, of such as lie between the *Aequator* and *Tropick of Cancer*; but beyond the Line, of such as are situate towards the other Tropick, and in all Places of such as at the Time of Observation are about four Hours distant from the Meridian.

2. *Eclipsis Luna*, Die 29 *Oct. An. 1697*. Observata est *Roterodami* per Te-
 lescopium quatuor fere pedum Parisiensium Oculari Convexo, in cujus Foco
 erant Fila Quatuor, sese in Axe interfecantia ad Angulos Rectos & Semirectos,
 ad Phases dimetiendas, Macularumque Lunarium situm determinandum hoc
 Telescopium impositum erat Fulcro habenti Axem situ parallelo Axi mundi con-
 stitutum, ut postquam ad Lunam directum esset ad unius Phasis Observationem,
 posset ad alias Phases observandas per Lunæ Semitam ad Occasum revolvj. Ita
 autem primo dirigebatur ad Lunam, ut eo immoto permanente Lunæ Limbus
 Borealis suo Motu ad Occasum raderet unum ex his Filis, quod ideo Parallelum
 dicimus, licet ob Motum Lunæ in Declinationem Motui Lunæ ad Occasum
 multo celeriori commixtum non nihil ab Aequatore declinaret dum Lunæ discus
 in reliqua tria Fila successive incideret. Horum trium Filorum intermedium An-
 gulos Rectos cum Parallelo efficiens, Rectum Perpendiculare & Verticale appel-
 lamus. Reliqua duo Obliqua, quorum Primum dicimus in quod prius Luna in-
 cidit, Secundum Obliquum in quod Luna incidit posterius. Initio Eclipsis,
 quando.

By M. Ja. Cist.
 lini. N. 236.
 P. 15.

Vid. infra.
 §. LIV. 2.

Fig. 127.

Fig. 128.

quando Lunæ Punctum Borealissimum nondum in Umbra erat Immersum, illud Filo aptavimus Parallelo. Deinde postquam tale Punctum Umbræ Immersum est, eidem Filo aptavimus Australissimum Lunæ Punctum. Unde factum est ut quod Filum initio fuerat Primum in aliarum Phasium determinatione fuerit Postremum, & Primum evaserit quod Postremum fuerat initio. Cum autem Lunæ Limbus Filum Parallelum percurreret, Lunæ Centrum intelligebatur describere Lunarem Semitam huic Filo Parallelam, quæ ab aliis tribus Filis secabatur. Portiones autem hujus Semitæ supponuntur proportionales Temporibus, quibus ipsas Lunæ Centrum percurrit, inæqualitas enim Motus Proprii universali Motui immixti exiguo Tempore imperceptibilis est. Cum igitur Lunæ Limbus Parallelum percurreret, observabatur beneficio Horologii Pendulo instructi, & diebus præcedentibus ad Solem conformati, Tempus adventus Lunæ Macularum aliquot & Lunarium Cornuum ad hæc tria Fila, & deprehensum est dictæ Eclipsis tempore Lunæ Discum transire per Filum Rectum, $2'. 24''$. per Fila vero Obliqua $3'. 24''$. ideoque Semidiametrum Lunæ transire per Rectum $1'. 12''$. per Obliqua vero $1'. 42''$. Differentia utriusque Transitus existente $30''$. Hinc observato uno Appulsu Lunæ ad quodvis horum Filorum, vel uno Egressu, dantur omnes alii ad reliqua Fila. Semidiameter Lunæ AB, jacens in Lunari Semita ABCDEF, pertransit per ejus punctum quodlibet dum Centrum A percurrit spatium sibi æquale AB, ut alia Semidiameter AK, Angulum Rectum efficiens cum alia recta Linea NCK ad punctum K, in quo proinde Lunam continget in K, ab ejus Semita declinans Angulo KCA, transit per ipsum Filum CK, dum Centrum Lunæ percurrit AC, Hypothenusam Trianguli Rectanguli AKC: estque Tempus Transitus Semidiametri AB, per Filum perpendiculare Lunam contigens in B, ad Transitus Semidiametri AK, per Filum Obliquum NCK, ut AB, vel AK, Sinus Anguli ACK, ad AC, Sinum Anguli Recti, sive Radium. Filo igitur NCK, faciente cum semita Lunæ Angulum KCA Semirectum, & Angulus KAC, in Triangulo Rectangulo, Semirectus erit, ideoque Latera CK, KA, æqualia, erit Transitus Rectus secundum AB, ad Transitus Semidiametri AK, per Filum Obliquum NK, ut Sinus Anguli Semirecti, ad Sinum Anguli Recti, ut 707 ad 1000, sive ut $72''$. ad $102''$. vel $1'. 42''$. fere, ut observabatur, Lunaris Centri Semita existente AH, Lunæ Semidiametro ipsi perpendiculari AM, ducta MNO, Parallela ipsi AH, ipsa congruet Filo quod Lunæ Limbus motu suo ad Occidentem radet, quod secabitur ab Obliquis NCK, NGI, & à Recto NEP, in puncto N, qua transit Axis Telescopii; facietque cum his Filis Lunaris Orbita duo Triangula Rectangula NEC, NEG, quæ supponuntur habere Angulos Semirectos ad puncta N, C, G: Sunt ergo Similia & Æqualia, habentque Latera CE, EG, EN, Æqualia Semidiametro Lunæ AM. Si hinc inde ab Interfectionibus, C, & G, accipiantur in Filis ipsi Semidiametro æquales CK, CS, GI, GR, & in orbita CA, CF, GD, GH, æquales CN, & jungantur AK, FS, DR, HI, erunt ipsæ omnes æquales inter se, efficientque ad Fila Angulos Rectos ad K, S, R, I. Quare Centro Lunæ existente in A, Luna Tanget Primum Obliquum in K, & postquam Centrum Lunæ venit ab A in C, ejus Semidiameter congruet Lineæ CE, ideoque Luna tanget Filum Rectum in E. Postquam autem Centrum Lunæ venerit ab A in D, tanget Secundum Obli-

Obliquum in R. Est autem AD, æqualis Diametro Lunæ, nam cum GD sit æqualis CA, addendo DC, habebitur AD æqualis GC, qui quidem est Diametro Lunæ æqualis. Sed cum GD sit æqualis CF, si ab his auferantur æquales GE, EC, erit FE æqualis ED, & erit DF dupla; tantumque erit à Contactu primo Secundi Obliqui in R, ad Contactum ultimum Primi Obliqui in S; & postquam Centrum Lunæ progressum fuerit in G ad distantiam Semidiametri unius EG, Luna continget ultimo Filum Rectum in E. Luna Centro progresso à G in H, ipsa tanget ultimo Secundum Obliquum in I. Supposito igitur transitu Recto Lunæ fieri 2'. 24". ut observatum est. Et,

| | | | Diff. | Contact. |
|--|---|----|-------|----------|
| Posito Centro in A, & contactu Primi Obliqui in K - - | 0 | 0 | 1 | 42 |
| Centrum Lunæ erit in C, & continget 1°. Rectum in E - | 1 | 42 | 0 | 42 |
| Centrum perveniet in D, & continget 1°. 2 Obliquum in R - | 2 | 24 | 0 | 30 |
| Lunæ Centrum erit in E, Filo intermedio Perpendiculari - | 2 | 54 | 0 | 30 |
| Centrum perveniet in F, & continget ultimo 1 Obliquum in S - | 3 | 24 | 0 | 42 |
| Centrum erit in G, & continget Ultimo Rectum in E - | 4 | 6 | 1 | 42 |
| Erit tandem in H, & continget ultimo 2 Obliquum in I - | 5 | 48 | | |

Hinc Calculo correspondebant ut plurimum Observationes in hac Eclipsi intra Secundum unum. Sufficiebat igitur in una Phasi observare duos in his Transiti-
bus in reliquis Phasibus unum, ut reliqui omnes innotescerent. Quod ad Luna-
res Maculas attinet; Comparatur Transitus Marginis præcedentis Lunæ &
Maculæ per Filum Rectum ad habendam Differentiam quam dicimus Longi-
tudinem Maculæ à Magine præcedenti: & Transitus Rectus Maculæ compara-
tur cum Obliquo ad habendam Differentiam quæ æqualis est distantia viæ Ma-
culæ à Semita Puncti Borealissimi vel Australissimi radentis Filum Parallelum.
Cum enim via Maculæ ABC, Parallela sit viæ Marginis DEF, eosdem cum
eisdem Filis Angulos facit Semirectos ad A & C, Rectos ad B, unde Angulus
ad A, æqualis est Angulo ad C, & Latus BA, æquale Latèri BE, Latitudini Ma-
culæ B à Filo FED. Data autem Longitudine & Latitudine Maculæ datur
ejus Situs in Luna. Descripto quippe circa ipsam Quadrato cujus Latus AB,
intelligatur congruere Filo Parallelo, & sit divisum in tot æquales partes quot
Secundis Luna per filum Rectum transit, Latera vero AC, BD, ipsi Filo per-
pendicularia sint in totidem similiter partes æquales divisa. Sumpta in Parallelis
Longitudine AE, CF, & ducta FE, & in Perpendicularibus Latitudine AG,
BH, quam æqualem dicimus viæ interceptæ inter Rectum & Obliquum, deter-
minatur Situs Maculæ M, in communi harum rectorum Intersectione.

Quod spectat ad Lunæ Cornua in Eclipsi, ipsa determinari possunt sola Lon-
gitudine, modo sciatur quor in Semicirculo Australi vel Boreali sint. Ut Cornu
I, per Longitudinem AE, vel CF, recta quippe FE, Lunæ Marginem secat
in duobus punctis L, & I, quorum unum est in Semicirculo Boreali, alterum
in Australi. Potest etiam determinari sola Latitudine AK, vel BM, modo scia-
tur quo in Semicirculo Orientali vel Occidentali sit Punctum I. Ex Lineis au-
tem Longitudinis & Latitudinis illa exactius Situm Cornu determinat, quæ
propior est Centro; ut hic punctum I, exactius determinatur Longitudine quam

Latitu-

Fig. 129.

Fig. 130.

Fig. 131.

Latitudine; è contra Punctum O exactius Latitudine quam Longitudine, idque ob minorem Obliquitatem Lineæ rectæ ad Circumferentiam, qua efficitur ut exigua variatio Distantiæ magis sit in Circumferentia sensibilis. Alia ratione per Obliquos Transitus determinatur Situs Macularum & Cornuum Lunæ, si Linea AD, Parallela Semitæ Lunari PQ, ipsius Marginem tangenti, fiat Diameter quadrati Lunæ Circumscripti quæ dividatur in tot æquales partes quot Secundis Luna per Filium Obliquum pertransit, ut in hac Eclipsi in partes 204. Hujus Quadrati duo Latera AC, BD, Primum Obliquum repræsentabunt, utpote illi Parallela, reliqua AB, CD, Secundum Obliquum, sumpta autem Differentia inter Transitum Marginis Præcedentis Lunæ & Maculæ M, per Obliquum, in Secundis horariis ab Angulo præcedente ab A in T, & ducta per T, Recta EF, Parallela Lateri, AC, & similiter sumpta ab eodem Angulo A, differentia inter transitum Marginis præcedentis K, & Maculæ M, per Secundum Obliquum AB, ut AV per punctum V, ducatur Recta GVH, Parallela Lateri AB, repræsentabit Secundum Obliquum secans Priorem in Puncto M, ibique Situm Maculæ determinabit. Eadem ratione determinabitur Situs Cornu E, per Differentiam ipsius Transitus & Marginis per Primum Obliquum sumptam in Diagonali ut AT, Situs Cornu H, per differentiam ipsius Transitus per Secundum Obliquum AB, ut AV, & ducta per V, Recta GH, Parallela Lateri AD, modo sciatur sitne Cornu in Semicirculo Præcedente aut Sequente.

An Eclipse of the Moon, 1665. Observed at Dantzick; by M. Hevelius. N. 19. p. 348.

XL. The Tables did not indicate an Eclipse of the Moon, July 27. (N.S.) 1665. but tho' the Sky here was very clear, yet the Moon was not at all obscured by the true Shadow, but entred only a little into the Penumbra, wherein it continued 50'. The Beginning of its touching the Penumbra did then almost happen, when *Aquila* was elevated 36°. 18'.

An Eclipse of the Moon, June 6. A. 1666; by M. Hevelius. Ibid. p. 348. N. 21. p. 371.

XLI. In the Eclipse of June 16, (N.S.) 1666, the first Phasis of 1 Dig. 45' appear'd in the Moon's Altitude of 2°. 30'. when the greatest Obscuration was already past. The End fell out 9^h. 27'. about 120°. from the Zenith Westward.

An Eclipse of the Moon, Sept. 19. A. 1670. by M. Hevelius. N. 61. p. 2023.

XLII. Die 29 Sept. (St. N.) 1670. mane Initium hujus Eclipsis incidit 2^h. 22'. quanquam id ipsum vix omnino accurate observari potuerit, ob Umbram Terræ Dilutissimam: Siquidem, durante Eclipsi, tota Umbra adeo Tenuis erat atque Diluta, ut omnes præcipuas Maculas per eam, meo viginti pedum Tubo, quin & brevioribus, optime conspiciere potuerim.

Maxima ejus Obscuratio incidit 3^h. 50'. Finis vero 5^h. 21'. Tota itaque Duratio fuit 2^h. 59'. & Quantitas vix amplius 9 Digitorum. Circa Medium hujus Eclipsis 3^h. 40'. Stellulam quandam Incognitam, ac solo Tubo conspicuam, à Luna circa Lacum Nigrum Majorem tectam, clarissime conspexi; sed exire eam non deprehendi. Deinde finita Eclipsi, jucundissimum quoque observatu erat, bina Luminaria simul supra Horizontem videre. Nam priusquam Luna occideret, Sol oriebatur. Cætera notatu digna ex particulari hoc Typo deprehendis.

| Ord. Phasium. | Phases Luna
Telescopio ob-
servata. | Altitudines Observata. | Tempora ex
Altitudini-
bus Correct. | Animadvertenda. |
|---------------|---|------------------------|---|--|
| | | | h ' " | |
| | | Caudæ Cygni 50°. 54. | 12 8 34 | |
| | | Caudæ Cygni 50 25 | 12 11 32 | |
| | | Pollucis II. 26 39 | 1 28 56 | |
| | | Pollucis II. 27 16 | 1 33 17 | |
| | Penumb. Init.
Initium Eclips. | | 2 10 0
2 23 0 | Cæpit Eclipsis in part. γ & sup.
circa Sin. Hyperb. |
| 1 | 1½ Dig. | | 2 31 0 | Umbra Sin. Apollinem, & M.
Porphy. Stringebat. |
| 2 | 2½ | | 2 36 0 | Lacus Niger Maj. tegi incipieb. |
| 3 | 3½ | | 2 41 40 | Umbra ad Inf. Corsicam per-
venerat. |
| 4 | 3½ paul. plus. | | 2 45 50 | Pal. Maoris & M. Ætna plane
tecta. |
| 5 | 4½ | | 2 49 0 | |
| 6 | 5 Dig. fere. | | 2 53 0 | |
| 7 | 5½ | | 2 57 0 | Inf. Besbic. & Inf. Macra
tecta. |
| 8 | 6 Dig. | | 3 3 0 | |
| 9 | 6½ | | 3 5 0 | |
| 10 | 6½ | | 3 8 30 | Inf. Melos tegi incipiebat. |
| 11 | 7½ | | 3 11 0 | |
| 12 | 7½ | | 3 16 0 | Inf. Rhodus tecta, |
| 13 | 7½ | | 3 20 0 | |
| 14 | 8½ | | 3 25 30 | |
| 15 | 8½ | | 3 29 0 | Inf. Major Caspii tegebatur. |
| 16 | 8½ | | 3 33 40 | |
| 17 | 8½ | | 3 37 50 | |
| | | | 3 40 0 | |

| | | | | |
|----|---------------|-----------------------|---------|--|
| 18 | 9 Dig. | | 3 47 0 | Max. Obscuratio per J. Leto. & M. Aber. se extend. |
| 19 | 9 Paulo min. | | 3 51 10 | Pal. Maræotis sub Umbra |
| 20 | 8½ | | 3 57 0 | Sectione Subsisteb. |
| 21 | 8½ | | 4 1 40 | |
| 22 | 8¼ fere. | Polluc. II. 49°. 50'. | 4 6 0 | |
| 23 | 8 paul. plus. | | 4 10 28 | |
| 24 | 7¾ | | 4 11 0 | Inf. Melos & Rhodus umbra |
| 25 | 7¾ | | 4 17 20 | exiverant. |
| 26 | 6¾ | | 4 22 25 | M. Porphyrites illustrari inci- |
| 27 | 6¾ fere. | | 4 27 35 | piebat. |
| 28 | 5¾ | | 4 33 40 | Umbra M. Ætnam jam reli- |
| 29 | 4½ | | 4 37 37 | querat. |
| 30 | 4 Dig. | | 4 44 46 | M. Argentar. & Inf. Besbic. |
| 31 | 3¾ | | 4 50 56 | illustrabantur. |
| 32 | 2¾ | | 4 54 0 | |
| 33 | 2¼ fere | | 4 57 0 | |
| 34 | 1¾ | | 5 1 23 | Umbra totam fere I. Caspian |
| 35 | 1¼ Dig. | | 5 4 45 | Maj. Transierat. |
| | | Cordis Ω 25°. 48'. | 5 7 43 | |
| | | | 5 10 50 | |
| | Finis Eclips. | | 5 21 25 | Finis circa Riphai Montes |
| | Penumb. Finis | | 5 24 0 | contingit. |
| | | Sirii. 18°. 54'. | 5 28 32 | |

An Eclips of the Moon, Sept. 8. XLIII. 1. Sept. 8. 1671. Circa Horam sextam vespert. Luna ascen-
 1671. at Ecton; debat totaliter Obscurata Ectonia in Comitatu Northamptoniano.
 by Mr. Palmer. Cœpit Emergere ex Umbra, Centro Luna Alto 9°. 35', five 7^h. 18'.
 n. 76. p. 2272. Finis, Arcturo Alto 16°. 30'. five 8^h. 16'. 20". Unde computatur Me-
 dium Eclipsis fuisse 6^h. 28'. 16".

At London; 2. The Emersion: Alt. of the upper Edge of the D. 10 30 | h. 7 21
 by Mr. Street. ib. The End of the Eclipse; Alt. of Arcturus. 16 20 | 8 16
 By Dr. Hook. 3. 7^h. 27^h ½; I first observed the Moon Eclipsed when it began to be
 77. p. 2296. enlightened, the Total Darkness being already past. The shadow passed
 through the middle of the Spot called by Hevelius, M. Porphyrius.

7^h. 49'. The Shadow passed through the middle of *M. Sinai*, through the middle of the Eastermost of the three Lakes called *Mare Adriaticum*, and just touched the Ridge of the *Appennine Mountains*.

7^h. 54'. It passed the middle of the *I. Besbicus* in the *Propontis*.

8^h. 0¹/₂'. It passed through the *Streights* of the *Pontus Euxinus*, at the Promontories *Acherusia* and *Aristes*.

8^h. 6¹/₂'. It touched the *Palus Maotis*, which *Palus Maotis* was then distant from the Limb of the Moon, next adjacent, one third part of its shorter Diameter or Breadth.

8^h. 17'. The Shadow went off the Body of the Moon upon the innermost Limb-line of *Hevelius's* large Chart of the Moon at the 29 Division, just without the *I. Major* of the *Caspian Sea*. The dusky *Penumbra* left not the Limb of the Moon quite without some kind of Darkness till 8^h. 29'; at which time I found that that side of the Moon which the shadow last left, was full as light and clear as the other.

About four or five Minutes after the Shadow was gone off, I perceived a faint Representation of Colours upon that part of the Body of the Moon, which was most affected with the *Penumbra*, somewhat resembling the Colours of a faint *Halo* about the Moon; this grew fainter and fainter, and after a few Minutes was no more Visible. It did not seem to be caused by any Clouds or Exhalations in the Air, the Sky near the Moon being very clear, and the said Colours not appearing any where, but upon the dusky part of its Phases. Possibly it might be caused by the Refraction of the Light from the Sun through the Atmosphere about the Earth.

4. Observatus est *Finis*, alto *Arcturo* ad occasum

Ablata Refractione, alto

Hinc datur *Finis* Observatus

Finis Penumbra alto *Arcturo*

Correct.

Unde datur

Observata est *Maotis* tota extra Umbram

Alto *Arcturo*

Correct.

| | ° | ' | " |
|---------|----|----|----|
| h | 13 | 41 | |
| " | 13 | 37 | 45 |
| 8 29 16 | 13 | 0 | |
| | 12 | 56 | |
| 8 33 40 | | | |
| 8 24 16 | | | |
| | 14 | 26 | 0 |
| | 14 | 23 | 55 |

At Paris; by
M. Bullialdus.
n. 76. p. 2273.

Exivit *Luna* ex Umbra e regione *Petræ Sogdiana*, *Hevelii*.

5. Hora 8. 30'. Per Nubes Dehiscentes, satis tamen crassas, *Lunam* sub obscure animadvertimus, & quidem tanto Lumine jam imbutum, ut dixisses Eclipsin jam esse præteritam. Exinde certum erat, Totalem Obscuracionem jam minimum esse eo tempore præteritam, imo aliquanto adhuc citius: Siquidem *Lunam* Rursus adesse nos omnes per Nubes illas satis dilucide deprehendimus; Sic ut Eclipsis in Cælo ultra dimidiam horam citius ingruerit, quam *Kepleri* Calculus id indicaverit. 8^h. 34'. Minimum ad integrum Digitum animadverti jam extra Umbram sese *Lunam* extricasse; & denuo 9^h. 41'. ad 1¹/₂'. Dig. Lumen *Lunæ* jam excrevisse, quantum id dijudicare circiter dabatur.

At Dantzick;
by M. Hevelius.
n. 78. p. 3028.

At Hamburg;
by Dr. Fogelius.
ib. p. 3033.

6. *Æque nos hic ac Hevelius, Gedani, Calculum Rudolphinum in nupera Eclipsi aberrasse deprehendimus. Emerfisse enim jam Lunam ex Umbra Terræ ante horam nonam etiam hic vidimus.*

An Eclips of the
Moon, Jan. 1.
1674-5. at
London; by
Dr. Hook. n.
111. p. 237.

XLIV. 1. *Initium veræ Umbrae*
Immerfio
Emerfio
Finis veræ Umbrae

| h. " | |
|------|----|
| 5 | 22 |
| 6 | 19 |
| 7 | 58 |
| 8 | 58 |

The Penumbra was seen to continue near half an Hour before it wholly quitted the Body of the Moon.

At Derby; By
Mr. Flamsteed.
ib.

2. Mr. Flamsteed observed the Beginning of the Entrance of the true Shadow h. 5. and 19'.

At Paris; by
M. Bullialdus.
ib. p. 238.

3. *Initium veræ Umbrae Alta Capella*
Immerf. Alta Capella
Emerf. Alt. Cap. Pollucis
Fin. veræ Umbrae, Alt. Sirio

| ° | ' | h. | ' | " |
|----|----|----|----|----|
| 52 | 26 | 5 | 32 | 29 |
| 62 | 8 | 6 | 33 | 3 |
| 43 | 46 | 8 | 9 | 30 |
| 20 | 47 | 9 | 10 | 0 |

At Paris; by
M. Cassini, M.
Picard, and M.
Roemer. ib.
p. 238. n. 112.
p. 257.

4. At 5^h. 12'. in the Evening, in the Royal Observatory, they began to perceive, that the Oriental part of the Moon, by little and little lost its Light; so that at 5^h. 25'. they saw a manifest Penumbra; then at 5^h. 32'. 50". the Limb over against the Spot called *Hevelius* grew so dark, that they all agreed, that this was the true Beginning of the Eclipse. At 8^h. 7'. one of the Observers believed the *Emerfion*, another at 8^h. 8'. and the third at 8^h. 9'. 30". but afterwards considering the *Emerfion* of the first Spots, they all esteemed it at 8^h. 8'. At 7^h. 21. the Southern Limb of the Moon was come close to a Telescopick Star, at 8^h. 9'. 20". another Star yet less than the former, came out of the darkest Side, almost over against the Spot *Langrenus*. At 9^h. 9'. 40". all the three Observers agreed, that the Moon then came out of the Shadow. The Diameter of the Moon being measured before the Eclipse was of 32'. 15".

The Times were noted by great *Pendulum Watches*, that had been adjusted by the Sun the same Day, and that were afterwards verified the next Day: Besides that, before the Eclipse at 4^h. 45'. 1". by the Watches, the Star *Capella* was 45 Degrees high towards the East.

Time.

| Time. | Phases. |
|---------|--|
| h. ' " | |
| 5 32 50 | <i>Beginning over against the Spot Hevelius.</i> |
| 5 36 00 | The first Spot of <i>Grimaldi</i> . <i>Palus Maræotis.</i> |
| 5 36 30 | The second Limb of <i>Grimaldi</i> . |
| 5 45 00 | The middle of <i>Aristarchus</i> . <i>Mons Porphyrites.</i> |
| 5 46 00 | <i>Mersennus.</i> |
| 5 48 30 | <i>Herigone.</i> |
| 5 53 00 | <i>Heraclides.</i> |
| 5 53 15 | The first Limb of <i>Copernicus</i> . <i>Ætna.</i> |
| 5 54 15 | The middle of <i>Copernicus</i> . |
| 5 54 40 | <i>Pitheas</i> , or <i>Hiera Insula</i> . |
| 5 55 05 | The second Limb of <i>Copernicus</i> . |
| 5 57 40 | The first Limb of <i>Timocharis</i> . <i>Coriscæ.</i> |
| 5 59 35 | The first Limb of the <i>Sinus Medius Æstuum</i> . <i>Adriatick Sea.</i> |
| 6 01 30 | The middle of the <i>Sinus Medius</i> . |
| 6 02 40 | The first Limb of <i>Tycho</i> , or <i>Sinai</i> : and the first Limb of <i>Plato</i> ,
or the <i>Lacus Niger Major</i> . |
| 6 03 50 | The second Limb of <i>Plato</i> , and the middle of <i>Tycho</i> . |
| 6 04 30 | The Center of the Disk. |
| 6 09 00 | The middle of <i>Manilius</i> , or <i>Mons Besbicus</i> . |
| 6 12 00 | The middle of <i>Menelaus</i> , or <i>Byzantium</i> . |
| 6 13 45 | <i>Dionysius Areop</i> , or <i>Mons Amanus</i> . |
| 6 14 30 | <i>Plinius</i> . |
| 6 15 45 | <i>Vitruvius</i> . |
| 6 20 35 | <i>Endymion</i> , or <i>Lac. Hyperbor. superior</i> . |
| 6 21 00 | <i>Promont. Heraclium</i> . |
| 6 24 50 | Betwixt <i>Alcuin</i> and <i>Taruntius</i> . |
| 6 26 00 | The first Limb of the <i>Caspian Sea</i> , <i>Mare Crisum</i> . <i>Palus Mæotis.</i> |
| 6 28 15 | The middle of the <i>Caspian Sea</i> . |
| 6 29 40 | The other Limb of the <i>Caspian Sea</i> . |
| 6 30 5 | The first Limb of <i>Langrenus</i> , or <i>Insula Maj.</i> |
| 6 3 5 | The middle of <i>Langrenus</i> . |
| 6 35 46 | Total Immersion, betwixt <i>Langrenus</i> and the <i>Caspian Sea</i> . |
| 8 8 00 | First Emerſion, towards <i>Grimaldus</i> . |
| 8 12 35 | The first Limb of <i>Grimaldus</i> . |
| 8 14 00 | The second Limb of <i>Grimaldus</i> . |
| 8 20 20 | <i>Mersennus</i> . |
| 8 24 5 | <i>Herigone</i> . |
| 8 24 35 | The middle of <i>Aristarchus</i> & the middle betwixt <i>Herigone</i> & <i>Morin</i> . |
| 8 26 30 | The middle of <i>Kepler</i> , or <i>Loca Paludosa</i> . |
| 8 28 30 | The first Limb of <i>Tycho</i> . |
| 8 29 50 | The second Limb of <i>Tycho</i> . |

Time.

| Time. | | | Phases. |
|-------|----|----|--|
| h. | ' | " | |
| 8 | 34 | 5 | The middle of <i>Copernicus</i> . |
| 8 | 35 | 35 | The second Limb of <i>Copernicus</i> . |
| 8 | 36 | 10 | <i>Pitheas</i> . |
| 8 | 36 | 30 | <i>Heraclides</i> . |
| 8 | 40 | 0 | The first Limb of <i>Timocharis</i> . |
| 8 | 42 | 35 | The first Limb of <i>Plato</i> . |
| 8 | 43 | 45 | The second Limb of <i>Plato</i> . |
| 8 | 49 | 30 | The middle of <i>Manilius</i> . |
| 8 | 52 | 10 | <i>Menelaus</i> and <i>Dionys. Areopag.</i> |
| 8 | 55 | 0 | <i>Possidonius</i> . |
| 8 | 56 | 6 | <i>Vitruvius</i> . |
| 8 | 59 | 30 | <i>Endymion</i> . |
| 9 | 6 | 20 | The first Limb of the <i>Caspian Sea</i> . |
| 9 | 7 | 10 | The middle of the <i>Caspian Sea</i> . |
| 9 | 8 | 40 | The other Limb of the <i>Caspian Sea</i> . |
| 9 | 9 | 40 | The <i>End</i> , between the <i>Caspian Sea</i> and <i>Langrenus</i> . |

At Dantzick; 5. Vigilantes oculos, per totam Eclipses Durationem tubo 20 Pedum, aliisque præstantioribus, ad quatuor Fixas (neglectis cæteris minoribus, quas optime etiam conspiciebam) inter quas Luna eo tempore versabatur, direxi. Ab *a* Stellula vix quatuor minutis Limbo suo Inferiori in σ , distabat; tres vero reliquas, utpote *b*, *c*, & *d*, Luna corpore suo omnino texit. Ex omnibus autem his quatuor insignioribus Stellulis, non nisi unica *c* ab Astronomis hætenus observata, Globisque adscripta est; nominatur *Informium inter π & σ Suprema a tergo Pollucis*; cujus cursus cum ingressu, via itineraria, atque egressu imprimis probe notandus. Quippe ex hujus generis observationibus, multo proclivius datur Motum Lunæ redintegrare, ejusque Nodos Latitudinemque restaurare, quam, meo quali judicio, ex nudis *Solis Eclipsibus*. Stellula *b*, ad montem *Eoum* circiter tecta est, & *d* ad ipsum Limbum Lunæ Inferiorem; illa per *Sinum Sirbonis*, *I. Rhodum*, & *S. Atheniensem*; hæc vero per *Desertum Mingui* transit.

Temp.

| Temp. sec.
Hor. Oscil.
ex altitu-
dinibus
Cor. | Ord. P. h. g. m. | Altitudines Fixarum.
non Animadversiones qua-
dam notatu dignae. | Nec
qua- | Per quas Maculas transverint Um-
bra Sectiones. |
|--|------------------|--|-------------|--|
| h. | | | | |
| 6 22 18 | | Altitudo Caudæ Cygni. | | |
| | | 39° 3' 0" | | |
| 6 25 4 | | Eadem Altitudo. | 38' | |
| | | 41' 0" | | |
| 6 35 0 | | Penumbra Luna subiit. | | |
| 6 41 50 | | Initium Eclipseos. | | Incepit circa 5° a puncto Nadir
Ortum versus. |
| 6 43 45 | 1 | Paludem Maræotidem at-
tigit. | | Tempore Initii, Sinus Sagaricus,
Inf. Besbica & Inf. Molos in Li-
nea Recta extiterunt. |
| 6 44 55 | 2 | Palus Maræotis omnino
obscur. | | |
| 6 49 0 | 3 | | | Sectio Umbræ per M. Pentadach-
lum & Eoum transiit. |
| 6 52 30 | 4 | Mons Porphyrites tectus. | | Per Loca Paludosa Montemque Ca-
taract. |
| 6 57 35 | 5 | | | Ad M. Barontum, per M. Perri,
Athor, & M. Troicum. |
| 7 2 15 | 6 | M. Ætnam tegere incepit. | | Ad Sin. Apollinis, Inf. Ficariam, ad
radices Montis Ætnæ, Inf. Didy-
ma, Lacumq; Meridionalem. |
| 7 2 55 | 7 | M. Ætna omnino tectus. | | Ad Inf. Sardiniam, per Inf. Hieram,
Insulamque Cretam. |
| 7 7 40 | 8 | | | Per M. Atlanticos Majores, Inf.
Vulcaniam, Rhodum, Montem-
que Anna. |
| 7 10 40 | 9 | M. Sinai tectus. | | Per Inf. Ophiusam, Inf. Cyprum &
M. Sinai. |
| 7 14 0 | 10 | Lucus Niger Major tectus. | | Ad L. Nigrum Majorem, per M. Si-
pylum, Libanum, Montemq; Seir. |
| 7 16 40 | 11 | | | Per L. Nigr. Minorem, Inf. Besbi-
cam, M. Olympum & Didymum. |

| | | | |
|---------|----|---|---|
| 7 20 20 | 12 | Fixa c, distitit a Limbo D ortum versus 30'. fere. | Seçtio Umræ transiit per M. Carpathos, Byzantium & Taurum. |
| 7 25 0 | 13 | | Per Lacum Borysthenem, Inf. Apolloniam, M. Moschum & Sogdianos. |
| 7 28 30 | 14 | | Per Montes Macrocmnios, Promont. Arietis, Herculeum, per Sinum extremum Ponti, Montemque Parapamifum. |
| 7 32 0 | 15 | | Per Paludes Hyperboreos, I. Corocondametis, Montemque Caucasum. |
| 7 34 10 | 16 | | Per Montes Ripheos, Palud. Maotidem, Sinum Inferiorem Maris Caspii. |
| 7 42 44 | | Totalis Immerfio contigit circa 50°. Limbi a puncto Zenith occafum versus. | |
| 7 55 30 | | Stellula a, in ipsa δ cum D ; Distitit a Limbo ejus inferiori 4 circit. Minut. | |
| 8 0 50 | | Stellula b, ad M. Eoum Teçta. | |
| 8 35 20 | | Stellula c, ad Lacum Merid. subiit. | |
| 8 51 20 | | Stellula d, ad ipsum Limbum Inferiorem occultata. | |
| 9 9 10 | | Stellula e, rurfusemerfit, sub Monte sc. Nevoso; ficut D ad 3'. 20". fubingreffa fuerit. Spectaculum fane erat Jucundiffimum, hæce omnia dilucidiffima, & quidem fub ipsa fere Maxima Obscuratione, & δ & D deprehendere | |
| 9 12 13 | | Emerfio Lunæ ex umbra contigit ad 30°. fere Limbi D a puncto Nadir Ortum versus. | |
| 9 20 20 | 17 | | Umbra inçeffit per Pal. Maotidem Fontes Amaros & M. Eoum. |
| 9 25 0 | 18 | | Per M. Audum, Ajacem & Troicum. |
| 9 30 20 | 19 | | Per M. Pentadaçylum, Carpathum, S. Sirbonis, & M. Lion. |

| | | | |
|----------|----|---|---|
| 9 32 40 | 20 | Mons <i>Porphyrites</i> exiit ex Umbra. | Per M. <i>Porphyrit. Cassium & Didymum.</i> |
| 9 36 5 | 21 | M. <i>Sinai</i> illuminari cœpit. | Per M. <i>Baronium</i> , ad I. <i>Siciliam</i> , per <i>Cretam</i> ad radices <i>Sinai</i> . |
| 9 42 15 | 22 | M. <i>Ætna</i> sub ipsa sectione Umbrae. | Per <i>Promontorium Apollinis</i> , M. <i>Ætnam</i> , per <i>Rhodium</i> Montefque <i>Scir</i> . |
| 9 47 10 | 23 | | Per <i>Ins. Vulcanicam</i> , M. <i>Masicyrum Cragum & Antiliban</i> . |
| 9 51 0 | 24 | | Ad L. <i>Nigrum Majorem</i> , S. <i>Pastanum</i> , M. <i>Didymum</i> , & Montes <i>Coibacaronos</i> . |
| 9 56 50 | 25 | <i>Insula Besbica</i> rursus prodit in Lucem. | Per S. <i>Nigrum Minorem</i> , ad I. <i>Besbicum</i> , per M. <i>Uxerios</i> . |
| 9 59 35 | 26 | | Per L. <i>Salmiderfam</i> , M. <i>Horminicum</i> , <i>Moschum</i> & <i>Lacum Trospitis</i> . |
| 10 5 0 | 27 | | Per <i>Pontum</i> , I. <i>Cyaneam</i> , S. <i>Atbeniensem</i> , & M. <i>Caucasum</i> . |
| 10 7 0 | 28 | | Per L. <i>Borysthenem</i> , ad I. <i>Apollo-niam</i> , per <i>Heracleum</i> ac M. <i>Tanconem</i> . |
| 10 19 35 | | Umbra nondum plane exiverat. | |
| 10 20 0 | | Finis Eclipseos. | |
| 10 23 0 | | Penumbra. | |
| 10 52 58 | | Altitudo γ . 37°. 12' | |
| 10 58 35 | | Alt. <i>Lucide Y</i> . 28 52 | |
| 11 11 33 | | Alt. <i>Capella</i> . 70 11 | |

6. Observatio hujus Eclipsis fuit admodum exacta; nam Sole prope Horizontem existente, per hujus Loci Altitudinem Minutorum tempora facile innotuerunt: Cætera meo Horologio (quod ne unum quidem Minutum ea die discrepabat) notavi, ita ut nulli dubio in observatione detur locus.

At Seville; by S. Professor of Mathematics. N. 118. p. 428.

| | | |
|---------------------|------|----------------------------|
| | h. | |
| Initium veræ Umbrae | 4 56 | |
| Immersio | 6 1 | |
| Emergio | 7 33 | |
| Finis | 8 39 | & aliquantulum productior. |

XLV. I.
*An Eclipse of the
 Moon, Jun. 27.
 1675. at Lon-
 don; by Mr.
 Flamsteed and
 Mr. Halley.
 N. 116. p. 371.
 N. 118. p. 432.*

| Hora Hora- | | | Phases. |
|--------------------------|----|-------|---|
| logii Oscil-
latorii. | | | |
| | h. | " | |
| 1 | 1 | 1 00 | Lunæ Diameter $3190 = 31'. 40''$ |
| 3 | 1 | 11 00 | Rursus Lunæ Di. $3191 = 31' 46''$ |
| 5 | 1 | 37 50 | Nullum Penumbrae vestigium: jamque Luna nubes subtervolantes subiit, sub quibus latuit usque |
| 6 | 1 | 46 40 | Dum per earum hiatus Penumbra densa, vel forsan ipsum Initium apparuit; sed certus esse non poteram. |
| 7 | 1 | 51 40 | E nudibus elevata Limbus, notabili fatis defectu laborare visus est, obscurato sextante vel minimum octante Peripheriæ. |
| 8 | 1 | 55 15 | <i>Pentadactylus</i> tectus. |
| 9 | 2 | 2 20 | <i>Porphyrites</i> tectus. |
| 10 | 2 | 5 30 | <i>Sina</i> Limbus primus. |
| 11 | 2 | 6 00 | <i>Ætnæ</i> Limbus proximus. |
| 12 | 2 | 8 40 | Partes residuæ illuminatæ $2071 = 20'. 38''$. |
| 13 | 2 | 12 00 | Fixa exigua Telescopica, minori tubo non visibilis, in majori apparuit dimidium fere capacitatis ejusdem, vel $15'$. à Limbo apparenter inferiori distans. |
| 14 | 2 | 17 45 | Partes illuminatæ residuæ, $1655 = 16'. 27''$. |
| 15 | 2 | 23 5 | <i>Besbici</i> Limbus prior. |
| 16 | 2 | 26 3 | <i>Horminius</i> tectus. |
| 17 | 2 | 29 00 | <i>Æmus</i> tectus. |
| 18 | 2 | 30 45 | Partes illustratæ residuæ $1047 = 10'. 24''$. |
| 19 | 2 | 35 00 | Partes lucidæ residuæ $865 = 8' 38''$; jamque tubo longiori fixa alia exigua apparuit, <i>Maculæ Caspiæ</i> Longitudinem à limbo Lunæ, Latitudinem ejus à Linea per cuspides ducta dextram versus, distans. |
| 20 | 2 | 37 15 | Umbra tegebat occidentale Lirtus <i>Ponti</i> . |
| 21 | 2 | 39 30 | Ipsa tetigit Limbum primum <i>Corocondometis</i> . |
| 22 | 2 | 45 00 | Paludem <i>Maotidens</i> tetigit. |
| 23 | 2 | 50 45 | <i>Maotis</i> tota tecta. |
| 24 | 2 | 56 10 | Dubium ad aliquid veri Luminis supererat. |
| 25 | 2 | 56 55 | <i>Immersio</i> : Certe enim Lux primaria Lunam penitus deseruerat, scil. è regione paulo supra Montes Ripheos & circa gradum Limbi <i>Heveliani</i> $330. 2^b. 37'. 30''$. Limbus Coloris Cineritii per tubum apparuit. |

Aer à septima observatione ad *Immerisionem* serenissimus extitit; & Lucula quædam Albicans per totum defectus tempus Cuspides obscuratæ Lunæ visa est

est insidere. quæ eam etiam post Immerfionem à parte, quæ ultimum in Umbram inciderat, reddebat conspicuam. Tenuis admodum erat Eclipsis hujus Penumbra, nec major quam *Sine* aut *Æne* Latitudo. Palus *Meotis* Lata apparuit admodum, & quam maxime fere potuit à Limbo Lunæ remota. *Meotus* è contra compressa admodum, nec plusquam dimidium Longitudinis ipsius à Limbo Lunæ distans.

2. *Initium* veræ Umbræ alta *Capella* ad ortum

Umbra attingit *Paludem Meot.* alta *Lyra* ad occ.

Immerfio Totalis alta *Lyra* ad occ.

| ° | ' | h. | |
|----|----|----|----|
| 18 | 45 | 1 | 55 |
| 50 | 51 | 2 | 55 |
| 48 | 50 | 3 | 6 |

At Paris, by
M. Bullialdus.
Ibid. 372.

| Temp. | Transitus Umbra. |
|---------|--|
| h. ' " | |
| 1 56 45 | <i>Initium</i> Infra <i>Grimaldum</i> . |
| 1 57 20 | Per primum Limbum <i>Grimaldi</i> . |
| 1 58 50 | Secundum Limbum <i>Grimaldi</i> . |
| 2 1 45 | <i>Galileum</i> . |
| 2 2 15 | Primum Limbum <i>Mersenni</i> . |
| 2 4 15 | <i>Initium</i> <i>Gassendi</i> . |
| 2 4 40 | Medium <i>Gassendi</i> . |
| 2 5 30 | Alterum Limbum <i>Gassendi</i> . |
| 2 6 15 | <i>Herigonum</i> & <i>Seleucum</i> . |
| 2 7 45 | <i>Morinum</i> . |
| 2 8 18 | Medium <i>Kepleri</i> . |
| 2 11 35 | <i>Aristarchum</i> & <i>Bullialdum</i> . |
| 2 12 40 | <i>Aristarchus</i> disparet. |
| 2 16 25 | <i>Initium</i> <i>Tychonis</i> . |
| 2 16 40 | <i>Initium</i> <i>Copernici</i> . |
| 2 17 25 | Medium <i>Tychonis</i> & <i>Copernici</i> . |
| 2 18 12 | Alterum Limbum <i>Tychonis</i> . |
| 2 21 45 | <i>Pytheam</i> & primum trium <i>Sinum Mediorum</i> . |
| 2 24 0 | Medium Secundi <i>Sinus Medii</i> . |
| 2 24 53 | <i>Heracidem</i> vel <i>Virginem</i> . |
| 2 26 10 | Primum Limbum <i>Timocharis</i> . |
| 2 26 40 | Medium <i>Timocharis</i> . |
| 2 31 40 | Promontorium inter <i>Verginem</i> & <i>Platonem</i> . |
| 2 32 20 | <i>Abulfedam</i> . |
| 2 34 15 | <i>Initium</i> <i>Manili</i> . |
| 2 36 15 | <i>Dionysium</i> <i>Arcopagitam</i> . |
| 2 38 10 | Littus Maris <i>Tranquillitatis</i> . |
| 2 38 45 | Primum Limbum <i>Menelai</i> & <i>Platonis</i> . |
| 2 39 20 | <i>Fracastorem</i> . |
| 2 39 53 | Medium <i>Platonis</i> . |

3.
By M. Cassini,
M. Picart, and
M. Roemer.
N. 117. p. 388.

| Temp. | Transitus Umbre. |
|---------|--|
| h. ' " | |
| 2 41 15 | Per Alterum Limbum Platonis. |
| 2 44 15 | Promontor. inter Conforinum & Bedam. |
| 2 50 20 | Primum Limbum Paludis Somnii. |
| 2 50 45 | Cornua erant verticalia. |
| 2 53 55 | Initium Langreni. |
| 2 55 20 | Primum Limbum Maris Caspii. |
| 3 1 10 | Alterum Limbum Maris Caspii. |
| 3 1 45 | Endymionem. |
| 3 3 15 | Mesbalam. |
| 3 7 45 | Finis five Totalis Immersio, supra Mare Caspium. |

Mare Caspium tunc distabat à Limbo Occidentali circiter $\frac{1}{2}$ suæ Latitudinis. Post *Immersiōnem Totalem* dignoscebatur adhuc totum Corpus *Lunæ*.

XLVI. 1.

An Eclipse of the
Moon Decemb.
22. 1675.
at Greenwich;
by Mr. Flam-
steed. N. 121.
p. 435.

| Hor. cor.
Horol. | Phases. |
|---------------------|--|
| h. ' " | |
| 2 29 30 | Inter Cuspides 2085 = 17'. 16". |
| 2 55 45 | <i>Hæmum</i> fere tetigit. |
| 3 0 30 | <i>Hæmum</i> certe tetigerat. |
| 3 11 30 | Cuspis dexter à <i>Maraotide</i> 1235 = 10'. 14". |
| 3 35 0 | Partes Lucidæ circiter 2800 = 23. 11. vel paulo forsan
amplius; difficile enim erat admodum, <i>Umbre</i> veræ Ter-
minos, per Aerem Vaporibus scædarum, definire. |
| 3 42 30 | <i>Umbra</i> prope <i>Macram</i> . |
| 3 52 45 | Inter Cuspides circiter 2288 = 18'. 57". |
| 4 7 15 | <i>Finis</i> : Limbus enim apparuit, & nihil videbatur in rotunditate
<i>Lunæ</i> desiderari. |
| 4 8 0 | Limbus admodum dilucide per Tubum conspectus. |
| 4 15 30 | <i>Penumbra</i> , quæ nudis Oculis Eclipsin referebat. |
| 4 19 30 | <i>Dæ</i> capta diameter 3757 = 31'. 5". Sed vix satis certa; quam
tamen haud multum à veritate abesse putem. |
| 4 23 0 | Etiannum, & postea, Limbus ab <i>Eclipsi</i> derelictus Obscurior vi-
debatur ac alter. |

Corfica à Limbo *Dæ* Remoto distantia 2732 = 22' 37".

Limbus ejus proximus à proximo *Luna* 1045 = 8 39.

Sine Limbus Remotior à *Luna* Proximo

599 = 4'. 58'. bona.

Lacus Nigri Majoris Medium à Limbo proximo

452 = 3 45

Notavi præterea, quod Umbra semper longe distinctior apparuit ad Cornua, quam alicubi in Facie Lunæ: In prima Observatione, vel paulo ante, Cornua fuere Horizonti Parallela.

Tunc etiam *Porphyrius*, & *Lacus Niger Major*, æqualiter ex Umbra extiteret, Longitudinem scilicet circiter *Mareotidis*.

Nunquam tamen *Porphyrium* superavit in hac Eclipsi; alte vero illum in Penumbramersum vidi.

In summa Eclipsi ad *Coriscam* fere Umbra pertigerit; nunquam tamen eam extinctam vidi, sed alte adeo in Penumbra immersam, ut ægre eam potuerim discernere.

Nec unquam Umbra vera *Insulam Macram* pervagabatur, sed Penumbra duntaxat densa, per quam difficile erat ipsam percipere.

4^h. 5¹/₂. Limbum videre non potui; nec 4^h. 6¹/₂; sed 4^h. 7'. videre me putabam Limbi Lucem sed languidissimam, & ægre admodum; 4^h. 7'. 15¹/₂. certior factus sum ex Umbris Emerxisse, nec aliquid in ejus rotunditate desiderari: Ergo tunc *Finem* observatum statuo.

Exibat Umbra juxta *Lacum Hyperboreum Superiorem* manente Penumbra, quæ Eclipsin nudis Oculis exhibebat usque 4^h. 15¹/₂; sed Limbus ab Eclipsi derelictus Limbi oppositi Claritudinem recuperavit non nisi 4^h. 28'. vel serius.

Tempora Phasium Correcta, ab Altitudinibus *Arcturi* & *Lucide Corona*, Quadrante Telescopico, pedum trium & amplius Radio, captis; quibus, clare aliquando in altera Cœli cardine emicantibus, captandis incubui, quoties Lunam subiere Nubes.

2. *Initium* Eclipsis accidit antequam ad Instrumenta veni: Quod tamen Londini in Vico *Wintoniensi* observavit *Edmundus Halleus*, cum Lunæ Limbus superior a Vertice

At London;
by Mr. Edm.
Halley. 1^o
p. 498.

| | °. | '. | h. | '. |
|------------|----|----|----------------------------|------|
| distaret | 39 | 51 | unde horam supputavit | 2 16 |
| alto eodem | 41 | 1 | Cornua Horizonti Parallela | 2 25 |
| | 54 | 12 | | 3 58 |

3. *Joh. Coelfonus* in Vico *Wappingensi*, ad *Aniachoresin*, (Angl. *the Hermitage*) Limbum Lunæ Deficientem aliquantulum vidit 2^h. 17'. At 4^h. 9'. 25". ex Umbra vera ipsum exiisse comperit, densa duntaxat Penumbra remanente.

By Mr. Colson,
1^o.

4. In hac Eclipsi Duo præcipua à nobis exacte determinata sunt, *Medium* scilicet Eclipsis tempus, ejusque Magnitudo. *Medium* deductum est non solum ex comparatione *Initii*, & *Finis*, sed etiam Duarum æqualium Phasium, determinatu facillimarum, quando scilicet distantia Cornuum æqualis erat Lunæ semidiametro, ante Eclipsin captæ, 15'. 28". Scilicet cum *Initium* Eclipsis existimatum fuerit 2^h. 24'. 35".

At Paris; by
M. Callini.
N. 123. p. 561.

Finis vero *Totalis*, relicta Penumbra simili ac fuerit in

determinatione *Initii*
Duratio totius Eclipsis provenit
Dimidia
Et Eclipsis *Medium*

| h. | ' | " |
|----|----|----|
| 4 | 15 | 25 |
| 1 | 50 | 50 |
| 0 | 55 | 25 |
| 3 | 20 | 0 |
| 2 | 38 | 5 |

Sexta vero circumferentiæ pars abscissa est.

Atque

| | h. | ' | " |
|----------------------|----|----|----|
| Atque iterum | 4 | 2 | 25 |
| Intervallum | 1 | 24 | 20 |
| Dimidium | | 42 | 10 |
| Hinc Medium Eclipsis | 3 | 20 | 15 |

Intra quartam minuti partem priori determinationi conveniens.

In Situ Umbræ & Eclipsis Magnitudine D. *Flamstedio* plane convenimus. Ab utrisque quippe Nostrum annotatum est, Umbram nunquam superasse *Porphyritem*, licet is alte in *Penumbram* fuerit Immerfus.

Porphyriti proximus est Mons parvus albicans, quem tunc *Aristarchi Comitem* appellavimus, eo quod ab ipso seu *Porphyrite* vix distet sui Diametro. Is Monticulus Immerfus est in Umbram 2^h. 51'. 15"; Emergit autem 3^h. 8'. 25". totoque tempore interjecto fuit Umbra *Porphyriti* proxima.

Uterque pariter annotavimus, in summa Eclipsi Umbram ad *Corficam* fere pertigisse, nunquam tamen ab ea fuisse tectam, sed relictum exiguum interval- lum, cujus termini distantia à Lunari Margine proximo capta est 8'. 17", cum *Flamstedius* Insulæ ipsius paulo remotioris distantiam ab eodem Limbo inve- nerit 8'. 39". *Insulam* quoque seu potius *Peninsulam Macram* utrique Um- bræ diutissime adjacentem conspeximus; nos id fieri cœpisse notavimus 3^h. 28'. 15". & per horæ quadrantem in eadem distantia perseverasse.

5.
By M. Bullial-
lus. N. 125.
p. 610.

| Capellæ
distant à
Vertice. | Phases. | Temp. m. |
|----------------------------------|---|----------|
| 0 | | h. ' " |
| 39 36 | <i>Penumbra tenuis.</i> | 2 6 12 |
| 40 42 | <i>Penumbra crassior.</i> | 2 12 7 |
| 42 30 | Initium sensibile è reg. <i>Sinus Hyperb.</i> circa gr. 70. | 2 23 32 |
| 42 50 | Digitum fere $\frac{1}{4}$. | 2 25 48 |
| 44 25 | Umbra attigit <i>Atlantem Minorem.</i> | 2 36 11 |
| 47 28 | Umbra paulo supra <i>Baronium</i> , supra <i>Ligustinum</i> , occupaverat <i>Macr. M.</i> | 2 56 27 |
| 48 56 | U. attigere fere <i>Catenam Mundi.</i> | 3 6 22 |
| 49 54 | U. attig. <i>Montunial.</i> | 3 12 54 |
| 50 30 | Attigit <i>Sinum Perontic. M. Pyram</i> & <i>Med. Palud. Hyp.</i> | 3 17 41 |
| 52 7 | Occupavit <i>Sinum Sagaricum</i> & <i>Peronticum</i> , atque <i>Præmont. Luna.</i> | 3 29 1 |
| 54 52 | <i>Leucopetra</i> extra Umbram. | 3 48 21 |

| | | | | | |
|----|----|---|---|----|----|
| 55 | 40 | <i>Sinus Peronsicus</i> extra Umbram. | 3 | 54 | 6 |
| 55 | 48 | <i>Sinus Sagaricus</i> extra Umbram. | 3 | 55 | 12 |
| 56 | 17 | <i>Sinus Cercinites</i> fere Emerferat. | 3 | 58 | 29 |
| 57 | 16 | Pars sub Umb. æqualis fere Latitudini <i>Paludis Meotidis</i> . | 4 | 5 | 38 |
| 58 | 30 | <i>Finis</i> versus è reg. <i>Mont. Macrotem</i> circa g. 355. | 4 | 13 | 59 |
| 59 | 6 | <i>Penumbra</i> . | | | |

Corficam non attingit Umbra, neque *Lacum Thrasymenum*, propterea Eclipsis non excedit 3 Dig. 30'. vel minus etiam. *Initium* uno scrupulo primo, vel 45'', antecessit adnotatum, ita ut statui exactius possit 2^h. 22'. 30''. Hinc tota *Duratio* satis præcise 1^h. 51'. 24''. Quare *Maxima Obscuratio* contingit 3^h. 18'. 14''.

| <i>Alto</i>
<i>Arcturo.</i> | <i>Tempor. m.</i> | <i>Phases.</i> |
|--|-------------------|---|
| 0 | h. ' '' | |
| 30 | 30 2 48 48 | <i>Initium.</i> |
| 36 | 0 3 20 8 | Umbra per <i>M. Porphyritem</i> & <i>Promont. Lune</i> . |
| 39 | 50 3 45 44 | Umbra strinxit <i>Lacum Thrasymen. Mont. Baronium</i> , & <i>Sinum Cercinitem</i> . |
| 44 | 15 4 13 20 | Umbra transit per <i>Prom. Lune</i> & <i>M. Cimmericum</i> . |
| 46 | 25 4 27 36 | Umbra tetigit <i>Lacum Nigrum Minorem</i> & <i>M. Carpathum</i> . |
| 48 | 30 4 41 44 | Defuit in regione <i>Hyperborea</i> media ad <i>Mare Hyperboreum</i> . |
| Tota itaque <i>Duratio</i> 1 ^h 52'. 56''. | | |

6.
At Strasburg;
by M. Richelt.
16.

7. In hac Eclipsi probe notandum est, quod omnes Sectiones nunquam *Montem Porphyritem* omnino texerint, sed ille per totam *Duratiorem*, etiam in ipsa *Maxima Obscuratione*, in ipso Umbræ Limbo conspicuus persistit.

At Dantzick;
by M. Hevelius.
N. 124. p. 589.

| <i>Alto</i>
<i>Arcturo.</i> | <i>Tempor. m.</i> | <i>Phases.</i> |
|--|-------------------|---|
| 0 | h. ' '' | |
| 30 | 30 2 48 48 | <i>Initium.</i> |
| 36 | 0 3 20 8 | Umbra per <i>M. Porphyritem</i> & <i>Promont. Lune</i> . |
| 39 | 50 3 45 44 | Umbra strinxit <i>Lacum Thrasymen. Mont. Baronium</i> , & <i>Sinum Cercinitem</i> . |
| 44 | 15 4 13 20 | Umbra transit per <i>Prom. Lune</i> & <i>M. Cimmericum</i> . |
| 46 | 25 4 27 36 | Umbra tetigit <i>Lacum Nigrum Minorem</i> & <i>M. Carpathum</i> . |
| 48 | 30 4 41 44 | Defuit in regione <i>Hyperborea</i> media ad <i>Mare Hyperboreum</i> . |
| Tota itaque <i>Duratio</i> 1 ^h 52'. 56''. | | |

XVII

Temp.

| Temp. ex
Altit. Cor. | Altitudines Fixarum. | Ordo Phasium. | Per quas Maculas transverint
Umbra Sectiones. |
|-------------------------|---|---------------|---|
| h. ' " | ° ' " | | |
| 2 28 38 | 26 12 Dext. Humeri Orionis. | | |
| 2 33 20 | 25 35 Dext. Hum. Orionis. | | |
| 2 36 40 | Initium Penumbrae. | | Ad Montem Baronium. |
| 3 8 10 | | | Penumbra densior. |
| 3 16 35 | | | Densa Penumbra. |
| 3 24 20 | | | Densissima Penumbra. |
| 3 30 0 | Initium Eclipseos. | 1 | Ad Sinum Apollinis. |
| 3 36 25 | | 2 | Per M. Alabast. & Sin. Hyperb. |
| 3 42 5 | | 3 | Ad Sinum Apoll. |
| 3 46 30 | | 4 | Per M. Baron. & L. Nigr. Majorem. |
| 3 52 10 | | 5 | Per Inf. Ophiussam. |
| 3 59 15 | | 6 | Ad M. Porph. & L. Nig. Min. |
| 4 7 45 | Stellula distabat in
Limb. 36'. vel 40'. | 7 | Per M. Porph. & M. Serr. |
| 4 11 45 | | 8 | Per Porph. Pr. & M. Carp. |
| 4 18 5 | | 9 | Per Inf. Corf. M. Arg. & M. Macroceras. |
| 4 26 0 | | 10 | Per M. Porphyrit. & Lac. Trismenum. |
| 4 56 20 | | 11 | Ad Sinum Apollinis, M. Christi, & Inf. Macr. |
| 5 55 28 | 32 33 Lucidæ Lyrae. | | Penumbra. |
| 5 57 33 | 32 50 Lucidæ Lyrae. | | Penumbra pene evanuit quoad
conicere potuimus ob
nubes. |
| 5 59 10 | | | |
| 6 8 20 | | | |

XLVII.

An Eclipse of the
Moon, Oct. 29.
(St. n.) 1678.
At Paris; by
M. Cassini.
N. 141. p. 1015.

Phases Luna & Macularum Secundum Denominationem Riccioli.

Incipit Umbra
Grimaldi Limbus Sequens
Galileus
Finis Galilei

| In Observatorio Regio. | | In Collegio
Claromontano. |
|------------------------|---------|------------------------------|
| Infra. | Supra. | |
| h. ' " | h. ' " | h. ' " |
| 6 43 30 | 6 43 40 | 6 43 54 |
| 6 45 0 | 6 45 0 | 6 45 29 |
| 6 46 0 | 6 46 0 | |
| 6 47 0 | | |

Merfennus

| | h. ' " | h. ' " | h. ' " |
|---|---------|---------|---------|
| <i>Mersennus</i> | 6 48 20 | | |
| Chorda Eclipsis Dig. 6. | | 6 49 00 | |
| <i>Gassendi</i> Initium | 6 50 50 | 6 50 50 | |
| <i>Gassendi</i> Medium | | 6 51 30 | 6 51 57 |
| <i>Schikardi</i> Initium | 6 51 43 | | |
| II. Digni Ecliptici | | 6 52 00 | |
| <i>Aristarchi</i> Initium | 6 52 50 | | |
| <i>Aristarchi</i> Medium | | 6 53 10 | 3 56 7 |
| <i>Aristarchi</i> & <i>Morini</i> Finis | 6 54 00 | | |
| <i>Capmanns</i> five <i>Oculus Draconis</i> | 6 56 00 | | |
| Digni III. | | 6 56 30 | |
| Chorda 9 Digitorum | | 6 57 00 | |
| Initium <i>Terra Pruna</i> & <i>Copernici</i> | 6 58 54 | | |
| <i>Copernici</i> Initium | | 6 59 10 | |
| <i>Copernici</i> Medium | 7 00 00 | 7 00 00 | 6 59 30 |
| <i>Copernici</i> Finis | 7 00 55 | 7 00 55 | |
| <i>Putea</i> Initium | 7 1 50 | 7 1 40 | |
| <i>Putea</i> Finis | 7 2 30 | | |
| <i>Caput Virginis</i> | 7 2 45 | 7 2 40 | 7 3 18 |
| <i>Harpalus</i> | 7 2 55 | | |
| <i>Tychonis</i> Initium | 7 4 20 | 7 4 20 | |
| <i>Tychonis</i> Medium | 7 5 00 | | 7 5 48 |
| <i>Tychonis</i> Finis | 7 5 55 | | |
| <i>Eratosthenes</i> | 7 6 20 | | |
| Digni V. | | 7 6 50 | |
| Promontorium inter <i>Virginem</i> & <i>Platonem</i> | 7 7 00 | | |
| <i>Insula</i> in ultimo <i>Sinu</i> <i>Mediorum</i> | 7 7 30 | | |
| <i>Clara</i> sequens <i>Tychonem</i> | 7 8 31 | | |
| Digni VI. | | 7 11 20 | |
| <i>Tymocharis</i> | 7 11 48 | | |
| <i>Platonis</i> Initium | | 7 13 20 | 7 13 29 |
| <i>Platonis</i> Medium | | 7 13 40 | |
| <i>Platonis</i> Finis & Initium <i>Manili</i> | 7 14 00 | | |
| <i>Platonis</i> Finis | | 7 14 40 | |
| <i>Manilius</i> | | 7 14 50 | 7 15 4 |
| Finis <i>Manili</i> | 7 15 12 | | |
| <i>Dionysii</i> Initium | 7 17 15 | | |
| <i>Dionysius</i> | | 7 17 25 | |
| <i>Menelaus</i> | | 7 18 10 | 7 17 59 |
| <i>Dionysii</i> Finis & <i>Menelai</i> Initium | 7 18 28 | | 7 18 11 |
| <i>Plinius</i> Incipit | 7 20 56 | | |
| <i>Plinius</i> | | 7 21 10 | 7 21 51 |
| <i>Picolomineus</i> seu <i>Clara</i> supra <i>Annulum</i> | 7 21 30 | | |
| Initium <i>Fracastorii</i> seu <i>Annuli</i> | 7 23 00 | 7 23 5 | |
| Initium <i>Possidonii</i> | 7 23 55 | | 7 24 38 |
| Finis <i>Fracastorii</i> | 7 24 25 | | |

| | h. / " | h. / " | h. / " |
|--|---------|---------|---------|
| <i>Clara ante Angulum Promontorii Acuti</i> | | 7 24 35 | |
| <i>Angulus Promontorii Acuti</i> | 7 25 10 | | |
| <i>Digiti IX.</i> | | 7 26 10 | |
| <i>Palus Somni</i> | 7 28 00 | | |
| <i>Initium Endymionis</i> | 7 29 20 | | |
| <i>Initium Tauranti</i> | 7 30 30 | | |
| <i>Angulus Cornuum cum Parallelo 77° 15'</i> | | 7 30 40 | 7 31 16 |
| <i>Hermetis Initium</i> | 7 31 5 | | |
| <i>Finis Tauranti sive Capitis Serpentis</i> | 7 31 40 | | |
| <i>Hermetis Finis</i> | 7 32 00 | | |
| <i>Proclus</i> | 7 32 20 | | |
| <i>Limbus Maris Caspii</i> | 7 32 29 | 7 33 3 | 7 33 10 |
| <i>Macula Inferior</i> | 7 33 3 | | |
| <i>Initium Langreni</i> | 7 34 24 | | |
| <i>Messala</i> | 7 34 40 | | |
| <i>Finis Langreni</i> | 7 35 40 | | |
| <i>Peninsula in Caspia</i> | 7 37 50 | | |
| <i>Finis Caspie</i> | 7 37 50 | 7 37 50 | 7 38 18 |
| <i>Macula Oblonga f.</i> | 7 39 16 | | |
| <i>Finis</i> | 7 40 41 | 7 41 00 | 7 41 28 |
| <i>Initium Emersonis</i> | 9 21 30 | 9 21 30 | 9 21 5 |
| <i>Grimaldus</i> | 9 22 44 | | 9 22 10 |
| <i>Grimaldi Finis</i> | 9 23 40 | 9 23 40 | |
| <i>Gallileus</i> | 9 24 35 | | |
| <i>Mare Humorum</i> | 9 28 15 | | |
| <i>Schikardi Medium</i> | 9 29 00 | | |
| <i>Aristarchi Initium</i> | 9 29 44 | | 9 29 41 |
| <i>Aristarchi Medium</i> | | 9 30 20 | |
| <i>Aristarchus Totus</i> | 9 30 25 | | |
| <i>Gassendus Totus</i> | 9 31 00 | | |
| <i>Kepleri Initium</i> | | 9 31 40 | 9 31 6 |
| <i>Keplerus Totus</i> | 9 32 24 | | |
| <i>Morinus</i> | 9 30 00 | | |
| <i>Finis Insulae Kepleri</i> | 9 34 22 | | |
| <i>Capuanus Totus</i> | 9 34 40 | | |
| <i>Initium Terra Pruina</i> | 9 35 43 | | |
| <i>Virgo</i> | 9 37 00 | | |
| <i>Aurea inter Pitheam & Keplerum</i> | | 9 37 20 | |
| <i>Harpalus & Clara ante Copernicum</i> | 9 37 34 | | |
| <i>Digiti III.</i> | | 9 37 40 | |
| <i>Bullialdus</i> | 9 38 30 | | |
| <i>Initium Copernici</i> | | 9 39 00 | 9 38 36 |
| <i>Tychonis Initium</i> | 9 40 00 | | 9 40 22 |
| <i>Copernicus Totus</i> | 9 40 18 | | |
| <i>Pitheas</i> | | 9 40 30 | |

| | h. ' " | h. ' " | h. ' " |
|---------------------------------------|----------|----------|----------|
| <i>Tychonis Medium</i> | 9 41 00 | 9 41 10 | 9 41 10 |
| <i>Tychonis Finis</i> | 9 42 20 | 9 42 20 | 9 42 20 |
| <i>Digiti IV.</i> | 9 42 40 | 9 42 40 | 9 42 40 |
| <i>Timocharus Totus</i> | 9 45 30 | | |
| <i>Plato Incipit</i> | | 9 45 30 | 9 45 00 |
| <i>Digiti V.</i> | | 9 47 00 | |
| <i>Plato Totus</i> | 9 47 00 | 9 47 30 | |
| <i>Insula sub Sinu Medio</i> | 9 50 00 | | |
| <i>Finis Sinus Medii</i> | 9 50 30 | | |
| <i>Archimedes</i> | 9 51 37 | | |
| <i>Digiti VI.</i> | | 9 52 00 | |
| <i>Manilius</i> | | 9 54 00 | 9 53 52 |
| <i>Fretum</i> | 9 54 22 | | |
| <i>Abifeldeæ Initium</i> | 9 54 40 | | |
| <i>Abifeldeæ Finis</i> | 9 55 30 | | |
| <i>Aristoteles & Eudoxus Toti</i> | 9 56 00 | | |
| <i>Digiti VII.</i> | | 9 56 20 | |
| <i>Dionysii Initium</i> | 9 57 12 | | |
| <i>Menelaus</i> | 9 57 50 | 9 57 40 | 9 57 16 |
| <i>Dionysii Medium</i> | | 9 58 30 | |
| <i>Menelaus Totus</i> | 9 59 00 | | |
| <i>Dionysius Totus</i> | 10 00 00 | | |
| <i>Digiti VIII.</i> | 10 1 30 | | |
| <i>Ostium Lacus Mortis</i> | | 10 2 00 | |
| <i>Plinius</i> | | 10 2 15 | 10 1 28 |
| <i>Promontorium supra Dionysium</i> | 10 3 00 | | |
| <i>Plinii Finis. Possidonium</i> | 10 3 40 | | |
| <i>Promontorium Acutum</i> | | 10 5 30 | |
| <i>Hermes</i> | 10 6 00 | | |
| <i>Digiti IX.</i> | | 10 6 15 | |
| <i>Chorda Umbræ 8 Digitorum</i> | | 10 9 30 | |
| <i>Messala</i> | 10 11 00 | | |
| <i>Initium Maris Caspii</i> | 10 13 30 | 10 12 10 | 10 11 30 |
| <i>Langrenus Totus</i> | 10 16 00 | | |
| <i>Chorda 6 Digitorum</i> | | 10 14 00 | |
| <i>Caspii Prior Finis</i> | | 10 16 50 | 10 16 44 |
| <i>Caspii Alter Finis</i> | 10 17 33 | 10 17 25 | |
| <i>Albedo in Mari Caspio</i> | 10 18 15 | | |
| <i>Finis Totalis</i> | 10 20 00 | 10 20 10 | 10 20 42 |

T t 2

Temp.

XLVIII. I.

An Eclipse of
the Moon, Aug.
19. m. 1681.
at Greenwich;
by Mr. Flam-
steed. Ph. Coll.
n. 3. p. 67.

| Temp. per.
Hor. Oscil. | | | Temp. inde
per Obser-
vat. Corr. | | | Observationes. |
|---------------------------|----|----|--|----|----|--|
| h. | ' | '' | h. | ' | '' | |
| 1 | 5 | 00 | 1 | 00 | 00 | Lunæ Diameter 6191 = 30'. 53''. |
| 1 | 46 | 30 | 1 | 41 | 30 | Penumbra notabilis. |
| 1 | 53 | 40 | 1 | 48 | 40 | Penumbra densa. |
| 1 | 55 | 40 | 1 | 50 | 40 | Initium. |
| 1 | 58 | 00 | 1 | 53 | 00 | Umbra ad Montem Climacum. |
| 2 | 3 | 20 | 1 | 58 | 20 | Sinus Sirbonis Incipit. |
| 2 | 5 | 50 | 2 | 00 | 50 | Mareotis Incipit. |
| 2 | 7 | 30 | 2 | 2 | 30 | Mons Cataractes Incipit. |
| 2 | 8 | 16 | 2 | 3 | 16 | Mareotis Tota tecta. |
| 2 | 10 | 50 | 2 | 5 | 50 | Sina Incipit. |
| 2 | 12 | 56 | 2 | 7 | 56 | Sina Tota tecta. |
| 2 | 18 | 00 | 2 | 13 | 00 | Audus. |
| 2 | 27 | 24 | 2 | 22 | 24 | Circinna. |
| 2 | 31 | 00 | 2 | 26 | 00 | Abulfeda Ricciolo. |
| 2 | 32 | 30 | 2 | 27 | 30 | Pentadactylus Incipit. |
| 2 | 33 | 32 | 2 | 28 | 32 | Ætna Limbus primus. |
| 2 | 34 | 26 | 2 | 29 | 26 | Pentadactylus Totus. |
| 2 | 35 | 6 | 2 | 30 | 6 | Ætna Medium. |
| 2 | 36 | 00 | 2 | 31 | 00 | Ætna Tota tecta. |
| 2 | 40 | 00 | 2 | 35 | 00 | Umbra per Med. Corphyritem. |
| 2 | 44 | 50 | 2 | 39 | 50 | Hæra Insula & Ficaria simul ad Umbram. |
| 2 | 46 | 10 | 2 | 41 | 10 | Umbra ad Horminum. |
| 2 | 50 | 10 | 2 | 45 | 10 | Mons Hercules Incipit. |
| 2 | 51 | 10 | 2 | 46 | 10 | M. Hercules intra. |
| 2 | 52 | 10 | 2 | 47 | 10 | Umbra per Med. Beshici. |
| 2 | 54 | 00 | 2 | 49 | 00 | per Med. Insula Majoris. |
| 2 | 55 | 30 | 2 | 50 | 30 | per Med. Byzantium. |
| 2 | 57 | 40 | 2 | 52 | 40 | per Med. Insula Cyanea. |
| 3 | 1 | 50 | 2 | 56 | 50 | ad Limbum primum Corcondometis. |
| 3 | 4 | 30 | 2 | 59 | 30 | ad Limbum primum Meotidis. |
| 3 | 13 | 00 | 3 | 8 | 00 | Macra Incipit obscurari: dub. |
| 3 | 14 | 00 | 3 | 9 | 00 | Meotis Tota tecta. |
| 3 | 19 | 12 | 3 | 14 | 12 | Partes Lucidæ residuæ erant 1092 = 5'. 27''. |

| <i>Digit
Eclipsed.</i> | <i>Phases.</i> | <i>Time.</i> | <i>2.
At Paris; by
M. Cassini.
Ibid. p. 66.</i> |
|----------------------------|---|--------------|---|
| d. ' " | | h. ' " | |
| | The Beginning at | 1 58 30 | |
| | The arrival of the Shadow at <i>Grimaldus</i> . | 2 1 30 | |
| | At <i>Tycho</i> . | 2 8 00 | |
| | At the Center of the Moon. | 2 37 30 | |
| | At the middle of <i>Copernicus</i> . | 2 39 30 | |
| | At <i>Aristarchus</i> . | 2 40 00 | |
| 8 16 00 | At the Middle of <i>Manilius</i> . | 2 54 30 | |
| | At <i>Plinius</i> . | 3 1 00 | |
| 9 2 00 | At the Lower part of the <i>Caspian Sea</i> . | 3 7 30 | |
| 9 31 20 | At the Upper side of the <i>Caspian Sea</i> . | 3 18 30 | |
| 10 1 00 | The Greatest Obscurity. | 3 35 45 | |
| 4 46 00 | { The Return of the Shadow } | 3 47 00 | |
| | { To <i>Aristarchus</i> . } | | |
| | To the Center of the Moon | 4 29 00 | |
| | To <i>Tycho</i> . | 4 41 30 | |
| | The End of the Eclipse. | 5 13 00 | |
| | The Whole Duration. | 3 14 30 | |

At 5h. 13'. 56''. the Apparent Altitude of the upper Edge of the Sun, was 17'. 10''. and that of the Moon, 1°. 11'. 30''.

| <i>Temp. juxta
Hor.</i> | <i>Phases.</i> | <i>Altitudines.</i> | <i>Temp. Correct.</i> | <i>3.
At Dantzick;
by M. Hevelius.
n. 5. p. 160.</i> |
|-----------------------------|---------------------------------|---------------------|-----------------------|--|
| h. ' " | | ° ' " | h. ' " | |
| 1 1 03 | Altitudo <i>Palilicii</i> . | 24 14 | 1 4 48 | |
| 2 4 30 | Eadem Altitudo. | 32 58 | 2 6 32 | |
| 2 9 00 | Rursus capta. | 33 30 | 2 10 32 | |
| 2 45 00 | Vestigium <i>Pennumbra</i> . | | 2 46 30 | |
| 2 53 00 | <i>Pennumbra</i> paulo Densior. | | 2 54 00 | |
| 2 59 00 | Adhuc Densior. | | 3 00 00 | |
| 3 1 00 | Densissima. | | 3 2 00 | |
| 3 2 00 | Initium Eclipseos. | | 3 3 00 | |
| 3 6 00 | Prima Phasis. | | 3 6 30 | |
| 3 12 00 | Secunda Phasis. | | 3 12 30 | |
| 3 14 00 | Tertia Phasis. | | 3 14 30 | |
| 3 20 00 | Quarta Phasis. | | 3 21 00 | |
| 3 22 30 | <i>Mons Sinai</i> Totus tectus. | | 3 23 00 | |
| 3 29 00 | Sexta Phasis. | | 3 30 00 | |
| 3 43 00 | Altitudo <i>Pollucis</i> . | 30 00 | 3 43 00 | |
| 5 2 00 | Centrum Solis Oritur. | | | |

Coloꝛ

Color hujus *Eclipsas* erat Cinericius sive Fuliginus. Tempore Observationis *Mons Porphyrites* & *M. Aëna* fere in eodem Perpendiculo existebant.

- Phasis I. Per Montem *Eonem* transibat.
 II. Ad *Paludes Arabic* incedebat.
 III. Inferiorem Ripam *Paludis Maraotidis*, & Extremitatem *Sinus Sirbonis* tangebatur.
 IV. Per Insulam *Letoam* incedebat ad ipsum *M. Sinai* usque, sed dictus Mons Totus adhuc erat conspicuus.
 V. *Sinum Syrticum* transire videbatur, sic ut totum *M. Sinai* Umbra jam tegeret.
 VI. Supra *Sinum Syrticum*, ultra Insulam *Cretam*, & per *Mare Mortuum* incedebat.

An Eclipse of the Moon, Feb. 11. 1682. by Mr. Flamsteed. n. 145. p. 89. XLIX. 1. Ann. 1682. Feb. 11. H. 8. 1'. Tubo pedum 16 *Lunæ* cepi Diametrum $6702 = 33'. 25''$. deinde distantiam Limbi ejus Proximi à Limbo Proximo *Maraotidis* $145 = 00'. 43''$. sed ejusdem Limbi Maculæ à Limbo *Lunæ* remotiore $6575 = 32'. 48''$. Hujus etiam Tubi ope Tempora, cum Obscuratio ad Centrum *Lunæ* pertigerit, & eum Radius ejus Arcus in Peripheria Deficientes vel Restitutos subtenderit, obtinui; è quibus medium derivari potest, forsan non minus accuratè, quam ab observatis *Initio* & *Fine*, *Immersione* & *Emergione*, collatis.



Phases

| Phases. | Tempora Phasium per Horologia
Oscillatoria correctâ. | | |
|---|---|-------------|---------------|
| | Grenovici in Observatorio. Londini. | | |
| | Mibi. | D. Halleio. | D. Haynesio. |
| | h. ' " | h. ' " | h. ' " |
| Famus ad Oram Inferiorem. Nudis Oculis | 8 48 38 | | |
| Umbra. ————— | 9 4 8 | | |
| Densa Penumbra. ————— | 9 11 44 | | |
| Initium. ————— | 9 12 32 | 9 13 04 | 9 12 18 |
| Medium Paludis <i>Maraotidis</i> tectum. | 9 14 02 | | forte citius. |
| Tota Palus tecta. ————— | | 9 14 39 | 9 13 48 |
| Sexta Pars <i>Peripheria Obscurata</i> — | 9 18 10 | | |
| <i>Circinna</i> intra Umbram ————— | 9 20 10 | | |
| <i>Porphyrus</i> Medius. ————— | | 9 21 1 | |
| Umbra stringit <i>Syrbonem</i> . — | | 9 21 9 | |
| Limbus 1 ^{us} . Montis <i>Cataractes</i> vel <i>Gassendi</i> | 9 21 32 | | |
| <i>Oculus Draconis</i> . ————— | 9 27 58 | | |
| Legebat Limbus primos <i>Greta</i> & <i>Arne</i> . | 9 28 22 | 9 28 44 | 9 28 19 |
| M. <i>Sina</i> Totus tectus. incipit <i>Hiera</i> | 9 29 48 | 9 30 16 | 9 30 12 |
| <i>Hiera</i> tot. ————— | | 9 30 44 | |
| Initium <i>Corfica</i> . ————— | 9 33 46 | | |
| Medium <i>Corfica</i> . ————— | | 9 34 48 | |
| <i>Sina</i> Mons incipit. ————— | 9 36 38 | 9 37 10 | 9 36 20 |
| <i>Sina</i> Medium. ————— | 9 37 22 | | |
| Totus <i>Sina</i> tectus ————— | 9 38 04 | 9 38 10 | 9 38 20 |
| Centrum <i>Luna</i> five <i>Digitis</i> VI. — | 9 38 48 | | |
| Incipit <i>Lacus Niger Major</i> . — | 9 39 58 | 9 40 36 | |
| Medium <i>Lacus Nigri Majoris</i> . | | | 9 40 48 |
| Totus tegitur <i>Lacus</i> . — | | 9 41 40 | |
| Initium <i>Besbici</i> . ————— | | 9 43 48 | |
| Medius <i>Besbicus</i> . ————— | 9 43 42 | | 9 44 13 |
| Totus <i>Besbicus</i> & stringit <i>Pont. Euxinum</i> . | | 9 44 36 | |
| Incipit <i>Byzantium</i> . ————— | 9 46 58 | | |
| Incipit <i>Horminius</i> . ————— | 9 47 22 | | |
| <i>Carpathes</i> . ————— | 9 47 50 | | |
| <i>Mons Serrorum</i> . ————— | 9 48 30 | | |
| <i>Apollonia</i> . ————— | 9 50 24 | 9 51 21 | |
| <i>Macra</i> . ————— | 9 52 44 | 9 53 33 | |
| <i>Mons Hercules</i> . ————— | 9 54 26 | | |
| <i>Macra</i> tota tecta ————— | | 9 54 53 | |
| <i>Mons Hercules</i> Totus ————— | | 9 55 33 | |
| <i>Corocondometis</i> Palus. ————— | 9 59 10 | | |
| Per Medium <i>Lacum Hyperbor. Superiorem</i> . | | 9 59 53 | |
| <i>Corocondometes</i> tota tecta. — | 10 00 38 | | |
| Umbra per Montem <i>Coracem</i> . — | | 10 1 17 | |
| Stringit <i>Maotida</i> . ————— | 10 2 20 | 10 2 38 | |

| | h. ' " | h. ' " | h. ' " |
|--|----------|----------|----------|
| <i>Per Lacum Hyperboreum Inferiorem.</i> — | | 10 3 53 | |
| <i>Per Medium Insula Majoris.</i> — | 10 5 14 | | |
| <i>Tota ferè Maotis tecta.</i> — | 10 6 54 | 10 8 00 | 10 6 48 |
| <i>Sexta pars Peripherie Lucida residua.</i> — | 10 7 28 | | |
| <i>Immersio.</i> — | 10 10 14 | 10 10 11 | 10 9 48 |
| <i>Emergio</i> — — — | 11 47 38 | 11 47 9 | 11 47 48 |
| <i>Sexta pars Peripherie Illuminata, cum</i> | 11 50 02 | 11 50 7 | |
| <i>Limbo 1°. Maraot.</i> — | | | |
| <i>Medium Maraotidis.</i> — | 11 50 42 | | 11 50 48 |
| <i>Tota Maraotis Illuminata</i> — | 11 51 6 | 11 50 58 | |
| <i>Umbra per Medium Porphyritidis</i> — | 11 52 50 | | |
| <i>Totus Porphyrites tectus.</i> — | 11 53 34 | | 11 52 54 |
| <i>Circinus tota.</i> — | 11 58 18 | | |
| <i>Stringit Sinum Syrbonis.</i> — | | 11 58 59 | |
| <i>Cataractes Syrbonis, vel Gassendus totus.</i> | 12 1 00 | | |
| <i>Hiera incipit.</i> — | | 12 3 14 | |
| <i>Ætna incipit emergere.</i> — | 12 3 20 | | |
| <i>Per Medium Lacus Nigri Majoris.</i> | | | 12 3 48 |
| <i>Per Medium Ætnæ.</i> — | | 12 4 19 | 45 |
| <i>Ætna totus reiectus.</i> — | 12 5 18 | 12 5 4 | 12 5 |
| <i>Creta Illuminatur.</i> — — | 12 7 34 | | 43 |
| <i>Sina incipit.</i> — — — | 12 13 38 | | 12 13 |
| <i>Sina Medium.</i> — — — | 12 14 28 | 12 14 12 | |
| <i>Sina totus.</i> — — — | 12 15 34 | 12 14 55 | |
| <i>Medium Besbici.</i> — — — | 12 17 34 | | 12 17 48 |
| <i>Centrum Lunæ, si Digiti VI reiecti.</i> — | 12 18 28 | | |
| <i>Per Medium Lacum Hyperbor. Superiorem</i> | | 12 19 3 | |
| <i>Umbra per Medium Byzantii.</i> — | | | 12 20 18 |
| <i>Byzantium extra.</i> — — — | 12 20 50 | 12 20 25 | |
| <i>Macra tota.</i> — — — | | 12 22 36 | |
| <i>Insula Apollonia tota, & Horminius.</i> — | 12 24 24 | 12 24 16 | |
| <i>Lacus Hyperboreus Inferior.</i> — — | | 12 26 6 | |
| <i>Mons Hercules.</i> — — — | | 12 30 56 | 12 31 18 |
| <i>Medium Corocondometidis ad Umbram.</i> | 12 31 38 | | |
| <i>Mons Corax.</i> — — — | | 12 32 37 | |
| <i>Maotis incipit illustrari.</i> — — | 12 33 18 | | 12 32 18 |
| <i>Maotis tota reiecta.</i> — — — | | 12 37 57 | 12 38 3 |
| <i>Sexta pars Peripherie Obscura residua.</i> — | 12 39 6 | | |
| <i>Media Insule Majoris Caspii.</i> — — | | 12 41 22 | 12 42 18 |
| <i>Tota Insula reiecta.</i> — — — | 12 42 18 | | |
| <i>Dubius Finis.</i> — — — | 12 45 10 | | 12 44 48 |
| <i>Finis certè.</i> — — — | 12 45 38 | 12 44 35 | 12 45 18 |

Ingruente *Eclipsi*, Distantias *Lune* à Fixis Sextante Ceperant D^s. *Halleius*
cum *Tho. Fabre* sequentes ;

| <i>Tempora per
Hor. Correct.</i> | | | <i>Observationes.</i> | <i>Distantie.</i> | | |
|--------------------------------------|----------|----------|--|-------------------|----------|----------|
| <i>h.</i> | <i>'</i> | <i>"</i> | | <i>o</i> | <i>'</i> | <i>"</i> |
| 8 | 53 | 07 | Limbi <i>Lune</i> Remotioris à <i>Regula</i> ——— | 8 | 05 | 20 |
| 8 | 55 | 07 | repet. ——— | 8 | 06 | 20 |
| 9 | 2 | 34 | Limbi <i>Lune</i> Rem. à <i>Lucid.</i> in <i>Lumbis</i> Leonis | 14 | 58 | 15 |
| 9 | 4 | 18 | repet. ——— | 14 | 57 | 55 |
| 9 | 8 | 42 | Limbi <i>Lune</i> Remot. à <i>Regulo</i> iterum — | 8 | 12 | 50 |
| | | | <i>Luna</i> in <i>Umbram</i> penitus mersa, D ^s . <i>Halleius</i>
mecum. | | | |
| 10 | 19 | 21 | Limbi <i>Lune</i> Proximi à <i>Regulo</i> ——— | 8 | 13 | 30 |
| 11 | 38 | 06 | Remoti à <i>Regulo</i> — | 9 | 23 | 50 |
| 11 | 41 | 48 | repet. ——— | 9 | 25 | 35 |
| | | | Et finita <i>Eclipsi</i> , | | | |
| 12 | 52 | 00 | Limbi Remotioris à <i>Regulo</i> iterum ——— | 9 | 58 | 20 |
| 12 | 55 | 10 | denuo. ——— | 10 | 00 | 00 |



2.
At Paris and
Copenhagen.
B. 146. p. 145.

| Phases. | Parisus. | | Hafnia. | |
|---------------------------------|------------------------|-----------------------------------|----------------------------|---|
| | In Observatorio Regio. | | In Collegio.
Claramont. | AD. Roemer,
reductione
Meridiano-
rum facta
Subtr.
41'. 40". |
| | AD. Cassino. | ADD. Pi-
card &
De la Hire. | AR. P. de
Fonteney. | |
| | h. ' " | h. ' " | h. ' " | h. ' " |
| <i>Initium</i> ————— | 9 20 55 | 9 21 58 | 9 21 25 | 9 21 50 |
| <i>Aristarchus Medius</i> — | 9 29 57 | 9 29 33 | 9 29 28 | 9 29 50 |
| <i>Medium Copernici</i> — | 9 38 45 | 9 38 25 | 9 38 40 | 9 38 48 |
| <i>Initium Tychois</i> ——— | 9 45 52 | 9 45 48 | 9 45 48 | 9 46 20 |
| <i>Finis Tychois</i> ——— | 9 47 40 | 9 47 36 | 9 47 20 | 9 47 50 |
| <i>Initium Platonis</i> ——— | 9 49 47 | 9 49 08 | 9 49 25 | 9 49 50 |
| <i>Centrum Luna</i> ——— | | 9 51 10 | | 9 50 20 |
| <i>Centrum ex Comparatione</i> | | | | |
| <i>Initii & Finis</i> ——— | 9 50 24 | 9 50 44 | 9 50 10 | 9 50 30 |
| <i>Manilius Medius</i> ——— | 9 53 16 | 9 53 30 | 9 51 18 | 9 53 20 |
| <i>Menelaus</i> ——— | 9 56 43 | 9 56 20 | 9 56 14 | 9 56 20 |
| <i>Dionysius</i> ——— | 9 57 09 | 9 57 40 | | 9 57 20 |
| <i>Promontorium Acutum</i> — | 10 04 31 | 10 04 15 | 10 04 12 | 10 04 20 |
| <i>Initium Caspie</i> ——— | 10 11 40 | 10 11 25 | 10 11 20 | 10 11 50 |
| <i>Finis Caspie</i> ——— | 10 16 27 | 10 16 30 | 10 15 50 | 10 16 20 |
| <i>Luna penitus Immerfa</i> — | 10 19 53 | 10 19 30 | 10 18 55 | 10 19 50 |
| <i>Initium Emerf. Grimaldum</i> | | | | |
| <i>inter & Galileum</i> ——— | 11 57 51 | 11 56 00 | 11 58 00 | 11 55 20 |
| <i>Aristarchus Medius</i> ——— | 12 02 00 | 12 01 55 | 12 03 09 | 12 01 50 |
| <i>Medium Platonis</i> ——— | 12 12 06 | 12 12 10 | 12 13 26 | 12 12 20 |
| <i>Medium Copernici</i> — — | 12 14 32 | 12 14 30 | 12 14 29 | |
| <i>Medium Tychois</i> ——— | 12 23 35 | 12 23 55 | 12 24 21 | 12 23 20 |
| <i>Centrum Luna</i> ——— | | 12 25 50 | | 12 26 20 |
| <i>Centrum ex Comparatione</i> | | | | |
| <i>Emerf. Limborum</i> — | 12 26 09 | 12 25 15 | 12 26 08 | 12 24 50 |
| <i>Manilius Medius</i> ——— | 12 26 22 | | 12 26 20 | 12 26 20 |
| <i>Menelaus Medius</i> ——— | 12 29 08 | 12 29 20 | 12 29 22 | 12 29 05 |
| <i>Dionysius</i> ——— | 12 32 30 | 12 33 15 | 12 32 20 | 12 32 50 |
| <i>Promontorium Acutum</i> — | 12 39 45 | 12 39 50 | | 12 39 50 |
| <i>Initium Caspie</i> ——— | 12 41 53 | 12 41 30 | 12 41 11 | |
| <i>Finis Caspie</i> ——— | 12 47 15 | 12 46 30 | 12 48 20 | 12 47 20 |
| <i>Finis Eclipsis</i> ——— | 12 54 27 | 12 54 30 | 12 54 17 | 12 54 20 |

Tempus

| <i>Tempus secund. Horol. Ambul.</i> | <i>Or. Phas.</i> | <i>Digiti Eclips.</i> | <i>Altitudines Stellarum.</i> | <i>Per quas Maculas transiverint Umbrae Sectiones, & quae insuper Notata fuerint.</i> | <i>Tempus ex Alt. Cor-rect.</i> | <i>3. At Dantzick; by M. Hevel. 16. p. 146.</i> |
|-------------------------------------|------------------|------------------------------|-------------------------------|--|---------------------------------|---|
| h. ' " | | | o ' " | | h. ' " | |
| 8 41 18 | | | <i>Palilicij</i> 39 53 | | 8 40 46 | |
| 8 44 9 | | | <i>Pal.</i> 39 27 | | 8 44 20 | |
| 8 48 14 | | | <i>Pal.</i> 39 0 | | 8 47 59 | |
| 10 8 28 | | | | <i>Initium Penumbrae Dilutissima.</i> | 10 9 0 | |
| 10 12 50 | | | | <i>Penumbra Densior.</i> | 10 13 20 | |
| 10 19 5 | | | | <i>Penumbra adhuc Crassior.</i> | 10 19 36 | |
| 10 23 50 | | | | <i>Penumbra Densissima.</i> | 10 24 24 | |
| 10 24 30 | | | | <i>Initium Eclips. circa 150° Limbi in 95° sc. à Puncto super. Lineae perpend. Nonagesimi Ortum versus contigit.</i> | 10 25 5 | |
| 10 29 45 | 1 | 1 $\frac{1}{4}$ Dig. | | <i>Incedebat per M. Germanician. ad Mare Syrtic. & Mont. Acabe.</i> | 10 30 30 | |
| 10 32 13 | 2 | 2 <i>ferè.</i> | | <i>Per M. Porphyriticem, loca Paludosa, Inf. Cercinna, Sinum Syrtic. per Mont. Sacr. & M. Caspium.</i> | 10 32 50 | |
| 10 36 30 | 3 | 2 $\frac{1}{2}$ <i>ferè.</i> | | <i>Ad M. Baronium, Inf. Ethusam, per medium Sinum Syrbonis & M. Pharan.</i> | 10 37 10 | |
| 10 43 25 | 4 | 3 $\frac{5}{8}$ | | <i>Per Sinum Apollinis, M. Aetn. Inf. Leto. ad Inf. Didymam.</i> | 10 44 10 | |
| 10 47 25 | 5 | 4 $\frac{5}{8}$ Dig. | | <i>Ad Inf. Majorc. Vulcaniam, Lemnos, Carpath. & M. Horeb.</i> | 10 48 15 | |
| 10 52 20 | 6 | 5 $\frac{3}{8}$ | | <i>Ad Lacum Nigrum Majorem, per M. Argentarium, Sipylum, Masicytam; Tabor, Sinai & Desertum Raphidim.</i> | 10 53 10 | |
| 10 55 45 | 7 | 6 $\frac{1}{2}$ | | <i>Per Scopulos Hyperboreos, Inf. Besbycam, M. Olympum, Didymam, & M. Antilibanum.</i> | 10 56 40 | |
| 11 0 33 | 8 | 7 $\frac{1}{4}$ Dig. | | <i>Ad Inf. Cyaneam, M. Hormin. M. Uxii & Montes Coibacaranos.</i> | 11 1 30 | |
| 11 4 30 | 9 | 8 $\frac{1}{2}$ | | <i>Per M. Ambonum ad Inf. Apolliniam & Sinum Atheniens. per M. Mosch. Uxii, & Coibacaranos.</i> | 11 5 30 | |
| 11 9 59 | 10 | 9 $\frac{1}{2}$ | | <i>Per Lacum Hyperbor. Superior. Pal. Byces, per Mont. Hercul. & Sinum extremum Ponti.</i> | 11 10 55 | |
| | | | U u 2 | | Per | |

| h. / " | | | | | h. / " |
|----------|---------------------------|------------------|--|---|----------|
| II 13 30 | II 10 Dig. | | | Per <i>Lacum Hyperboreum Inferiorem</i> , M. <i>Cammerium</i> , & ad <i>Sinum Inferiorem Maris Caspii</i> . | II 14 27 |
| II 16 30 | II 10 $\frac{1}{2}$ ferè | | | Per M. <i>Ripheos</i> , Pal. <i>Meotidem</i> , Inf. <i>Alopeciam</i> , ad Inf. <i>Majorem Caspii</i> , per M. <i>Nerosum</i> . | II 17 17 |
| II 19 10 | II 11 $\frac{1}{4}$ | | | Ad <i>Ripam Paludis Meotidis</i> & <i>Mont. Hippoci</i> . | II 20 10 |
| II 22 0 | II 11 $\frac{1}{2}$ | | | Per M. <i>Alaunum</i> , & <i>Lacum Occident. Minorem</i> . | II 23 0 |
| II 26 30 | II 12 Dig. | | | <i>Totalis Obscuratio</i> circa 297° <i>Limbi</i> in 94° sc. à <i>Puncto Superioris Lineæ perpendicularis Nonagesimi Ortum</i> versus <i>deprehensa</i> . | II 27 30 |
| II 35 36 | | Pollucis 52° 13' | | | II 37 4 |
| II 37 35 | | Pollucis 52 0 | | | II 38 42 |
| II 38 50 | | Pollucis 51 41 | | | II 39 35 |
| II 58 20 | | | | <i>Recuperatio Luminis</i> circa 118° <i>Limbi</i> in 88° sc. à <i>Puncto Superioris Lineæ perpendicularis Nonagesimi Occalum</i> versus <i>extitit</i> . | I 0 8 |
| I 0 40 | I 5 $\frac{1}{2}$ Dig. | | | Ad M. <i>Alabastrinum</i> , M. <i>Pentadactylum</i> , M. <i>Audum</i> , & Pal. <i>Maraotidem</i> . | I 2 31 |
| I 3 35 | I 16 $\frac{1}{2}$ | | | Per M. <i>Porphyritem</i> , ad <i>Mare Syriticum</i> , & per M. <i>Eoum</i> . | I 5 32 |
| I 8 10 | I 17 2 ferè | | | Per M. <i>Baronium</i> , loca <i>paludosa</i> , Inf. <i>Cercinna</i> , inter <i>Mare Syriticum</i> & <i>Aegyptiacum</i> , ad S. <i>Syrbonis</i> . | I 10 32 |
| I 11 48 | I 18 2 $\frac{1}{2}$ ferè | | | Per <i>Sinum Apollinis</i> , Inf. <i>Taraciniam</i> , Inf. <i>Aethusam</i> , <i>Sinumque Syrbonis</i> . | I 13 54 |
| I 16 20 | I 19 3 $\frac{1}{2}$ | | | Ad Inf. <i>Majorem</i> , per M. <i>Aetnam</i> , M. <i>Neptunum</i> , <i>Jachynum</i> , Inf. <i>Letoam</i> , Inf. <i>Didyman</i> & M. <i>Lyon</i> . | I 18 33 |
| I 20 50 | I 20 4 $\frac{1}{2}$ Dig. | | | Ad Inf. <i>Corficam</i> , per Inf. <i>Vulcaniam</i> , & <i>Carpathos</i> , ad <i>Mare Mortuum</i> , per <i>Desertum Sin.</i> | I 23 6 |
| | | | | | Ad |

| h. ' " | | | | h. ' " | | | |
|--------|----|----|----|----------------------|--|---|-------|
| 1 | 24 | 39 | 21 | 5 $\frac{1}{4}$ Dig. | Ad M. <i>Argentarium</i> , <i>Mare Pamphili</i> um, Inf. <i>Cyprum</i> , per M. <i>Horminum</i> . | 1 | 27 0 |
| 1 | 26 | 25 | 22 | 6 <i>ferè</i> | Per Mont. <i>Apenninum</i> , ad <i>Lacum Trasimenum</i> , ad Mont. <i>Sipulum</i> per M. <i>Infer. Libanum</i> . | 1 | 28 40 |
| 1 | 28 | 32 | 23 | 6 $\frac{1}{2}$ | Ad M. <i>Carpas</i> bas, per Inf. <i>Besbicam</i> , M. <i>Olympum</i> , <i>Didymum</i> & M. <i>Dalangueros</i> . | 1 | 31 0 |
| 1 | 31 | 12 | 24 | 7 $\frac{1}{2}$ Dig. | Per M. <i>Perce</i> , <i>Byzantium</i> , ad M. <i>Horminum</i> , & per M. <i>Antitaurum</i> . | 1 | 33 44 |
| 1 | 35 | 59 | 25 | 8 $\frac{1}{4}$ | Per <i>Sinum Circini</i> tem, Inf. <i>Macram</i> , Inf. <i>Apolloniam</i> , Medium Montem <i>Moschum</i> , atque <i>Montes Sogdianos</i> . | 1 | 38 37 |
| 1 | 39 | 33 | 26 | 9 Dig. | Per <i>Lac. Hyperboreum Inferiorem</i> , Pal. <i>Byces</i> , M. <i>Sirobolum</i> , per Sin. <i>Extremum Ponti</i> ac M. <i>Paropamisum</i> . | 1 | 42 15 |
| 1 | 41 | 45 | 27 | 9 $\frac{1}{2}$ Dig. | Inter Pal. <i>Byces</i> & <i>Lacum Corocondometis</i> , per M. <i>Herculis</i> & M. <i>Caucasum Inferiorem</i> . | 1 | 44 30 |
| 1 | 44 | 19 | 28 | 10 | Per M. <i>Cimmerium</i> , M. <i>Tancon</i> , & M. <i>Nerosum</i> . | 1 | 47 9 |
| 1 | 47 | 20 | 29 | 10 $\frac{1}{2}$ | Per Pal. <i>Maotidem</i> , Inf. <i>Minorem Maris Caspii</i> , Montemque <i>Nerosum Superiorem</i> . | 1 | 50 14 |
| 1 | 51 | 44 | 30 | 11 paulo plus. | Per M. <i>Alaunum</i> , M. <i>Sanctum</i> ; <i>Montesque Hippoci</i> . | 1 | 54 42 |
| 1 | 56 | 12 | 31 | 12 Dig. | <i>Finis</i> , circa 294° <i>Limbi</i> , in 97 sc. gradu à <i>Puncto superiori Lineæ perpendicularis Nonagesimi Occasum</i> versus contigit. | 1 | 59 17 |
| 1 | 57 | 10 | | | <i>Densissima Penumbra</i> . | 2 | 0 17 |
| 2 | 1 | 15 | | | <i>Satis adhuc Crassa</i> . | 2 | 4 27 |
| 2 | 2 | 30 | | | <i>Paulo Dilutior</i> . | 2 | 5 45 |
| 2 | 10 | 20 | | | <i>Penumbra Dilutissima</i> . | 2 | 13 42 |
| 2 | 12 | 30 | | | <i>Finis Penumbrae</i> . | 2 | 15 0 |
| 2 | 15 | 39 | | | <i>Alt. Pol. 29° 12'</i> | 2 | 19 7 |
| 2 | 17 | 35 | | | <i>Ead. Alt. 28 46'</i> | 2 | 22 8 |

Ingruente

Ingruente *Eclipsi* Umbra erat valde diluta. Limbusque ejus quasi Anfractu-
sus, & minime terminatus, sic ut difficulter admodum ab Initio Phases deter-
minari potuerint, nec accuratè distingui per quas Maculas Umbra transibat,
successu tamen temporis crescente *Eclipsi*, distinctiùs omnia deprehendebantur.
Color ab *Initio* videbatur satis Tristis, Obscurus, & Fuliginosus, ac si *Eclipsi*,
eadem ratione, circa *Maximam Obscuracionem*, ut illa Anno 1642. mens. *April.*
adeò sese Obumbratam sistere vellet, quo vix conspiceretur; sed res planè ali-
ter cecidit, siquidem Luna cum jam omnino esset Eclipsata, Totus tamen ejus
Discus satis clarè in oculos incurrebat: Color namque ejus tum omninò Ru-
bidus sive Sanguineus aut Rubiginosus erat, qui eousque perseverabat, donec
Luna ad medietatem Lumen suum recuperasset, atque tum rursus satis Obscura
& Fuliginiosa apparuit.

At *Lisbon*; by Mr. *Jacobs*. lb. 8. 31'. p. m. 4. The Beginning of this *Eclipse* was observed at *Lisbon* by Mr. *Jacobs* at

p. 151. L. Difficillima fuerit parvæ hujus *Eclipsi* observatio, propter obliquam Lu-
An Eclipse of næ in Umbram Terræ incidentiam, Umbræque ipsius Tenuitatem, per quam
the Moon, Limbum Lunæ, media etiam *Eclipsi*, satis distinctè cernere potuimus. Par-
June 17. m. tes diametri tunc ab Umbra vera deficientes, propter confusos ejus terminos
1684. at *Green- wick*; by Mr. accuratè definire non licuit. Distantiam ergo cœpi inter Cuspides malè defi-
Flamsteed. nitos, circa Medium Defectus, è qua dictæ partes facile deduci possint, &
n. 162. p. 689. Erroris minore periculo.

| Tempora per
Horolog. Of-
cillator. | Tempora Ve-
ra ab Obs.
Correct. | Observationes. | |
|--|---------------------------------------|--|-------|
| h. ' " | h. ' " | | |
| 1 40 40 | 1 31 8 | <i>Eclipsi</i> Minæ | ' " |
| 2 4 30 | 2 54 58 | Lunæ Diameter Tubo ped. 16. erat, 1605 = | 31 57 |
| 2 6 40 | 2 57 8 | rep. 6430 = | 32 4 |
| 2 12 0 | 2 2 20 | <i>Penumbra</i> densa, forsan <i>Initium</i> | |
| 2 16 00 | 2 6 28 | Umbra Limbum supra <i>Sinam</i> temeraverat | |
| 2 18 00 | 2 8 28 | Umbra certè intra discum | |
| 2 21 36 | 2 12 4 | Chorda <i>Peripherie</i> Obscuratæ — 1670 = | 8 28 |
| 2 26 00 | 2 16 28 | rep. 2010 = | 10 1 |
| 2 30 00 | 2 26 28 | iter. 2290 = | 11 25 |
| 2 42 00 | 2 32 28 | Decrevit <i>Eclipsi</i> sensibilibiter | |
| 2 43 00 | 2 33 28 | Inter Cuspides Obscur. iter. 1895 = | 9 20 |
| 2 50 00 | 2 40 28 | <i>Finis</i> ; sed Dubius Mihi | |
| 2 54 00 | 2 44 28 | <i>Finis</i> certè, Ministro <i>Fabro</i> Consentiente. | |
| 2 57 00 | 2 47 28 | <i>Penumbra</i> Densa. | |
| 3 11 00 | 3 1 28 | Etiam adhuc. | |
| 3 20 00 | 3 10 28 | Limbus <i>Austrinus</i> haud adhuc Limbi <i>Borei</i>
Claritudinem recuperaverat. Sed lux
ejus hebetior quam in Limbo <i>Boreo</i> , ut
in prima Observatione apparuit. | |

LI. Artis profecto Laborisque haud exigui erat Tubos, etsi breviores *ſc.* 5, 6 & 7, pedum, adeo firmiter continue ad *Lunam* retinere, ut *Penumbram* recte discernere potuerimus, Phasesque omnes exacte designare, per quas nimirum *Maculas* transirent, vel quas omni tempore attingerent. Attamen pro viribus, quousque severa Tempestas atque transeuntes *Nubeculae* permittebant, rem peregi. Notandum imprimis habes, quod *Densissima Penumbra* genuinum *Initium Eclipsos* praecesserit, ita ut vix ac ne vix verum *Initium* discernere quiverim. Colorem quod attinet, hunc in hac *Eclipsi* maxime notandum habes; quippe talem diversitatem raro admodum in ipso Colore deprehendi: modo enim erat *Ferrugineus* vel *Mustelinus*; *Ingruenti* vero *Totali Obscuratione*, *Limbus Lunae* circumcirca erat *Sublividus*, ex parte *Sublustris*, & *Rubicundus*; verum in *Lunae Medio*, quasi satis *Denso*, & *Obscura Nubecula* conspiciabatur, ut vix *Maculas* recte in *Lunae* distinguere potuerimus; quae *Nigricantior Umbra* paulatim successive versus *Dextram*, & *Paludem Maotidem* promovebatur, sic ut circa *Initium Recuperationis Luminis* tota genuina *Umbra* admodum *Obscura*, & *Nigricans* appareret, atque circa ultimam annotatam *Phasin* reliqua pars adhuc *Obscurata Lunae*, sive ejus *Limbus*, neutiquam deprehendi vel minimum potuerit.

An Eclipse of the Moon, November 30. (ſt. n) 1685. at Danzick; by M. Hevelius. n. 178. p. 1256.

| Ord. Ph. | Tempus ſec.
Horolog. ex
Alt. Correſt. | Notanda. | Altitudines
☉, ☽, &
Fixarum. | Per quas Maculas tranſiverint
Umbræ Sectiones. |
|----------|---|--|------------------------------------|--|
| | | | 0 / // | |
| | | Altitudo ☉
Meridiana. | 12 39 0 | |
| | Hor. ' " | | | |
| | 3 19 0 | Altitudo ☉ | 1 54 0 | |
| | 4 28 0 | Altitudo ☽ | 9 0 6 | |
| | 5 2 30 | Altitudo Capella. | 3 1 0 0 | |
| | 5 8 0 | Altitudo Capella. | 3 1 2 0 | Dub. |
| | 9 32 25 | Penumbra. | | |
| | 9 42 35 | Penumbra
Densior. | | |
| | 9 48 5 | Penumbra
Densissima. | | |
| | 9 50 10 | Init. quantum collig.
dabatur. | | |
| 1 | 9 53 20 | Pal. Marsotia jam tota
erat tecta. | | Ad M. Andum, & Pal. Maraotidem. |
| 2 | 9 57 0 | | | Per Germanicianum ad Mare Syrticum, & M. Acabe. |
| 3 | 10 0 15 | M. Porphyrit. nond. plane
tect. erat. | | Ad M. Porphyritem, per Mare Syrticum, Montemque Aja-
cem. Per |

| | | | | | |
|----|----|----|----|--------------|--|
| 4 | h. | 10 | 3 | 5 | |
| 5 | 10 | 6 | 25 | | |
| 6 | 10 | 10 | 25 | | |
| 7 | 10 | 14 | 55 | Insula Melos | |
| | | | | jam tota | |
| | | | | recta erat. | |
| 8 | 10 | 19 | 55 | | |
| 9 | 10 | 24 | 5 | | |
| 10 | 10 | 27 | 10 | | |
| 11 | 10 | 31 | 27 | | |
| 12 | 10 | 36 | 0 | | |
| 13 | 10 | 41 | 20 | | |
| 14 | 10 | 45 | 35 | | |
| 15 | 10 | 48 | 40 | | |
| 16 | 10 | 52 | 10 | | |

Per M. Petri, inter Inf. Circin-
nam & Inf. Tarracinniam, per
Sinum Syrbonis, & Paludes A-
rabie.

Ad Inf. Ficariam, Maltam, M.
Athos, per M. Pharan & M.
Troicum.

Ad Sinum Apollinis, per Inf. Erro-
ris, per M. Aetnam, Inf. Letoam
& Didymam.

Supra Sin. Apollinis, ad Lacum Her-
culeum, Inf. Sicilia, per M. Par-
thenium & Taigetum, ad Inf.
Melos & Mont. Lion.

Ad Inf. Minorcam, per Sinum Pa-
stannum, M. Micalo, Inf. Cyprium,
M. Hor. ad M. Sinai & Deser-
tum Raphidim.

Ad M. Ligustinum, Paryaum, inter
M. Sipulum & Didymum, per M.
Libanum & Seir.

Supra M. Nigrum Minorem, per
M. Appenninum, Inf. Besbicam,
M. Cimeum, Montemq; Calcha-
stan.

Ad Montes Serrorum, & Carpathes,
ad Byzantium, per M. Hermi-
nium Taurum, & M. Delangue-
ros.

Per Montes Macrocmnios, Inf. Cy-
ancam, M. Amanum & Antitau-
rum.

Per Inf. Macram, per Erichtenios
Scopulos, Sin. Atheniensem, & Pal.
Arcefam.

Per Lac. Hyperbor. Superiorem. Pal.
Byces, Inf. Acam, M. Herculis,
Sin. Extremum Ponti, Montem-
que Parapomifum.

Ad Lacum Hyperbor. Inferiorem,
M. Immerium, per M. Coracem,
Taucan, Sin. Inferiorem Maris
Caspui.

Per Montes Rhiphaeos, Pal. Maori-
dem, Inf. Majorem Caspii, & M.
Nerosum.

Luna

| | | | |
|----------|----------|--|---|
| h. ' " | | Luna nond.
plene Ob-
scurata. | |
| 10 | 56 35 | | |
| 10 58 25 | | Totalis Im-
mersio. | |
| h. ' " | | Altitud. Pro-
cyonis. | o ' " |
| 11 | 20 45 | | |
| 11 25 45 | | Eadem Alti-
tudo. | 31 38 0 |
| 11 36 0 | | Obscura illa
Umbra vel
Nubec. pla-
ne in Med.
Disco Lun.
consistebat. | |
| 12 43 0 | | Init. Emer-
sionis seu Re-
cuper. Lu-
minis. | |
| 17 | 12 46 40 | Palus Mara-
otis jam pe-
nitus tecta. | Ad Mont. Audum, haud pro-
cul à Palude Maraotide. |
| 18 | 12 51 0 | | Per Mare Eoum, Mare Syri-
cum, Montemque Casium. |
| 19 | 12 57 0 | | Per M. Atlantem Minorem, in-
ter Inf. Circinnam & Inf. Tara-
cinian, per Mare Egyptiacum,
Sin. Sirbonis, Montemq; Phara- |
| 20 | 1 5 0 | | Ad Inf. Majore. Sardin. & Hier.
per M. Aetnam, Inf. Melos,
ad Mont. Anna & Hajalon. |

Initium Eclipseos paulo supra Paludem *Maraotidem* extitit.

Totalis Obscuratio, ad Montem *Sanctum*, infra Paludem *Maraotidem* accidit.

Recuperatio Luminis fere *Ibidem* contigit ubi *Initium* coepit.

2.
At Nurem-
burg; by M.
G. C. Eimmart.
n. 182. p. 146.

h. ' ''
9 19 00

The *Penumbra* was very Obscure, and the *Beginning* of the *Eclipse* was at hand.

9 23 30

The *Eclipse* was *Begun*, the Quantity almost half a *Digit*, and the distance between the *Cusps* was about 42 Degrees of the Moon's Limb, and *Palus Maraotis* was just all *Eclipsed*; hence we may conclude the *Beginning* about 9^h. 21' 30''.

10 23 30

As near as I can collect, was the time of the *Total Immersion* into the Shadow, to verify which, the Azimuth of the Moon's Center was observed to the East, 41° 18'; 2' 12'' of time after the said *Immersion*.

12 13 0

Or 10' 13'' before the *Culmination* of the *Right Shoulder* of *Orion*, was the *Emergence* or first Appearance of the Moon out of the *Total Darknefs*.

1 14 fer

Was the just *End* of the *Eclipse*, being 2' 20'' before the *Culmination* of *Syrus*.

Whence the *Middle* of this *Eclipse* should have happened at 11^h. 18' p. m. at *Nuremburg*: the *Total Duration* 3^h 52' 30'', and the *Total Darknefs*: 1^h. 49' 30''.

The Meridian Altitude of the Moon's upper Limb was observed 63° 23' 50'', and the Moon's apparent Diameter while totally *Eclipsed* was found 30' 07''.

By M. J. Ph.
Wurzelbaur.
Eb. p. 147.

3. M. Wurzelbaur made use of a Pendulum-Clock corrected by Altitudes: According to his Observation,

h. ' ''

9 23 30

was the *Beginning* of the *Eclipse*, at about 119 Degrees of the Limb of the Moon in *Hevelius's Selenography*.

9 24 50

Palus Maraotis was all covered.

10 25 20

The *Total Immersion*, about the 299th Degree of the Limb of the Moon.

12 11 30

The Moon began to *Emerge* out of the Shadow, about the 112th Degree of her Limb.

1 14 30

The *End* of the *Eclipse* about the 295th Degree of the Limb.

By these Observations the *Middle* of the *Eclipse* ought to have been about 11^h. 19' p. m. at *Nuremburg*, differing but one Minute from M. Eimmart's Observation.

The *Duration* will be 3^h. 51', and the *Total Darknefs* 1^h. 46'.

At Lisbon; by
Mr. Jacobs.
n. 184. p. 206.

4. Initium

Immerſio

Emerſio

Finis

h. ' ''

8 02

9 06

10 50

11 57

| <i>Phasis.</i> | <i>Temp.</i> |
|---|--------------|
| <i>Penumbra Notabilis</i> | h. ' " |
| <i>Initium</i> ob interpositas Nubes præcise determinare haud licuit, ideoque incertius pono <i>Eclipsin</i> incepisse ad 9 ^h . 25'. | 9 15 00 |
| vel | 9 27 00 |
| <i>Umbra ad Paludem Maræotim</i> | 9 38 20 |
| <i>Palus Maræotis</i> tecta | 9 40 20 * |
| <i>Mons Sinai</i> tegitur | 9 46 30 * |
| <i>Mons Thambes</i> tectus | 10 01 20 * |
| <i>Mons Andus</i> tegitur | 10 08 00 * |
| <i>Mons Neptunus</i> tectus | 10 14 00 * |
| <i>Umbra ad Montem Sipylum</i> | 10 15 10 * |
| <i>Insula Circinna</i> tegitur | 10 16 40 |
| <i>Ad Montem Didymum</i> | 10 18 20 * |
| <i>Mons Didymus</i> tegitur | 10 20 10 |
| <i>Emergit M. Andus</i> | 10 23 00 |
| <i>Umbra ad Paludem Maræotim</i> | 10 48 30 |
| <i>Emergit Maræotis</i> | 10 52 00 |
| <i>Ad Montem Sinai</i> | 11 38 00 |
| <i>Emergit Sinai</i> | 11 39 30 |
| <i>Finis Eclipsis</i> | 11 04 00 |

LII.

An Eclipse of the Moon, November 19. 1686. at Dublin; by Mr. Will. Molineux. n. 185. p. 236.

Notanda. Tempora sunt Horologii Oscillatorii ad Stellas fixas rectificati. Quæ *Asterismo* notantur Observationes, per hiantes Nubes captæ sunt, adeoque accuratas haberi nolo. Quantitatem hujus Eclipsis sex puto *Digitorum*.

LIII. Half a Quarter of an Hour after 7 in the Evening, the Moon arose clear, but of a deep Red Colour without any Sign of Eclipse: At 7^h. $\frac{1}{2}$ the Moon went into a thick Cloud, but was again clear at 7^h. 38'. when the Under-side of this Body of the Moon was *Begun* to be *Obscured*, in a clear Sky; she being then in the 25th Degree of *Libra*, and 6^o. $\frac{1}{2}$ above the Horizon. (Suppose the Center.) At 9^h. the whole Under-side of the Moon was *Eclipsed*, and about 8' after 9^h, it was at the height, or rather seemed to decrease. At 9^h. $\frac{1}{2}$ there was still a third Part of the Moon *Eclipsed*. (Suppose of her Circumference.) About 10^h. it *decreased* apace, and at 10^h. $\frac{1}{2}$ there was but little to be seen: At 10^h. 45'. it was certainly Ended, the Moon being then about 22° High.

An Eclipse of the Moon. April 5. 1688. at Moscu; by M. Timmerman. n. 192. p. 453.

LIV. r.
An Eclipse of
the Moon,
Octob. 19. 1697.
at Chester; by
Mr. Ed. Halley.
n. 235. p. 784.

| <i>Immersion.</i> | <i>Times.</i> | <i>Emersions.</i> | <i>Times.</i> |
|-----------------------------------|--------------------|-------------------------------|--------------------|
| | h ' " | | h ' " |
| The Beginning | 6 8 $\frac{1}{2}$ | Porphyrites and the Middle of | |
| Porphyrites Immersed | 6 16 | M. <i>Ætna</i> | 8 7 00 |
| North-part of <i>Maoris</i> | 6 21 $\frac{1}{2}$ | <i>Horminius</i> | 8 17 30 |
| <i>Lacus Niger Maj.</i> and South | | <i>Mons Herculis</i> | 8 18 30 |
| End of <i>Ma.</i> | 6 26 | <i>Besbicus</i> | 8 21 00 |
| <i>Besbicus</i> | 6 46 $\frac{1}{2}$ | <i>Apollonia</i> | 8 26 15 |
| <i>Apollonia</i> | 6 49 $\frac{1}{2}$ | <i>Byzantium</i> | 8 29 00 |
| <i>Byzantium</i> | 6 53 | <i>Lacus Niger Major</i> | 8 32 $\frac{1}{2}$ |
| <i>Horminius</i> | 6 59 | South-part of <i>Maoris</i> | 8 35 |
| North-part of <i>Maoris</i> | 7 2 $\frac{1}{2}$ | North-part of <i>Maoris</i> | 8 43 |
| <i>Mons Corax</i> | 7 3 $\frac{1}{2}$ | The End | 8 49 $\frac{1}{2}$ |
| <i>Mons Herculis</i> | 7 10 | | |
| South-part of <i>Maoris</i> | 7 12 $\frac{1}{2}$ | | |

About the *Middle* there remained 9' 26". of the Luminous part, and consequently the *Digits* Eclipsed 8 $\frac{2}{3}$.

2.
At Rotterdam;
by M. J. a. Cas-
fini. n. 236.
p. 20.
Vid. *sup.* §.
XXXIX. 2.

| | <i>Temp. Cor.</i> | <i>Observationes.</i> |
|-----|-------------------|--|
| | h. ' " | |
| A | 6 20 36 | Promontorium Acutum ad Primum Obliquum. |
| B | 6 21 27 | Præcedens <i>Luna</i> Margo ad Perpendiculare. |
| C | 6 22 3 | Promontorium Acutum ad Perpendiculare. |
| C-B | 0 00 36 | Differentia Transitus per filium Perpendiculare, quæ est Longitudo Promontorii Acuti à Margine Præcedente. |
| C-A | 0 1 27 | Differentia Transitus Promontorii Acuti inter 1. Obliquum & Perpendiculare, quæ ejus est Latitudo à Margine Boreali. |
| | 6 32 34 | <i>Luna</i> inter Nubes conspecta adhuc apparuit integra. |

In Prima Phasi.

| | h. | ' | " | |
|----------------|----|----|----|--|
| A | 6 | 41 | 23 | Initium Maris <i>Crisi</i> ad 1. Obl. |
| B | 6 | 41 | 50 | Promontorium Acutum ad 1. Obl. |
| C | 6 | 42 | 12 | <i>Plinius</i> ad 1. Obl. |
| D | 6 | 42 | 25 | <i>Menelaus</i> ad 1. Obl. |
| E | 6 | 42 | 33 | <i>Manilius</i> ad 1. Obl. |
| F | 6 | 42 | 43 | Primus Margo ad Perpendiculare. |
| G | 6 | 43 | 00 | <i>Proclus</i> ad Perpendiculare. |
| H | 6 | 43 | 20 | Promontorium Acutum ad Perpendiculare. |
| I | 6 | 43 | 26 | <i>Margo Sequens</i> ad 1. Obliquum. |
| K | 6 | 43 | 30 | <i>Menelaus</i> ad Perpendiculare. |
| L | 6 | 44 | 00 | Cornu præcedens <i>Luna</i> ad Perpendiculare, ipsa tangit
Filum Horiz. |
| M | 6 | 44 | 21 | Cornu <i>Sequens</i> ad 1. Obl. |
| N | 6 | 44 | 35 | <i>Menelaus</i> ad 2. Obl. |
| O | 6 | 44 | 57 | Cornu <i>Sequens</i> ad Verticale. |
| P | 6 | 45 | 07 | <i>Margo Sequens</i> ad Perpendiculare. |
| Q | 6 | 45 | 33 | Cornu <i>Sequens</i> ad 2. Obl. |
| R | 6 | 45 | 55 | <i>Grimaldus</i> ad 2. Obl. |
| S | 6 | 46 | 49 | <i>Sequens Margo</i> ad 2. Obliquum. |
| P-F | 0 | 02 | 24 | Transitus <i>Luna</i> Perpendiculare. |
| S-I | 0 | 03 | 23 | Transitus <i>Luna</i> per 2. Obl. |
| H-F | 0 | 00 | 37 | Promontorii Acuti Longitudo à Margine Præcedente. |
| H-B | 0 | 01 | 30 | Promontorii Acuti Latitudo à Margine Boreali. |
| K-F | 0 | 00 | 47 | <i>Menelai</i> Longitudo à Margine Præcedente. |
| K-D }
N-K } | 0 | 01 | 05 | <i>Menelai</i> Latitudo à Margine Boreali. |
| L-F | 0 | 01 | 17 | Cornu Præcedentis Longitudo à Margine Præcedente. |
| O-F | 0 | 00 | 00 | Latitudo nulla. |
| O-M | 0 | 02 | 14 | Cornu Sequentis Longitudo à Margine Præced. |
| Q-O | 0 | 00 | 36 | Latitudo ejusdem Cornu à Margine Boreali. |

In Secunda Phasi.

| | h. | ' | " | |
|-----|----|----|----|---|
| A | 7 | 6 | 40 | <i>Margo Præcedens</i> ad Perpendiculare. |
| B | 7 | 7 | 8 | <i>Cornu Præcedens</i> ad Verticale. |
| C | 7 | 7 | 47 | <i>Cornu Sequens</i> ad 1. Obl. |
| D | 7 | 8 | 19 | <i>Margo Sequens</i> ad 1. Obl. |
| E | 7 | 9 | 4 | <i>Cornu Sequens</i> ad Verticale. |
| | 7 | 12 | 22 | <i>Umbra</i> ad <i>Manilius</i> . |
| B-A | 0 | 0 | 28 | <i>Cornu Præced.</i> Longitudo à Margine præcedente. |
| E-A | 0 | 2 | 24 | <i>Cornu Sequentis</i> Longitudo à Margine Orientali. |
| E-C | 0 | 1 | 17 | <i>Cornu Sequentis</i> Lat. à Margine Australi. |

In Tertia Phasi.

| | h. | ' | " | |
|-----|----|----|----|---|
| A | 7 | 19 | 30 | <i>Cornu Præced.</i> ad 1. Obliquum. |
| B | 7 | 21 | 9 | <i>Margo Præced.</i> ad Verticale. |
| C | 7 | 21 | 19 | <i>Cornu Præced.</i> ad Verticale. |
| D | 7 | 21 | 51 | <i>Margo Præced.</i> ad 2. Obliquum. |
| E | 7 | 22 | 24 | <i>Cornu Sequens</i> ad 1. Obliquum. |
| F | 7 | 22 | 47 | <i>Margo Sequens</i> ad 1. Obl. |
| G | 7 | 23 | 9 | <i>Cornu Præcedens</i> ad 2. Obl. |
| H | 7 | 23 | 31 | <i>Cornu Sequens</i> ad Verticale. |
| I | 7 | 24 | 40 | <i>Cornu Sequens</i> ad 2. Obliquum. |
| | 7 | 26 | 4 | <i>Umbra</i> ad <i>Diomysium</i> . |
| C-B | 0 | 0 | 10 | <i>Cornu Præcedentis</i> Longitudo à Margine Orientali. |
| C-A | 0 | 1 | 49 | <i>Cornu Præced.</i> Latitudo à Margine Australi. |
| G-C | 0 | 1 | 50 | Eadem Latitudo. |
| H-B | 0 | 2 | 22 | <i>Cornu Sequentis</i> Long. à Margine præcedente. |
| H-E | 0 | 1 | 7 | <i>Cornu Sequentis</i> Lat. à Margine Australi. |
| I-H | 0 | 1 | 9 | Eadem Latitudo. |

In Quarta Phasi.

| | | h. | ' | " | |
|--------------|---|----|----|-----|---|
| A | | 7 | 40 | 24 | <i>Cornu Præcedens</i> ad 1. Obliquum. |
| B | | 7 | 41 | 35 | <i>Cornu Præced.</i> ad Verticale. |
| C | | 7 | 42 | 18 | <i>Margo Præced.</i> ad 1. Obl. |
| D | | 7 | 42 | 44 | <i>Cornu Præced.</i> ad 2. Obl. |
| E | | 7 | 42 | 51 | <i>Cornu Sequens</i> ad 1. Obl. † |
| F | | 7 | 43 | 14 | <i>Margo Sequens</i> ad 1. Obl. |
| G | | 7 | 43 | 58 | <i>Margo Sequens</i> ad Verticale. |
| H | | 7 | 45 | 04 | <i>Cornu Sequens</i> ad 2. Obl. |
| G - 2' 24" = | I | 7 | 41 | 34 | <i>Margo Præced.</i> ad Verticale. |
| B — | I | 0 | 00 | 01 | <i>Cornu Præced. Long.</i> à Margine Præced. |
| B — | A | 0 | 01 | 11 | { <i>Cornu Præced. Lat.</i> à Margine Australi. |
| D — | B | 0 | 01 | 09 | |
| H — | E | 0 | 2 | 13 | <i>Differentia Transitus Cornu Sequens</i> inter Obl. |
| | K | 0 | 1 | 6½ | <i>Dimidium, Lat. Cornu Seq.</i> à Margine Australi. |
| E + | K | 7 | 43 | 57½ | <i>Cornu Sequens</i> ad Verticale. |
| E + K | I | 0 | 2 | 23½ | <i>Longitudo Cornu Seq.</i> à Margine Præcedente. |

In Quinta Phasi.

| | | h. | ' | " | |
|--------------|---|----|----|----|---|
| A | | 7 | 50 | 22 | <i>Cornu Præced.</i> ad 1. Obl. |
| B | | 7 | 51 | 04 | <i>Promontorium Acutum</i> in Umbra. |
| C | | 7 | 51 | 18 | <i>Cornu Præcedens</i> ad Verticale. |
| D | | 7 | 51 | 58 | <i>Margo Præcedens</i> ad 2. Obl. |
| E | | 7 | 52 | 29 | <i>Cornu Sequens</i> ad 1. Obl. |
| F | | 7 | 52 | 56 | <i>Margo Sequens</i> ad 1. Obl. |
| D - 2' 24" = | I | 7 | 49 | 34 | <i>Margo Præcedens</i> ad 1. Obl. |
| B — | I | 0 | 01 | 30 | <i>Longit. Obl. Promontorii Acuti</i> à 1. Obliq. |
| D - 0' 42" = | L | 7 | 51 | 16 | <i>Margo Præcedens</i> ad Perpendiculare. |
| C — | L | 0 | 00 | 02 | <i>Cornu Præced. Long.</i> à Marg. Præced. |
| C — | A | 0 | 00 | 56 | <i>Cornu Præced. Lat.</i> à Margine Australi. |
| E — | I | 0 | 02 | 55 | <i>Long. Obliq.</i> à 1 obl. ad Max. Long. 3'. 24". |
| | | 0 | 00 | 29 | <i>Complementum.</i> |
| F — | E | 0 | 00 | 27 | <i>Idem Complementum. Medium.</i> 0' 28". |

In Sexta Phasi.

| | h. | ' | " | |
|----------------|----|----|----|--|
| A | 7 | 54 | 06 | <i>Cornu Præcedens</i> ad 1. Obl. |
| B | 7 | 54 | 44 | <i>Promont. Acutum</i> ad 1. Obl. |
| C | 7 | 54 | 59 | <i>Cornu Præced.</i> ad Perpend. |
| D | 7 | 55 | 38 | <i>Margo Præced.</i> ad 2. Obl. |
| F | 7 | 55 | 52 | <i>Cornu Præced.</i> ad 2. Obl. |
| G | 7 | 56 | 04 | <i>Cornu Seq.</i> ad 1. Obl. |
| H | 7 | 56 | 36 | <i>Margo Seq.</i> ad 1. Obl. |
| I | 7 | 57 | 20 | <i>Cornu Sequens</i> ad Verticale. |
| <hr/> | | | | |
| D - 2' 24" = L | 7 | 53 | 14 | <i>Margo Præced.</i> ad 1. Obl. |
| D - 0 42 = K | 7 | 54 | 56 | <i>Margo Præced.</i> ad Perpend. |
| C — K | 0 | 00 | 03 | <i>Cornu Præced. Long.</i> à Marg. Præced. |
| C — A | 0 | 00 | 53 | <i>Cornu Præced. Lat.</i> à Marg. Australi. |
| F — C | | | | |
| I — K | 0 | 02 | 24 | <i>Longitudo Cornu Sequentis</i> à Marg. Præced. |
| I — G | 0 | 01 | 16 | <i>Latitudo Cornu Seq.</i> à Marg. Australi. |
| B — L | 0 | 01 | 30 | <i>Longitudo obliqua Prom. Acuti</i> à 1. Obl. |

In Septima Phasi.

| | h. | ' | " | |
|----------------|----|---|----|---|
| A | 8 | 6 | 58 | <i>Cornu Præced.</i> ad 1. Obl. |
| B | 8 | 7 | 39 | <i>Cornu Præced.</i> ad Perpend. |
| C | 8 | 8 | 15 | <i>Margo Præced.</i> ad 2. Obl. |
| D | 8 | 8 | 21 | <i>Cornu Præced.</i> ad 2. Obl. |
| E | 8 | 8 | 55 | <i>Medium Umbra</i> ad Perpend. ferè. |
| F | 8 | 9 | 14 | <i>Margo Sequens</i> ad 1. Obl. |
| G | 8 | 9 | 55 | <i>Cornu Sequens</i> ad Verticale. |
| H | 8 | 9 | 58 | <i>Margo Sequens</i> ad Verticale. |
| <hr/> | | | | |
| C - 2' 24" = I | 8 | 5 | 51 | <i>Margo Præced.</i> ad 1. Obl. |
| C - 0 42 = K | 8 | 7 | 33 | <i>Margo Præced.</i> ad Perpend. |
| B — K | 0 | 0 | 06 | <i>Cornu Præced. Long.</i> à Margine Præced. |
| B — A | 0 | 0 | 41 | } <i>Cornu Præced. Lat.</i> à Margine Australi. |
| D — B | 0 | 0 | 42 | |
| G — K | 0 | 2 | 22 | <i>Cornu Sequentis Long.</i> à Marg. Præced. |

In Octava Phasi.

| | | h. 7 " | |
|---|------------|---------|--|
| A | | 8 23 53 | Cornu Præcedens ad 1. Obl. |
| B | | 8 24 24 | Cornu Præced. ad Perpend. |
| C | | 8 24 55 | Margo Præced. ad 2. Obl. |
| D | | 8 24 57 | Cornu Præced. ad 2. Obl. |
| E | | 8 25 55 | Margo Seq. ad 1. Obl. |
| F | | 8 26 26 | Cornu Sequens ad Perpend. |
| G | | 8 26 38 | Margo Sequens ad Vert. |
| H | | 8 28 20 | Cornu Sequens ad 2. Obl. |
| C | 0' 42" = K | 8 24 13 | Margo Præced. ad Perpend. |
| B | — K | 0 00 11 | Longitudo Cornu Præced. à Marg. Australi. |
| B | — A | 0 00 31 | { Latitudo Cornu Præced. à Marg. Australi. |
| D | — B | 0 00 33 | |
| F | — K | 0 02 13 | Longitudo Cornu Seq. à Marg. Præcedenti. |
| H | — F | 0 01 34 | Latitudo Cornu Seq. à Marg. Australi. |

In Nona Phasi.

| | | | |
|---|------------|---------|--|
| A | | 8 49 25 | Cornu Præced. ad 1. Obl. |
| B | | 8 49 42 | Eclipsis Concavitas ad 1. Obl. |
| C | | 8 50 08 | Cornu Præced. ad Verticale. |
| D | | 8 50 41 | Margo Præced. ad 2. Obl. |
| E | | 8 50 51 | Cornu Præced. ad 2. Obl. |
| F | | 8 51 34 | Cornu Seq. ad Verticale. |
| G | | 8 51 43 | Margo Seq. ad 1. Obl. |
| H | | 8 52 26 | Margo Seq. ad Vert. |
| H | 2' 24" = I | 8 50 01 | Cornu Præced. ad Perpend. |
| C | — I | 0 00 06 | Longitudo Cornu Præced. à Marg. Præced. |
| C | — A | 0 00 43 | { Latitudo Cornu Præced. à Marg. Australi. |
| E | — C | 0 00 43 | |
| F | — I | 0 01 32 | Long. Cornu Seq. à Marg. Præced. |

In Decima Phasi.

| | h. | ' | " | |
|----------------|----|----|----|--|
| A | 9 | 4 | 29 | <i>Cornu Præcedens ad 1. Obl.</i> |
| B | 9 | 5 | 4 | <i>Umbra recedit à Plinio.</i> |
| C | 9 | 5 | 53 | <i>Cornu Seq. ad 2. Obl.</i> |
| D | 9 | 6 | 17 | <i>Cornu Præced. ad 2. Obl.</i> |
| E | 9 | 7 | 2 | <i>Margo Seq. ad 1. Obl.</i> |
| F | 9 | 7 | 47 | <i>Margo Seq. ad Vert.</i> |
| E - 2' 42" = H | 9 | 5 | 20 | <i>Margo Præced. ad Verticale.</i> |
| D — A | 0 | 1 | 48 | <i>Transitus Cornu Præced. inter Obliquos.</i> |
| M | 0 | 0 | 54 | <i>Dimidium Lat. Cornu Præced. à Marg. Australi.</i> |
| D — M = I | 9 | 5 | 23 | <i>Cornu Præced. ad Verticale.</i> |
| I — H | 0 | 0 | 3 | <i>Longitudo Cornu Præced. à Marg. Præced.</i> |
| E - 3' 24" = L | 9 | 3 | 38 | } <i>Margo Præced. ad 1. Obl.</i> |
| F - 4 06 = L | 9 | 3 | 41 | |
| C — L | 0 | 2 | 25 | <i>Long. Obliqua Cornu Seq. à 1. Obliquo.</i> |
| | 9 | 9 | 4 | <i>Umbra recedit à Langreno.</i> |
| | 0 | 9 | 19 | <i>Finis Maris Tranquillitatis.</i> |
| | 0 | 13 | 40 | <i>Aristoteles.</i> |
| | 0 | 14 | 39 | <i>Finis Maris Crisii.</i> |
| | 0 | 21 | 34 | <i>Finis.</i> |

*Tempus*

LV.

A Transit of
the Moon
above *Venus*.
O.H. 11. (f.n.)
1670. at *Dant-*
zick; by M.
Hevelius. n. 66.
p. 2026.

| <i>Tempus</i>
<i>juxta Hor.</i>
<i>Amb. manè.</i> | | <i>Distantie</i>
& <i>Altit.</i> | <i>Tempus ex</i>
<i>Alt. Corr.</i> |
|---|---|-------------------------------------|---------------------------------------|
| h. ' " | | o ' " | h. ' " |
| 3 47 20 | Altitudo <i>Lucide</i> γ | 41 45 0 | 3 54 10 |
| 3 50 35 | Eadem Altitudo. | 41 21 0 | 3 57 28 |
| 5 47 44 | <i>Distantia</i> φ à $\mathcal{U}$ | | 5 54 44 |
| 5 48 35 | <i>Dist.</i> $\mathcal{D}$ Limb. Orient. à $\mathcal{U}$ | 35 25 30 | 5 55 35 |
| 5 51 59 | <i>Dist.</i> φ à $\mathcal{U}$ | 35 50 0 | 5 59 0 |
| 5 53 40 | <i>Dist.</i> $\mathcal{D}$ Limb. Orient. à $\mathcal{U}$ | 35 29 10 | 6 0 40 |
| 6 12 30 | <i>Dist.</i> φ à $\mathcal{U}$ | 35 50 20 | 6 6 0 |
| 6 14 3 | <i>Dist.</i> $\mathcal{D}$ Limb. Orient. à $\mathcal{U}$ | 35 37 35 | 6 8 0 |
| 6 37 0 | <i>Dist.</i> φ à $\mathcal{D}$, quantum nudo oculo
dijudicari potuit. | 0 6 0 | 6 51 0 |
| 7 12 0 | <i>Dist.</i> φ à $\mathcal{D}$ Limbo Infer. | 0 5 0 | 7 7 0 |
| 7 17 0 | <i>Dist.</i> φ à $\mathcal{D}$ Limb. Infer. | 0 4 0 | 7 12 0 |
| 7 35 0 | <i>Venus</i> clare apparuit. | | 7 30 0 |
| 9 11 0 | <i>Venus</i> permanfit conspicua. | | 9 12 0 |
| 9 15 48 | Altitudo <i>Solis</i> . | 19 8 0 | 9 17 20 |
| 9 17 39 | Altitudo <i>Solis</i> . | 19 58 0 | 9 19 4 |

LVI. Initium Occultationis accidit 3^h. 38' 27". mane circa Montem *Ger-* An Occultati-
manicianum. Linea Itineraria, quantum ex solo Ingressu haud obscure colligere of *Saturni*
licuit, transiit per M. *Aetnam*, Centrum ferè *Lunæ*, per M. *Florminium*, M. by the Moon,
Herculis, & superiorem partem *Maris Caspii*. Ego quantum memini, bis tan- *June* 1. (f.n.)
tum, si hujus Anni Observationem excipias, intra 41 Annos, *Saturnum* à *Lu-* 1671. at *Dant-*
na tectum vidi; Anno nimirum 1630. Die 29 *Junii*, vesp. h. 11. cum in *zick*; by M.
Freto Danico circa Insulam *Huennan* versarer: Rursus Anno 1661. Die 3. *Hevelius*. n. 78.
Augusti hic *Dantisci* h. 7. 58'. 20". Vesp. p. 3027.

LVII. Sudo admodum Cælo, *Lunam* protinus Orientem, nec non paulò post A Transit of
Jovem, Tubo 20. pedum summâ Aviditate excepimus, atque deprehendimus the Moon
Ostante nostro permagno Orichalcico, 9 ferè pedum Radio, *Jovem* à *Lunæ* above $\mathcal{U}$ *Sept.*
Limb. Orientali adhuc 1° 23' 40". esse remotum; Jovialesque omnes qua- 30. (f.n.) 1671.
tuor à Dextra, à qua *Luna* accedebat, adesse. Ipsum quidem Conjunctionis at *Dantzick*;
momentum insperatus quidam Casus infeliciores deprehendere prohibuit. Cum by M. *Hevelius*.
enim *Jupiter* ad Limbum *Lunæ* Orientalem ad 3' jam accideret, atque ad 6 lb. p. 3031.
duntaxat à Linea Conjunctionis, per utrumque Cornu ducta, distaret, ecce
Y y 2 super

supervenientes Nebeculas, quæ tam *Jovem* quam ipsam *Lunam*, nobis à conspectu eripuerunt. Tabulæ *Rudolphina* Occultationem, eamque multò citiùs promissere, nulla tamen omninò fuerit, sed, arctissimus solummodò ad duos propemodum Digitos, Transitus extiterit, *Horà* scil. 7 26' 0".

An Occultat.
of the *Pleiades*
by the *Moon*,
Feb. 23. 1671.
at *Derby*; by
Mr. *Flamsteed*.
n. 86. p. 5034.

Fig. 116.

LVIII. Alta *Luna* 20° 50'. cepi ipsius Diametrum 32' 48"; & Alta ipsâ 19° 23' rursus eam cepi, 32' 47". Ergo *Luna* in Horizonte Semidiameter erat vera 16' 19". Plus tamen etiamnum ab Occidentali Stella *Pleiadum* absuit quam commode caperet Telescopium. At 11h 19', p. m. Alta * a b, Occidentali *Pleiadum* 9 50'; ejusdem Stellæ distantiam cepi à Cornu *Luna* Proximo 11' 58". divertens deinde subito ad ** altitudinem (ostensam Quadrante, 20 digitorum Radio, ad Tubi latus affixo) notandam, & continuo reversus, Stellam (quippe tunc à *Luna* Tectam) non comperi. Interea *Luna* descenderat Minuta 10, simulque tantundem Stella, quam subisse *Lunam* h. 11. 20' ex sequente Phasi conjicio: Etenim 11h 30'. Alta * a c. 8° 43'. Stellam c, à *Luna* tectam conspexi. Ejus cum cepissem à Cornu Proximo Distantiam 16' 35'. spatium Temporis inter hujus & præcedentis Occultationem, editis supputationibus, constitui 9' 37"; quæ Tempori hujus Phasæ sublatæ, dant utique præcedentis Occultationis Tempus ut constitui.

H. 11. 37', alta * a c. 11° 37', ipsa *Luna* subit, me interea distantiam ejus dimetiente 22' 36'. à Cornu *Luna* apparenter Inferiori, sed Superiori verè. Erat, Stella evanescente, *Luna* Semidiameter apprens 16' 21'. quæ propterea occultata erat 87° 25'. Peripheria *Lunaris* à Cuspide Superiori, cujus erat Reclinatio (à Linea per Centrum ejus, *Eclipticæ* ducta perpendiculari) 1° 37'. Sic subingressus Stellæ fuit 4° 12'. supra Lineam per Centrum *Luna* *Eclipticæ* ductam Parallelam, & *Luna* Centrum in Antecedentia * a 16' 18". cum minori Latitudine 1' 12".

Fixæ Locus Authori *Carolino* 25° 1' 24"; Latitudo perpetua 4° 20' 39", quamobrem *Luna* Locus Apparens hora Apparenti *Derbie* 11h 37', p. m. erat 24° 45' 6', & Latitudo visa 4° 19' 27". Bor.

Notatu præterea dignissimum quod etiamsi omnes ferè omnium Astronomorum Hypotheses, *Luna* Plene Perigæ in Quadraturis Majorem tribuant Diametrum, & proinde Minorem à Terra Distantiam quam in Syngiis aut Oppositionibus Perigæis: contrarium tamen Galæus fieri & evenire, *Luna* etenim Plena Perigæa transiens juxta *Pleiadas*, Nov. 6. 1671. Majorem habebat Diametrum quam in hoc Transitu, quando in eodem ferè loco à Sole distitit gradus 70, *Luna* Semidiameter Horizontalis

| | | | | | |
|--------------------------|---------|----------|---------|-----------|---------|
| Nov. 6. 1671. Bullialdo. | 71' 00" | Streetio | 16' 30" | Observata | 17' 00" |
| Feb. 23. 1672. | 17 50 | | 17 13 | | 16 19. |
| | <hr/> | | <hr/> | | <hr/> |
| | + 50 | | + 43 | | - 41. |

Amplius

Amplius non nunc miramur *Lunam* tam diu numerorum recusasse vincula & de Tabulis supputata apparentiarum tempora usque adeo expectationes nostras fecellisse, à falsis quandoquidem Hypothesibus ipsas plerumque constructas fuisse liquet.

LIX. *April 2d. (st. n.) 6^h 50'. v.* A Line drawn through the Horns of The Moon's the Moon passed through the Star that is at the Point of the Northern Horn of *Taurus*, and the Distance of this Star to the Northern Horn of the Moon, was by a Minute greater than the Semidiameter of the Moon. Place, Mar. 23. 1674; by M. Cassini, n. 82. p. 4047.

LX. *Immersio Stelle Sequentis Diurnum in Sinistro Pede posteriori Leonis fuit* An Occultation of a Fix'd Star by the Moon, *1^h 19' 34".* Immersionis plaga fuit *juxta Finem Sebiciardi versus Phocilidem* in Selenographia *Riccioli.* Feb. 29. (st. n.) 1676. at Paris; by M. Cassini. n. 123. p. 564.

Emergio vero fuit 1^h 16' 40". in equali à recta distantia à *Vendelino & Petavio.*

Per puncta *Immerisionis & Emerisionis,* diligenter notata, ducta recta Linea Diametrum illi perpendicularem abscondit in ratione 6' 45", ad 26' 5".

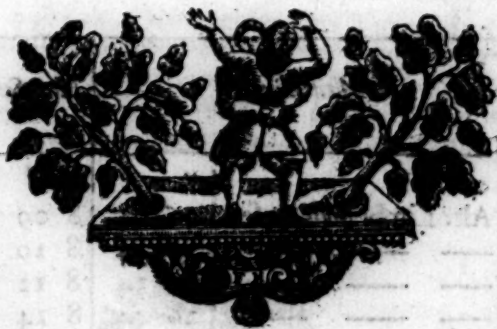
Fuit autem Diameter *Lune* ad Meridianum accedentis 32' 50".

H. 12. 29'. *Margo Lune Superior* fuit in eodem Parallelo cum Stella, quæ tunc præcedebat *Lunam* minuto horario 1' 50".

H. 12. 40' 18". Stella præcedebat Marginem Occidentalem *Lune* minutis horar. 2' 11". *Lune* Diameter pertransibat 2' 14".

H. 12. 52' 35". Stella præcedebat eundem Marginem 2' 25".

Altitudo Meridiana Limbi Inferioris *Lune* capta est 39° 25' 25".



LXI.

A Transit of
the Moon
above *Jupiter*,
Feb. 28. m.
167 $\frac{1}{2}$, at
Greenwich; by
Mr. Flamsteed.
Ib. p. 566.

| Hora Horologii
correcta. | | | Alt. & Distantia. | | |
|-----------------------------|----|----|--|----|----|
| h. | ' | " | | ' | " |
| 4 | 20 | 15 | $\mathcal{U}$ à Limbo <i>Lune</i> Lucido — | 26 | 9 |
| 4 | 47 | 0 | $\mathcal{D}$ à Capite <i>Diameter</i> — | 31 | 30 |
| 4 | 49 | 30 | $\mathcal{U}$ à Cuspide Proximo — | 26 | 28 |
| 4 | 52 | 15 | $\mathcal{U}$ Rectam per Cuspides ductam præterierat decim ^a
parte Distantiæ vel 3' circiter, oculari per Tubum
conjecturâ. | | |
| 4 | 56 | 0 | $\mathcal{U}$ à Cuspide — — | 27 | 33 |
| 5 | 1 | 15 | — à Recta per Cuspides — | 7 | 53 |
| 5 | 3 | 30 | — à Cuspide — — | 28 | 22 |
| 5 | 7 | 25 | — à Recta — — | 9 | 58 |
| 5 | 10 | 50 | — ab eadem — — | 11 | 55 |
| 5 | 15 | 50 | — à Cuspide — — | 30 | 27 |
| 5 | 21 | 20 | — à Limbo Remotiori dub. — | 62 | 4 |
| 5 | 26 | 0 | — à Cuspide Proximo — | 33 | 0 |
| 5 | 31 | 25 | — à Recta per Cuspides — | 20 | 9 |
| 5 | 37 | 0 | — à Cuspide — — | 36 | 15 |
| 5 | 41 | 10 | — $\mathcal{D}$ à Altæ 10 $\frac{1}{2}$ gr. <i>Diameter</i>
circ. — — | 31 | 53 |
| 5 | 48 | 30 | Differentia Altit. Limbi $\mathcal{D}$ à
Inferioris & $\mathcal{U}$ — — | 23 | 1 |
| 5 | 52 | 40 | $\mathcal{U}$ à Cuspide Proximo aberat. | 41 | 40 |
| 6 | 9 | 40 | — à Cuspide dub. | 47 | 29 |

An Occultati-
on of *Mars* by
the Moon,
Aug. 21. 1676.
at Greenwich;
by Mr. Flam-
steed. n. 129.
p. 723.

LXII. 1. Aug. 21. A. 1676. ante meridiem pro correctione Horologii has
Limbi Solaris Altitudines acceperam.

| Hora Ho-
rologii. | | | Altitudines. | Hor. Supp. | | Hor. Err. | |
|----------------------|----|----|--------------------------------|------------|------------------|-----------|---|
| h. | ' | " | | h. | ' | " | ' |
| 8 | 04 | 31 | Alt. Limbi <i>Solis</i> Infer. | 26 | 04 | | |
| 8 | 5 | 42 | — — — | 26 | 14 | | |
| 8 | 7 | 58 | — — — | 26 | 34 | | |
| 8 | 9 | 10 | — — — | 26 | 44 $\frac{1}{2}$ | | |
| 8 | 10 | 15 | — — — | 26 | 54 | | |
| 8 | 17 | 15 | — — — | 27 | 54 | | |
| 8 | 09 | 26 | † | 4 | 55 | | |
| 8 | 10 | 35 | † | 4 | 53 | | |
| 8 | 12 | 53 | † | 4 | 55 | | |
| 8 | 14 | 03 | † | 4 | 53 | | |
| 8 | 15 | 12 | † | 4 | 57 | | |
| 8 | 22 | 09 | † | 4 | 54 | | |

Deinde post Meridiem, cælo serenissimo.

Hor.

| Hor. Horol. | | | Correcta. | | | Distantia. | | |
|-------------|---|---|-----------|---|---|--------------------------------------|-------|-------|
| h. | ' | " | h. | ' | " | | | |
| 10 45 03 | | | 10 49 58 | | | Mars à Limbo Lucido <i>Lunæ</i> — | 5125= | 42 08 |
| 11 06 11 | | | 11 11 05 | | | Eadem Distantia — — — | 3829= | 31 29 |
| 11 20 00 | | | 11 24 55 | | | Iterum — — — — — | 3007= | 24 44 |
| 11 35 57 | | | 11 40 52 | | | Denuo — — — — — | 1982= | 16 18 |
| 11 57 31 | | | 12 02 26 | | | ♂ Z. five Diff. Alt. Limb. Inf. ♂ | 1912= | 07 35 |
| | | | | | | Jamq; tubo ped. 16 ♂ à Limbo | 1158= | 5 47 |
| 12 05 00 | | | 12 09 55 | | | Planeta nudis oculis diutius con- | | |
| | | | | | | spici non potuit. — — — | | |
| 12 9 44 | | | 12 14 39 | | | ♂ Lux cum lumine <i>Lunæ</i> con- | | |
| | | | | | | fusa. ♂ Z. — — — — — | 1185= | 9 44 |
| 12 10 03 | | | 12 14 58 | | | ♂ penitus Teet. Cusp. Boreo. | 3475= | 17 20 |
| 12 18 38 | | | 12 23 33 | | | 414 ♂ in Recta per Cuspides | | |
| | | | | | | ducta apparuit — — — | | |
| 12 20 36 | | | 12 25 31 | | | 414 ♂ à Limbo vel Cusp. Tu- | | |
| | | | | | | bo breviori — — — — — | 3912= | 32 10 |
| 12 24 58 | | | 12 29 53 | | | 414 ♂ à Cuspide iterum eodem | | |
| | | | | | | Tubo — — — — — | 3935= | 32 21 |
| 12 46 00 | | | 12 50 55 | | | <i>Lunæ</i> Diameter longiori tubo — | 5971= | 29 47 |
| 1 04 30 | | | 1 09 25 | | | Iterum eodem tubo. | 5973= | 29 48 |
| 1 10 56 | | | 1 10 51 | | | ♂ Emerf. forfan 4" vel 5" citius | | |
| 1 13 29 | | | 1 18 24 | | | ♂ à Cuspide Boreo — — — | 3675= | 18 20 |
| 1 18 15 | | | 1 23 10 | | | Eadem Distantia — — — — — | 4035= | 20 08 |
| 1 22 00 | | | 1 26 55 | | | <i>Lunæ</i> Altæ 23° Tubo longiori | | |
| | | | | | | Diameter — — — — — | 5988= | 29 55 |
| 1 39 00 | | | 1 39 55 | | | <i>Lunæ</i> Diameter breviori Tubo — | 3645= | 29 58 |

414 ♂ secundum *Tychonem* Locus nunc est ♂ 17° 58'. Latitudo 1° 20' Australis; unde cum *Lunæ* tum *Martis* Locus accurate deduci potest.



Temp.

2.
At Oxford; by
Mr. Halley.
Ib. p. 724.

| Temp. Cor. | | Distant. | |
|------------|--|----------|-------|
| h. m. s. | | | |
| 11 43 30 | The Center of <i>Mars</i> from the nearest Limb of the <i>Moon</i> | 719" = | 12 40 |
| 11 49 2 | Again | 571" = | 10 3 |
| 11 54 48 | Again | 409" = | 7 12 |
| 12 3 25 | The Center of <i>Mars</i> from the North Cusp of γ . | 1118" = | 19 41 |
| 12 10 28 | The Gibbous part of <i>Mars</i> touched the <i>Moon's</i> Limb. | | |
| 12 10 42 | <i>Mars</i> was wholly covered, being distant from the Cusp | 963" = | 17 14 |
| 1 10 41 | <i>Mars</i> did Emerge. I suppose, his Center. | | |
| 1 12 45 | <i>Mars</i> was distant from the Northern Horn of γ | 1018" = | 71 55 |
| 1 31 10 | <i>Mars</i> passed over a Point noted in the Telescope. | | |
| 1 33 15 | The Southern Limb of <i>Aetna</i> passed by the same Point. | | |
| 1 34 00 | The Lucid Limb passed over the same Point. | | |
| 1 32 35 | The <i>Moon's</i> Diam. observed $1692 = 30' 1''$.
Alt. γ 131° circ. | | |
| 1 57 52 | <i>Mars</i> from the Northern Horn of the <i>Moon</i> . | 2042" = | 36 5 |
| 2 2 53 | <i>Mars</i> from the Southern Horn of the <i>Moon</i> . | 2266" = | 40 3 |

3.
At Danzick;
by M. Hevelius.
Ib. p. 721.

| Temp. Sec.
Horol. Oscil. | Fixarum
Nomen. | Aleis-
dines. | Tempus ex
alt. corr. | Animadvertenda. |
|-----------------------------|--------------------|------------------|-------------------------|--|
| h. m. s. | | | h. m. s. | |
| 1 1 25 | <i>Cauda Cygni</i> | 57 10 | 1 0 24 | |
| 1 9 45 | | | 1 8 45 | σ Distabat ferè tanto inter-
stitio à Limbo γ Lucido,
quanto M. <i>Porphyrites</i> à M.
<i>Aetna</i> removetur. |
| 1 36 39 | | | 1 33 42 | <i>Mars</i> à <i>Luna</i> omninò reclus. |
| 1 45 25 | <i>Cauda Cygni</i> | 51 47 | 1 44 7 | |
| 2 47 54 | | | 2 46 29 | <i>Mars</i> emicuit; Finis nempe
Occultationis. |
| 3 19 50 | <i>Scheat Peg.</i> | 45 3 | 3 18 19 | |

Mars obtectus est circa Montem *Audum*, incedens quasi per *Loca Luna Paludosa*, per *M. Aetnam*, infra Insulam *Beshicam*, supra Paludem *Acherusiam*, supra *M. Coracem*, per Paludem *Meotidem*, & paulò supra Insulam *Alopeciam*, & ipsum *Luna Centrum*; sicque rursus ad Lacum *Majorem Occidentalem* exiens.

Si quæras, unde viam itinerariam hanc adedò accuratè mihi determinare licuerit, & quidem ad partem *Lune Obscuram*, scias, eò evenisse, quod Tubis illis meis præcipuas Maculas majores in parte *Lune Umbrosâ* satis distinctè deprehendere potuerim; atque ita dilucidè conspexerim, *Martem* circa medium ferè *Paludis Meotidis* Emicuisse.

LXIII. Observavit *Bullialdus Initium*, Alto sup. Horiz. ad occasum *Capite An Occultation Andromeda* $18^{\circ} 11'$. unde datur à Meridie $7^h. 20'$ T. A. sed Med. $7^h. 29'$ of Saturn by $55''$. Finem vero vidit, Alta ad Occas. *Cinguli Androm. Australiori Mag.* $21^{\circ} 17'$. unde à Meridie colligitur T. A. $8^h. 30' 22''$. *Feb. 27. st. n. 1678, at Paris; by M. Bullialdus. n. 139. p. 969.*

Monere hic necessum est Tabulas *Philolaicas* h. Promotiorem in Longitudine ostendere, quam in Cœlo apparet, scrupulis primis ut minimum 19. ita ut h. tunc fuerit in Cœlo in $3^{\circ} 28'$. & Lat. Aust. $1^{\circ} 38'$.

In hac porrò Observatione adhibita Illust. Viri *Joh. Hevelii* *Lunaris Disci* descriptione, in illa Limbi parte, quæ in recta linea à medio Montis *Berosi* per Montes *Ripheos* ducta, paulò supra *Alanum* Montem, infra Terminos *Australes Paludum Hyperborearum*, sita est, Saturnum Emeruisse aspeximus.

LXIV. 1. Etiam si nunc per quinquaginta Annos (pro quo D. O. M. im- *An Occultation of Jupiter, Jun. 5. (st. n.) 1679. at Dantzick; by M. Hevelius. Ph. Coll. n. 1. p. 29.* mortales & debeo & habeo gratias) Observationibus Rerum Cœlestium operam dederim, non nisi tamen semel tantummodò *Foram à Luna* vidi obtectum, Anno nempe 1646. die 24 *Decembris, St. n. vesperti, Sole* scilicet existente sub Horizonte. Gratulor igitur mihi magnopere, quod hanc observationem, non solum Cœlo perquam sereno, sed etiam ex voto & quidem cum gratissimo meo Hospite, Clarif. & Doctissimo Domino *Edmundo Halleio* observare potuerim.

Lunam intraverit ad *M. Audum*, & quantum conjicere dabatur ex *Foris* Exitu, viam carpsit per *Loca Paludosa* Insulæ *Cercinæ*, supra Montem *Aetnam*, per Inf. *Beshicam*, per *Byzantinum*, Inf. *Apolloniam* & superiorem partem *Paludis Meotidis*; sic ut paulò supra *Centrum Lune* incesserit, *Luna* habenti aliqualem Latitudinem Austrinam. Deinde, quod rarissimum, *Jovis* apparentem Diametrum in hac observatione accuratissimè (ut mihi videor) dimensum. Memini quidem me aliquoties *Jovis* Diametrum per Maculas Lunares observasse, eandem videlicet ad $55''$. plus minus accedere; sed hac vice diameter *Jovis* longè extitit minor. Cognitâ enim totâ Duratione hujus Occultationis $58' 10''$, atque datâ simul Diametro *Lunari* $32' 40''$, protinus innotescit, ex illa temporis Mora, cum scilicet primum *Jupiter* Limbo suo *Luna* Limbum attingeret, & cum rursus Occultaretur (id quod factum est spatio $55''$) Diameter *Jovis*, $30'' 53''$.

| Or. Observ. | Temp. juxta
Horol. Oscil. | | | Altitudines Fixarum & Solis, cum Distantiis
Jovis à Luna Limbo. | | | | | | | Temp. ex
Alt. corr. | | | |
|-------------|------------------------------|----|----|--|---|---|---|----|----|----|------------------------|----|----|----|
| | h. | ' | " | | | | ° | ' | " | | h. | ' | " | |
| | 1 | 18 | 55 | Altit. Capitis <i>Andromedæ</i> | — | — | 1 | 24 | 52 | 0 | 1 | 20 | 54 | |
| | 1 | 29 | 0 | Altit. <i>Arcturi</i> | — | — | — | 1 | 31 | 3 | 00 | 1 | 31 | 24 |
| 1 | 1 | 52 | 0 | <i>Luna</i> Oriebatur | — | — | — | — | — | — | 1 | 54 | 25 | |
| 2 | 3 | 26 | 0 | <i>Sol</i> Oritur | — | — | — | — | — | — | 3 | 28 | 25 | |
| 3 | 3 | 33 | 0 | Dist. <i>Jovis</i> à Limbo <i>Lunæ</i> æquabat fere Dia- | | | | | | | | | | |
| | | | | metrum <i>Lunarem</i> | — | — | — | — | — | — | 3 | 35 | 0 | |
| 4 | 3 | 41 | 0 | <i>Jupiter</i> à <i>Luna</i> Limbo in tanta dist. aberat | | | | | | | | | | |
| | | | | quanta est dist. <i>M. Porph.</i> à Mont. <i>Sinai</i> | | | | | | | | | | |
| 5 | 3 | 58 | 0 | <i>Jupiter</i> à Limbo <i>Lunæ</i> Orient. ut <i>M. Ætna</i> à | | | | | | | | | | |
| | | | | <i>Palude Maraotide.</i> | — | — | — | — | — | — | 4 | 00 | 0 | |
| 6 | 4 | 13 | 50 | <i>Jupiter</i> distabat à Limbo > duab. Diam. 4— | | | | | | | | | | |
| 7 | 4 | 14 | 40 | <i>Jupiter</i> ad unicam Diam. <i>Jovis</i> removebatur— | | | | | | | | | | |
| 8 | 4 | 15 | 40 | <i>Jupiter</i> stringebat <i>Lunæ</i> Limbum Orient.— | | | | | | | | | | |
| 9 | 4 | 16 | 9 | <i>Jupiter</i> dimidiâ parte Occultabatur — | | | | | | | | | | |
| 10 | 4 | 16 | 39 | <i>Jupiter</i> Totus omnino Tectus — | | | | | | | | | | |
| 11 | 5 | 14 | 0 | <i>Jup.</i> nunc notabili particula prodire videbat— | | | | | | | | | | |
| 12 | 5 | 14 | 20 | Dimidius <i>Jupiter</i> jam exiverat — | | | | | | | | | | |
| 13 | 5 | 14 | 45 | Totus <i>Jupiter</i> omnino prodierat — | | | | | | | | | | |
| | 10 | 22 | 30 | Altitudo ☉ — | — | — | — | 53 | 34 | 44 | 10 | 25 | 2 | |
| | 10 | 27 | 16 | Altitudo ☉ — | — | — | — | 53 | 59 | 45 | 10 | 29 | 42 | |
| | 10 | 30 | 8 | Altitudo ☉ — | — | — | — | 54 | 14 | 0 | 10 | 32 | 26 | |
| | 10 | 38 | 0 | Altitudo ☉ — | — | — | — | 54 | 53 | 40 | 10 | 40 | 26 | |
| | 10 | 45 | 28 | Altitudo ☉ — | — | — | — | 55 | 27 | 20 | 10 | 47 | 46 | |

At Paris; by
M. Cassini.
ib. p. 33.

2. At 3^h 0' 11". the first *Satellite* was hid by the East Limb of the *Moon*. At 3^h 2' 0" the East-side of the *Moon* touched the West-side of *Jupiter*; then I took the Height of *Jupiter*, which was 8° 01'. at 3^h 2' 51". At 3^h 2' 57". *Jupiter* was intirely hid by the *Moon*. It entered at equal Distance from the two Spots *Grimaldi* and *Aristarchus*; the last of which was in the Section of the *Moon*, which distinguished the Light from the Dark part. At 3^h 5' 1". the Second *Satellite* was hid by the East-side of the *Moon*. At 3^h 5' 48". the third *Satellite* was hid. At 3^h 56'. we perceived by the Eye that *Jupiter* was parted from the obscure Side of the *Moon*.

M. De la Hire took the Height of *Jupiter* two Minutes after parting, and found it, 17° 17'.

Temp.

LXV.

An Occultation
of the Bull's
Eye, at Green-
wich, Sept. 4.
1680. by Mr.
Flamsteed.
Ph. Coll. n. 4.
p. 99.

| Temp. Corr.
mane. | Observat. | | | |
|----------------------|--|------|---------|-------|
| h. ' " | | | ' " | |
| 2 49 52 | <i>Lunæ</i> Cuspis Austrina à <i>Palilicio</i> — | 5300 | = 26 26 | |
| 2 53 10 | <i>Palilicium</i> à Limbo Proximo — | 1028 | = 05 08 | |
| 2 56 54 | à Cuspide dicta — | 4830 | = 24 06 | |
| 3 01 16 | — rep. — | 4370 | = 21 48 | |
| 3 05 42 | <i>Immersio</i> , à Cuspide — | 4049 | = 20 12 | vel |
| 4 08 20 | <i>Lunæ</i> Diameter — | 6595 | = 32 54 | paulo |
| | ab Amanuensii repetita. — | 6588 | = 32 52 | minus |
| 4 14 02 | <i>Palilicium</i> Emerferat. | | | |
| 4 15 12 | distitit à Cuspide Aust. — | 3411 | = 17 01 | |
| 4 18 52 | rep. — | 3725 | = 18 35 | |
| 4 22 00 | iter. — | 3980 | = 19 51 | |
| 4 23 52 | denuo — | 4098 | = 20 56 | |

3^h 05' 40". Limbo *Lunæ* Lucido Fixa videbatur adhærere, & post duo scrupula secunda Temporis, nihil in Limbo apparuit. Locus *Immerſionis* erat juxta Australissimam revera trium Macularum parvularum medio jacentium inter Paludem *Mareotida* & Montem *Climacem*.

4^h 13' 45" Fixa non *Emerſerat*; tunc, vel paulo postea, nescio qua occasione amovi oculum à Telescopio, at cum iterum adhibui 4^h 14' 2". *Emerſam* vidi & plena Luce effulgentem.

| h. ' " | | | ° ' " |
|---------|--|----------|-------|
| 4 53 20 | <i>Lunæ</i> Limb. Prox. à Pede Lucido <i>Orionis</i> — | 26 12 25 | |
| 4 56 20 | Pes <i>Orionis</i> Lucidus à <i>Palilicio</i> — | 26 29 25 | |
| 4 58 20 | <i>Lunæ</i> Limbus Prox. à Pede <i>Orionis</i> iter. — | 26 12 00 | |
| 5 04 02 | <i>Lunæ</i> Limbus Prox. à <i>Polluce</i> — | 44 09 10 | |
| 5 06 33 | — rep. — | 44 08 15 | |
| 5 11 55 | Lucidus Pes <i>Orionis</i> à <i>Palilicio</i> — | 26 29 40 | |

LXVI.

An Occultation
of the Bull's
Eye at Green-
wich, Oct. 28.
1680; by Mr.
Flamsteed.
Ph. Coll. n. 5.
p. 100.

| Temp. Corr. | Observat. | | | |
|-------------|------------------------------------|------|---------|--|
| h. ' " | | | ' " | |
| 7 11 52 | <i>Lunæ</i> Diameter — | 6745 | = 33 39 | |
| 7 16 26 | — rep. — | 6744 | = 33 39 | |
| 7 19 46 | <i>Palilicium</i> à Limbo Lucido — | 5895 | = 29 24 | |
| 7 24 44 | — rep. — | 5095 | = 25 25 | |
| 7 40 14 | — — — | 3203 | = 15 59 | |
| 7 51 22 | — — — | 1817 | = 09 04 | |

Z z z

Atin-

2. Mr. Benj. Harry in Ballasore Road Observed that the Moon past to the Northward of the Bull's Eye, and that the Star and the Moon's under-Limb were in equal Altitude when they were both $13^{\circ} 45'$ high to the West, which gives the time $14^h 49'$ and when the South Horn of Taurus was $23^{\circ} 30'$ high, which makes the Time $15^h 13'$ the Western Limb of the Moon was in a Line with Capella and the Bull's Eye.

3. The Correct Time of the Immersion was $6^h 18' 22''$, and the Emer- sion at $7^h 19' 46''$. At Ballasore;
by Mr. Ben.
Harry. Ph. Col.
n. 5. p. 125.

LXVIII. Sept. 27. ft. n. 1682. hora 3. m. Lunam cum tres reliques Planetas nudo quidem conspexi Oculo; sed Luna eo tempore adhuc ad 7. circiter Gradus removebatur, s. s. s. Occasum versus.

Quantum autem ex Inclinatione Cornuum Luna quoad Planetarum ductum colligere licuit, protinus prospiciebam, nullas fore Occultationes, sed tantum Transitus; sic ut Luna infra illos Superiores Planetas incederet. In qua Opinio magis magisque etiam sum confirmatus: Cum Die subsequente, 28 Sept. m. nec Regulus fuerit à Luna Tectus, quæ Stella ratione utriusque Latitudinis potius Occultari debuisset. Regulus namque in ipsa Conjunctione, $4^h 6'$. Distabat à Superiori Cornu Boream versus adhuc $31' 17''$. id quod optimo Micrometro Tuboque egregio, accuratè observatum est; adeo ut nulla prorsus fuerit Occultatio Reguli, sed tantummodo Luna Transitus. Ita pariter accedit die 25 Octob. Nam Jupiter & Saturnus nec non & Die 26. Octob. ft. n. minime fuerunt à Luna Obtekti; sed Luna satis longè infra Planetas incessit.

A Transit of the
Moon below
the 3 Superior
Planets, and
Regulus. 1682.
at Dantzick;
by M. Heveli-
us. n. 143.
p. 17. n. 151.
p. 325.
n. 143. p. 18.
min. 31. sec. 17.
n. 151. p. 326.
min. 31. sec. 55.

LXIX. Feb. 11. ft. n. 1683. h. 9. cum primum Luna in Oculos incurret, Regulus satis longè Occasum versus removebatur; ita ut ipsa Conjunction (quantum ruditer colligere dabatur) Oriente circiter Luna, scil. 5^h vel 6^h contigerit: Utrum autem Regulus omnino Tectus sit, an vero tantummodo Transitus fuerit, haud adeo accurate deprehendere licuit.

An Occultation
of Regulus by
the Moon,
Feb. 11. ft. n.
1683. at Dant-
zick; by M.
Hevelius.
n. 151. p. 331.



LXX.

An Occultation
of Two Fixt
Stars by the
Moon, and a
Transit above a
Third, April 2.
st. n. 1683. at
Dantzick; by
M. Hevelius.
ib.

| Temp. Sec.
Horol. Am-
bul. | | Dist. & Al-
titud. |
|----------------------------------|--|-----------------------|
| h. / " | | |
| 9 53 30 | Initium Occultationis Stellulæ
Majoris A. 5. Mag. | |
| 10 08 30 | Conjunctio <i>Lunæ</i> & Stellulæ
C, distabat à <i>Lunæ</i> Cornu
Inferiori. — — — | 0 / "
0 04 00 |
| 10 29 36 | Initium Occultationis Stellulæ
B, 6. Magn. | |
| 10 52 50 | Finis Occultationis Stellulæ
A. | |
| 11 45 30 | Altitudo <i>Lyre</i> — — — | 31 44 00 |
| 11 46 30 | Altitudo eadem — — — | 31 55 00 |
| 11 47 50 | Denuo — — — — | 32 06 00 |

Sectio Luminis & Umbræ hac Die per Montes *Serrorum* & *Carphatos*, per *Sinum Peronticum*, inter *Byzantium* & *Inf. Cyaneam*, per M. *Amanum*, *Taurum*, *Urijque Montes* incidit.

Prior Stellula A, in Catalogo *Tychonico* non invenitur; sed in meo *Novo* Vocatur sub Cornu *Tauri* Sequens 5^a magn. Versatur hoc tempore in Π 19° 11' 35". & in Lat. 4° 43' 44". Austr. Altera vero B, quantum ex hac observatione colligere potui, existit in Π 19° 17' 00" & in Latit. 4° 47' 0". Austr. At tertia C, quæ forte nudis oculis non conspicitur, degit modo in Π 19° 9' 0". & in Latit. 5° 2' 0" Austr. Cæterum Stella A, *Lunam* subingressa est ad montem *Audum*, transiit per *Insulam Cercinnam*, per M. *Neptunum*, Mare *Adriaticum*, inter M. *Horminium* & M. *Amanum*, per M. *Herculis*; sic ut inter *Paludem Marotidem* & *Inf. Majorem Caspii* rursus Emerferit: unde liquidum est hanc Stellulam A, fere Centralem cum *Luna* celebrasse *Conjunctionem*.

Altera vero Stella B, 6^a Magn. *Lunam* ingressa est ad *Paludem Marotidem*, Transiit per *Sinum Syriticum*, ad M. *Athos*, per M. *Latium*, inter Montes *Sipylum* & *Macysitum*, infra *Centrum Lunæ*, per Superiorem M. *Moschum*, per *Fretum Ponticum*, atque sic infra *Insulam Majorem Caspii*.

LXXI. Maii 2. ft. n. 1683. 11h 0' 0'' vesp. Luna supra Stellulam in radice Caudæ *Canceri* transibat, quæ modo versatur in $\odot$ 27° 53' 37''. & in Lat. 2° 18' 42''. Austr. sic ut in ipsa Coniunctione non nisi ad 12' à *Cornu* Infer. abesset.

12h 0' 0''. etiam alia Fixa, sed minutissima, tecta est, quæ in Catalogo alias non habetur. Quantum conijcere dabatur, hærebat in $\odot$ 28° 30' & in Latit. 1° 54' Austr.

An Occultation
of a Fixed Star,
and a Transit
above another,
May 2. ft. n.
1683. at Dant-
zick; by M.
Hevelius. ib.

LXXII. Ipsum Momentum *Immerſionis* accuratissime notavi, id quod incidit 11h 17' 20''. vesp. secundum Horolog. Ambulat. Linea itineraria incessit per Mare *Pamphilium*, infra Insulam *Carpachos*, Inf. *Cyprum*, infra Sinum Extremum *Pontii*, & Sinum Inferiorem Maris *Caspii*. 11h 24' 42'' secundum Horol. Ambul. Altitudo Lucidæ *Lyre* Observata est 44° 39' 0''; ex qua Initium Occultationis corrigi potest. Sectio Luminis & Umbræ per *Lacum Nigrum Majorem*, ad Inf. *Corſicam*, M. *Nyconium*, per *Lacum Strymonicum*, ad Inf. *Rhodus*; per M. *Sinai* & M. *Techisandam* incidit.

An Occultation
of Regulus by
the Moon,
May 4. ft. n.
1683. at Dant-
zick; by M.
Hevelius. ib.

LXXIII. 1. At 9h 26' the under Limb of the Moon was just Risen, and soon after *Jupiter* appeared near the Eastern Limb of the Moon, within a few Minutes of being Eclipsed.

An Occultation
of Jupiter by
the Moon,
March 31. 1686.
at London; by
Dr. Hook, and
M. Halley.
n. 181. p. 85.

9h 33' as near as could be guessed, was the Time of the Central Immerſion, which was very difficult to be observed by reason of the Asperity of the Moon's Limb, which Undulated and Sparkled very much, as it appeared through the Vapours near the Horizon: The Ingreſs happened much about the length of the Spot, called by *Hevelius*, *Palus Maraotis*, to the North of the said Spot, or about the 124th Degree of the outer Limb of his *Selenography*, nearly in the same Latitude with the Moon's Center.

10h. 30'. The Western Edge of *Jupiter* began to Emerge out of the dark Limb of the Moon.

10h 31' 20''. The whole Disk of *Jupiter* was entire, so that he was about a Minute and a third in coming out from behind the Moon.

The Emerſion was exactly in a right Line with the Moon's Center and the Northern part of *Palus Meotis*, or about the 324th Degree of the Inner Limb of the *Selenographick Table* of *Hevelius*.



2.
At Greenwich;
by Mr. Flam-
steed. n. 184.
p. 206.

| Temp. Corr.
per ho. Oscil. | | | | |
|-------------------------------|----|----|--|-------------|
| h. | ' | " | | |
| 9 | 32 | 30 | ♃'s Limbus tangebatur Limbum <i>Luna</i> Lucidum, <i>Ma-</i>
<i>raotidis</i> Diametrum à termino ejus Boreo. | |
| 9 | 33 | 42 | ♃ Totus tectus erat. Quantum per vapores Hori-
zontis, & undulationem Limbi valde turbidam, li-
cuit conjicere. | |
| 10 | 11 | 12 | Differentia Declinationum Limbi <i>Luna</i> verè Auftri-
ni & loci <i>Ingressus</i> , Tubo ped. 8 & <i>Micrometro</i> | ' " |
| | | | 1546= | 12 42 |
| 10 | 30 | 30 | <i>Jovis</i> particula <i>Emerferat</i> è regione Borealis Limbi
<i>Meotidis</i> | |
| 10 | 31 | 36 | <i>Jupiter</i> Totus liber. | |
| 10 | 35 | 50 | Differentia Declinationum Centri <i>Jovis</i> & Limbi
<i>Luna</i> Austrini | |
| | | | 3436= | 28 15 |
| 10 | 41 | 40 | Repet. | 3585= 29 28 |
| 10 | 44 | 00 | <i>Luna</i> Diameter. | 3906= 32 07 |
| 10 | 45 | 44 | Repet. | 3915= 32 11 |

At Nurem-
burg; by
M. Zimmer-
man. n. 183.
p. 177.

3. At 10^h 19' 56". M. I. *Ja. Zimmerman* observed the first Contact of the Limbs of ♃ and the ☿, and at 10^h 20' 47". ♃ was all Eclipsed.

At 11^h 22' 51". ♃ was wholly Clear from the Eclipse.

The Immersion was about the 117th, the Emergence at the 321st. Degree of the Limb, in the Chart of *Hevelius*.

At 11^h 31' 06". The third Satellite of ♃ Emerged. These times were collected from the Culminations of fixt Stars, and the Vibrations of a Pendulum.

By M. Wurt-
zelbauer. ib.

4. At 10^h 20' 50". ♃ applied to the Limb of the ☿, over-against the *Loca Paludosa Insula Cercinæ*.

At 10^h 22' 00". he appeared about half Eclipsed.

At 10^h 22' 30". he was wholly Hid.

At 11^h 19' 40". ♃ began to Emerge.

At 11^h 21' 20". he was quite Free from the Interposition of the ☿. The point of the Emergence was somewhat to the North of the *Palus Meotis*. No Spot in the ☿ was so near the apparent Magnitude of ♃'s Disk at the *Insula Besbicus Hevelii*.

At 11^h 40' 00". the Altitude of *Procyon* was 8° 37'. whence the Pendulum Clock, which had been set by Altitudes of the ☉, the afternoon preceeding, may be examined.

5. Etiamfi hucusque per 56 Annos nullam Observationem alicujus Mo-
menti neglexerim, non nisi tres *Jovis* Eclipses rite deprehendere & annotare
potuerim: Utpote primam Anno 1646, die 24 Decemb. vesperti, sed tantum-
modo ejus *Finem*; Secundam Anno 1679, die 5 Junii ante meridiem de die,
quo tempore res omnis feliciter successit; Tertiam hoc Anno corrente 1686,
die 10 April. vesperti.

Inter alia autem notandum occurrit, quod hæcce *Occultatio* non *Luna* om-
nino existente *Plena*, sed altera die circiter post ipsum *Plenilunium* vesperti ac-
ciderit; & quidem eodem Tempore (quod permirum sanè accidit, & est ca-
sus, qualis haud facile unquam continger) eademque facie, ut illa *Occultatio*
Anni 1646, die 24 Decemb. vesperti visa est, quo Tempore *Luna* jam ad bi-
dium pariter decreverat, & sine dubio eandem *Librationem* etiam exhibuit,
quam in hac nostra ultima *Observatione*. Nam Sectio Luminis atque Umbrae
plane fuit eadem, & per easdem Maculas transiit (quod satis admirari nequeo)
nimirum ad *Lacum Hyperboreum Majorem* & *Minorem*, tum ad *Montes Riphaeos*,
per *Paludem Meotidem*, per *Lacum Majorem Maris Caspii* & *Sinum* ejus *Infe-*
riorem, ad *Montem Nerosum*.

E contrario, *Jovis Occultatio* Anno 1679 à me habita, plane extitit diversa,
siquidem illa non circa *Plenilunium*, sed *Novilunium* accidit, tertia circiter die
ante *Conjunctionem* ipsam.

| Horol. am-
bular. | | Altit. Qua-
drant. capte. | Tempus ex
Ali. Cor. |
|----------------------|--|------------------------------|------------------------|
| h. ' " | | Gr. ' " | H. ' " |
| 5 10 10 | Altitudo Solis. | 13 47 0 | 5 11 43 |
| 5 12 30 | Altitudo Solis. | 13 28 0 | 5 13 55 |
| 5 17 40 | Altitudo Solis. | 12 41 0 | 5 19 21 |
| 5 23 50 | Altitudo Solis. | 11 46 0 | 5 25 43 |
| 8 7 10 | Altitudo Arcturi. | 29 55 0 | 8 12 50 |
| 8 11 15 | Altitudo Arcturi. | 30 32 0 | 8 17 4 |
| 8 15 10 | Altitudo Arcturi. | 30 59 0 | 8 20 51 |
| | <i>Luna</i> oritur circit. | | 9 24 0 |
| 9 44 50 | <i>Jupiter</i> distabat ab Inf. <i>Cercinna</i> 43. circit. mi-
nut. | | 9 52 50 |
| 10 21 30 | <i>Jovis</i> distantia erat tanta, quanta distantia M. Si-
nai à <i>Palude Maeotide</i> . | | 10 31 30 |
| 10 40 35 | <i>Jovis</i> distantia erat ferè æqualis distantiae inter M.
<i>Ætnam</i> & M. <i>Porphyritem</i> . | | 10 51 5 |

| h. | ' | " | | h. | ' | " |
|----|----|----|--|------------|----|-------|
| 10 | 51 | 30 | <i>Jovis</i> Limbus à <i>☽</i> limbo distabat tanto interstitio, quanto <i>Pal. Marcoris</i> à <i>Limbo Luna.</i> | 11 | 2 | 0 |
| 10 | 56 | 9 | <i>☿</i> Limbo suo Tangere incipiebat <i>Luna</i> Limbum, atque sic <i>Initium Occultationis</i> accidit. | 11 | 7 | 9 |
| 10 | 56 | 54 | Dimidius <i>Jupiter</i> Occultabatur. | 11 | 7 | 54 |
| 10 | 57 | 39 | Totus <i>Jupiter</i> omnino à <i>☽</i> Tectus. | 11 | 8 | 39 |
| 11 | 8 | 31 | Occultatio <i>Comitis Jovis</i> ultimi ad <i>M. Alabastrinum</i> accidit.
Duo tantummodo <i>Comites</i> à parte Orientali conspecti sunt. | | | |
| 11 | 15 | 54 | Altitudo <i>Lyrae</i> | 32° 59' 0" | 11 | 26 0 |
| 11 | 19 | 0 | Insula <i>Besbica</i> & <i>Rhodus</i> reperiuntur sub eodem perpendiculo; id quod ad 35 gr. circ. à Linea <i>☽</i> verticali removebatur. | | | |
| 11 | 21 | 37 | Altitudo <i>Lyrae.</i> | 33 50 0 | 11 | 32 15 |
| 11 | 24 | 57 | Altitudo <i>Lyrae.</i> | 34 24 0 | 11 | 36 24 |
| 11 | 38 | 15 | <i>Emersionis</i> Initium <i>Jovis.</i> | | 11 | 49 15 |
| 11 | 39 | 0 | Dimidius <i>Jupiter</i> Emergebat. | | 11 | 50 0 |
| 11 | 39 | 45 | Totus <i>Jupiter</i> apparebat.
Diameter <i>Luna</i> Micrometro observata erat 31' 0". | | 11 | 50 45 |
| 11 | 54 | 10 | Distantia <i>Jovis</i> à confinio <i>Lucis</i> & <i>Umbrae</i> erat æqualis distantia <i>M. Aetna</i> à <i>M. Porphyrite.</i> | 12 | 5 | 40 |
| 11 | 57 | 20 | Distantia <i>Jovis</i> à Confinitio <i>Lucis</i> & <i>Umbrae</i> elongabatur intervallo inter Insulam <i>Besbica</i> & <i>M. Aetnam.</i> Et <i>Comes ☿</i> Remotissimus à <i>Fove</i> tantum aberat, quantum ipse <i>Comes</i> à dicto Confinitio <i>Lucis</i> | 12 | 9 | 20 |
| 12 | 6 | 9 | Altitudo <i>Lyrae.</i> | 40 19 0 | 12 | 18 39 |
| 12 | 9 | 18 | Eadem Altitudo denuo. | 40 46 0 | 12 | 21 40 |
| 12 | 13 | 20 | Altitudo <i>Luna.</i> | 16 15 0 | | |

Primo liquidum est ex ipsa Observatione, quod *Orbita*, seu Linea *Jovis* Itineraria, per *Montem Alabastrinum*, per *M. Christi*, *M. Carpathes*, infra *M. Macroceronios*, & per *Lacum Hyperboreum Inferiorem* incesse- rit. Secundo quod Insula *Besbica* & Insula *Rhodus* sub uno eodemque perpendiculo, tempore *Occultationis*, circiter 11^h 30' extiterit; sic ut 35 gradus *Luna* Limbi culminaverit. Intravit itaque *Jupiter* Limbum *Luna* Illuminatum circa 61 gra- dum,

dum, à Linea scil. perpendiculari Nonagesimi atque puncto Zenith, Ortum versus; Exivit vero circa 31 gradum à dicta Linea Perpendiculari Nonagesimi occasum versus, ad Limbum *Lunæ* obscuratum. Proinde linea *Jovis* itineraria fuit subtensa 104. ferè graduum, attenta videlicet parte *Lunæ* Boreali.

Præterea etiam maximè notatu dignum, quod ex hac observatione Diametrum *Jovis* exquisitè elicere potuerim; nimirum 51'' 42''; & tantæ magnitudinis extitit etiam Diameter *Jovis* 50'' circ. quoties illam per *Maculas Lunæ* dimensus sum. Quod autem Anno 1679. die 5 Jun. cum similem *Jovis* Eclipsim observarem, longè ea extitit minor, nimirum tantum 30'' 53''. id ex eo evenisse puto, quod observatio illa, tempore Diurno, Splendente *Sole* fuerit observata; quo Radii Stellarum & Planetarum adventitii magis à Luce *Solis* abstergantur, quam Tempore Nocturno, nocte obscura. Quod si autem quæras, quamnam Diametrum apparentem veriore existimem? Scias illam, quam Anno 1679. 5 Jun. de die, *Sole* splendente observavi. Non equidem ex eo quod non æquè diligenter hanc quam illam determinaverim; sed quod tempore Nocturno Radii Adventitii magis obstant, sicuti diximus, quam tempore Diurno.

6. At 9^h 31' 6''. *Jupiter* was in a Perpendicular falling on the Limb of the *Moon* over-against the Northern-part of the Spot *Grimaldi*, (*Maræotis*) near to *Riccioli* (*Stag. Miris*) and distant from the Limb about 4 times as much as the said Spot. At Paris; by Mr. Cassini. 1713. p. 175.

9^h 40' 21''. *Jupiter* touched the Circumference of the *Moon*, which undulated by reason of the Vapours near the *Horizon*.

9^h 41' 20''. He quite disappeared in the Inequalities of the *Moon's* Limb, the *Total Immersion* might be some Seconds later. So the *Central Immersion* was at 9^h 40' 51''. *Jupiter* entered over-against that Part of *Grimaldi* next *Riccioli*.

10^h 30' 2''. The *Outermost Satellite* which preceeded *Jupiter* appeared over against the Middle of the *Caspian Spot* (*Pal. Meotis*) through which the Section of *Light and Darkness* passed, and made nearly an Equilateral Triangle, with the Extremities of that Spot.

10^h 40' 24''. The first Limb of *Jupiter* began to come out of the Dark side of the *Moon*, over-against the North-part of the *Caspian Spot*, about *Cleomedes*, (*ad Montes Rhipæos*.)

10^h 40' 56''. The Center of *Jupiter* did Emerge. It was difficult to distinguish the Moment when *Jupiter's* Disk was fully clear, but at 10^h 41' 36''. the Eclipse was certainly past.

At the Emerision of the Center, the Altitude of *Jupiter* was 11° 31'.

At 10^h 42' 49''. The *Second Satellite* being the nearest of the three that followed the Planet, Emerged.

At 10^h 45' 1''. The *Innermost Satellite*, being near its greatest Elongation, Emerged.

At 10^h 50' 40''. The *Third*, or *Penextimus Satelles*, being likewise near its greatest Elongation, began to appear over-against the Northern-Edge of the *Caspian Spot*,

At 11^h 45'. The Diameter of the Moon was 32' 27". and according to the Calculus of M. Cassini, her Parallax was 61 Min.

At Avignon;
by R. P. Bonfa.
ib. p. 176.

7. The Central Immersion was at 9^h 42' 13". and the Central Emerision at 10^h 45' 26'. over-against the Southern-part of the Caspian Spot.

An Occultation
of Jupiter by
the Moon,
Apr. 28. 1686.
n. 181. p. 87.

LXXIV. 1. The Immersion was seen at Torteridge (which Place is about 9 Miles from London, and nearly 25" of Time to the Westwards thereof) by Mr. Ed. Haines: who between a Gap of the Clouds observed the Contact of the Moon's Limb and Jupiter's, at 3^h. 3' $\frac{1}{2}$.

The Clouds closing again permitted him to observe no more: however, from this we may conclude the Central Immersion at London, to have been 3^h 4' $\frac{1}{2}$ *mane*.

The Emerision was observed at London, by Mr. Edm. Halley, to fall out at 3^h 49'. for at 3^h 50'. Jupiter was all out, and the Limbs so little separated, that he judged, that a Minute before, the Center of Jupiter had been upon the Moon's Edge: The Point of the Emerision was over-against the Southern-part of the Spot, call'd by Hevelius *Insula Macra*, or at the 34th Division of the Inner Limb of his Map of the Moon.

At Avignon;
by R. P. Bonfa.
n. 183. p. 177.

2. The Immersion of the Center happened at 3^h 37' 23". on the East-side of the Spot *Xenophanes*. The Emerision was at 4^h 28' 24". between *Seneca* and *Berosus*, according to Riccioli, or ad *Montes Alanos*, Hevelii, a little to the Northward of the *Palus Maoris*.

3.
At Dantzick;
by M. Hevelius.
ib. p. 184.

| Secund. Ho-
rol. Amb. | | Temp. Cor. | |
|--------------------------|---|------------|---------|
| h. ' " | | h. ' " | |
| 3 23 20 | Altitudo <i>Arcturi</i> . | 3 16 0 | 3 20 12 |
| 3 24 25 | Eadem Altitudo. | 3 04 0 | 3 21 35 |
| 3 44 30 | Jupiter à Limbo <i>Lune</i> distabat Majori adhuc Intervallo quam M. <i>Sinai</i> à M. <i>Ætna</i> . | | 3 41 30 |
| 3 47 00 | Jovis distantia erat tanta, quanta M. <i>Porphyritidis</i> à <i>Byzantio</i> . | | 3 44 00 |
| 3 52 00 | Jovis à Limbo <i>Lune</i> Distantia erat æqualis Distantiæ <i>Insulæ Sardinie</i> & <i>Paludis Maræotidis</i> . | | 3 49 00 |
| 3 59 00 | Jupiter à Limbo <i>Lune</i> paulo plus distabat quam <i>Pol. Maræotis</i> ab <i>Ætna</i> . | | 3 56 00 |
| 4 16 40 | Distantia Jovis à Limbo <i>Lune</i> æquabatur serè Distantiæ M. <i>Porphyritidis</i> ab Inf. <i>Cercinna</i> . | | 4 13 40 |
| | Planetarum Occasus factus est. | | 4 17 00 |

| Hora Horolog. ma. | | | | Temp. Corr. | LXXV. 1. |
|-------------------|---|-------|----------|-------------|--|
| h. / " | | | o / | h. / " | An Occultation of Saturn by the Moon, Mar. 19. 1689. at Totteridge; by Mr. Ed. Haines. n. 186. p. 268. |
| 12 06 10 | Altitud. Pollucis. | 28 35 | 12 12 14 | | |
| 12 16 47 | | 26 59 | 12 22 47 | | |
| 1 18 00 | Luna Limbus tangebatur Ansum Occid. | | 1 24 00 | | |
| | h ⁿⁱ . | | | | |
| 1 18 30 | Immersio Centri Saturni paulo infra Pal. Maraton. | | 1 24 30 | | |
| 1 19 00 | Jam Saturnus omnino latuit. | | 1 25 00 | | |
| 4 01 25 | Altitudo Centri Solis. | 20 00 | 7 07 17 | | |
| 4 09 06 | | 18 55 | 4 14 43 | | |

2. March 18. At Night I observed here the Occultation of Saturn by the Moon, which happened at 12^h 13^m 55^s. it passed directly under the Midst of the Moon's Discus.

In Ireland; by Dr. Ash, Bp. of Cloyne. n. 243. p. 293.

LXXVI. 1. Oct. 13. 1665. at six of the Clock, with a very good Telescope near 38 Foot long, and a double Eye-glass, Saturn appeared to me somewhat otherwife than I expected, thinking it would have been Decreasing, but I found it as full as ever, and a little hollow above and below.

Phases of Saturn, An. 1665. at Mainhead near Exeter; by Mr. Will. Ball. n. 9. p. 152.

2. Jun. 29. 1666. between 11 and 12 at Night, I observed the Body of Saturn through a 60 Foot Telescope, and found it exactly of the Shape Represented in the Figure. The Ring appeared of a somewhat brighter Light than the Body; and the black Lines *aa*, crossing the Ring, and *bb*, crossing the Body (whether Shadows or not I dispute not) were plainly visible; whence I could manifestly see, that the Southermost-part of the Ring, was on this side of the Body, and the Northern-part, behind or covered by the Body.

Fig. 132. An. 1666. at London; by Dr. Hook. n. 14. p. 246. Fig. 133.

3. Aug. 17. 1668. at 11^h these Parisian Observers, imploying a Telescope of 21 Foot, saw the Globe of Saturn in the Middle manifestly appearing above and below, beyond the Oval of his Anses; which was hardly discernible the last Year. They measured divers ways the Inclination of the greater Diameter of the Oval to the Equator, and found it of about 9 Degrees. By this Observation and other like ones of this and the preceeding Year, Mr. Hugen finds, that, instead of 23° 30', the Angle of the Planes of the Ring and of the Ecliptick must be of 31°, or thereabouts.

An. 1668. at Paris; by Mr. Hugen & Mr. Piccart. n. 45. p. 900.

4. Aug. 26. st. n. 1670. Telescopium illud 50 Pedes longum, quod non ita pridem mihi transmisistis, Faciem Saturni utut Luna fuerit præfens nitidissime ac clarissime detegebat. Quali autem mihi apparuit, adjecta Delineatio M. Hevelii. commonstrabit; alia planè Facie, quam Cl. Hugenio, tum vobis An. 1666.

An. 1670. at Danzick; by M. Hevelius. n. 65. p. 2089. Fig. 134.

tum Parisiensibus 1668, videbatur. Siquidem *Annulus*, qui *Saturnum* circumdat, multo nunc arctior, compressiorque animadversus quam illo tempore, quasi obliquiori, respectu *Terra*, nunc via incedit.

At Paris; by
Mr. Hugen.
ib. p. 1093.

5. This Summer Mr. *Hugen* observed *Saturn* with his Telescope of 22 Feet, and saw his Figure to be very conformable to what it should be according to his *Hypothesis*; viz. the *Ansa* or Arms to be very narrow, in so much that their opening appeared not but very obscurely.

At London;
by Dr. Hook.
ib.

6. Sept. 16. Dr. *Hook* observed the Phase of *Saturn* as here represented. Fig. 135.

An. 1671. at
Paris; by Mr.
Cassini. n. 78.
p. 3024.

7. *Saturn* according to the *Hypothesis* of Mr. *Hugen* was to have retaken his Round Figure in the Months of *July* and *August* 1671. But this Appearance hath been perceived ever since the End of *May*, at a Time when he was distant enough from the *Sun* and the *Horizon*, to be well Observed. He hath remained in this Figure unto the 11th of *August*, and Mr. *Cassini* did then observe him thus; but three Days after he saw him with *Arms*, though very Narrow ones.

By Mr. Hugen.
ib. p. 3025,
3026.

8. Our *Philosophers* here, know very well, that as soon as Mr. *Cassini* had told me that the *Arms* of *Saturn* were returned in *August*, I said that assuredly they would disappear before the End of this Year. I still observed them, Nov. 6, st. n. in the Evening, but they were so faint and obscure, that it was hard to discern them; so that within a few Days they will appear no more at all. This confirms altogether my *Hypothesis* of the *Ring*, which now disappears in Proportion that the Rays of the *Sun* do obliquely illuminate the flat Surface of it, Obverted to our Sight.

At Dantzick;
by Mr. Hevelius.
ib. p. 3032.
Fig. 136.

9. Quali Facie nuper, Diesc. Sept. 11. st. n. apparuerit, quàm resistimè & diligentissimè delineavi atque hic transmissi. An vero Mense Jun. Jul. & Aug. eum planè Rotundum conspexeritis, ut volunt *Parisienses*, vix mihi imaginari possum. Etenim utut *Brachia Saturni* ad latera apparuerint arctissima, etiam Tubo 60 vel 70 Pedum; haud tamen credo, ea omnino evanuisse, ita ut ne Vestigium aliquod fuerit reliquum. Fortasse *Parisienses* Telescopiis brevioribus in ipso Crepusculo, Luna præsentè, *Saturnum* contemplati sunt.

At Derby by
Mr. Flamsteed.
ib. p. 3034.

10. Oct. 12. with my less Tube I thought I saw something on each side of *Saturn*, amidst the Colours of my Glass, and the Spurious Rays of his Body. Directing my longer Tube (of 14 Feet.) to him, I could see his *Ansa* somewhat more distinctly, but very slender, and to one, that thought not of them, scarce discernible.

Nov. 30. I observed him with my 14 Foot Telescope, the Aperture being 1½ Inch, and its Eye-Glass drawing two Inches. He appeared perfectly Round, free from Rays and Colours, and no *Ansa* to be seen. Mr. *Townley* in his last (Nov. 20.) tells me, that he looked at him one Night, and could hardly distinguish his Line of the *Ansula*, but plainly saw a dark Line thro' him near his upper Part.

At Paris; by
Mr. Cassini.
n. 92. p. 5180.
An. 1675. at
London; by
Mr. Flamsteed.
n. 116. p. 372.
At Dantzick;
by Mr. Hevelius.
n. 127.
p. 661.

11. Dec. 16. we found that *Saturn* had retaken his Round Figure.

12. Jun. 27. m. 1675. *Saturnum* Patulis auctum *Ansis* vidimus.

13. In *August*, The Figure of *Saturn* appear'd, as Fig. 137.

14. Ex Schemate *Saturni* à *Cl. Hevelio* ante Annum observato video, eum An. 1676. at
 Telescopiis, nostris longè inferioribus, uti. Tunc enim Temporis (ut & Paris; by M.
 nunc (Aug. 1776.) cernebatur nobis in *Saturni* Globo Zona subobscura, pau- Cassini. n. 128.
 lo Australior Centro, instar Zonarum *Jovialis*. Deinde Latitudo *Annuli* P. 690.
 dividebatur bifariam, Lineâ obscurâ apparenter Ellipticâ, revera Circulari quâsi Fig. 137.
 in duos *Annulos* Concentricos, quorum Interior Exteriori Lucidior erat. Fig. 138.
 Hanc Phasim statim post Emerfionem *Saturni* è Solaribus Radiis per totum
 Annum usque ad ejus Immerfionem conspexi; primo quidem, Telescopio Pe-
 dum 35; deinde minori, Pedum 20.

LXXVII. 1. Vidi *Saturnum* Aug. 26. st. n. 1670. alta *Aquila* $24^{\circ} 32'$ Places of Saturn
 o". in Distantia ab Extrema *Alæ Pegasi* $33^{\circ} 48' 0''$. & ab *Ore* ejus $24^{\circ} 31'$ Observed An.
 40". in $4^{\circ} 11'$. *Piscium*, in $1^{\circ} 53'$. Latit. Austr. in ipsâ nempe Oppositi- 1670. at Dant-
 one *Solis* existentem. zick; by Mr.
 Hevelius. n. 63.

2. Dec. 29. st. n. 1677. $8^{\circ} 58'$. Vidimus in eodem Azimutho inque No- P. 2089.
 nagesimo Eclipticæ Gradu ab Horizonte *Saturnum* & *Boreum Oculum* δ , An 1677. at
 infra *Saturnum* erat, unde Planetam & Fixam eandem in *Zodiaco* Longitudi- Paris; by Mr.
 nem obtinere deprehendimus, viz. juxta *Tychonem* $11^{\circ} 3^{\circ} 58' 53''$. Bullialdus. n.
 139. P. 973.

LXXVIII. 1. About the End of *October* 1671. we discovered, by a Te- The Outermost
 lescope of 17 Feet, 11 small Stars near *Saturn*, one of which by its particular Satellite of Sa-
 Motion shew'd it self to be a true Planet: which we found by comparing it turn discovered
 not only to *Saturn* and his ordinary Satellite, discovered 1655. by M. *Hugens*, by Mr. Cassini.
 but also to the Fix'd Stars. The Motion of it was very manifest in respect n. 92. p. 5178.
 of the Fix'd Stars, but less sensible in respect of *Saturn*; yet it appear'd that
 from *October* 25. unto *November* 1. his Distance from *Saturn* increased West-
 ward, and from that time unto *November* 6. it diminished: So that his
 greatest Digression from *Saturn* happened in the Beginning of *November*. -

Dec. 16. We found that on the East of *Saturn*, there was a small Star, far
 distant in a streight Line to *Saturn*, and to his ordinary Satellite, which was
 Oriental also but little distant from *Saturn*. And Dec. 24th. we saw this Sa-
 tellite in the West, and a Star, Oriental likewise, less distant from *Saturn* than
 that we had seen the 16th.

Dec. 13. and 17. 1672. We perceived, with an Excellent Telescope, (of
 35 Foot made by *Campani*) an Occidental Star, remote from *Saturn*, which
 in both those Observations had a Southern Latitude in respect of the Line of
 his Wings; but in the first it was further distant from *Saturn* than in the Se-
 cond: so that, if this was the same Star, as I suppos'd it to be, it mov'd to-
 wards *Saturn* on the East, and consequently (supposing it to be his Satellite)
 it was in the Superior part of his Circle.

Feb. 6. 1673. We began to see him again, and we observed him almost all
 the Days following till the 20th. This New Planet did more and more remove
 from *Saturn* till the 19th of Feb. when we measured the Difference between
 his Passage and that of the Center of *Saturn* to be $30''$ of an Hour, which
 gave at least 10 Diameters of *Saturn*, and on the 20th the Distance was judg-
 ed by Estimate to be yet greater.

This

26. p. 5179.
5184.

This Digression, being treble to that of the *Ordinary Satellite*, enabled us at first to judge the Time of this Revolution to be Quintuple, applying to the *Satellites* that Proportion, which *Kepler* hath noted in the *Principal Planets*, between the Periodical Times and their Distances. We were afterwards confirmed in this Opinion; For by the Apparent Swiftneſs of his Motion it was eaſie to ſee that this *Planet* had been in Conjunction with *Saturn* Feb. 3. 1673. and by his Motion on the Weſt it appears, that he was in the Inferior part of his Circle: And becauſe during this time of 17 Days he remov'd more and more from *Saturn*, 'tis certain that he remained in the ſame Quadrant of the Inferior Occidental Circle above 17 Days, and that his whole Revolution is more than 68 Days. He was theſe laſt Days at a Diſtance almoſt equal to that which he had about the end of Octob. 1671; ſo that in 480 days or thereabout he made a certain Number of Intire Revolutions, which can be no more than 7; ſince each of them is without Queſtion of more than 68 days. If you ſhould count 7 of them, each would be 68¹ Days; if you count 6, each would be 80 Days; and if you count but 5, each would be 96 Days. But this laſt Suppoſition can by no means be made to agree with the two Obſervations of Dec. 1672, and the firſt doth not agree ſo well with them as the Second.

n. 133. p. 831.

Mr. Caſſini has ſince found that this *Outtermoſt Satellite*, is diſtant from the Center of *Saturn* 10¹ Diameters of his Ring; that the Period of his Revolution in 80 Days is ſo juſt, that he doth not Anticipate 9 Revolutions, which are made in two Years, but by one whole Day; and that in the Conjunctions with *Saturn*, his Latitude encreaſes according as the Ring of *Saturn* enlargeth it ſelf; though the Line of his Motion is not Parallel to the Circumference of the Ring.

Mr. Caſſini hath alſo diſcovered, after many Revolutions, that this *Satellite* hath a Period of *Apparent Augmentation and Diminution*, by which Period he becomes viſible in his greateſt Occidental Digreſſion, and Inviſible in his greateſt Oriental Digreſſion; he begins to *Appear* two or three Days before his Conjunction in the Inferior part, and to *Diſappear* two or three Days after his Conjunction in the Superior part: So that he remains Inviſible in every Revolution of 80 Days for a whole Month together.

This Viciffitude of *Phaſes* makes it ſeem probable, that one part of his Surface is not ſo capable of Reflecting to us the Light of the Sun, which maketh it viſible, as the other part is. Whence we may conjecture, that the Globe of the *Satellite* hath ſome Diverſity of Parts Analogous to that of the Earth, the one part of whoſe Surface is cover'd by the Sea, which is not ſo fit to Reflect from all parts the Light of the Sun, as the Continent which maketh up the other part: So that this *Planet* by a Converſion about his *Axis*, or by an Expoſition of the ſame Hemisphere to *Saturn* (much after the manner of the Hemisphere of the *Moon* to the *Earth*.) ſometimes turns to us the part Analogous to the Continent, ſometimes that part which answers to the Sea.

LXXIX. Dec. 23. 1672. We found a small Star Westward of *Saturn*, between him and his Ordinary Satellite, which was on the West also, almost at a double Distance, Dec. 30. We saw a little Star, on the East of him and his Ordinary Satellite, which had passed also to the East of him.

The Third Satellite of Saturn
discovered by
M. Cassini. n.
91. p. 5181.

Jan. 10. 1673. This little Star appeared to have returned almost to the same Position in respect of *Saturn* and his Ordinary Satellite, where it had been Dec. 23. Jan. 15. the Ordinary Satellite was Oriental, and the New one Occidental, as it had been in the precedent, but a little nearer to *Saturn*. We had that Evening time enough attentively to Observe this Planet for a whole Hour together, during which we perceived, it approached to *Saturn* on the West, and consequently was in the Superiour part of his Circle: Which did fully confirm us in the Supposition we were inclined to, that it was an Interior Satellite.

Comparing the Observations together, we began to find the Rule of the Motion of the new Interior Satellite. For the two last shewed us, that in 5 Days he had made more than a whole Revolution. The first Observation compared with the 3d made us judge, that in 18 Days he had made a Number of Revolutions almost whole ones, which certainly were 4; each of them was of $4\frac{1}{2}$ Days: So that between the 10th and 15th it might be, that there had been one Revolution of $4\frac{1}{2}$ Days, or two Revolutions of $2\frac{1}{2}$ Days each. But the Combination of the first with the 2d, made us conclude the Period of $2\frac{1}{2}$ Days. We therefore Judged by these Observations; that this last Planet finishes his Revolution about *Saturn* in $4\frac{1}{2}$ Days; that the Semidiameter of his Circle is $\frac{1}{3}$ of the Diameter of *Saturn's* Ring; and that he was towards his greatest Occidental Digression the 23d of December, and Jan. 1. about 7 a Clock in the Evening. We have since found, that his greatest Digression from the Center of *Saturn* is only $\frac{1}{3}$ of his Ring, and the Period of his Revolution is 4 Days 12^h and $27'$. His Latitude Augments also according as the Ring enlargeth, and at the present that the largeness of the Ring is greater than the Diameter of the Globe of *Saturn*, he is to pass in the Conjunctions without touching either *Saturn* or his Ring. Yet notwithstanding we have not yet been able to distinguish him in the Conjunctions either in the upper or lower part of his Circle; but only in his Greatest, as well Oriental, as Occidental Digressions.

n. 133. p. 833.

LXXX. These two Satellites were first of all seen in Mar. An. 1684. by two excellent Object Glasses of 100 and 136 Feet, and afterwards by two others of 93 and 70 Feet, all made by S. Campani, after the Discovery of the 3d and 5th Satellites, which had been made by others of his Glasses of 47 and 34 Feet. We have since seen all these Satellites with that of 34 Feet, and continued to observe them with Glasses of M. Borelli of 40 and 70 Feet, and by those which M. Arrouquel hath lately made, of 80, 155, and 220 Feet. It was easy for us to see these two Satellites by these different Sorts of Glasses, after having found the Rules of their Motion, whereby we might with more particular Attention look upon the Places where they ought to be.

Two Interior Satellites of Saturn discovered by M. Cassini. n. 181. p. 79.

The *First Satellite* was observed 45° Distant from its *Perigee*, moving toward the West, Mar. 11. 1686. *ft. n.* at $10^h 40'$. at Night, and returned to the same Position on the 14th of April at the same Hour.

The *Second* was 36° Distant from the *Perigee* to the West, the 30th of Mar. 1686. *ft. n.* at 8 a Clock in the Evening.

The *First* or *Innermost Satellite*, is never distant from *Saturn's Ring* above $\frac{2}{3}$ of the apparent length of the same *Ring*; It makes one *Revolution* in $1^d 21^h 19'$ and the Circle of its Orb is nearly in the same Plain with the *Ring*.

The *Second* or *Penultimate Satellite* is $\frac{1}{4}$ of the Length of the *Ring* distant therefrom, and makes his *Revolution* in $2^d 17^h 43'$.

After a great Number of Choice Observations, it was concluded, that the Proportion of the *Digressions* of the *Second* to that of the *First*, (counting both from the Centre of *Saturn*) is as 22 to 17; and of its *Revolution* as $24\frac{1}{2}$ to 17. This is that very same Proportion which *Kepler* Observes between the *Distances* and *Period* of the *Primary Planets*, and which we have found between the other *Satellites* of *Saturn*, and is verified in the *Satellites* of *Jupiter*. There is nothing that better shews the admirable Harmony of the Particular *Systems* with the Great *System* of the World.

The antient Astronomers having translated the Names of their Heroes among the Stars, whose Names have continued down to us Unchanged, notwithstanding the Endeavour of the following Ages to alter them; and *Galileo* after their Example, having honoured the House of the *Medici* with the Discovery of the *Satellites* of *Jupiter*, made by him under the Protection of *Cosmus II.* (which Stars will be always known by the Name of the *Sidera Medicea*.) Wherefore the Discoverer Concludes that the *Satellites* of *Saturn*, being much more exalted and more difficult to Discover, are not unworthy to bear the Name of *Louis Le Grand*, under whose Reign, and in whose *Observatory* the same have been detected, which therefore he calls *Sidera Ludovica*. not doubting but to have Perpetuated the Name of that King, by a Monument much more lasting than those of Brass and Marble, which shall be erected to his Memory.

M. Hugen's
Theory of the
Fourth Satellite
of Saturn corrected;
by Mr. Ed. Halley.
n. 145. p. 82.
Apr. An. 1683.

LXXXI. The *Fourth* or *Penultimate Satellite* of *Saturn*, first Discovered by M. Hugen 1655, I have of late frequently Observed with a 24 Foot Telescope: And I perceived that M. Hugen's Numbers were considerably run out, and about 15° in 20 Years too swift; this made me resolve more nicely to enquire into it's *Period*, and accordingly I waited till I had gotten a Competent Number of Observations, the most Considerable whereof are these.

1682. Nov. 13. $13^h 00'$. p. m. the *Satellite* appeared on the North side of *Saturn*, and a Perpendicular let fall from it on the Transverse Diameter of the *Ring*, fell upon the middle of the Dark space of the following *Anse*; and the same Night $19^h 00'$. it had past the Conjunction, and the Perpendicular fell exactly on the Western Edge of the *Globe* of *Saturn*; The Northern Latitude, and Retrograde Motion, made it evident that the *Satellite* was then in *Perigee*.

Again

Again, Nov. 21. 16^h 15'. this *Satellite* of *Saturn* was on his South-side, the Perpendicular on the Line of the *Ansa* fell on the Middle of the dark Space of the Western *Ansa*, and the same Night 19^h 00', the Perpendicular fell precisely on the Center of *Saturn*, and the Distance therefrom was somewhat less than one Diameter of the *Ring*. By this it was evident that the *Satellite* was in *Apogeo*.

I observed it in *Apogeo* again on the 24th of Jan. 1683. at 8^h 00' p. m. the Perpendicular on the Line of the *Ansa* fell exactly on the Western Limb of the *Globe* of *Saturn*, and at 9^h 30' p. m. the said Perpendicular fell within the *Globe* more than half way to the Center, and the Distance from the Line of the *Ansa* towards the South, seemed much about one Diameter of the *Ring*.

Lastly, Feb. 9. 1683. 8^h 10' p. m. it was again in *Apogeo*, and I could by no means discern towards which side it inclined most, nor whether the Transverse Diameter of the *Ring*, or the Distance of the *Satellite* therefrom, were the greater; so that at that time it was precisely *Apogeon*.

To compare with these, I chose two out of those of *Hugens*, which seemed the most to be confided in; the first made 1659. March 14. st. n. 12^h 00' at the *Hague*; when the *Satellite* appeared about one Diameter of the *Ring* under *Saturn*, but it was gone so far to the Westward, that he concluded, that about 4 Hours before, or 7^h 40' at *London*, it had been in *Perigee*.

Again, March 22. 1659. 10^h 45'. the *Satellite* was a whole Diameter above the Line of the *Ansa*, and the Perpendicular thereon fell nearly upon the Extremity of the Eastern *Ansa*.

By the First of my Observations it appears that the *Satellite* was in *Perigee* 1682. Nov. 13. 17^h 00'. circiter, at which time *Saturn* was 30^s 29' 39" from the first Star of *Aries* in the *Ecliptick*, but the *Earth* reduced to *Saturn's* *Equinoctial*, and the *Satellite* was 9^s 23' 46". à 1^a * γ. And March 4. 1659, 7^h 40'. *Saturn's* Place in the *Ecliptick* was 6^s 0' 41". but the *Earth* reduced, and consequently the *Satellite*, in 11^s 28' 18". à *Prima Stella Arietis*. The Interval of Time is 8655 Days, 9^h 20'; in which the *Satellite* had made a certain Number of Revolutions to the Fix'd Stars, and besides 9^s 25' 28". or 295 Degrees 28', whose Complement to a Circle 64° 32' is 2 Days 20^h 36' Motion of the *Satellite*, according to *Hugens*. So that 8655 Days 5^h 56', or 12467846 Minutes of Time, is the time of some Number of intire Revolutions; and dividing that Interval by 15 Days 22^h 39', or 22959' (the Period of *Hugens*) the Quotient 543 shews the Number of Revolutions; and again dividing 12467876 by 543, the Quotient 22961¹/₂ min. or 15 Days, 22^h 41' 6". appears to be the true Time of this *Satellite's* Period. Hence the Diurnal Motion will be 22° 34' 38" 18"', and the Annual besides 22 Revolutions 10^s 20' 43". Having made Tables to this Period, I found that in the *Apogeon* Observation of *Hugens*, the *Satellite* was above 3 Degrees faster than by my *Calculus*; and that in the three other Observations of my own, being likewise in the Superior part, it was 2¹/₂ Deg. slower than by the same Calculation. Now 'tis evident that the Differences must arise from some *Eccentricity* in the Orbit of this *Satellite*, and that in Mar. 1659.

the *Apocronion* (as I may call it) was somewhere in the Oriental Semicircle; and that in Nov. 1682. it was in the Western Semicircle, and supposing the *Apocronion* fix'd, it must necessarily be between $9^{\circ} 23' 46''$, and $11^{\circ} 28' 18''$, à $1^{\circ} * \gamma$, that being the common Part between those two Semicircles; and because the Difference was greater in *Hugens's* Observation than in Mine, 'twill follow that the *Linea Absidum*, or *Apocronion*, should be nearer to $9^{\circ} 23' 46''$, than to $11^{\circ} 28' 18''$. I will suppose $10^{\circ} 22' 00''$ à *Prima Stella Arietis*, (which happens to be also the Place of *Saturn's* Equinox) and the greatest Equation about $2\frac{1}{2}$ Degrees. Upon the Score of this Inequality the mean Motion of the *Satellite* will be found about $2^{\circ} 45'$. slower in $23\frac{1}{2}$ Years, or 7 Minutes in a Year, whence I state the Annual Motion $10^{\circ} 20' 36''$. above 22 Revolutions, and the Correct *Epocha* for the last Day of December 1682. at Noon in the Meridian of London $9^{\circ} 10' 15''$ à $1^{\circ} * \gamma$, from which *Elements* I compose the following Table.



Tabula

Tabula Motus Medii Satellitis Saturnii ab Hugenio inventi, à Prima * v.

| Anno
Christi
Curr. | Epoche.
s. o i | Anno.
s. o i | Mot. Med.
s. o i | Debita. | Mot. Med.
s. o i | Hor. Min. | Mot. Med.
o i | Min. | Mot. Med.
i ii |
|--------------------------|---------------------|-----------------|---------------------|---------|---------------------|-----------|------------------|------|-------------------|
| 1641 | 8 29 17 | 1 | 10 20 36 | 1 | 0 22 35 | 1 | 0 56 | 31 | 29 10 |
| 1661 | 10 14 10 | 2 | 9 11 12 | 2 | 1 15 9 | 2 | 1 53 | 32 | 30 6 |
| 1681 | 11 29 3 | 3 | 8 1 48 | 3 | 2 7 44 | 3 | 2 49 | 33 | 31 3 |
| 1682 | 10 19 39 | 4 | 7 14 59 | 4 | 3 0 18 | 4 | 3 46 | 34 | 31 59 |
| 1683 | 9 10 15 | 5 | 6 5 35 | 5 | 3 22 53 | 5 | 4 42 | 35 | 32 55 |
| 1684 | 8 00 51 | 6 | 4 26 11 | 6 | 4 15 28 | 6 | 5 39 | 36 | 33 52 |
| 1685 | 7 14 2 | 7 | 3 16 47 | 7 | 5 8 2 | 7 | 6 35 | 37 | 34 48 |
| | | 8 | 2 26 57 | 8 | 6 0 37 | 8 | 7 32 | 38 | 35 45 |
| Mens.
Ann.
Com. | Mot. Med.
s. o i | | | | | | | | |
| | | 9 | 1 20 23 | 9 | 6 23 12 | 10 | 9 24 | 40 | 37 38 |
| | | 10 | 0 11 9 | 10 | 7 15 46 | | | | |
| | | 11 | 11 1 45 | 11 | 8 8 21 | 11 | 10 21 | 41 | 38 34 |
| Jan. | 0 0 0 | 12 | 10 14 56 | 12 | 9 0 55 | 12 | 11 17 | 42 | 39 31 |
| Feb. | 11 9 53 | | | | | 13 | 12 14 | 43 | 40 27 |
| Mar. | 8 12 2 | 13 | 9 5 32 | 13 | 9 23 30 | 14 | 13 10 | 44 | 41 24 |
| April. | 7 21 56 | 14 | 7 26 8 | 14 | 10 16 5 | 15 | 14 7 | 45 | 42 20 |
| | | 15 | 6 16 44 | 15 | 11 8 39 | | | | |
| Maii | 6 9 14 | 16 | 5 29 54 | 16 | 0 1 14 | 16 | 15 3 | 46 | 43 17 |
| Jun. | 5 19 7 | | | | | 17 | 16 0 | 47 | 44 13 |
| Jul. | 4 6 26 | 17 | 4 20 30 | 17 | 0 23 48 | 18 | 16 56 | 48 | 45 10 |
| Aug. | 3 16 19 | 18 | 3 11 6 | 18 | 1 16 23 | 19 | 17 52 | 49 | 46 6 |
| | | 19 | 2 1 42 | 19 | 2 8 58 | 20 | 18 49 | 50 | 47 3 |
| Sept. | 2 26 12 | 20 | 1 14 53 | 20 | 3 1 32 | | | | |
| Oct. | 1 13 31 | | | | | 21 | 19 45 | 51 | 47 59 |
| Nov. | 0 23 24 | | | | | 22 | 20 42 | 52 | 48 56 |
| Dec. | 11 10 43 | | | | | 23 | 11 38 | 53 | 49 52 |
| | | | | | | 24 | 22 35 | 54 | 50 49 |
| | | | | | | 25 | 23 31 | 55 | 51 45 |
| | | | | | | | | | |
| | | | | | | 25 | 6 24 25 | 26 | 24 27 56 52 42 |
| | | | | | | 26 | 7 17 00 | 27 | 25 24 57 53 38 |
| | | | | | | 27 | 8 9 35 | 28 | 26 20 58 54 35 |
| | | | | | | 28 | 9 2 9 | 29 | 27 17 59 55 31 |
| | | | | | | | | 30 | 28 13 60 56 27 |
| | | | | | | 29 | 9 24 44 | | |
| | | | | | | 30 | 10 17 18 | | |
| | | | | | | 31 | 11 9 53 | | |
| | | | | | | 32 | 0 2 28 | | |

In Anno Bissextili post Februarium ad-
de unum diem, motumque ei com-
petentem.

I here

I here suppose the *Linea Apfidum* fix'd, as having no Argument from Observation to prove the contrary, tho' it be very probable that as the *Apogeon* of our *Moon* has a Motion about the *Earth* in about 9 Years, so that of this *Satellite* ought to have about *Saturn*, but with a much Longer Period, which future Observation may discover.

The distance of this *Satellite* from the Center of *Saturn* seems to be much about 4 Diameters of the *Ring*, or 9 of the *Globe*, and the Plane wherein it moves very little or nothing differing from that of the *Ring*, that is to say intersecting the Orb of *Saturn* $4^{\circ} 22'$ and $10^{\circ} 22'$ à 1^{re} * γ , with an Angle of $23\frac{1}{2}$ Degrees, so as to be nearly Parallel to the *Earth's Equator*, whence the *Latitude* of the *Apogeon* Semicircle from $4^{\circ} 22'$ to $10^{\circ} 22'$ of *Saturn's Longitude* from the First Star of γ , will be Northern, and of the other Semicircle Southern; and the contrary in the other half of *Saturn's Longitude*, to wit, from $10^{\circ} 22'$ to $4^{\circ} 22'$ of his distance from the First Star of γ .

It follows now to shew how by the help of this Table to compute the place of this *Satellite*, to any time required.

First we must have the true *Longitude* of *Saturn* from the *Earth*, and numbred from the First Star of γ , (or rather the Place of the *Earth* viewed from *Saturn* together with its *Latitude* from the Orb of *Saturn*, but that being never fully $\frac{1}{2}$ of a degree we neglect it as a Nicety) and therefrom substract $10^{\circ} 22'$, there remains the distance of *Saturn* from this *Equinoctial* Point, with which distance as with the *Longitude* of the *Sun*, take out the *Right Ascension* and *Declination* thereto ($23\frac{1}{2}$ degrees being the *Obliquity* common to both) and to the *Right Ascension* adding $10^{\circ} 22'$ the Summ shall be the *Longitude* of the *Satellite's Apogeon*. Then say, as *Radius* to *Sine* of the *Declination*, so 8 to the greatest *Latitude* in *Apogeo*, or *Perigao* in the parts of the Semidiameter of the *Ring*.

Next Collect the *Middle Motion* of the *Satellite*, and from it Substract $10^{\circ} 22'$ the remainder shall be the mean *Anomaly*, with which in the Table of the *Moon's Primary Equation*, take out the *Equation* answering thereto, and the half thereof added or substracted to or from the *Middle Motion*, according to the Table, gives the True Motion of the *Satellite*, from which substract the *Apogeon*, and if the remainder be more than 6 *Signs*, the *Satellite* is Occidental, if less Oriental; and as *Radius* to *Sine* of the remainder, so 8 to the Semidiameters of the *Ring*, or 18 to the Semidiameters of the *Globe*, that the *Satellite* is to the Eastward or Westward of the Center of *Saturn*, according to the foregoing Precept.

Lastly, as *Radius* to *Co-sine* of the said Remainder, so is the greatest *Latitude* from the Line of the *Ansa*, to the *Latitude* sought.

Here Note, that I purposely neglect the Inequality of the distance arising from the *Eccentricity*, as being too small to be any way observable.

Lastly to clear all difficulties that may arise to them that are but little versed in this sort of Calculation, I have added an Example of the Work, that where the precept may seem obscure it may be thereby illustrated.

An. 1657. Maii 19. ft. n. M. *Hugens* observed the *Satellite* very near *Saturn* on the Western side, and very little above the Line of the *Ansa*. I suppose this about 10^h p. m. Let us now Calculate to that time.

| | | | | | |
|---|---------------------|----------------------------|----|----|----|
| 1657 Maii 9 ^d 9 ^h 10' Londini | | Mot. Med. Satel. | s. | ° | ' |
| <i>Saturni Locus</i> | $\cap$ 28° 57' | 1641 | 8 | 29 | 17 |
| $\cap$ a 1a * γ . | 5 ^s 0 32 | 16 | 5 | 29 | 54 |
| <i>Equinoct. Sub.</i> | 10 22 00 | Maii | 6 | 9 | 14 |
| | | 9 ^d | 6 | 23 | 12 |
| $\cap$ ab <i>Equinoct.</i> | 6 8 32 | 9 ^h 40' | 0 | 9 | 5 |
| <i>Ascen. Recta</i> | 6 7 50 | | | | |
| | | Long. Med. Satel. | 4 | 10 | 42 |
| <i>Apogeon.</i> | 4 29 50 | <i>Apocron.</i> | 10 | 22 | 00 |
| <i>Declin. Aust.</i> | 3 23 | | | | |
| | | <i>Anomalía Med.</i> | 5 | 18 | 42 |
| | | | | | |
| | | <i>Equatio Sub.</i> | 0 | 0 | 31 |
| | | | | | |
| | | Long. Vera Satel. | 4 | 10 | 11 |
| | | <i>Apogeon</i> | 4 | 29 | 50 |
| | | | | | |
| | | <i>Residuum</i> | 11 | 10 | 21 |
| | | <i>h. e. ante Apogeon.</i> | 0 | 19 | 39 |

Ergo $2\frac{1}{2}$ Semid. *Annuli* ad occasum & $\frac{1}{2}$ ad Boream; Agreeing exactly with the Description and Figure of M. *Hugens*.

I here call the Plane of this *Satellite's* Orb, which hitherto I suppose the same with that of the *Ring*, *Saturn's Equinoctial*, not that any discovery hath been able to prove that the *Axis* of that *Globe* is at right Angles thereto, but because it hath pleased M. *Hugens* to call it so, and likewise because it is so nearly Parallel to our *Globe's Equinoctial*; Nevertheless to speak my Opinion, I believe that the *Axis* is Inclined, and that not a little, to the Plane of the *Ring*; for as the Reflection of the *Sun's* Light from the *Ring* is a great convenience to that Hemisphere of *Saturn*, which beholds its Illuminate side; so that the other Hemisphere is very much Incommoded by the Shadow of the *Ring*, which for many Months, and in some Parallels for several Years, occasions a continual Night by the Interception of the *Sun's* Beams, which is a consequence that Demonstratively follows the Position of the *Ring*, in the Plane of *Saturn's Equator*. Now this great inconvenience would be in some measure relieved by the Oblique Position of the *Axis*, for then the Parallels of Latitude intersecting the Plane of the *Ring*, many, and in most cases all of them, might for some time in every Diurnal Revolution of the *Globe*, free themselves from this *Eclipse*, which otherwise were sufficient to render this *Globe* of *Saturn* unfit for any settled Habitation; but this is but Conjecture.

The

The other two *Satellites* of *Saturn* discovered by *S. Cassini* at *Paris*, *An.* 1672. and 1673. I must confess I could never yet see; I have been told that they Disappear for about $3\frac{2}{3}$ of *Saturn*'s Revolution, and were only to be seen when the *Anse* were very small, it being supposed that the Light which proceeds from the *Anse*, when considerably opened, might hide these *Satellites*.

*The Theory of
the 5 Satellites
of Saturn cor-
rected; by M.
Cassini, n. 187.
p. 299.*

LXXXII. 1. La *Distance* du *Premier Satellite* du Centre de *Saturn* m'a paru variable, & son *Mouvement* sensiblement Inegal, plus viste, en ce temps, dans le Demicercle Occidental, que dans l'Oriental. J'ay dernièrement determiné sa Moyenne *Distance* de $\frac{3}{4}$ du *Diametre* de l' *Anneau* de *Saturne*, son *Mouvement* Journalier de $6^s 10^o 41' 31''$. Ainsi si son *Mouvement* estoit Egal, la durée de sa Conjonction avec *Saturne*, c' est à dire, tout le temps qu'il met à parcourir son *Anneau*, seroit de $7^h 46'$. Elle m' a paru plus grande par les observations immediates, mais il est à remarquer que je n'ay jusqu'à present pu voir ce *Satellite* plus près de *Saturne*, que d' un quart d' un *Anse*.

J'ay Calculé l' *Epoque* de son *Mouvement*, pour le dernier *Decembre* 1685. à Midi au Meridien de *Paris* en $\vee 24^o 50'$.

La *Distance* du *Second Satellite* du Centre de *Saturne* m'a paru plus Uniforme. Je l' ay determinée d'un *Diametre* de l' *Anneau* & $\frac{1}{4}$: Son *Mouvement* paroît aussi plus Egal. J' ay Calculé le Journalier de $4^s 11^o 31' 30''$. Ainsi la *Durée* de sa Conjonction devroit estre de $8^h 36'$. Je n'ay pas non plus vu jusqu'à present ce *Satellite* plus proche de l' *Anneau* de *Saturne* que d'un $\frac{1}{4}$ d' un *Anse*. Comme ce *Satellite* se voioit la plus part du temps dedans les Confins de la *Distance* du *Premier*, au quel il est Egal en *Grandeur*, & Semblable dans la *Couleur*, la difficulté de distinguer l' un de l' autre a esté extreme, de fort que sans un assiduité particuliere aux Observations, & sans une grande multitude de Combinaisons je n'en serois pas venu à bout.

J' ay determiné l' *Epoque* de ce *Satellite* pour le 31 *Decembre* 1685. à Midien $\vee 9^o 10'$.

La *Distance* du *Troisieme* du Centre de *Saturne* paroît d' un *Diametre* de l' *Anneau* & $\frac{3}{4}$. Son *Mouvement* Journalier $2^s 18^o 41' 50''$. Ainsi sa Conjonction doit durer 10 heures. L' *Epoque* de son *Mouvement* pour le Midi du Dernier de l' Année 1685 $\vee 9^o 39'$.

La *Distance* du *Quatrieme Satellite* au Centre de *Saturne* paroît de 4 *Diametres* de l' *Anneau*. Son *Mouvement* Journalier de $22^o 34' 38''$. la *Durée* de sa Conjonction $15^h 6'$. L' *Epoque* de son *Mouvement* au mesme temps & lieu que les autres, en $\vee 18^o 1'$.

La *Distance* du *Cinquieme Satellite* au Centre de *Saturne* de 12 *Diametres* de l' *Anneau*. Son *Mouvement* Journalier de $4^s 32' 17''$. Ses Conjonctions durent 24 heures. L' *Epoque* de son *Mouvement* au mesme temps & lieu, en $\vee 16^o 19'$. Sur ces Principes on peut construire les *Tables*, & les *Ephemerides*.

By ib.
p. 300.

2. The following Tables are Calculated from these Elements, and Reduced to the Meridian of London.

Tabula

Tabula Motus Medii Intimi Satellitis Saturni, à Cassino Detecti Anno 1686.

| Anno Christi Curr. | Epoche | Anni. | Mot. Med. | Diebus. | Mot. Med. | Hor. Min. | Mot. Med. | Min. | Mot. Me. |
|--------------------|----------|-------|-----------|---------|-----------|-----------|-----------|------|----------|
| | s. ° ' | | s. ° ' | | s. ° ' | | s. ° ' | | s. ° ' |
| 1681 | 19 34 | 1 | 4 2 34 | 1 | 6 10 42 | 1 | 0 7 57 | 31 | 4 6 |
| 1685 | 10 30 | 2 | 8 5 7 | 2 | 0 21 23 | 2 | 0 15 53 | 32 | 4 14 |
| 1686 | 13 4 | 3 | 0 7 41 | 3 | 7 2 5 | 3 | 0 23 50 | 33 | 4 22 |
| 1687 | 15 37 | 4 | 10 20 56 | 4 | 1 12 46 | 4 | 0 31 47 | 34 | 4 30 |
| 1688 | 18 11 | 5 | 2 23 30 | 5 | 7 23 28 | 5 | 0 39 44 | 35 | 4 38 |
| 1689 | 1 26 | 6 | 6 26 4 | 6 | 2 4 9 | 6 | 0 47 40 | 36 | 4 46 |
| 1701 | 4 14 | 7 | 10 28 38 | 7 | 8 14 15 | 7 | 0 55 37 | 37 | 4 54 |
| | | 8 | 9 11 52 | 8 | 2 25 32 | 8 | 1 3 34 | 38 | 5 2 |
| Mens. Anni Com. | Mot. Med | | | | | | | | |
| | s. ° ' | 9 | 1 14 26 | 9 | 9 6 14 | 9 | 1 11 31 | 39 | 5 10 |
| | | 10 | 5 17 00 | 10 | 3 16 55 | 10 | 1 19 28 | 40 | 5 18 |
| | | 11 | 9 19 34 | 11 | 9 27 36 | 11 | 1 27 24 | 41 | 5 26 |
| Jan. | 0 0 0 | 12 | 8 2 48 | 12 | 4 8 18 | 12 | 1 35 21 | 42 | 5 34 |
| Feb. | 5 1 27 | | | | | | | | |
| Mar. | 3 0 49 | 13 | 0 5 22 | 13 | 10 19 00 | 13 | 1 43 18 | 43 | 5 42 |
| April. | 8 2 16 | 14 | 4 7 56 | 14 | 4 29 42 | 14 | 1 51 15 | 44 | 5 50 |
| | | 15 | 8 10 29 | 15 | 11 10 23 | 15 | 1 59 11 | 45 | 5 58 |
| Maii | 6 23 2 | 16 | 6 23 43 | 16 | 5 21 4 | 16 | 2 7 8 | 46 | 6 5 |
| Jun. | 11 24 29 | | | | | | | | |
| Jul. | 10 15 15 | 17 | 10 26 17 | 17 | 0 1 46 | 17 | 2 15 5 | 47 | 6 13 |
| Aug. | 3 16 42 | 18 | 2 28 51 | 18 | 6 12 28 | 18 | 2 23 1 | 48 | 6 21 |
| | | 19 | 7 1 25 | 19 | 9 23 9 | 19 | 2 30 58 | 49 | 6 29 |
| Sept. | 8 18 9 | 20 | 5 14 39 | 20 | 7 3 50 | 20 | 2 38 55 | 50 | 6 37 |
| Oct. | 7 8 54 | | | | | | | | |
| Nov. | 0 10 21 | | | 21 | 1 14 32 | 21 | 2 46 52 | 51 | 6 45 |
| Dec. | 11 1 7 | | | 22 | 7 25 13 | 22 | 2 54 49 | 52 | 6 53 |
| | | | | 23 | 2 5 55 | 23 | 3 2 45 | 53 | 7 1 |
| | | | | 24 | 8 16 36 | 24 | 3 10 42 | 54 | 7 9 |
| | | | | 25 | 2 27 18 | 25 | 3 18 39 | 55 | 7 17 |
| | | | | 26 | 9 7 59 | 26 | 3 26 35 | 56 | 7 25 |
| | | | | 27 | 3 18 41 | 27 | 3 34 32 | 57 | 7 33 |
| | | | | 28 | 9 29 22 | 28 | 3 42 28 | 58 | 7 41 |
| | | | | 29 | 4 10 3 | 29 | 3 50 25 | 59 | 7 49 |
| | | | | 30 | 10 20 45 | 30 | 3 58 22 | 60 | 7 57 |

In Anno Bissextili post Februarium adde unum diem, motumque ei competentem.

Tabula Motus Medii Penintimi Satellitis Saturni, à Caffino Detecti Anno 1686.

[illegible]

Tabula Motus Medii Satellitis Saturnii Medii, à Cassino detecti Anno 1673.

| Anno Christi Curr. | Epocha.
s. o. 1 | Annus. | Mot. Med.
s. o. 1 | Diebus. | Mot. Med.
s. o. 1 | Hor. Min. | Mot. Med.
Sex. o. 1. " | Min. | Motus Med.
o. 1 |
|--------------------|----------------------|--------|----------------------|---------|----------------------|-----------|---------------------------|------|--------------------|
| 1661 | ♈ 22 50 | 1 | 9 14 29 | 1 | 2 18 42 | 1 | 0 3 17 | 31 | 1 41 |
| 1681 | ♏ 16 3 | 2 | 6 28 58 | 2 | 5 7 24 | 2 | 0 6 33 | 32 | 1 45 |
| 1685 | ♈ 2 41 | 3 | 4 13 27 | 3 | 7 26 5 | 3 | 0 9 50 | 33 | 1 48 |
| 1686 | ♎ 17 10 | 4 | 4 16 38 | 4 | 10 14 47 | 4 | 0 13 7 | 34 | 1 52 |
| | | 5 | 2 1 8 | 5 | 1 3 29 | 5 | 0 16 24 | 35 | 1 55 |
| 1687 | ♏ 1 39 | | | | | | | | |
| 1688 | ♎ 16 9 | 6 | 11 15 37 | 6 | 3 22 11 | 6 | 0 19 40 | 36 | 1 58 |
| 1689 | ♎ 19 20 | 7 | 9 00 6 | 7 | 6 10 33 | 7 | 0 22 57 | 37 | 2 1 |
| 1701 | ♎ 9 15 | 8 | 9 3 17 | 8 | 8 29 35 | 8 | 0 26 14 | 38 | 2 5 |
| | | 9 | 6 17 46 | 9 | 11 18 16 | 9 | 0 29 31 | 39 | 2 8 |
| Mens. Ann. Com. | Mot. Med.
s. o. 1 | 10 | 4 02 15 | 10 | 2 6 58 | 10 | 0 32 47 | 40 | 2 11 |
| | | 11 | 1 16 45 | 11 | 4 25 40 | 11 | 0 36 4 | 41 | 2 14 |
| | | 12 | 1 19 55 | 12 | 7 14 22 | 12 | 0 39 21 | 42 | 2 18 |
| Jan. | 0 0 0 | 13 | 11 4 24 | 13 | 10 3 4 | 13 | 0 42 38 | 43 | 2 21 |
| Feb. | 9 9 37 | 14 | 8 18 54 | 14 | 0 21 46 | 14 | 0 45 55 | 44 | 2 24 |
| Mar. | 10 23 8 | 15 | 6 3 23 | 15 | 3 10 27 | 15 | 0 49 11 | 45 | 2 28 |
| April | 8 2 45 | | | | | | | | |
| | | 16 | 6 6 34 | 16 | 5 29 9 | 16 | 0 52 28 | 46 | 2 31 |
| Maii | 2 23 40 | 17 | 3 21 3 | 17 | 8 17 51 | 17 | 0 55 45 | 47 | 2 34 |
| Jun. | 0 3 17 | 18 | 1 5 32 | 18 | 11 6 33 | 18 | 0 59 1 | 48 | 2 37 |
| Jul. | 6 24 12 | 19 | 10 20 1 | 19 | 1 25 15 | 19 | 1 2 18 | 49 | 2 40 |
| Aug. | 4 3 49 | 20 | 10 23 12 | 20 | 4 13 57 | 20 | 1 5 35 | 50 | 2 44 |
| | | | | | | | | | |
| Sept. | 1 13 25 | | | 21 | 7 2 39 | 21 | 1 8 52 | 51 | 2 47 |
| Oct. | 8 4 20 | | | 22 | 9 21 20 | 22 | 1 12 8 | 52 | 2 50 |
| Nov. | 5 13 57 | | | 23 | 0 10 02 | 23 | 1 15 25 | 53 | 2 54 |
| Dec. | 0 4 52 | | | 24 | 2 28 44 | 24 | 1 18 42 | 54 | 2 57 |
| | | | | 25 | 5 17 26 | 25 | 1 21 59 | 55 | 3 00 |
| | | | | | | | | | |
| | | | | 26 | 8 6 8 | 26 | 1 25 15 | 56 | 3 4 |
| | | | | 27 | 10 24 50 | 27 | 1 28 32 | 57 | 3 7 |
| | | | | 28 | 1 13 32 | 28 | 1 31 49 | 58 | 3 10 |
| | | | | 29 | 4 2 13 | 29 | 1 35 6 | 59 | 3 13 |
| | | | | 30 | 6 20 55 | 30 | 1 38 22 | 60 | 3 17 |

In Anno Bissextili post Februarium adde unum diem, motumque ei competentem.

Tabula Mors Medii Penextimi Satellitis Saturni, ab Hugenio inventi Anno 1673.

| Anno Christi Curr. | Epoche. | Annus. | Mot. Med. | Diebus. | Mot. Med. | Hor. Min. | Mot. Med. | Min. | Motus Med. |
|--------------------|-----------|--------|-----------|---------|-----------|-----------|-----------|------|------------|
| | s. o. i. | | s. o. i. | | s. o. i. | | o. i. ii. | | ' " |
| 1641 | VS 24 43 | 1 | 10 20 41 | 1 | 0 22 35 | 1 | 0 56 | 31 | 29 10 |
| 1661 | ✠ 11 19 | 2 | 9 11 22 | 2 | 1 15 9 | 2 | 1 53 | 32 | 30 6 |
| 1681 | Υ 27 56 | 3 | 8 2 3 | 3 | 2 7 44 | 3 | 2 49 | 33 | 31 3 |
| 1685 | ✓ 13 15 | 4 | 7 15 19 | 4 | 3 0 18 | 4 | 3 46 | 34 | 31 59 |
| 1686 | III 3 56 | 5 | 6 6 00 | 5 | 3 22 53 | 5 | 4 42 | 35 | 32 55 |
| 1687 | III 24 37 | 6 | 4 26 41 | 6 | 4 15 28 | 6 | 5 39 | 36 | 33 52 |
| 1688 | II 15 19 | 7 | 3 17 22 | 7 | 5 8 2 | 7 | 5 35 | 37 | 34 48 |
| 1689 | II 28 34 | 8 | 3 00 39 | 8 | 6 0 37 | 8 | 7 32 | 38 | 35 45 |
| 1701 | II 14 32 | 9 | 1 21 20 | 9 | 6 23 12 | 9 | 8 28 | 39 | 36 41 |
| | | 10 | 0 12 1 | 10 | 7 15 46 | 10 | 9 24 | 40 | 37 38 |
| Mens. Ann. Com. | Mot. Med. | | | | | | | | |
| | s. o. i. | 11 | 11 2 42 | 11 | 8 8 21 | 11 | 10 21 | 41 | 38 34 |
| | | 12 | 10 15 58 | 12 | 9 0 55 | 12 | 11 17 | 42 | 39 31 |
| | | 13 | 9 6 39 | 13 | 9 23 30 | 13 | 12 14 | 43 | 40 27 |
| Jan. | 0 0 0 | 14 | 7 27 20 | 14 | 10 16 5 | 14 | 13 10 | 44 | 41 24 |
| Feb. | 11 9 54 | 15 | 6 18 1 | 15 | 21 8 39 | 15 | 14 7 | 45 | 42 20 |
| Mar. | 8 12 3 | | | | | | | | |
| April. | 7 21 57 | 16 | 6 1 17 | 16 | 0 1 14 | 16 | 15 3 | 46 | 43 17 |
| | | 17 | 4 21 58 | 17 | 0 23 48 | 17 | 16 0 | 47 | 44 13 |
| Maii | 6 9 16 | 18 | 3 12 40 | 18 | 1 16 23 | 18 | 16 56 | 48 | 45 10 |
| Jun. | 5 19 10 | 19 | 2 3 21 | 19 | 2 8 58 | 19 | 17 52 | 49 | 46 6 |
| Jul. | 4 6 29 | 20 | 1 16 36 | 20 | 3 1 32 | 20 | 18 49 | 50 | 47 3 |
| Aug. | 3 16 22 | | | | | | | | |
| | | | | 21 | 3 24 7 | 21 | 19 45 | 51 | 47 59 |
| Sept. | 2 26 16 | | | 22 | 4 16 42 | 22 | 20 42 | 52 | 48 56 |
| Oct. | 1 13 35 | | | 23 | 5 9 16 | 23 | 21 38 | 53 | 49 52 |
| Nov. | 0 23 29 | | | 24 | 6 1 51 | 24 | 22 35 | 54 | 50 49 |
| Dec. | 11 10 48 | | | 25 | 6 24 25 | 25 | 23 31 | 55 | 51 45 |
| | | | | 26 | 7 17 00 | 26 | 24 27 | 56 | 52 42 |
| | | | | 27 | 8 9 35 | 27 | 25 24 | 57 | 53 38 |
| | | | | 28 | 9 2 9 | 28 | 26 20 | 58 | 54 35 |
| | | | | 29 | 9 24 44 | 29 | 27 17 | 59 | 55 31 |
| | | | | 30 | 10 17 18 | 30 | 28 13 | 60 | 56 27 |

In Anno Bissextili post Februarium adde unum diem, motumque ei competentem.

Tabula

Tabula Mediorum Motuum Extimi Satellitis Saturnii, à Cassino Detecti Anno 1671.

| Anno Christi
Curr. | Epocha.
s. ° ' | Annis.
— | Mot. Med.
s. ° ' | Diebus.
— | Mot. Med.
s. ° ' | Hor. Min.
— | Mot. h. e.
° / " | Min.
— | Mot. Me.
° / " |
|-----------------------|---------------------|-------------|---------------------|--------------|---------------------|----------------|---------------------|-----------|-------------------|
| 1661 | ☿ 24 45 | 1 | 7 6 23 | 1 | 0 4 32 | 1 | 0 11 | 31 | 5 51 |
| 1681 | ♄ 25 15 | 2 | 2 12 47 | 2 | 0 9 5 | 2 | 0 23 | 32 | 6 3 |
| 1685 | ♄ 25 21 | 3 | 9 19 10 | 3 | 0 13 37 | 3 | 0 34 | 33 | 6 14 |
| 1686 | ♄ 1 44 | 4 | 5 00 6 | 4 | 0 18 9 | 4 | 0 45 | 34 | 6 25 |
| | | 5 | 0 6 29 | 5 | 0 22 41 | 5 | 0 57 | 35 | 6 37 |
| 1687 | ♄ 8 7 | — | — | — | — | — | — | — | — |
| 1688 | ♄ 14 31 | 6 | 7 12 53 | 6 | 0 27 14 | 6 | 1 8 | 36 | 6 48 |
| 1689 | ♄ 25 27 | 7 | 2 19 16 | 7 | 1 1 46 | 7 | 1 19 | 37 | 7 00 |
| 1701 | ♄ 25 45 | 8 | 10 0 12 | 8 | 1 6 18 | 8 | 1 31 | 38 | 7 11 |
| | | 9 | 5 6 35 | 9 | 1 10 50 | 9 | 1 42 | 39 | 7 22 |
| | | 10 | 0 12 59 | 10 | 1 15 23 | 10 | 1 53 | 40 | 7 34 |
| Mens.
Ann.
Com. | Mot. Med.
s. ° ' | — | — | — | — | — | — | — | — |
| | | 11 | 7 19 22 | 11 | 1 19 55 | 11 | 2 5 | 41 | 7 45 |
| | | 12 | 3 0 18 | 12 | 1 24 27 | 12 | 2 16 | 42 | 7 56 |
| Jan. | 0 0 0 | 13 | 10 6 41 | 13 | 1 28 59 | 13 | 2 27 | 43 | 8 8 |
| Feb. | 4 20 41 | 14 | 5 13 5 | 14 | 2 3 32 | 14 | 2 39 | 44 | 8 19 |
| Mar. | 8 27 45 | 15 | 0 19 28 | 15 | 2 8 4 | 15 | 2 50 | 45 | 8 30 |
| April | 1 18 25 | — | — | — | — | — | — | — | — |
| | | 16 | 8 0 24 | 16 | 2 12 36 | 16 | 3 1 | 46 | 8 42 |
| Maii | 6 4 34 | 17 | 3 6 47 | 17 | 2 17 8 | 17 | 3 13 | 47 | 8 53 |
| Jun. | 10 25 15 | 18 | 10 13 11 | 18 | 2 21 41 | 18 | 3 24 | 48 | 9 4 |
| Jul. | 3 11 23 | 19 | 5 19 34 | 19 | 2 26 13 | 19 | 3 35 | 49 | 9 16 |
| Aug. | 8 2 4 | 20 | 1 0 30 | 20 | 3 0 46 | 20 | 3 47 | 50 | 9 27 |
| | | — | — | — | — | — | — | — | — |
| Sept. | 0 22 45 | — | — | 21 | 3 5 18 | 21 | 3 58 | 51 | 9 38 |
| Oct. | 5 8 53 | — | — | 22 | 3 9 50 | 22 | 4 9 | 52 | 9 50 |
| Nov. | 9 29 34 | — | — | 23 | 3 14 22 | 23 | 4 21 | 53 | 10 1 |
| Dec. | 2 15 43 | — | — | 24 | 3 18 54 | 24 | 4 32 | 54 | 10 12 |
| | | — | — | 25 | 3 23 27 | 25 | 4 43 | 55 | 10 24 |
| | | — | — | — | — | — | — | — | — |
| | | — | — | 26 | 3 27 59 | 26 | 4 55 | 56 | 10 35 |
| | | — | — | 27 | 4 2 31 | 27 | 5 6 | 57 | 10 46 |
| | | — | — | 28 | 4 7 4 | 28 | 5 17 | 58 | 10 58 |
| | | — | — | 29 | 4 11 36 | 29 | 5 29 | 59 | 11 9 |
| | | — | — | 30 | 4 16 8 | 30 | 5 40 | 60 | 11 21 |

In Anno Bissextili post Februarium ad-
de unum diem, motumque ei com-
petentem.

I shall only add that the Proportion of the Squares of the *Times* of the *Periods* to the Cubes of the *Distances*, (which is proposed as probable by *Kepler*, but now demonstratively found true by Mr. *Newton*,) gives us nicely the Proportion of the *Distances* of these *Planets* from the Center of *Saturn*; and supposing the *Satellite* of *Hugens* four *Diameters* of *Saturn's Ring* distant from him, we shall find by the *Periods* the *Distances* as follows.

| | Periodus. | | | Distantiæ. |
|------------------------|-----------|----|------------------|------------|
| | d. | h. | ' | Diam. |
| <i>Intimi</i> ----- | 1 | 21 | 18 $\frac{1}{2}$ | 0, 964 |
| <i>Penintimi</i> ----- | 2 | 17 | 41 $\frac{1}{2}$ | 1, 235 |
| <i>Medii</i> ----- | 4 | 13 | 47 $\frac{1}{2}$ | 1, 740 |
| <i>Penextimi</i> ----- | 15 | 22 | 41 | 4, 000 |
| <i>Extimi</i> ----- | 79 | 7 | 54 | 11, 621 |

These *Distances* may be used, as more accurate than those obtained by Observation, which yet differ but little therefrom.

The Phasis of
Jupiter; by
Dr. Hook.
n. 14. p. 245.
Fig. 139.

LXXXIII. An. 1666. Jun. 26. between 3 and 4 of the Clock in the Morning, I observed the Body of *Jupiter* through a 60 Foot Glass, and found the apparent Diameter of it through the Tube, to be somewhat more than two Degrees, that is about 4 times as big as the Diameter of the *Moon* appears to the naked Eye.

I saw the Limb pretty round, and very well defined without Radiation. The parts of the *Phasis* of it had various Degrees of Light. About *a*, and *f*, the North and South Poles of it, somewhat darker, and by degrees it grew brighter towards *b* and *e*, two Belts or Zones; the one of which, *b*, was a small dark Belt crossing the Body Southward; Adjoining to which was a small Line of a somewhat lighter Part; and below that again, Southwards, was the Great black Belt *c*. Between that, and *e*, the other smaller black Belt, was a pretty large and bright Zone, but the Middle *d*, was somewhat darker than the Edges.

The Revolution
of Jupiter upon
his Axis; by
S. Campani.
n. 1. p. 3.
By Dr. Hook.
ib.

LXXXIV. 1. S. Campani affirms that, by the Goodness of his Glasses, he hath Observed certain *Protuberances* and Inequalities of *Jupiter*: and he is now Observing whether they do not change their Situation.

2. An. 1664. May 9. about 9 a Clock at Night, Mr. Hook with an Excellent 12 Foot Telescope observed a small Spot in the biggest of the three Obscure Belts of *Jupiter*, and Observing it from time to time, he found that within two Hours after, the said Spot had moved from East to West, about half the Length of the Diameter of *Jupiter*.

3. Eustachio de Divinis pretends, that the Permanent Spot in *Jupiter* hath been first of all discover'd with his Glasses; that P. Götignies is the first that hath

hath thence deduced the Motion of *Jupiter* about his *Axis*; and that M. *Cassini* at first opposed it: But that Spot was observed in England a good while before.

4. There are two Sorts of Spots at certain times to be seen in the Disque of *Jupiter*. One Sort are nothing but the Shadows of his Satellites: but the other have some resemblance to those that are seen in the Moon: and they are perhaps of the same Nature with those that are called Belts. They Move from the Eastern to the Western Limb; their apparent Motion is unequal and swifter near the Center than the Circumference; and they never are so well seen as when they approach to the Center; they being very narrow and almost imperceptible when they approach to the Circumference; which makes us believe, that they are flat and superficial to *Jupiter*.

Among these Spots there is none so sensible, as that which is situated in the Northern-part of the Southern-Belt. Its Diameter is about the tenth part of that of *Jupiter*; and at the time that its Center is nearest to that of *Jupiter*, it is Distant from it about the third part of the Semidiameter of that Planet.

M. *Cassini*, after he had made many Observations of this Spot during the Summer of the Year 1665, found, that the Period of its apparent Revolution is of 9^h 56'. He continued to Observe it till the Beginning of 1666, when *Jupiter* approach'd to the Beams of the Sun: But after he was got free of the Sun-Beams it was difficult to be discern'd. This gave grounds that it might be of the Nature of the Spots of the Sun (which after having appeared for a while, disappear for ever) M. *Cassini* ceased at length to observe them.

But Jan. 19. 1672. (st. n.) when he observ'd *Jupiter*, at 4^h in the Morning, he perceived in the same Place of his Disque the Figure of the same Spot, adhering to the same Southern-Belt. It was already gone beyond the Moiety of this Belt, and he saw it advance little by little towards the Western Limb, to which it seemed to be very near at 6^h.

By the Celerity of its Motion near the Center, and by the Place where he had begun to see it, he judged that it might have been in the midst of the Belt at 4^h 35' in the Morning. And as he prepared himself to make Ephemerides of its Motion for that Year 1672, he perceived that in those he made for the Year 1666, this Spot had been in the midst of *Jupiter* the same Day, namely the 19th of Jan. at the same Hour. So that in six Years, of which one is a Bissextile, it is found to have made, in respect of the Earth, at least 5294 Revolutions, each of 9^h 55' 85'', compensating one Revolution by another; and at most 5294 Revolutions of 9^h 55' 51''; forasmuch as he was assured of the preciseness of one Mean Revolution to one eighth of a Minute.

Until then he had never seen an immediate Return of this Spot after 9 hours and 56 minutes, because it had not happened, that *Jupiter* after the Apparition of the Spot had stayed, in one and the same Night, long enough above the Horizon, at least a sufficient height, to observe him with due Distinctness. He had only concluded the time of this Revolution by Returns observed after about 20, 30, and 50 hours; and he had more precisely limited it by Observations more distant. But the Night after Mar. 1, at 7^h in the Evening, he saw this Spot in the midst of the Belt; and the same Night at

5^h 26'. in the Morning, he saw it again returned precisely to the same Place. Mar. 3. He together with M. Buor and M. Mariotte, began to see at 8^h 4' the Spot already somewhat removed from the Oriental Limb, but yet obscure and small. At 8^h 47'. they saw it very distinctly advancing towards the middle of the Belt. At 9^h 15' 40". until 9^h 8'. they saw it in the midst of the Belt. At 9^h 15'. it was past the middle, and was come nearer to the Occidental Limb. And a little after the Heavens being over-cast, he could observe it no further.

Places of Jupiter observed;
by Mr. Flamsteed at Derby.
n. 82. p. 4036.

LXXXV. 1. An. 167 $\frac{1}{2}$. Feb. 16. 7^h 44 $\frac{1}{2}$ '. Alto Jove 18° 10'. ejus Distantiam à Fixa Lucis 4^{re} (cujus Lat. 1° 40' Bor. Locus Mihi π 14° 7' 16". at Streetio, 14° 3' 54") Tubo Longiori dimensus sum, 16' 33". & Differentiam Altitudinum Centrorum μ & π 1' 1".

17 Feb. 7^h 25'. p. m. Alto μ 15° 54'. ipse à Fixa distatit 21' 50"; Altitudinum differentia erat 8' 40".

18 Feb. 7^h c'. Fixa distantia à Centro μ erat 28' 15"; Altitudinum differentia circ. 15' 29". In utraque Observatione Erro Altior erat Fixa, à qua semper Meridianum versus Stetit.

Inito dein Calculo ad dies singulos & horas observationum, investigavi

| | Februar. | d. h. ' | d. h. ' | d. h. ' |
|---|----------|--------------------|---------|---------|
| | 16 | 7 44 $\frac{1}{2}$ | 17 | 7 25 |
| Jovis à Fixa Longitudinem in Antecedentia | 0 | 7 11 | 0 | 1 11 |
| Latitudinem ad Austrum ab ea | 0 | 9 16 | 0 | 17 22 |
| Ergo μ Latitudo Borealis | 0 | 13 30 | 0 | 13 14 |
| Locus verus π { Mihi | 1 | 26 30 | 1 | 26 46 |
| { Streetio. | 13 | 58 0 | 13 | 49 54 |
| | 13 | 54 38 | 13 | 46 32 |
| | | | 13 | 38 42 |

2. Mar. 15. v. Observare coepi Jovis distantias & Positiones à Stella ϵ 38. cujus Latitudo 10° 20 $\frac{1}{2}$ ' Bor. Locus Streetio π 9° 54' 0"; mihi vero π 9° 57' 20". 7^h 25'. p. m. Alto μ 32° 52'. Distantia Centri Ipsius ab ipsa 33' 50". Altitudinum Differentia circit. 20' 42".

Mar. 16. 7^h. 8'. Alta Fixa 36°. Jovis ab ea distantia erat 27'. 7". Altitudo minor 16' 3".

Mar. 19. Alto μ 29° 35'. i. e. 6^h 45'. Fixa Altior erat quam Planeta 2' 24"; à quo 6^h 55'. distatit. 10' 21".

Hor. 7. 11. Saetles 4^{tus} à Fixa, 7' 28". Etiamnum Erro semper altior apparuit, sed verè fuit depressior quam Fixa: postea humilior visus est, sed revera fuit Altior.

Mar. 20. melius præparato ad Altitudinum Differentias capiendas Micrometro, observationes habui (sic putem) accuratissimas, quæ sequuntur.

Jov.

| <i>Jov. Alt.</i> | <i>Temp.</i> | | |
|------------------|-----------------------|---|------|
| | h. | | |
| 30 | 0 6 44 | <i>Jovis Centrum vere altius erat Stella</i> | 2 13 |
| 30 | 47 6 51 $\frac{1}{2}$ | <i>Altitudinum eadem differentia rursus capta</i> | 2 14 |
| 32 | 0 6 59 | <i>Jovis Centrum à Fixa distat</i> | 7 0 |
| 38 | 30 7 54 $\frac{1}{2}$ | <i>Centrum Jovis verè altius Fixa</i> | 3 14 |
| 40 | 50 8 18 $\frac{1}{2}$ | <i>Altitudinum differentia denuo capta</i> | 3 42 |
| | | <i>Et deinde Centrorum distantia erat</i> | 7 5 |

Ad Locum *Jovis* ex his Observationibus acquirendum supputavi, ad

| | h. ' " | h. ' " |
|--|---------|---------|
| | 6 51 30 | 8 18 40 |
| | 0 ' " | 0 ' " |
| <i>Angulos Circuli Verticalis cum Ecliptica</i> | 35 39 — | 46 15 — |
| <i>Jovis erat à Fixa distantia</i> | 0 7 0 | 0 7 0 |
| <i>Altitudinum differentia</i> | 0 2 14 | 0 3 42 |
| <i>Ergo μ erat in consequentia Fixe</i> | 0 2 3 | 0 1 44 |
| <i>Cum Latitudine majori</i> | 0 6 42 | 0 6 47 |
| <i>Quare Jovis Latitudo vera</i> | 1 27 12 | 1 27 17 |
| <i>Locus verus μ { Streetio</i> | 9 56 3 | 9 55 44 |
| <i>Alibi</i> | 9 59 23 | 9 59 4 |

| <i>Maii.</i> | <i>Temp.</i> | <i>Alt. Jov.</i> | | |
|----------------|--------------|------------------|--|---------|
| d. | h. | | | |
| 24 | 10 00 | 14 10 | <i>Humilior erat Jovis Centrum quam</i> | 0 ' " |
| | | | <i>Stella dicta Ω 38'</i> | 0 7 46 |
| | | | <i>A qua semel ejus cepi distantiam</i> | 0 20 00 |
| | | | <i>deinde</i> | 0 19 54 |
| 26 | 8 46 | 33 30 | <i>Distantia centri μ à Fixa</i> | 0 10 4 |
| <i>Caro</i> | 9 00 | 31 50 | <i>Fixa altior erat quam μ</i> | 0 6 30 |
| <i>sereno.</i> | 9 20 | 29 10 | <i>Altitudinum differentia erat</i> | 0 6 38 |
| | 9 33 | 27 20 | <i>Differ. Azymuthorum μ & Fixe</i> | 0 7 19 |
| | 9 36 | 27 00 | <i>Distantia μ à Fixa denuo capta</i> | 0 10 2 |

N. 86. p. 5037.

*

| | | | | | | | | | |
|--------------------|----|----|----|----|--------------------------------------|---|----|----|---|
| 27. | h. | ' | " | ' | Centri ν à <i>Fixa</i> distantia | o | ' | " | * |
| <i>Ventis</i> | 8 | 59 | 31 | 24 | Azymuthorum differentia | 0 | 1 | 50 | |
| <i>inter-</i> | 9 | 7 | 30 | 30 | Eadem rursus differentia | 0 | 1 | 52 | * |
| <i>dum tu-</i> | 9 | 16 | 29 | 13 | Centri μ à <i>Fixa</i> distantia | 0 | 6 | 1 | |
| <i>bam</i> | 9 | 23 | 28 | 10 | | | | | |
| <i>moven-</i> | | | | | | | | | |
| <i>tibus.</i> | | | | | | | | | |
| 28. | h. | ' | " | ' | A <i>Fixa</i> distantia centri μ | 0 | 6 | 2 | * |
| <i>Ventis</i> | 8 | 57 | 31 | 8 | Differentia Azimuthorum centri, &c | 0 | 3 | 29 | |
| <i>valide</i> | 9 | 19 | 28 | 10 | Isthæc Differentia rursus | 0 | 3 | 32 | |
| <i>flantibus</i> | 9 | 31 | 26 | 30 | Distantia rursus | 0 | 6 | 7 | |
| <i>& T le-</i> | 9 | 34 | 26 | 8 | | | | | |
| <i>scopium</i> | | | | | | | | | |
| <i>concuti-</i> | | | | | | | | | |
| <i>entibus.</i> | | | | | | | | | |
| 30. | h. | ' | " | ' | <i>Jovis</i> distantia à <i>Fixa</i> | 0 | 15 | 38 | |
| <i>Cælo sa-</i> | 9 | 36 | 26 | 15 | | | | | |
| <i>tis fere-</i> | | | | | | | | | |
| <i>no.</i> | | | | | | | | | |

Hæc & præcedente nocte in Consequentia *Fixa* erat, antea semper in Antecedentia.

Harum observationum certissimas habeo, quas asterisco (*) notavi Azymuthorum differentias; diebus 27 & 28 observatas, nimis strictas acceptas æstimo, ob vacillationem Tubi, quapropter, quando non ut volui eas accuratè dimeriri licebat, ne nimis amplas caperem curabam.

Ad *Jovis* locum ab his observationibus obtinendum supputavi.

| | d. | h. | ' | " | d. | h. | ' | " | d. | h. | ' | " |
|---|-------|----|----|----|---------|----|----|---------|----|----|----|---|
| | Maii. | 26 | 0 | 0 | 27 | 9 | 7 | | 28 | 9 | 19 | |
| <i>Angulos Parallacticos sive Circuli Ver-</i> | | | | | | | | | | | | |
| <i>itcalis cum Ecliptica.</i> | | 80 | 47 | — | 79 | 49 | — | 78 | 36 | — | | |
| <i>Jovis à Fixa distantia observata</i> | | 9 | 10 | 4 | 9 | 6 | 2 | 0 | 6 | 3 | | |
| <i>Differentia abs. Alt.</i> | | 0 | 6 | 30 | Azym. 1 | 52 | | Azym. 3 | 30 | | | |
| <i>Ergo Angulus Positionis</i> | | 31 | 1 | — | 62 | 38 | — | 66 | 3 | 0 | | |
| <i>Et Planeta in antecedentia Fixa</i> | | 0 | 8 | 38 | 0 | 2 | 51 | conf. 2 | 27 | | | |
| <i>Cum minori Latitud. Boreali</i> | | 0 | 5 | 11 | 0 | 5 | 19 | 0 | 5 | 32 | | |
| <i>Fixa Latitudo Tythonica Bor.</i> | | 1 | 20 | 30 | 1 | 20 | 30 | 1 | 20 | 30 | | |
| <i>Fixa Locus</i> π <i>Streeterio</i> | | 9 | 54 | 9 | 9 | 54 | 9 | 9 | 54 | 9 | | |
| <i>Mihi</i> | | 9 | 57 | 30 | 9 | 57 | 30 | 9 | 57 | 30 | | |
| <i>Jovis itaque Locus</i> π <i>Streeterio</i> | | 9 | 45 | 31 | 9 | 51 | 18 | 9 | 56 | 36 | | |
| <i>Mihi</i> | | 9 | 48 | 52 | 9 | 54 | 39 | 9 | 59 | 57 | | |
| <i>Cum Latitudine Boreali</i> | | 1 | 15 | 19 | 1 | 15 | 11 | 1 | 14 | 58 | | |

3. *An.* 1673. *Mar.* 13. vesp. *Jupiter* *Aphelium* *Pronus* ad *Phasin* *Acroni* n. 64 p. 6033-
cam, & *Limitem* *Orbitæ* *Boreum* paululum transgressus, *Retrogradus* incescit
versus 9am M *Lucis* *quarta*, à qua (Alto eo sex circiter gradus) *Limbi* ejus
Remotissimi distantiam, septempedalibus *Tubo* & *Micrometro* *Townleiano* cepi
4560 52' 34".

Mar. 17. circa horæ dimidiam post exortum *Jovis* ejus eodem *Tubo*,
Limbi *Remotissimi* à *Fixa* cepi iterum distantiam 2073 = 23' 54".

Mar. 20. Sequentes habui observationes, *Primam* *Breviori* *Tubo*, *digito-*
rum tantum 85, reliquas *Longiori*, viz. 164. dig.

| | Fix. Alt. | Hor. Sup. | | Limbo. ph. | Cent. |
|---|-----------|-----------|---|------------|-------|
| | ° / | h. / | | / // | / // |
| 1 | 6 0 | 7 14 | <i>Limbi</i> U remotior. à <i>Fixa</i> dist. 850 | 9 48 | 9 24 |
| 2 | | | Eadem dist. <i>Tubo</i> Long. capta 1650 | 9 52 | 9 28 |
| 3 | 14 40 | 8 16 | <i>Limbo</i> U Inf. depress. ac <i>Fixa</i> 784 | 4 41 | 4 17 |
| 4 | 15 40 | 8 23 | Alt. eadem repetita <i>Differen.</i> 786 | 4 41 | 4 17 |
| 5 | | | <i>Jovis</i> <i>Diameter</i> ———— 135 | 0 48 | |
| 6 | 16 25 | 8 26 | <i>Limbo</i> . iterum capta distantia 1665 | 9 57 | 9 33 |
| 7 | | | <i>Denuo</i> ———— 1658 | 9 54 | 9 30 |
| 8 | 19 00 | 8 50 | <i>Differ. Alt. Limbi Jov. & Fixe</i> 138 | 5 00 | 4 36 |

Mar. 26. *Vesp.* Alto *Jove* 15° 50'. *Limbi* sui remotioris à *Fixa* distan-
tiam, eodem minori *Tubo* dimensus sum 4205 = 48' 30".

Ad *Planeta* *Locum* ex his annotationibus eliciendum, structis supputationi-
bus, invenio.

| | d. h. / | h. / |
|--|---------|---------|
| <i>Mar.</i> | 20 8 16 | — 8 50 |
| | ° / // | ° / // |
| <i>Angulum</i> <i>Parallacticum</i> ———— | 34 44 0 | 37 30 0 |
| <i>Centrum Jovis</i> à <i>Fixa</i> distat ———— | 0 9 38 | 0 9 30 |
| <i>Altitudinum</i> <i>differentia</i> erat ———— | 0 4 17 | 0 4 36 |
| Ergo, <i>Angulus</i> erat <i>Positionis</i> ———— | 80 6 0 | 78 21 0 |
| Et <i>Jupiter</i> in antecedentia <i>Fixe</i> ———— | 0 1 38 | 0 1 55 |
| Cum <i>Latitudine</i> <i>Minori</i> ———— | 0 9 19 | 1 9 18 |

Fixa mihi *Locus*, accepto motu *Annuo* 50'', erit $\approx$ 13° 37' 11". quem
vult *Author Carolinus* 13° 33' 47". *Latitudo* ejus *Borea*. 1° 45'. *Locus*
ergo vetus *Jovis* erit *Mihi*.

8h 16' $\approx$ 132 35' 33" } *Latitudo* { 10 35' 40"
8 50 $\approx$ 13 35' 16" } vera { 1 35' 42"

At *Fixa* concesso Loco *Carolino*, prodibit μ^{us} Locus $\approx 13^{\circ} 32' 09''$.

Loco sic *Planeta* & Latitudine perceptis *Orbita* *Jovialis* & *Terrestri* *Orbita* Planum Inclinationem inde eruere conabimur.

Huic equidem inveniendae, una cum Loco *Solis*; ejusdem, *Jovis* & *Terra* intermutuae distantiae postulantiur: quas à Tabulis quibuscvis probatoribus tutissime haurire licet: Ego Tabulis utor plerumque *Carolinis*; quippe quas, accuratiores, & faciliores cæteris omnibus comperi; ex quibus ad 8^{a} 16 p. m. deprompsi;

| | | |
|---------------------------------------|-------|------------------------------------|
| <i>Solis</i> locum verum | _____ | γ . $10^{\circ} 40' 18''$. |
| distantiam à <i>Terra</i> | _____ | 100084. |
| <i>Jovis</i> à <i>Sole</i> distantiam | _____ | 544921. |
| à <i>Terra</i> | _____ | 444952. |

Fig. 140.

Jam in apposita Figura sint, S *Sol*, T *Terra*, μ *Planeta*, S E Radius *Eclipticae*, ad μ^{us} *Orbitam* protensa, & Angulus μ T S, visa *Planeta* à *Terra* Latitudo $1^{\circ} 35' 40''$.

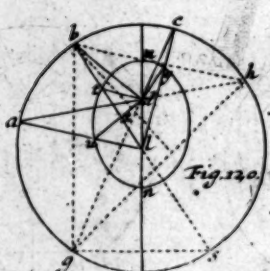
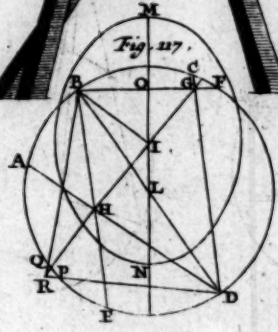
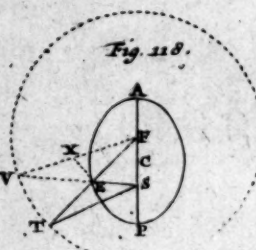
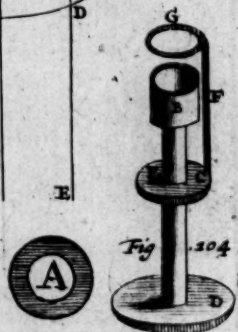
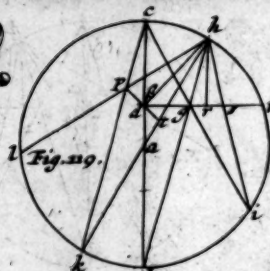
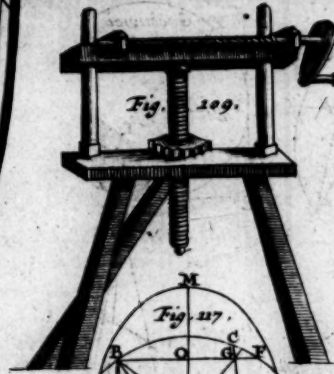
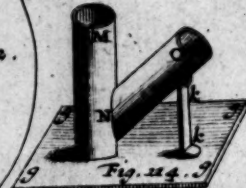
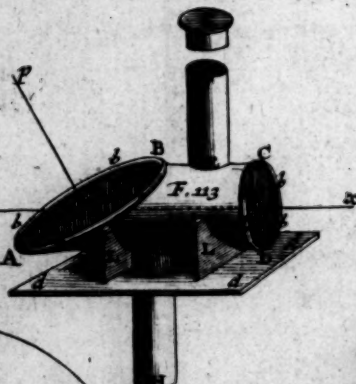
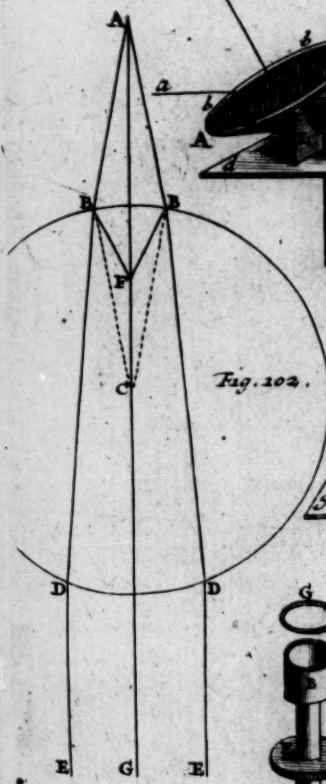
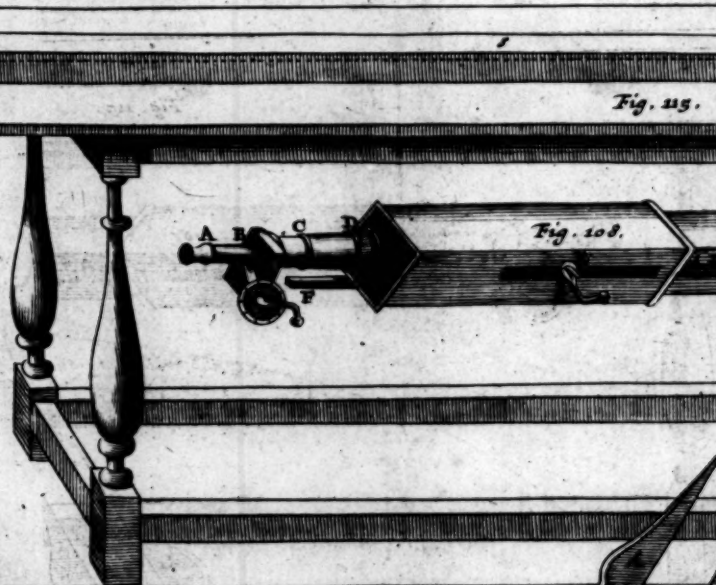
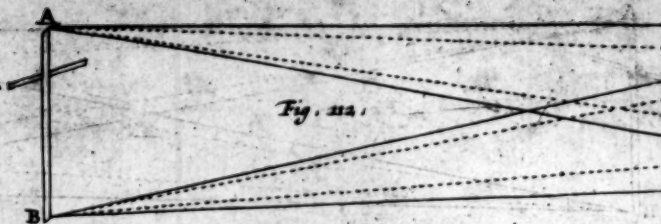
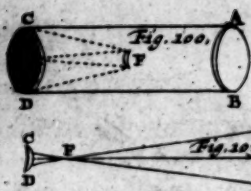
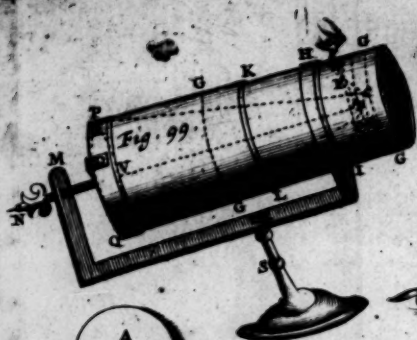
Ex datis (in Triangulo μ S T) Angulo, μ T S, visa Latitudinis ad Circulum complemento, μ S, & μ T, *Planeta* à *Sole* & *Terra* Distantiis, ut supra repertis, eruetur Angulus μ S E, Latitudo sive Inclinatione *Planeta* à *Sole* conspecta $1^{\circ} 18' 7''$.

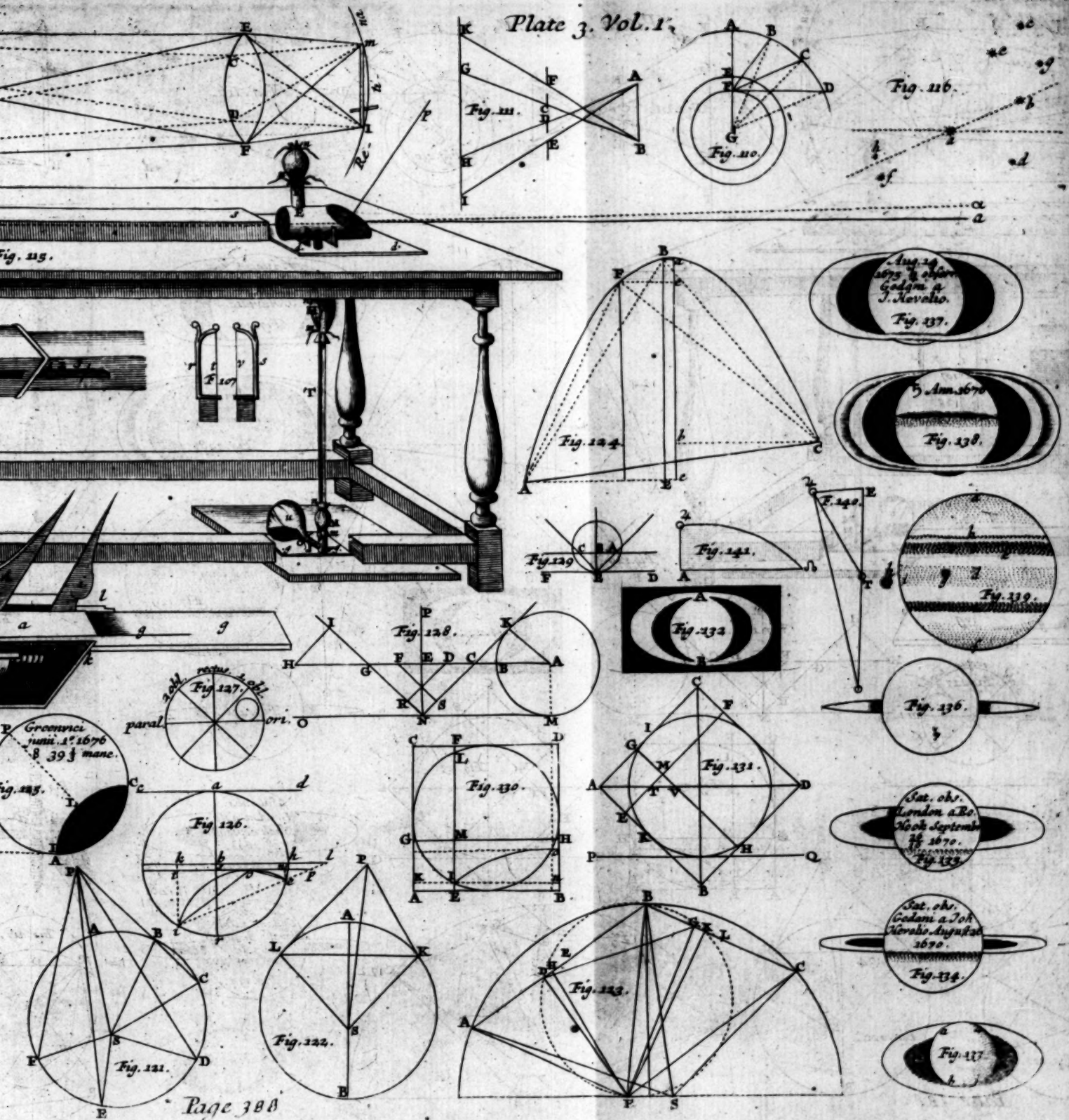
Jovis Locus Geo-centricus erat, $\approx 13^{\circ} 35' 33''$, ab iis ergo datis μ & *Terra* à *Sole* distantis, inveniatur Locus Helio-centricus *Planeta* $\approx 13^{\circ} 03' 33''$; de quo subductis figillatim iis *Nodi* Locis, quo Autores, quorum Nomina in sequenti Tabella exaravimus, assumpserunt, annexa produnt Argumenta Latitudinis; de quibus videre est, nullis plus *Jovem* à Limite promotum haberi quam $6^{\circ} 29' 59''$, nec minus quam $3^{\circ} 58' 59''$. quæ quantavis videtur differentia in maximâ *Orbita* Inclinatione investiganda, Errorum scrupulis secundis $23''$ Majorem inferre nequit.

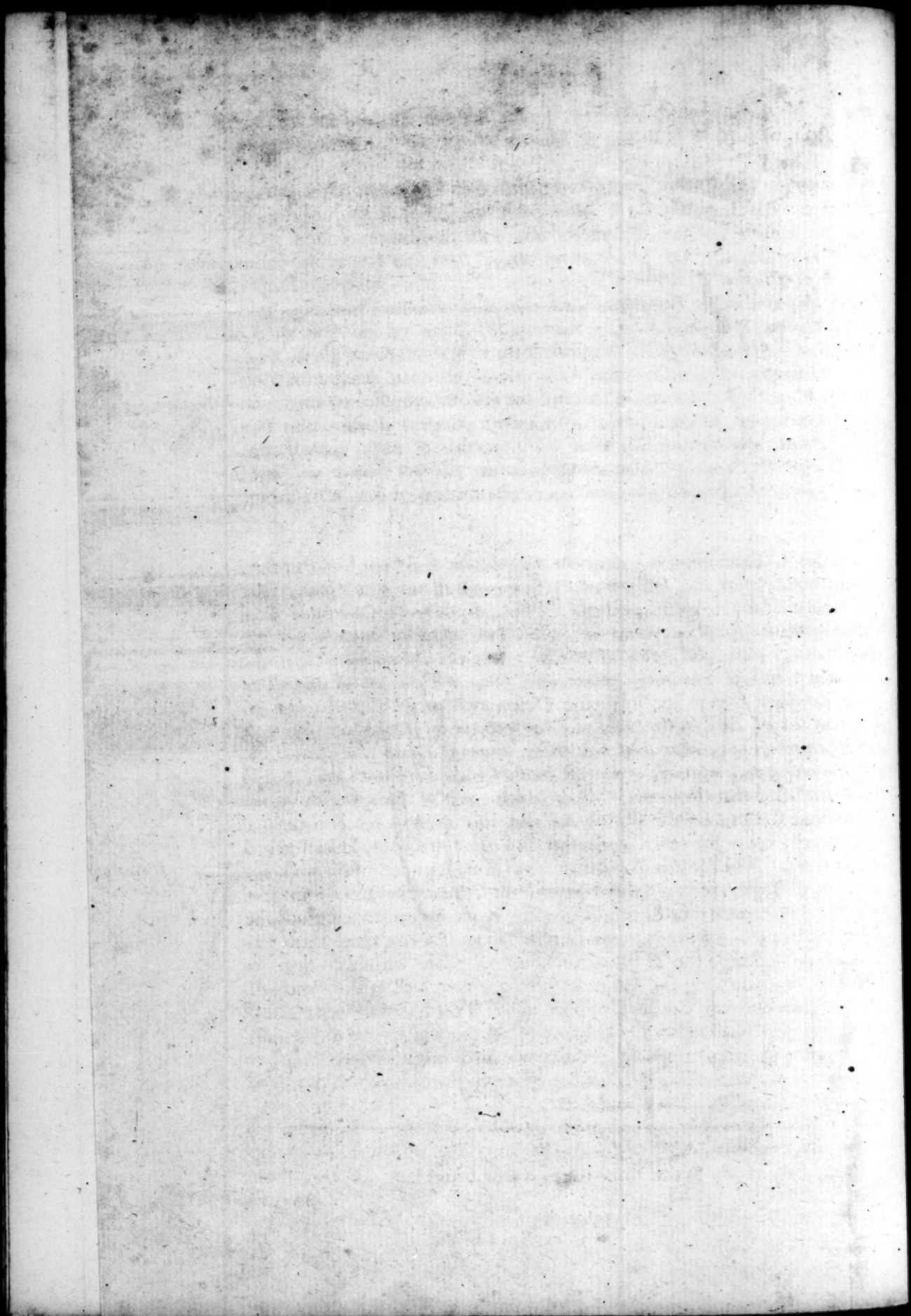
| Auctores. | Ω Loca. | | | Argumenta Latitudinis. | | |
|-------------|----------------|---|-------|------------------------|---|-------|
| | s. | o | '' | s. | o | '' |
| Keplerus. | 3 | 6 | 33 37 | 3 | 6 | 29 56 |
| Streetius. | 3 | 6 | 33 47 | 3 | 6 | 29 46 |
| Wingius. | 3 | 7 | 11 39 | 3 | 5 | 21 54 |
| Ricciolus. | 3 | 7 | 18 00 | 3 | 5 | 45 33 |
| Cassinus. | 3 | 8 | 45 00 | 3 | 4 | 18 33 |
| Bullialdus. | 3 | 9 | 4 34 | 3 | 3 | 58 59 |

Fig. 141.

Iste *Nodi* Locus, quem Cl. *Cassinus* elegerit mihi etiam si aliquantulum justo promotior videtur, Magis tamen cæteris variis de causis, placet: sumptis propterea, Triangulo μ A Ω , Argumento Latitudinis, Ω A, $94^{\circ} 18' 33''$. & Inclinatione μ A, $1^{\circ} 18' 07''$, eruetur Angulus Inclinationis Plani *Orbita* *Jovialis*







Fovialis ad *Eclipticam* $1^{\circ} 18' 20''$. quem statuunt *Keplerus* $1^{\circ} 19' 00''$. *Streetius*, $1^{\circ} 20' 00''$. *Beillivaldus* & *Wingius* $1^{\circ} 21' 48''$. omnes justo non-nihil Majorem.

Tantamque esse Inclinationem, vel saltem non Majorem, cum hesternæ noctis, tum Mensium *Februarii*, *Martii*, & *Maii*, *Anni Elapsi*, observationes fuadent. Interea verò non dissimulandum, posse & majorem (scilicet $1^{\circ} 20' 20''$.) à transitu $4'$ prope 8^{mo} *Mo*. *Anno* 1649. *Maii*. 29 & 30. *St. Juliano* *Almag. Nov.* *Bonia* & *Majorca* à *Ricciolo* & *Muto*, Viris Doctissimis, observato Demon-
strari: id quod nobis (siquidem Orbitalium Inclinationes ab omnibus inva-
riabiles habentur.) videtur innuere, Errorem vel huic, vel illis *Fixarum* *Le-*
titudinibus à *Tychono* assignatis, inesse aliquem: quæ propterea donec accu-
ratius restituantur, à præcisa hujus Inclinationis quantitate determinanda me-
rito nos arcent: Hoc tantum, quoniam *Fixarum* ex Latitudines etiam Immu-
tabiles reperiuntur, ausim affirmare; Angulum Maximæ Inclinationis *Plani*
Orbitæ Fovialis ad *Eclipticam* Minorem esse scrupulis $26' 40''$. quam Latitu-
do *Stellæ* 9^{m} *Lucis* $4^{\text{æ}}$ quæ *Tychoni* dicitur, *Ultima* *quatuor* in *sinistra*
Ala Virginis: quæ propterea si quando correctæ dabitur, eadem certa dabitur
Inclinatio.

LXXXVI. Examining our Ancient *Ephemerides* I do not find that three *The Conjunctions*
Conjunctions of *Saturn* and *Jupiter* have ever happened in one Year's space, since *ons of Saturn*
they were first in use to this present. Those of *Moleti* Calculated from *and Jupiter.*
the *Alphonse* *Tables* indeed make three in the space of eight Months betwixt *An. 1682. and*
August 1563, and *April* 1564. Inclusive; but the *Ephemerides* of *Stradi* *1683. at*
Calculated from the *Prutenick*, make only one, and the 26 of *August* of *Greenwich;*
which *Junctinus* gives us the following Observation in the Preface to his *by Mr. Flam-*
Astronomical Tables, *An. 1563. Aug. 24 14 30. p. m. Aurange*, *Jupiter* à *steed. n. 149.*
parte Septentrionis cooperiebat quasi Saturnum, qui erat à parte Meridionali, et *p. 244.*
traque autem horum cellarum, in fine 28 gradus Canceri deprehendebatur. *Riccioli*
hence concludes, that the Planet covered some part of *Saturn* at this time.
But without Reason, for the Words *quasi cooperiebat* intimate not that the one
did corporally cover the other, but rather that there was some small Interval
betwixt them. The *Caroline Tables* make the visible Latitude of *Saturn* now,
 $11^{\circ} 45'$ of *Jupiter* $20' 10''$. both North, the Conjunction being some few
days past: but because their Latitudes alter slowly we may hence conclude the
Difference $8' 25''$. to have been nearly their Distance at that time, these Ta-
bles being grounded on the *Tychonick* Observations made within less than 40
Years after, and shewing the Latitudes of the *Planets* well at this time near
100 Years later we may Conclude to have answer'd them as well then; and if
we consider how small a space the Distance of $8'$ minutes appears to the nak-
ed Eye in the Heavens, especially betwixt two such bright Planets as *Saturn*
and *Jupiter* are, that the *Caroline* Distance agrees very well with the Words of
Junctinus, and that *Riccioli* was mistaken.

Their next Conjunction according to *Maginus's Ephemerides* founded on
the *Prutenick* Numbers, was *April* 29. 1583 in 21° Deg. of γ , the *Sun* being
then in 17° Deg. of γ , so that that Planet's Rising before him in Signs of short
Ascensi-

Ascension and with South Latitude this *Congress* could not be observed by the Noble *Tycho* who was mindful of it, as appears by this Note in Page 55 of his *Historia Cælestis*. May 30 A. M. *quo primum post Conjunctionem Saturnum vidimus, capta sunt distantia inter Jovem & Saturnum per Radium.*

1^h 47^m 13^s 24^l | 1^h 50^m 3^s 24^l | 18

The same *Ephemerides* shew the next *Conjunction* of *Saturn* and *Jupiter* 1603. December 14 at Noon in $9^{\circ} 36'$ of $\mathcal{A}$, but the Ingenious *Kepler* and our Sir *Chr. Heydon* found it by Observation 7 days sooner, or the 7 Day of the same Month in the Morning, in near 8 Degrees of $\mathcal{A}$, the Planets being but then newly Emerged from the Rays of the *Sun*.

The *Ephemerides* of the Learned *Kepler* Calculated from his own *Rudolphine Tables*, make the next *Conjunction* 1623. betwixt the 7th and 8th of July in $6^{\circ} 46'$ of $\mathcal{S}$ the Planet of *Saturn* being then only 4 Minutes to the North of *Jupiter*, but this first *Conjunction* in the *Fiery Trigon* happening under the *Sun's* Beams was not observable.

By the same *Tables*, and *Ephemerides* of *Eichstade* Calculated from them; these Planets met again in the 25° of $\mathcal{K}$, betwixt the 15 and 16 of Febr. 1643. with a degree Difference of Latitude.

By the joint Consent of *Eichstade's* and our *Wing's Ephemerides* the same Planets were in *Conjunction* again 1663. on the 10 of October at Noon in $13^{\circ} 30'$ of $\mathcal{A}$ with one degree of Difference of Latitude; this *Conjunction* was observable after *Sun-set* in our Latitude, but I hear not that any one observed it.

In every one of these Years there happened only one *Conjunction* of the two *Superiors*, nor is it possible that there should be more, except the *Heliocentrical Conjunction* fall near the *Opposition* of the *Sun*; for then there may be three, two *Direct*, and one *Retrograde*, as has been within the Space of 7 Months, betwixt October and May last inclusive, of which the true times are determined from the following Observations.

| An. 1682. | | | | | |
|-----------|----|----|----|--|---------|
| d. | | | | | |
| Oct. | 5 | 17 | 51 | Betwixt the Centers of <i>Sat.</i> and <i>Jup.</i> | 0 34 54 |
| | | 17 | 54 | rep. | 0 34 48 |
| 12 | 13 | 49 | | Betwixt their Centers | 0 16 2 |
| | 13 | 54 | | rep. | 0 16 4 |
| | 14 | 3 | | Betwixt their Next Limbs | 0 15 22 |

17. Oct. 17.

| Oct. | h. | m. | | | |
|------|----|----|---------------------------------|-------|----------|
| 17 | 14 | 10 | Betwixt their Centers | — | 0 20 9 |
| | 14 | 17 | — | rep. | 0 20 12 |
| | 14 | 21 | — | again | 0 20 14 |
| | 14 | 25 | Betwixt their next Limbs | — | 0 19 44 |
| | 14 | 33 | Betwixt their remoter Limbs | — | 0 20 37 |
| | 15 | 9 | Saturn from the Heel of Castor | — | 48 32 25 |
| | 15 | 14 | Jupiter from the same Star | — | 48 45 5 |
| | 15 | 17 | — | rep. | 48 45 20 |
| | 15 | 20 | Saturn from the same Star again | — | 48 32 20 |
| | 15 | 50 | Betwixt their Centers again | — | 0 20 30 |
| 19 | 15 | 41 | Betwixt their Remote Limbs | — | 0 26 2 |
| | 15 | 45 | Their Centers | — | 0 25 37 |
| | 15 | 47 | Their Next Limbs | — | 0 25 11 |
| 22 | 18 | 25 | Betwixt their Centers | — | 0 33 19 |
| | 18 | 29 | — | rep. | 0 33 26 |

The Distances betwixt the Planets were measured with the Micrometer and 16 Foot Glass, from the Fixt Stars with the Sextant: those of the 12th by my Assistant, the rest by my self.

On the 22 day, the Planet Jupiter was in Consequence of Saturn something less Distant from him than he had been observed on the 5th day near the same hour. Hence the Middle Time, betwixt these Observations is pointed out for the time of their true Conjunction, but to determine it more accurately, I shall Examine the Observations made with the Sextant on the seventeenth day, which being nearest the time are most proper for this purpose.

The Correct Longitude of the Heel of Castor now $\Phi 0^{\circ} 50' 42''$. its Latitude $51^{\circ} 40''$. South. The Latitude of Saturn by the Caroline Tables, $96^{\circ} 20''$ of Jupiter $41^{\circ} 30''$. both North.

By the assumed Latitude of Saturn $56^{\circ} 20''$. and his Distance from the Heel of Castor observed and corrected $48^{\circ} 32' 30''$. I found their Difference of Longitude $48^{\circ} 30' 37''$. therefore Saturn in $\Omega 19^{\circ} 21' 19''$.

By the Latitude of Jupiter assumed $41^{\circ} 30''$. and his Distance from the Star $48^{\circ} 45' 20''$. their Difference of Longitude $48^{\circ} 43' 56''$. and Jupiter's Place in $\Omega 19^{\circ} 34' 39''$.

Hence Jupiter's Place in consequence of Saturn's $13^{\circ} 20''$. with which and the Distance of their Centers observed, the same night $20' 12''$. I find the true Difference of their Latitudes $15' 20''$ but half a Minute Different from what I assumed it on the Authority of the Tables.

The Apparent Motion of Jupiter from the 14 to the 18 day of October by an Ephemeris exactly calculated and made agreeable to these observations is $29' 16''$. of Saturn $15' 01''$. both Direct, hence the Motion of Jupiter from Saturn in four days is $14' 15''$. I say therefore as 4 days Motion, or $14'$

$15''$. 15

14^h. is to 4 Days, or 96 Hours; so is 13^h 20^m (which *Jupiter* is past the Conjunction of *Saturn*;) to 90 Hours or 3 Days 18 Hours; the time inter-lapfed since the Conjunction; which taken from the 17 Day 15 Hours the time of my Observation, gives the true time of the Conjunction of the two Planets on the 13 Day, one and twenty Hours after Noon, or according to the common Account, the 14 Day, at 9 a Clock in the Morning.

At which time *Saturn* is with *Jupiter* in $\odot$ 19° 07 $\frac{1}{2}$ with 15 $\frac{1}{2}$. more Northern Latitude.

The *Acta Eruditorum Lipsiensia* p. 388. make this Conjunction to have hap-pened the same Day in the same Longitude with the *Eleventh Star of Leo*; whose Place they state in $\odot$ 19° 04' Lat. 0° 16'. N. with 14 Minutes dif-ference of Latitudes betwixt the two Planets. But their Observation seems to have been made only by the Judgment of the bare Eye, without an In-strument, which considered, I wonder not that it differs at all, but rather that the Difference is so small from this Determination.

On the 19 of *January* following, viewing then the Planets both *Retrograde* with the 16 Foot Glas, I found them approached within a measurable Di-stance of each other.

| Jan. 1685. | | | |
|------------------------|------|--|---------------|
| d. | h. | | |
| 19 | 6 41 | Betwixt their Centers | 0 33 28 |
| | 6 45 | | rep. 0 33 24 |
| | 6 49 | Betwixt their remote Limbs | 0 33 52 |
| 28 | 6 3 | Betwixt their Centers | 0 15 8 |
| | 6 7 | | rep. 0 15 6 |
| Bois being in 8 to the | 7 00 | Betwixt their remote Limbs | 0 15 31 |
| Sun. | 7 8 | By T. Smith | rep. 0 15 9 |
| | 7 12 | Betwixt their Centers | 0 15 5 |
| | 7 14 | | rep. 0 15 2 |
| | 7 17 | Betwixt their next Limbs | 0 14 29 |
| | 7 20 | | rep. 0 14 31 |
| | 7 21 | | again 0 14 26 |
| | 9 24 | <i>Jupiter</i> from the Heel of <i>Caster</i> | 46 18 10 |
| | 9 26 | | rep. 46 18 3 |
| | 9 28 | <i>Saturn</i> from the said Heel | 46 8 50 |
| | 9 30 | | rep. 46 9 55 |
| | 9 37 | <i>Jupiter</i> from the bright Star of the Li-on's Head E. | 8 42 5 |
| | 9 39 | | rep. 8 42 5 |
| | 9 40 | <i>Saturn</i> from the same Star | 8 29 35 |
| | 9 42 | | rep. 8 29 40 |

| | h. | | | o | ' | " |
|----------|-------|---|------|----|----|----|
| | 9 48 | Jupiter from the Lions Heart | — | 8 | 18 | 00 |
| | 9 50 | — — — | rep. | 8 | 17 | 55 |
| | 9 52 | Saturn from the same | — | 8 | 29 | 35 |
| | 9 54 | — — — | rep. | 8 | 29 | 35 |
| | 9 59 | The Lions Heart from E in the Head | — | 12 | 58 | 50 |
| | 10 3 | The Heel of Castor from the Lions Heart | — | 54 | 34 | 20 |
| Jan. 30. | 10 8½ | The Heel of Castor from E ♄ | — | 46 | 24 | 45 |
| | 5 28 | Betwixt their Centers | — | 0 | 11 | 36 |
| | 5 30 | — — — | rep. | 0 | 11 | 33 |
| | 5 34 | Betwixt their Remote Limbs | — | 0 | 11 | 58 |
| | 5 36 | — — — | rep. | 0 | 12 | 1 |
| | 5 38 | Betwixt their Next Limbs | — | 0 | 11 | 1 |
| | 5 41 | — — — | rep. | 0 | 11 | 00 |
| Feb. 7 | 7 37 | Between their Centers | — | 0 | 28 | 35 |
| | 7 40 | — — — | rep. | 0 | 28 | 34 |

From Observations formerly made, I have determined the true Places and Latitudes to this present time of,

| | s. | o | ' | " | o | ' | " |
|-------------------------------|----|----|----|----------|------|---|--------------|
| The Heel of Castor | — | 50 | 0 | 51 10 | Lat. | 0 | 51 40 South. |
| Bright * in the Lions Head, E | — | ♄ | 16 | 15 27 | | 9 | 41 07 North. |
| Lions Heart | — | — | ♄ | 25 24 45 | | 0 | 26 20 North. |

And from the above recited Measures, the true Distances of the Planets from these Stars January the 26th at 9h 40' p. m. as follows.

| | o | ' | " |
|--|----|----|----|
| Saturn from the Heel of Castor | 46 | 09 | 00 |
| Jupiter from the same | 46 | 18 | 10 |
| Saturn from the Lions Heart | 8 | 29 | 40 |
| Jupiter from the same | 8 | 18 | 00 |
| Saturn from the bright * in the Lions Head E | 8 | 29 | 40 |
| Jupiter from the same | 8 | 42 | 10 |

Whence I collect the true Places at this time.

| | s. | o | ' | " | o | ' | " |
|---------------------|----|----|----|----|---------|----|-------|
| Of Saturn | ♄ | 16 | 57 | 10 | Lat. | 1 | 13 10 |
| Of Jupiter | ♃ | 17 | 07 | 10 | Lat. | 1 | 01 30 |
| Difference of Long. | — | 10 | 00 | 00 | of Lat. | 11 | 40 |

The *Retrograde* Motion of *Jupiter* from *Saturn* in 4 Days, betwixt the Twenty sixth and thirtieth of this Month, by my correct *Ephemeris* is $12' 15''$. I say therefore as $12' 15''$. is to 4 Days or 96 Hours; so is $10' 00''$. (the Difference of the *Planets* present Longitudes,) to 78 Hours or three Days six Hours, which therefore added to the time of that Observation *Jan.* 26^h 9^h $\frac{2}{3}$, gives the true time of the *Conjunction* *Jan.* 29^d 16^h. or according to the common account *Jan.* 30 at 4^h in the Morning. At which time both the *Planets* are in Ω $16^{\circ} 51' \frac{1}{2}$. with $11' \frac{1}{2}$. Difference of Latitude or Distance from each other, which is further confirmed by the Measured Distances of the *Planets* on the 30th at Night before recited.

On the 26th Day at 9^h 40' the *Sun's* True Place was by my Tables in $17^{\circ} 21' \frac{1}{2}$, so that he was now about $\frac{1}{2}$ of a Degree past their Opposition.

Towards the Latter end of the following *April*, the Planet *Jupiter* began to approach *Saturn* again both being now Direct; the 28 at Night with the 16 foot Glafs and *Micrometer* I measured the Distances.

| An. 1683. | | | | | | |
|-----------|----|----|-----------------|---|-----------------------|----------|
| | d. | h. | ' | | | o ' " |
| Apr. | 28 | 10 | 21 | Betwixt their Centers | — | 0 32 35 |
| | | 10 | 23 | — | rep. | 0 32 33 |
| | | 10 | 24 | Betwixt their Next Limbs | — | 0 32 4 |
| | | 10 | 26 | — | rep. | 0 32 2 |
| | | | | Betwixt their Remote Limbs but not accurately, Clouds interposing | — | |
| May | 7 | 10 | 29 | — | — | 0 33 22 |
| | | 8 | 59 | <i>Jupiter</i> from the <i>Lions Heart</i> | — | 10 59 00 |
| | | 9 | 1 | — | rep. | 10 59 00 |
| | | 9 | 3 $\frac{1}{2}$ | <i>Saturn</i> from the <i>Lions Heart</i> | — | 10 58 50 |
| | | 9 | 5 | — | rep. | 10 58 50 |
| | | 9 | 11 | <i>Jupiter</i> from <i>E</i> in the <i>Lions Head</i> | — | 8 55 35 |
| | | 9 | 15 | — | rep. | 8 55 40 |
| | | 9 | 17 | <i>Saturn</i> from the same Star | — | 8 39 40 |
| | | 9 | 18 | — | rep. | 8 39 40 |
| | | 9 | 30 | Betwixt their Centers | — | 0 15 38 |
| | | 9 | 33 | — | rep. | 0 15 37 |
| | | 9 | 35 | Betwixt their Next Limbs | — | 0 15 3 |
| | | 9 | 36 | — | rep. | 0 15 00 |
| | | 9 | 40 | Betwixt their Remote Limbs | — | 0 16 2 |
| | | 9 | 42 | — | rep. | 0 15 58 |
| | | 10 | 20 | <i>Jupiter</i> from <i>B</i> in Ω | — | 38 11 45 |
| | | 10 | 23 | — | rep. | 38 11 45 |
| | | 10 | 26 | <i>Saturn</i> from the same Star | — | 38 10 55 |
| | | 10 | 28 | — | rep. | 38 10 45 |
| May | 11 | 9 | 28 | Betwixt their Centers | — | 00 20 2 |
| | | | 31 | — | rep. | 00 20 2 |
| | | 16 | 9 | 22 | Betwixt their Centers | 00 34 4 |

From these Observations I state the Distances of the Planets from the Fixed Stars May 7^h at 9^h 5'. p. m. as follows.

| | | 0 | ' | " |
|---------------------------------|-------|----|----|----|
| Saturn from the Lyons Heart | ----- | 10 | 58 | 50 |
| Jupiter from the same | ----- | 10 | 59 | 00 |
| Saturn from E in the Lyons Head | ----- | 8 | 39 | 40 |
| Jupiter from the same | ----- | 8 | 55 | 35 |
| | 0 ' " | 0 | ' | " |

Hence the true Longitude of Sat. $\odot$ 14 27 42. Lat. 1 12 46. North.
of Jupiter $\odot$ 14 26 37. Lat. 0 56 43. North.
Difference of Longitude ----- 1 4. Lat. 0 16 3.

The Difference of Latitudes something exceeds the Distance measured with the Micrometer, by reason that the Wind then shaking the Sextant permitted us not to be so exact as usually, but the Difference, being less than half a Minute, I esteem inconsiderable.

The Diurnal Motion of Jupiter from Saturn was now 3' 15". It holds therefore as 3' 15". (one days Motion,) is to one Day or 24 Hours; so 1' 4". (the Distance of Jupiter from the $\odot$ with Saturn) to 8 Hours, the Interval betwixt the Observation and following Conjunction, which was therefore 17^h after Noon, or according to the vulgar reckoning, May 8. at 5 a Clock in the Morning. At which time the true Place of the Planet is $\odot$ 14^o 28¹³, the Difference of their Latitudes, 15' 40". Saturn being so much more Northerly than Jupiter.

In all, best esteemed, Astronomical Tables extant, the Mean Motions of the Planet Saturn are too Swift, of Jupiter too Slow considerably: Hence it came to pass that they made the Direct Conjunctions some Days Later, the Retrograde earlier than they were found by Observation.

2. Oct. 26. ft. n. 1^h 40'. mane Situm Jovis & Saturni Tubo & Microme- At Dantick;
tro ex voto deprehendere mihi obigit: quo tempore simul Fixa quædam fa- by M. Hevelius.
tis conspicua (quod notatu dignum) dictis Planetis satis prope adhærebat. n. 143. p. 18. &
Jupiter sese cum tribus Comitibus tum offerebat, forte & Quartus adfuit, sed n. 151. p. 326.
ob Nubeculas haud fuit conspectus. Saturnus distabat à Jov. 16' 44"; Jupi-
ter à Stella (ni fallor in Armo Dextro $\odot$) 27' 55"; Rursus Saturnus à di-
cta Stella 38' 1". Stella dicta versatur modo juxta nostrum Catalogum in
19^o 2' 9". $\odot$ & Latit. $\odot$ 20' 45". B. Die 30 Octob. 5^h mane Distantia
 $\odot$ & $\odot$, 25' 5" extitit; unde certo colligere licuit Conjunctionem jam ante
complures dies celebratam esse quam Ephemerides Calculusque primum die 3
Novem. exhibent.

Id quod subsequentes Observationes adhuc clarius demonstrant. Nam Loco, quod Distantia $\odot$ & $\odot$ de die in diem (si $\odot$ instaret) paulatim Minor fieri debebat, continuo Aucta est. Die 1 Nov. hor. 2 mane, ope Micro-
metri nostri dicta Distantia extitit 31' 35". Et Die. 2. Nov. eandem Distan-
tiam rursus reperi 35' 21". Die 3. Nov. mane hor. 1. jam 39' 9"; Die 4.
Nov. celo perquam sereno adhuc paulo Major dicta Distantia inter $\odot$ & $\odot$
deprehenfa.

Alteram Conjunctionem, quod attinet quæ ex Retrogressionibus horum Planetarum incidere debebat secundum Ephemeridum Scriptores die 26 Januarii hujus Anni currentis 1683. Observationes nonnullas ac præcipuas habitas hic apponam.

| An. 1683. | Temp. ex
alt. corr.
vesp. | | | Observationes. | | Distantia. | | |
|-----------|---------------------------------|----|----|---|------|------------|----|----|
| | h. | ' | " | | | o | ' | " |
| Feb. 1 | 6 | 40 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> inventa est | 3300 | 0 | 35 | 5 |
| Feb. 2 | 9 | 30 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 2900 | 0 | 22 | 3 |
| Feb. 3 | 9 | 00 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 2500 | 0 | 19 | 0 |
| Feb. 4 | 10 | 00 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 2300 | 0 | 17 | 29 |
| Feb. 5 | 8 | 30 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 2100 | 0 | 15 | 59 |
| Feb. 6 | 7 | 51 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 1850 | 0 | 14 | 6 |
| Feb. 7 | 8 | 17 | 19 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 1700 | 0 | 12 | 55 |
| Feb. 8 | 6 | 10 | 0 | Dist. <i>Satur.</i> & <i>Jov.</i> ——— | 1600 | 0 | 12 | 10 |

Feb. 9. vesp. 9^h 0' 8". *Planetæ* *Tubo* conspexi per dehiscentes densissimas nubes; Oculoque fugitivo deprehendebam *Conjunctionem* ipsam jam celebratam esse nocte præcedente inter 8 & 9 *Febr.* Nam dicta Distantia paulo amplior modo apparebat. Prout etiam 11 *Febr.* vesp. 9^h 0' 0" factum est: nam Distantia inter *Saturnum* & *Jovem* erat 2000, hoc est 0° 15' 12" *Micrometro*, quæ die 8 *Feb.* tantum inventa est 0° 12' 0".

Ad hæc *Conjunctionem* jam esse celebratum, exinde certò constabat, quod uterque *Planeta* cum *Ventre Ursa Majoris* non amplius, ut quidem die 8 *Feb.* continebat, in linea subsisteret recta; tum etiam quod *Saturnus* non amplius ad *Orbitam Jovialium* sub Angulo Recto commoraretur.

Quo autem hoc ipsum eo evidentius redderetur, Observationes nonnullas diebus subsequenibus continuatas hic apponam.

| | h. | ' | " | | | | ' | " |
|---------|----|----|---|---|-------|------|----|----|
| Feb. 12 | 9 | 0 | | Dist. <i>Satur.</i> & <i>Jov.</i> capta est | — | 2200 | 17 | 6 |
| 13 | 7 | 15 | | ————— | ————— | 2550 | 19 | 24 |
| 14 | 9 | 0 | | ————— | ————— | 2900 | 22 | 3 |
| 17 | 6 | 0 | | ————— | ————— | 3750 | 28 | 30 |
| 20 | 9 | 0 | | ————— | ————— | 5250 | 30 | 12 |

Ex hisce igitur Observationibus continuatis satis superque liquet, cum de die in diem *Planeta* ab invicem magis magisque discesserint, quod *Conjunctio* ipsa inter 8 & 9 *Febr.* revera jam fuerit observata.

Denique adjiciendum pariter censeo *Astrophilorum* gratia, quidnam circa *Tertiam eorum Conjunctionem*, Mense *Maii* observaverim.

| An. 1683. | Temp. Sec.
Hor. ambul.
vesp. | Observationes. | Distantie |
|-----------|------------------------------------|--|-----------|
| | h. ' " | Dist. Saturni & Jov. inventa
est Micrometro — | ' " |
| Maii 8 | 9 6 | 4300 | 32 41 |
| 10 | 9 14 | 3750 | 28 30 |
| 11 | 9 10 | 3450 | 26 13 |
| 12 | 8 45 | 3050 | 23 10 |
| 13 | 9 15 | 2800 | 21 17 |
| 14 | 9 45 | 2550 | 19 23 |
| 15 | 9 30 | 2400 | 18 15 |
| 16 | 9 30 | 2250 | 17 6 |
| 17 | 9 40 | 2150 | 16 10 |
| 18 | 10 00 | 2100 | 15 58 |
| | | Hac die extitit Vera Conjunctio. | |
| 20 | 11 45 | 2450 | 18 37 |
| 21 | 9 15 | 2650 | 20 9 |
| 22 | 9 20 | 2900 | 22 3 |
| 23 | 9 5 | 3250 | 24 43 |
| 24 | 10 6 | 3600 | 27 22 |
| 25 | 9 30 | 4000 | 30 25 |
| 26 | 11 0 | 4453 | 33 50 |
| 27 | 9 25 | 4900 | 37 15 |
| 28 | 9 56 | 5325 | 40 29 |

Ex quibus Observationibus cuilibet nunc liquidum est, cum Distantia Saturni & Jovis de die in diem continuè decreverit ad 18 Maii, & ab hac die rursus creverit, Conjunctionem horum Planetarum eadem ipsa die etiam accidisse, & quidem, (uti ex observationibus diei 15, & 20. Maii patet) hora antemeridiana 10 : quæ secundum Ephemeridum Scriptores die 26. primum ingruere debuit. Sic ut hæc tertia & ultima hujus Anni Conjunctio Magna pariter haud mediocriter Tabulas eluserit ; ita quidem quod citius ultra 8 integros dies revera contigerit.

De cætero, hanc ultimam Conjunctionem jam celebratam die sc. 18 Maii vesp. hor. 10. fuisse, ex eo liquet, quod Saturnus jam non amplius eo tempore ad Orbitam Jovialium sub Angulo versabatur Recto ; deinde etiam (uti ex subsequentium dierum observationibus videre est,) quod à die 18. Maii ad diem 28 quousque Micrometro Distantiam Saturni & Jovis dirimere potui, continuo aucta fuit.

Ultimo, notandum quoque occurrit, cum die 21 Maii vesp. inter reliquas observationes etiam Distantiam utriusque Planeta à Stella Superiori in Pede Anteriora

Anteriore Dextro Urse Majoris sextante obtinuerim, atque eadem dicta Stella cum utroque Planeta in eadem simul fere recta tum confisterent linea, quod promptum sit cuilibet Rerum Cælestium cultori dijudicare, an observationes meæ, Sextante nostro novo simul obtentæ, cum observationibus Micrometro captis omnino etiam conveniant. Sextante Distantia Jovis à dicta Stella erat $32^{\circ} 38' 40''$, & Saturni $32^{\circ} 19' 45''$; sic ut genuina utriusque Planete Distantia extiterit $18' 55''$; Micrometro vero ea ipsa Distantia inventa est ea ipsa die 21 sc. Maii $20' 9''$; sic ut Sextante obtenta $1' 14''$. Minor extiterit. Non est autem, mi Amice, quod existimes, hoc vel illo Instrumento me aberrasse, minime profecto; quippe Saturnus & Jupiter cum dicta Stella non omnino in linea recta substiterunt; prout cuilibet ex Globo & Calculo patet, hincque necessario illa distantia, Sextante derivata, etiam paulo minor oportuit esse,

The Mean Conjunctions of Saturn and Jupiter; by Mr. Flamsteed. n. 149. p. 254. July An. 1683.

LXXXVII. Riccioli in the Second Part of the first Tome of his *Almagest*, has given us a Table of all the mean Conjunctions of the two Superiors from the Creation till the Year of Christ 2358, but very coarse and incorrect. I have therefore made a new one for 43 Revolutions which are completed in 853 Julian Years, and 235 Days from their mean Motions, which I have corrected by very late Observations. This being the Period of the Greatest Conjunctions, after which Space of Time they return to the same Place of the Zodiac within $\frac{1}{2}$ of a Degree.

The Ordinary Conjunctions happen once in Twenty Years, or more precisely in 19 Julian Years, and 312 Days, in which time Saturn's Mean Motion is $8^{\circ} 20' 48''$, Jupiter's the same above one Revolution.

These are commonly termed the Lesser of the Great Conjunctions, which continue in Signs of the same Triplicity for 10 Revolutions to each other, or 198 Years: each Conjunction according to the Mean Motions being $8^{\circ} 20' 48''$ removed from the preceeding, so that if any Conjunction was made upon the first Point of γ , the next following shall be in $20' 48''$ of ζ , and all the following for 198 Years shall fall in γ , Ω and ζ , Signs of the same Triplicity.

But the 11 Conjunction after, shall happen in the first Degree of η , and the following ten Conjunctions in δ , η , and ν , Signs of the same Triplicity. Of these the first is called by our Astrologers the Greater Conjunctions.

But the Greatest is, when after 43 Conjunctions completed in 853 Years, 235 Days, the Mean Conjunctions having been made in all the Signs return to that Point of the Ecliptick from whence they began: tho' I must confess, had I been to name them, I would have called those the Greatest which happen in the Signs, Θ and Ω , because then the Planets Rise highest, and are longest visible in our Horizon, as also being near their North Nodes, they approach nearest, and if they have any extraordinary Influence (which *Naboyd* thinks either they have not, or if they have, we understand not) it must according to their Axioms be strongest.

Those which happen in ν and η I should call the Greater or Middle, because the Planets being then near their South Nodes, may approach each other again

again very nearly tho' they rise not high in our *Horizon*, being in Southern Signs; the rest might be accounted the *Lesser* or *Ordinary*.

The *Mean Conjunction* of *Saturn* and *Jupiter* this Year 1683. was on the 14th Day of *January* old *Stile*, at 12 Hours after Noon in the Meridian of *London*, at which time the Mean Motions of both the *Planets* were $4^{\circ} 11' 45''$. This may be the *Radix* of the following *Table*.

A Table of the *Mean Conjunctions* of *Saturn* and *Jupiter*.

| Intervals. | | | Intervals. | | |
|-----------------------|---------|---------|-----------------------|---------|----------|
| Revolutions Complete. | Time. | Motion. | Revolutions Complete. | Time. | Motion. |
| y. d. | s. ° ' | | y. d. | s. ° ' | |
| 1 | 19 312 | 8 2 48 | 23 | 456 219 | 6 4 31 |
| 2 | 39 258 | 4 5 37 | 24 | 476 165 | 2 7 19 |
| 3 | 59 204 | 0 8 25 | 25 | 496 111 | 10 10 8 |
| 4 | 79 150 | 8 11 13 | 26 | 516 57 | 6 12 56 |
| 5 | 99 96 | 4 14 1 | 27 | 536 3 | 2 15 44 |
| 6 | 119 42 | 0 16 50 | 28 | 555 315 | 10 18 32 |
| 7 | 138 353 | 8 19 38 | 29 | 575 261 | 6 21 21 |
| 8 | 158 299 | 4 22 26 | 30 | 595 207 | 2 24 9 |
| 9 | 178 245 | 0 25 15 | 31 | 615 153 | 10 26 57 |
| 10 | 198 191 | 8 28 3 | 32 | 635 99 | 6 29 45 |
| 11 | 218 137 | 5 00 51 | 33 | 655 45 | 3 2 34 |
| 12 | 238 83 | 1 3 40 | 34 | 674 356 | 11 5 22 |
| 13 | 258 29 | 9 6 28 | 35 | 694 302 | 7 8 10 |
| 14 | 277 340 | 5 9 16 | 36 | 714 248 | 3 10 59 |
| 15 | 297 286 | 1 12 4 | 37 | 734 194 | 11 13 47 |
| 16 | 317 232 | 9 14 53 | 38 | 754 140 | 7 16 35 |
| 17 | 337 178 | 5 17 41 | 39 | 774 86 | 3 19 24 |
| 18 | 357 124 | 1 20 29 | 40 | 794 32 | 11 22 12 |
| 19 | 377 70 | 9 23 18 | 41 | 813 343 | 7 25 00 |
| 20 | 397 16 | 5 26 6 | 42 | 833 289 | 3 27 49 |
| 21 | 416 327 | 1 28 54 | 43 | 853 235 | 0 00 37 |
| 22 | 436 273 | 10 1 43 | | | |

By

By this *Table* to find the Time of any *Mean Conjunction*, Past or Future, nearest to any Place of the *Zodiack*; for Times Past, subtract the Longitude of the given Place from the Longitude of the *Radix* $4^{\circ} 11' 45''$. the residue seek in the last Column of the *Table*; if you find not the precise Number take the next to it, against this you have in the second Column the Years and Days, and in the first the Number of *Conjunctions* past since any was made in that Place, subtract the Years and Days from 1683. *Jan.* 14. and the Motion from $4^{\circ} 11' 15''$. so have you the true Time of the *Mean Conjunction*, and Longitudes of the *Planets* then.

But for Times to come subtract the *Radix* from the given Place, seek the Residue as before in the last Column; if you find it not, take that you find nearest it; against which, as before, you have in the second Column, the Years and Days; in the first, the Revolutions future; for Example,

If it were required to know when the last *Conjunction* happen'd in the first Deg. of π , Subtracting π or Ten Signs from $4^{\circ} 11' 15''$. the Residue is $6^{\circ} 11' 15''$. which seeking I cannot find in the third Column of the *Table*, but I find $6^{\circ} 12' 56''$. which is not two Degrees more, and against them 516 Years, 57 Days, and in the first Column 26, for the Number of *Conjunctions* interlapsed. Subtracting 516 Years, 57 Days from 1683. *Jan.* 14. there remains 1166 Years, 322 Days, which shews me that the *Conjunction* was in the Year 1166. *Nov.* 18. and subtracting the Motion $6^{\circ} 12' 56''$, from $4^{\circ} 11' 45''$. it points me to the Place in $9^{\circ} 28' 49''$.

Or if the Time of the first *Conjunction* in π to come were demanded. I subtract the *Radix* $4^{\circ} 11' 45''$. from 6 Signs, the Residue is $1^{\circ} 17' 15''$. I seek in the *Table* but find it not, I take therefore the next to it $1^{\circ} 20' 29''$. against which stands 357 Years, 124 Days, these added to 1683. *Jan.* 14. give me the Year 2040, and 138 Days, *May* 18, for the Time of this *Conjunction*, and adding the $1^{\circ} 20' 29''$. to the *Radix* $4^{\circ} 11' 45''$. it makes $6^{\circ} 2^{\circ} 14''$. for the true mean Longitude of this *Conjunction*.

From the *Mean Conjunction* the Apparent may be found by the help of a Planetary Instrument, or the usual Astronomical Tables.

*The Shadows of
Jupiter's Satel-
lites observed;
by S. Campani.
n. 1. p. 3.
By M. Cassini
and others. n. 8.
p. 143. n. 10.
p. 171. n. 82.
p. 4039.*

LXXXVIII. 1. *S. Campani* affirms, that he hath remarked in the Belts of *Jupiter*, the Shadows of his *Satellites*, and followed them, and at length seen them Emerge out of his *Disk*.

2. *M. Cassini*, after he had discover'd (by the means of those excellent Glasses of 35 Foot made by *S. Campani*) the Shadows cast by the *Satellites* of *Jupiter* upon his *Disk* when they happen to be between the *Sun* and *Him*; and after He had also distinguish'd their Bodies upon the *Disk* of *Jupiter*; made some Predictions when they should appear, to the End that the Curious might be convinc'd of this Matter by their own Observations.

Some of these Predictions have been verified, not only at *Rome*, and in other Places of *Italy*, but also at *Paris* by *M. Anzout*, and in *Holland* by *M. Hugen*; particularly *Sept.* 26. 1665. at half an Hour after seven a Clock, one of these Shadows was seen both in *France* and in *Holland*.

The *Spots*, have this Peculiar, which distinguisheth them from all others, that they are found precisely in that Place of *Jupiter*, where some *Satellite* is seen by the *Sun*; that they go from the Oriental Limb to the Occidental of the Disk of *Jupiter*, with a Motion always equal to that of the *Satellite*; that in respect to us they precede the *Satellite*, before the Opposition of *Jupiter* to the *Sun*, and follow him after the Opposition; that the further *Jupiter* is distant from the Opposition, the greater is the apparent Distance of the same *Satellite*; that at divers Times of the Year, this Distance changeth in Proportion of the Annual *Parallax* of the *Satellite*, according as he is differently seen by the *Sun*, and by the Earth; and that at one and the same time of the Year, when divers *Satellites* happen to be between *Jupiter* and the *Sun*, the *Spots* correspondent to them are distant from them in Proportion of the Semidiameters of the Circles of the same *Satellites*.

3. *An.* 1666. *Jan.* 26. about $3^h 15'$ in the Mornings, I perceived (with *By Dr. Hook.*
a 60 Foot Glass) near the Middle of the Zone *d*, a very round *Spot*, like *n.* 14. p. 245.
that represented at *g*, which was not to be perceived about half an Hour *Fig.* 139.
before; and I observed it in about $10'$. time to be gotten almost to *d*, keeping equal Distance from the *Satellite* *h*, which moved also Westwardly, and was joined to the Disk at *i*, at $3^h 25'$, so that it was sufficiently evident that this black *Spot* was nothing else, save the Shadow of the *Satellite* *h*, Eclipsing a part of the Face of *Jupiter*. The other three *Satellites* in the time of the *Eclipse* were Westwards of the Body of *Jupiter*.

LXXXIX. 1. Anni duo & amplius elapsi sunt ex quo Eruditissimus *Ri-* *The Elongations*
chardus Townleius Armiger, mihi Maximas *Jovialium* Siderum a Centro *Jo-* *of Jupiter's Sa-*
vis Digressiones, à seipso observatas, necnon & Motus cujusque Medios, *Mo-* *telles; by Mr.*
tuumque illorum Radices, ab Observationibus ejus deductas, *Townleio* suo *Flamsteed. n.*
accommodatas, communicavit. Ab eodem deinceps *Ephemerides* tuas, *Clarif-* *81. p. 4036.*
sime *Cassine*, *Mediceorum Siderum* *An.* 1668. impetravi; quibus quando cum *4037. n. 94.*
Motus tum Motuum Radices, necnon & summas Elongationes à Te consti- *p. 6033. n. 96.*
tutas, nonnihil à *D. Townleii* inventis dissidere comperi, & Ego, quod ipse *p. 6094.*
impensius hortatus est, nonnullas primâ quoque Occasione Observationes in-
stituere Operæ fore pretium duxi. *Micrometro* itaque & *Tubo* 14. *Ped.* *An.*
1672. Mense *Mar.* *Stril.* *Jul.* sequentia qua potui Cura Experimenta pri-
ma feci, Observationibus, in majorem Certitudinem, identidem quaque
Nocte iteratis.

An. 1672. *Mar.* 19^d 7^h 11'. Limb. *Jovis* remotior à 430. *Satellite* dist.

$$1601^* = 9' - 34'' * 9' 37''.$$

27. 8. Limb. remotior ab eodem 430. *Satellite* *n.* 28. p. 4037.

$$1591 = 9 - 30$$

28. 8. Eadem Distantia — 1598 = 9 — 33

Jovis Diameter pluribus observationibus reperta 128. Ergo Semidiameter ejus 64; quâ divisis Distantiis observatis, apparentes fient *Satellitæ* à Limbo *Jovis* Remotiori Distantiæ in Semidiametris ejus.

| d. | fd. | | | fd. | |
|----------|-----|----|----------------------------------|-----|----|
| Mar. 19. | 25 | 1 | Sublatà semidiam. à centro fient | 24 | 1 |
| 27. | 24 | 51 | | 23 | 51 |
| 28. | 24 | 58 | | 23 | 58 |

Cujus tunc Motus à *Fove* & distantiz à centro ipsius fuere, secundum numeros tuos, ut hic.

| d. | h. | ' | s. | o | ' | fd. | ' |
|---------|----|----|----|----|----|-----|----|
| Mar. 19 | 7 | 11 | 8 | 25 | 33 | 22 | 56 |
| 27 | 8 | .. | 2 | 19 | 35 | 22 | 37 |
| 28 | 8 | .. | 3 | 11 | 13 | 22 | 34 |

Aberat ergo *Satelles* ab extrema Elongatione, in prima observatione, tantum 4'; in secunda 23'; in tertia 26': Semidiametri scrupulos sexagenarios; quos propterea si observatis *Elongationibus* modo debito adjiciamus, fient maximæ Digressiones, hujus 4^{ti} *Satellitis*, à centro *Fovæ* per primam Observationem 24^{ta} 5'; per secundam 24^{ta} 14'; per tertiam, 24^{ta} 24'; quam *Tu* statuisti tantum 23^{ta}. *R. Fournleins* 24^{ta} 72.

Harum Elongationum posteriores duas accuratiores existimo, quippe quibus investigandis observationes commodas, omni qua cura poteram, peregi; priorem deinde inter plura noctis 19^{te} Adversaria inveni quam perinde exquisitè captam haud ausim affirmare. Utcunque tamen Observationem adjeci, quippe qui non adeò à sequentibus dissentit, quin eas possit confirmare, necnon ostendere, perparùm (si quicquam) minùs, Sinistram quam dextram versus, hunc Satellitem à *Fove* Elongari.

Sed tamen inter observandum sensu, Aeris & Ventri motum,, quatiendo vel agitando Tubum (ad erectam Abietem, ope funis, & trochleæ sub dio pensilem) observationem reddere difficilem; quin & frequenter efficere, ut nimis strictas acciperem Distantias. Quamobrem à pluribus hujusmodi observationibus, quæ summam curam & præcisionem deposcunt, eo usque supersedere constitui, donec commodiorem iis instituendis locum aptarem, quem tandem paravi. In Fenestra quadam Ligneam Machinam, brevis adinstar Scalæ, aptari curavi; cujus operi impositus Tubus quaquaversum converti potuit, nec à ventis, nisi admodum turbidis, hinc inde, ut sub dio fuit, agitari. Huic imposito Tubo, *An.* 1683. *Apr.* 4. Vesp. Meipsum observationibus omni diligentia peragendis accinxi, nec frustra quidem: Etenim, cælo admodum sereno, omnes quatuor *Satellites*, per Tubum Lentium Convexarum, conspexi, & eorum infra scriptas à Limbo *Fovæ*, cuique remotiori, Distantias Dimensus sum; scil.

3 985 iterum — 988
 2 628 — 636
 1 425 — 427
 4 272 — 272 Altitudo *Jovis* Quadrante ferè bipedali capta
 24° — 00'. Ergo hora apparens *Derbia* 8h 26' p. m. & tunc 4^{us} *Satelles* infra
 lineam utrinque per extimos *Satellites*, apparuit; sed vix plenam, ni fallor.
 Semidiametrum.

Jovis Diameter, identidem repetitis observationibus, reperta 133; Simidia-
 meter ergo 66 $\frac{1}{2}$, quæ observatis sublata Distantiis, fient interstitia inter Cen-
 trum *Jovis* & *Comitis Primi*, 360; *Secundi* 569; *Tertii*, 921; *Quarti*, 205;
 quibus per 66 $\frac{1}{2}$ divisis, prodibuat visæ Elongationes à centro *Jovis*, in ip-
 sius Semidiametris.

| Id. ' | s. ° ' | Id. ' |
|--|---------------------|-------|
| 1 5 25 Motus <i>Satellitum</i> à <i>Jove</i> & | 1 9 4 52 4 59 | |
| 2 8 33 Remotiones apparentes se- | 2 2 22 47 7 57 | |
| 3 13 51 cundum <i>Tuas Tabulas, Derbie</i> | 3 2 20 26 12 48 | |
| 4 3 5 reductas, fuere; | 4 4 23 49 2 29 | |

Defecit ergo *Satelles Primus*, 1' tantum; *Secundus* 3; *Tertius* 12; Scrupulis Semidiametri sexagenariis à summa Elongatione, quos propterea si obser-
 vatis addamus, fient extremæ Digressiones.

| Id. ' | Id. | Id. ' |
|--|-----|-------|
| <i>Primis</i> , 5 26 Quas <i>Tu</i> 5 Attamen D. <i>Townleius</i> , 5 31 | | |
| <i>Secundi</i> , 8 37 ponis, 8 ut in Schedis aliquibus 8 47 | | |
| <i>Tertii</i> , 14 2 13 reperio, 13 28 | | |

Commoda rursus prævisâ dari opportunitas *Apr. 11. vesperti*: quamobrem,
 cum non ab uno aut altero Experimento distantias has duxerim definiendas, ha-
 bitis tunc etiam Observationibus, ulterius mecum inquirere institui; quas cum
 primum auspicabar, Cælum circa *Jovem* raris adeo nubibus tectum erat, ut sub-
 obscurius non nisi aliquando *Satellites* potuerim conspiciere; quorum tamen à
 limbo *Jovis* remotiori, ut tulit aer, cepi distantias; nimirum,

Hor 7 $\frac{1}{2}$ p. m. 3 947 iterum 932

2 628 — 614

1 405. Facto tamen Cælo protinus ad votum sereno,

accuratius notavi.

3 947

2 622

1 403

4 942. Iterum 957, *Alto 4^{ve}. 24° 00'* ergo hora apparens 7 $\frac{1}{2}$ 56'.
Satelles Quartus paulo supra lineam, per *Primum* & *Secundum* ductam,
 apparuit;

F f f 2

apparuit; *Tertius* infra eam, sed & aliquando existimavi in ea. *Jovis* capta diameter 132, Semidiameter ergo 66, observatis quæ subducta Distantiis interstitium dabit inter centrum *Jovis* & *Primi*, 339; *Secundi*, 556; *Tertii*, 881; *Quarti*, 891; quibus sigillatim per 66 divisis, prodeunt Elongationes apparentes à centro *Jovis* in Semidiamentris ejusdem, *Primi* quidem 5^d 8'; *Secundi*, 8^d 25'; *Tertii*, 13^d 21'; *Quarti*, 13^d 30'.

Satellium Motus Medii à *Pleni-Mediceis*, cum Distantiis eorum à centro *Jovis*, secundum numeros tuos fuere, ut in hac tabellula exarantur. Unde videre est, *Primum* à summa Elongatione abesse scrupulos Semidiametri 10'; *Secundum*, 26'; *Tertium*, 38'; quos propterea si observatis Elongationibus adjiciamus, fient *Maximæ* hinc deducendæ Digressiones.

| | s. | o | ' | sd. | ' |
|---|----|----|----|-----|----|
| 1 | 8 | 15 | 35 | 4 | 50 |
| 2 | 2 | 10 | 59 | 7 | 34 |
| 3 | 2 | 12 | 2 | 12 | 22 |
| 4 | 10 | 25 | 8 | 13 | 15 |

Primi | 5 18 |

Secundi | 8 51 |

Tertii | 13 59 | perparum ab iis, quas observationibus noctis *Quarta* deduximus, dissentientes.

His tamen utrisque vicibus *Intimus Sattel* ad *Lævam*, *Secundus* & *Tertius* ad *dextram*, à *Jove* apparuere; sed *Apr. 15. vesp. Tertium* à sinistra, in maxima Elongatione apparitutum prævidi, cui propterea Phænomeno invigilare operæ fore pretium duxi, nimirum ut perspicerem, num eadem esset ejusdem *Satellitis* ad manum utramque à centro *Jovis*, summa Remotio. Cælum nocte observationi antedicta sudum erat; sic pro voto observavi circa *hor. 7.* *Tertii* Distantiam 955; & *Jovis* Diam. 131. *Jovis* ergo Semidiameter 56½, observatæ quæ subducta Distantiæ, fit interstitium inter centrum *Jovis* & *Satellitem*, 889; quod per eandem Semidiametrum divisum visibilem dat Elongationem *Comitis* à Centro *Jovis* in Semidiamentris ipsius 13^{sd} 35'. Motus *Satellitis* medius erat 3' 04" 9'. Locus *Jovis* verus ≈ 10° 27'. Ergo *Planeta* à *Pleni-Mediceo* 9' 3" 42' aberat, à summa Elongatione tantum scrupulos 3; quos si observatæ digressioni 13^{sd} 35' adjiciamus, fiet maxima ad *Sinistram* hac vice 13^{sd} 38'. parte nimirum tertia Semidiametri minor quam ad *Dextram*, bis conspirantibus notis, observavimus: Quod mihi videtur innuere, esse aliquam *Centri Orbitæ* hujus *Planeta* à Centro *Jovis* *Excentricitatem*.

An Instrument
for finding the
distances of Ju-
piter's Sate'llites
from his Axis;
by Mr. Flam-
steed, n. 178.
p. 1262. Dec.
An. 1685.
Fig. 142.

2. The Little Circle in the Middle represents the Planet *Jupiter*, the four Concentrick Circles, the proper Orbits of his four *Satellites*, duly proportioned to the Breadth of his Body; the Distances betwixt the parallel Lines intersecting them, being each equal to one of his Semidiameters.

The 4 divided Circles next without these, are distinguished into so many parts as there are days and hours in each *Satellite's* Revolution; the innermost of them serving for the *First* or *Innermost Satellite*; that next it, for the 2^d; that next without this for the 3^d; and outermost for the 4th; above which is a small divided Arch of 15 Degrees.

By this to find the Distances of the *Satellites* from *J's Axis* to a proposed Time.

1. Find

1. Find the *Parallax* of *Jupiter's* Orb to the Time proposed, and note whether it be to be Added or Subtracted.

2. Extend the Thread from the Center of the *Instrument* over the *Parallax* numbred in the small Arch, it cuts off in the four divided Circles, so many hours as each *Satellite* spends in passing from the *Axis* of the Shadow to the *Axis* of Ψ viewed from our Earth; these I call the Simple *Parallaëtick Intervals*, which if the *Parallax* was to be added, are also Additional, if to be subtracted, Subductive.

3. To these *Parallaëtick Intervals* add the Times of half the Duration of the *Eclipse* of each *Satellite*, which for the *First* may be assumed $1^h 10'$; for the *2d*, $1^h 30'$, (greater Exactness being needless); but for the *3d*, and *4th* when *Eclipsed*, (their *Immersion* into the Shadow and *Emergence* from it being commonly given in the *Catalogues*) take half the Difference of these Times at the next *Eclipse*, to the Time proposed, for half the Duration, and Add them to the Simple *Parallaëtick Intervals*, so have you them Augmented. But as often as the *4th Satellite* is not *Eclipsed*, (which is two Years in every six) its Interval needs no Augmentation.

4. Find in the *Tables* the Times of the *Eclipses* of each *Satellite* next preceding the Time proposed, and when the *4th* is not *Eclipsed*, of its passing the *Axis* of the Shadow, to which if the *Parallaëtick Intervals* Augmented were Additional, Add them to, if Subductive, Subtract them from, each the Time of its proper *Satellite's Eclipse*, so have you very near the Apparent Times when each *Satellite* last past over the *Axis* of Ψ viewed from our Earth.

5. Subtract each of the Times thus got from the Time proposed, the Remainders are the Intervals of the Motion of each *Satellite* from Ψ 's *Axis*.

6. Extend the Thread, from the Center over each of these Intervals of Motion numbred severally in the divided Circles belonging each to its proper *Satellite*, where it cuts the proper *Orbit* of that *Satellite*, whose Interval was numbred in its peculiar Circle, it shews amongst the Parallels, how many Semidiameters of Ψ , that *Satellite* is distant from him, and on which side of him 'tis Posited.

Note further, that the Thread as it lay extended over the *Parallax* of the *Orb* numbred in the small Arch, where it cut the several proper *Orbits* of each *Satellite*, shewed amongst the Parallels, how many Semidiameters of Ψ , the Center of the Shadow was distant from the Center of Ψ , viewed from our Earth. And that if the *Parallax* of the *Orb* were Additional, the Shadow lies on the Right-hand from Ψ , if Subductive on the Left.

To explain these Precepts, I shall give two brief Examples. Let it be then proposed to know how far each *Satellite* appears distant from Ψ on the 26th of Dec. this present Year 1685. at $6^h 52'. p. m.$ when the *Third Satellite* falls into the Shadow; also on Jul. 16. 1686. at $10^h 00'. p. m.$ when there is no *Eclipse*.

An. 1685.

Fig. 143. An. 1683. December 26^l 16^h 52'. p. m. the Parallax of the Orb is 9° 20' Additional; Therefore

| | 1 | 2 | 3 | 4 |
|---|-----------------|-------------------------------|-----------------|--------------------------------|
| | d. h. / | d. h. / | d. h. / | d. h. / |
| The Simple Parallaetick Intervals add. ——— | 0 1 5 | 0 2 10 | 0 4 25 | 0 10 20 |
| The half Durations of the Eclipses to be added ——— | 0 1 10 | 0 1 30 | 0 1 18 | 0 0 0 |
| The Parallaetick Intervals augmented ——— | 0 2 15 | 0 3 40 | 0 5 43 | 0 10 20 |
| Last Immersions, and ϕ . Dec. ——— | 25 9 37 | 25 5 47 | 19 12 58 | 10 00 30 |
| Times of last passing Jupiter's Axis, Dec. — | 25 11 52 | 25 9 27 | 12 18 41 | 10 10 50 |
| Subtracted from the Time proposed. Dec. — | 26 16 52 | 26 16 52 | 26 16 52 | 26 16 52 |
| Leaves the Intervals of Motion. ——— | 1 5 00 | 1 7 25 | 6 22 11 | 16 6 2 |
| Over which numbred in their peculiar Circles the Thread being severally laid, cuts the proper Orbis of each at their visible Distances from Jupiter ——— | fd.
5. dext. | fd.
6 $\frac{1}{2}$. fin. | fd.
3. dext. | fd.
4 $\frac{1}{2}$. dext. |

Again,

Again, *An.* 1686. *Jul.* 16. 10^h p. m. the *Parallax* of the *Orb* is 10° 46' *Fig.* 144.
Subductive. Hence

| | 1 | 2 | 3 | 4 |
|---|---------------------------------------|--------------------------------------|---------------------------------------|---------------------------------------|
| | d. h. / | d. h. / | d. h. / | d. h. / |
| The <i>Simple Parallaetick</i>
<i>Intervals Sub.</i> ——— | 0 1 12 | 0 2 35 | 0 5 10 | 0 12 00 |
| Half <i>Duration</i> of the
<i>Eclipses add.</i> ——— | 0 1 10 | 0 1 30 | 0 1 21 | — — — |
| The <i>Parallaetick Inter-</i>
<i>vals augmented.</i> ——— | 0 2 22 | 0 4 5 | 0 6 31 | 0 12 00 |
| The next last <i>Emersi-</i>
<i>ons</i> , and passing the
<i>Axis</i> of the <i>Shadow</i> ,
<i>Jul.</i> ——— | 15 5 55 | 15 22 2 | 15 9 19 | 15 17 52 |
| The Time of last pas-
sing the visible <i>Axis</i>
of <i>Jupiter</i> ——— | 15 3 33 | 15 17 57 | 15 2 48 | 15 5 52 |
| The Time proposed — | 16 10 00 | 16 10 00 | 16 10 00 | 16 10 00 |
| The <i>Intervals</i> of <i>Mo-</i>
<i>tions</i> ——— | 1 6 27 | 0 16 3 | 1 7 12 | 1 4 8 |
| Therefore <i>Distan.</i> from
<i>Jupiter's Axis</i> ——— | fd.
5 $\frac{1}{2}$. <i>Dext.</i> | fd.
8 $\frac{1}{2}$. <i>Sin.</i> | fd.
13 $\frac{1}{2}$. <i>Sin.</i> | fd.
10 $\frac{1}{2}$. <i>Sin.</i> |

And the *Satellites* stand at the two proposed Times as in the two *Fi-*
gures.

In drawing of which, tho' I have considered their *Latitudes* from the *Line*
of their utmost *Elongations* passing through *Jupiter's* Center; yet I give no
Rules for determining it, the Contrivances and Directions necessary on that
Account, being too many and troublesome to be inserted here: My Design
is only to shew the Ingenious Observer, how to find at what Distance from
4, each *Satellite* appears, that so he may not mistake one for another when
he is to Observe any of their *Eclipses*.

XC. 1. *An.* 1688. The *French Astronomers* have made these *Observations*
by a 14 Foot Telescope.

Octob. 7. 10^h 32' p. m. The *First Satellite* (called *Pallas*) Entred upon
the Face of *Jupiter*.

Octob. 8.

Eclipses and
Places of the
Satellites of
Jupiter Obser-
ved, at Paris;
by
" 44. p. 892.

Octob. 8. 8^h 11'. The *Second Satellite* (called *Juno*) Went out behind *Jupiter*.

Octob. 9. 8^h 54'. The *Second Satellite* went out from the Face of *Jupiter*.

Octob. 16. 10^h 4'. The *Second Satellite* entred upon the Face of *Jupiter*.

Octob. 22. 10^h 41' 33". The *First Satellite* entred into the Shadow of *Jupiter*.

Octob. 23. 8^h 32'. The *First Satellite* entred upon the Face of *Jupiter*.

Nov. 12. 10^h 40'. The *Second Satellite* entred into the Shadow of *Jupiter*.

Nov. 20. 2^h 38' 30". After Midnight, the *Third Satellite* (called *Themis*) entred into the Shadow of *Jupiter*.

At Dantzick;
by M. Hevelius.
n. 78. p.
3029.

2. Cum An. 1671. die 25. Sept. st. n. manè ex conducto Dn Cassinus Parissius, & Dn Picard Uraniburgi, suscepissent, ad Occultationem *Primi Jovialis* attendere, volui haud minus ego huic Phænomeno diligenter invigilare. Itaq; Hor. 4. 27'. ubi primum *Jupiter* emicuit, deprehendi adhuc *Joviales* omnes adesse, tres sc. à Læva & unum ad dextram. Duo illi propinquiores ad Sinistram haud procul videbantur à Limbo *Jovis*, non minus ille qui ad Dextram apparebat, aliorum Comitum Minimus. Ad quintam usque & 7' ferè, omnes quatuor (utut Cælum jam livesceret) distincte apparebant. Præter tamen omnem spem, hor. 5. 12' videbatur mihi propinquior ille *Comes* ad Lævam (respectu Tubi mei, qui inverso ordine objecta exhibet) penitus evanescere, remanentibus illis tribus, quanquam ille dexterior magis magisque etiam ad *Jovem* accedebat. An planè momentum ipsum temporis id fuerit *Immerfusionis* illius *Comitis*, vix ausim adeo certò affirmare; nihilominus tardius non incidit illa *Occultatio*; sed anne unico minuto ferè, adhuc citius forte, ingruerit, facile concesserim.

| Sec. Horol.
amb. mane. | Observationes. | Distantia
& Altitudines. | Temp. cor.
rect. |
|---------------------------|----------------------------------|-----------------------------|---------------------|
| h. / " | | q / 1 / 2 | h. / " |
| 4 36 25 | <i>Jupiter</i> primum cœnspectus | — — — | 4 32 0 |
| 5 7 25 | Altitudo <i>Procyonis</i> | 34 43 0 | 5 2 7 |
| 5 16 35 | Primus <i>Jovialis</i> Evanuit | — — — | 5 12 0 |
| 5 26 5 | Altitudo <i>Procyonis</i> | 36 39 0 | 5 23 27 |

At Derby; by Mr. Flamsteed. 3. An. 1671. Feb. 17. 7^h 25'. p. m. Alt. Ψ erat 15° 54'. At 8^h 59'. p. m. vel forsan 1. min. maturius, *Satelles Primus* ad dextram Ψ is, in ipsius n. 82. p. 4036. Umbram incidit, adeo tamen Evanescentis exigua erat à Limbo Distantia, ut quanta fuerit dijudicare non potuerim.

Mart.

Mart. 19^d. 6^h 45'. *Alt.* *Jov.* 29° 35' *Satelles Primus* ad *Limbum Jovis* ib. 4037. appropinquabat, cui 7^h 51'. jungebatur.

An. 1672. *Ap.* 15. 7^h 43'. *vesp.* *Alt.* 25° 00'. *Satelles Primus* mox n. 96. p. 6099. *Jovem* à tergo subiturus $\frac{1}{2}$ circiter diametri à Limbo ejus apparuit.

8^h 6'. *Alt.* 27° 20'. subivit *Jovem*. *Alt.* *Jove* 27° 26' certe non conspicietur.

4. *Jul.* 6. (st. n.) 1675. ante mediam noctem, hora scil. 11. & 16. Se- *Ar* Paris; by *cundis præcisè, Secundus Jovis Satelles* Egredi incipiebat ex Planetæ hujus, *M. Cassini.* *qui ipsum obscuraverat, umbra.* n. 117. p. 389.

5. *An.* 1679. *Jun.* 5. st. n. 3^h. m. I discovered 3 *Satellites of Jupiter*: *Ph. Col. n. 1.* *The First* was distant Westward of the Limb of *Jupiter*, a little less than a *p. 33.* *Diameter*; the *Second* was distant, on the East-side, a little more than a Dia- *Fig. LXIV. 2.* *meter.* The *Third* was more Eastward than the *Second*, by somewhat less than a *Diameter* of *Jupiter*.

XCI. 1. Let A be the *Sun*, B *Jupiter*, C the *First Satellite* of *Jupiter*, *The Equation of* which enters into the *Shadow* of *Jupiter*, to come out of it at D; and let *Light; by M.* EFGHLK be the *Earth* placed at divers Distances from *Jupiter.* *Romer.*

Now suppose the *Earth*, being in L towards the 2^d Quadrature of *Jupiter*, *n. 136. p. 893.* hath seen the *First Satellite* at the Time of its *Emersion*, or issuing out of the *Fig. 145.* *Shadow*, in D; and that about 42 $\frac{1}{2}$ Hours after (*vid.* after one Revolution of this *Satellite*) the *Earth* being in K, doth see it returned in D; it is manifest, that if the *Light* require time to traverse the Internal LK, the *Satellite* will be seen returned later in D, than it would have been if the *Earth* had remained in L, so that the Revolution of this *Satellite* being thus observed by the *Emersions* will be retarded by so much time, as the *Light* shall have taken in passing from L to K, and that on the contrary, in the other Quadrature FG, where the *Earth* by approaching goes to meet the *Light*, the Revolutions of the *Emersions* will appear to be shortened by so much, as those of the *Emersions* had appeared to be lengthened.

This new *Equation* of the Motion of *Light*, which hath been established by the *Royal Academy*, and in the *Observatory* for the Space of 8 Years, was confirmed by the *Emersion* of the *First Satellite* observ'd at *Paris* 1676. *Nov. 9.* *The Theory of* 5^h 35' 45", at Night, 10' later than it was expected, by deducing it from *Jupiter's Sa-* those that had been Observed in the Month of *August*, when the *Earth* was *tellites; by M.* much nearer to *Jupiter.* *Cassini.* n. 128. p. 681. Sept. An. 1676. *M. Cassini's*

XCII. 1. *M. Cassini*, having formed a new Hypothesis for the *Satellites* of *Jupiter*, different from that of *Galileo*, thinks that the Plain of their Orbs is *Tables, for the* Inclined to the Plain of the *Ecliptick*; He settles their *Nodes* with the Orbs of *Eclipses of the* *Jupiter* towards the 13° of *Leo* and *Aquarius*; and finds that the *Obliquity* of *First Satellite* their Circles to the Orbit of *Jupiter*, is almost double to the *Obliquity* of this *Orbit* to the *Ecliptick.* *of Jupiter, A-* *duced, and Re-* *Meridian of* *London; by*

2. *M. Cassini*, in the last Treatise of a Book, Entitul'd, *Recueil d'Observations faites en Plusieurs Voyages, &c.* has employed his Skill, to make easy *Mr. Edm.* *Halley.* Vol. I. G g g the n. 21. p. 238. Nov. An. 1694.

the Calculation of the Eclipses of the First Satellite of Jupiter, which is otherwise Operose even to the Skilful. The Tables have for Principles, that this Satellite Revolves to the Sun in $1^d 18^h 28' 36''$, so precisely, that in 100 Years the Difference is not sensible; That in the Time of the Revolution of Jupiter to his Aphelion, which he supposes in $4332^d 14^h 52' 48''$, this Satellite makes exactly 2448 Months or Revolutions to the Sun; and dividing the Orbit of Jupiter into 2448 parts, he has in a Large Table of Equation shewn what is the Inequality of the Motion of Jupiter in each Revolution reduced to time, assuming, Thirdly, the greatest Equation of Jupiter $5^{\circ} 32' 40''$. whence the hourly Motion of the Satellite from Jupiter being $8^{\circ} 28'$, it follows that the greatest Inequality (Jupiter passing the Signs of Cancer and Capricorn,) amounts to $39' 8''$ of Time, to be added in Cancer, subtracted in Capricorn. Lastly, As to the Epocha, or beginning of this Series of Revolutions, he has determined the Aphelion of Jupiter about 1° Degree forwarder than *Astronomia Carolina*, and above two Degrees more than the *Rudolphine Tables*, viz. precisely in 9° of *Libra*, in the beginning of this Century, which perhaps he finds the proper Motion of Jupiter about the Sun at this time to require; and the Number of Revolutions since Jupiter was last in Perihelio, is here stiled Num. I.

A Second Inequality is that which depends on the Distance of the Sun from Jupiter, which he says Mr. Romer did most ingeniously explain by the Hypothesis of the Motion of Light; to which yet Cassini by his manner of Calculus seems not to assent, though it be hard to imagine how the Earth's Position in respect of Jupiter should any way affect the Motion of the Satellites. This Inequality he makes to amount to two Degrees in the Satellite's Motion, or $14' 10''$ of Time, wherein he supposes the Eclipses to happen so much sooner when Jupiter Opposes the Sun, than when he is in Conjunction with him. The Distribution of this Inequality he makes wholly to depend on the Angle at the Sun between the Earth and Jupiter, without any Regard to the Excentricity of Jupiter, (who is sometimes $\frac{1}{2}$ a Semidiameter of the Earth's Orb farther from the Sun than at other times) which would occasion a much greater Difference, than the Inequality of Jupiter and the Earth's Motion, both of which are accounted for in these Tables with great Skill and Address. But what is most strange, he Affirms that the same Inequality of two Degrees in the Motion, is likewise found in the other Satellites, requiring a much greater Time; as above two Hours in the 4th Satellite: which if it appeared by Observation would overthrow M. Romer's Hypothesis entirely. Yet I doubt not herein to make it Demonstratively plain, that the Hypothesis of the Progressive Motion of Light is found in all the other Satellites of Jupiter to be necessary, and that it is the same in all; there being nothing near so great an Annual Inequality as M. Cassini supposes in their Motions, by his Table, p. 9. and his *Præcepta Calculi*. The Method however used to compute this is very Curious; for having found that whilst the Sun Revolves to Jupiter there pass $398^d 21^h 13'$. wherein are made $225\frac{1}{2}$ Revolutions of the Satellite to Jupiter, the Number of Revolutions since

since *Jupiter* was last in Opposition to the *Sun*, is what he calls *Num. II.* in which the Inequality of the *Earth's* Motion is allowed for in the Months, and that of *Jupiter's* Orb by a Table of the *Equation* of *Num. II.* amounting in all to $3\frac{1}{2}$ Revolutions of the *Satellite* to *Jupiter*. This in the Tables following I have thought fit to leave out, shewing how to find it by the help of the former Equation of *Num. I.* The Numbers are in effect the same with *M. Cassini's*, only Reduced to our *Stile* and *Meridian*, and the Form of them Abridged, and 'tis hoped Amended.



| | | | | |
|-------|-----|-----|-----|-----|
| 0 000 | 000 | 000 | 000 | 000 |
| 0 001 | 001 | 001 | 001 | 001 |
| 0 002 | 002 | 002 | 002 | 002 |
| 0 003 | 003 | 003 | 003 | 003 |
| 0 004 | 004 | 004 | 004 | 004 |
| 0 005 | 005 | 005 | 005 | 005 |
| 0 006 | 006 | 006 | 006 | 006 |
| 0 007 | 007 | 007 | 007 | 007 |
| 0 008 | 008 | 008 | 008 | 008 |
| 0 009 | 009 | 009 | 009 | 009 |
| 0 010 | 010 | 010 | 010 | 010 |
| 0 011 | 011 | 011 | 011 | 011 |
| 0 012 | 012 | 012 | 012 | 012 |
| 0 013 | 013 | 013 | 013 | 013 |
| 0 014 | 014 | 014 | 014 | 014 |
| 0 015 | 015 | 015 | 015 | 015 |
| 0 016 | 016 | 016 | 016 | 016 |
| 0 017 | 017 | 017 | 017 | 017 |
| 0 018 | 018 | 018 | 018 | 018 |
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| 0 027 | 027 | 027 | 027 | 027 |
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| 0 031 | 031 | 031 | 031 | 031 |
| 0 032 | 032 | 032 | 032 | 032 |
| 0 033 | 033 | 033 | 033 | 033 |
| 0 034 | 034 | 034 | 034 | 034 |
| 0 035 | 035 | 035 | 035 | 035 |
| 0 036 | 036 | 036 | 036 | 036 |
| 0 037 | 037 | 037 | 037 | 037 |
| 0 038 | 038 | 038 | 038 | 038 |
| 0 039 | 039 | 039 | 039 | 039 |
| 0 040 | 040 | 040 | 040 | 040 |
| 0 041 | 041 | 041 | 041 | 041 |
| 0 042 | 042 | 042 | 042 | 042 |
| 0 043 | 043 | 043 | 043 | 043 |
| 0 044 | 044 | 044 | 044 | 044 |
| 0 045 | 045 | 045 | 045 | 045 |
| 0 046 | 046 | 046 | 046 | 046 |
| 0 047 | 047 | 047 | 047 | 047 |
| 0 048 | 048 | 048 | 048 | 048 |
| 0 049 | 049 | 049 | 049 | 049 |
| 0 050 | 050 | 050 | 050 | 050 |
| 0 051 | 051 | 051 | 051 | 051 |
| 0 052 | 052 | 052 | 052 | 052 |
| 0 053 | 053 | 053 | 053 | 053 |
| 0 054 | 054 | 054 | 054 | 054 |
| 0 055 | 055 | 055 | 055 | 055 |
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| 0 057 | 057 | 057 | 057 | 057 |
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| 0 060 | 060 | 060 | 060 | 060 |
| 0 061 | 061 | 061 | 061 | 061 |
| 0 062 | 062 | 062 | 062 | 062 |
| 0 063 | 063 | 063 | 063 | 063 |
| 0 064 | 064 | 064 | 064 | 064 |
| 0 065 | 065 | 065 | 065 | 065 |
| 0 066 | 066 | 066 | 066 | 066 |
| 0 067 | 067 | 067 | 067 | 067 |
| 0 068 | 068 | 068 | 068 | 068 |
| 0 069 | 069 | 069 | 069 | 069 |
| 0 070 | 070 | 070 | 070 | 070 |
| 0 071 | 071 | 071 | 071 | 071 |
| 0 072 | 072 | 072 | 072 | 072 |
| 0 073 | 073 | 073 | 073 | 073 |
| 0 074 | 074 | 074 | 074 | 074 |
| 0 075 | 075 | 075 | 075 | 075 |
| 0 076 | 076 | 076 | 076 | 076 |
| 0 077 | 077 | 077 | 077 | 077 |
| 0 078 | 078 | 078 | 078 | 078 |
| 0 079 | 079 | 079 | 079 | 079 |
| 0 080 | 080 | 080 | 080 | 080 |
| 0 081 | 081 | 081 | 081 | 081 |
| 0 082 | 082 | 082 | 082 | 082 |
| 0 083 | 083 | 083 | 083 | 083 |
| 0 084 | 084 | 084 | 084 | 084 |
| 0 085 | 085 | 085 | 085 | 085 |
| 0 086 | 086 | 086 | 086 | 086 |
| 0 087 | 087 | 087 | 087 | 087 |
| 0 088 | 088 | 088 | 088 | 088 |
| 0 089 | 089 | 089 | 089 | 089 |
| 0 090 | 090 | 090 | 090 | 090 |
| 0 091 | 091 | 091 | 091 | 091 |
| 0 092 | 092 | 092 | 092 | 092 |
| 0 093 | 093 | 093 | 093 | 093 |
| 0 094 | 094 | 094 | 094 | 094 |
| 0 095 | 095 | 095 | 095 | 095 |
| 0 096 | 096 | 096 | 096 | 096 |
| 0 097 | 097 | 097 | 097 | 097 |
| 0 098 | 098 | 098 | 098 | 098 |
| 0 099 | 099 | 099 | 099 | 099 |
| 0 100 | 100 | 100 | 100 | 100 |

Tabula

*Epocha Revolutionum Primi Satellitis ad Jovis Umbrae
sub Meridiano Londinensi.*

| <i>Anno
Jul.
Curr.</i> | <i>d. h. m. s.</i> | <i>Nam. I.</i> | <i>Nam. II.</i> |
|--------------------------------|--------------------|----------------|-----------------|
| 1660 | 0 11 5 48 | 968 | 200 6 |
| 1661 | 0 1 17 24 | 1174 | 181 2 |
| 1662 | 1 9 57 36 | 1381 | 162 2 |
| 1663 | 1 00 9 12 | 1587 | 143 5 |
| 1664 | 1 8 49 24 | 1794 | 125 1 |
| 1665 | 0 23 1 00 | 2000 | 105 7 |
| 1666 | 0 13 12 36 | 2206 | 86 4 |
| 1667 | 0 3 24 12 | 2412 | 67 0 |
| 1668 | 0 12 4 24 | 171 | 48 6 |
| 1669 | 0 2 16 00 | 377 | 29 2 |
| 1670 | 1 10 56 12 | 584 | 10 9 |
| 1671 | 1 1 7 48 | 790 | 216 9 |
| 1672 | 1 9 48 00 | 997 | 198 5 |
| 1673 | 0 23 59 36 | 1203 | 179 1 |
| 1674 | 0 14 11 12 | 1409 | 159 7 |
| 1675 | 0 4 22 48 | 1615 | 140 3 |
| 1676 | 0 13 3 00 | 1822 | 121 9 |
| 1677 | 0 3 14 36 | 2028 | 102 5 |
| 1678 | 1 11 54 48 | 2235 | 84 1 |
| 1679 | 1 2 6 24 | 2441 | 64 7 |
| 1680 | 1 10 46 36 | 200 | 46 4 |
| 1681 | 1 00 58 12 | 406 | 27 0 |
| 1682 | 0 15 9 48 | 612 | 7 6 |
| 1683 | 0 5 21 24 | 818 | 213 6 |
| 1684 | 0 14 1 36 | 1025 | 195 3 |
| 1685 | 0 4 13 12 | 1231 | 175 9 |
| 1686 | 1 12 33 24 | 1438 | 157 5 |
| 1687 | 1 3 5 00 | 1644 | 138 1 |
| 1688 | 1 11 45 12 | 1851 | 119 7 |
| 1689 | 1 1 56 48 | 2057 | 100 4 |

*Epoche Revolutionum Primi Satellitis ad Jovis Umbram
sub Meridiano Londinensi.*

| <i>Anno
Jul.
Curr.</i> | | | | | <i>Num.
I.</i> | <i>Num.
II.</i> |
|--------------------------------|----|----|----|----|--------------------|---------------------|
| | d. | h. | ' | " | | |
| 1690 | 0 | 16 | 8 | 24 | 2263 | 81 0 |
| 1691 | 0 | 6 | 20 | 00 | 21 | 61 6 |
| 1692 | 0 | 15 | 00 | 12 | 228 | 43 3 |
| 1693 | 0 | 5 | 11 | 48 | 434 | 23 9 |
| 1694 | 1 | 13 | 52 | 00 | 641 | 5 5 |
| 1695 | 1 | 4 | 3 | 36 | 847 | 211 5 |
| 1696 | 1 | 12 | 43 | 48 | 1054 | 193 1 |
| 1697 | 1 | 2 | 55 | 24 | 1260 | 173 7 |
| 1698 | 0 | 17 | 7 | 00 | 1466 | 154 4 |
| 1699 | 0 | 7 | 18 | 36 | 1672 | 135 0 |
| 1700 | 0 | 15 | 58 | 48 | 1879 | 116 6 |
| 1701 | 0 | 6 | 10 | 24 | 2085 | 97 3 |
| 1702 | 1 | 14 | 50 | 36 | 2292 | 78 9 |
| 1703 | 1 | 5 | 2 | 12 | 50 | 59 5 |
| 1704 | 1 | 13 | 42 | 24 | 257 | 41 1 |
| 1705 | 1 | 3 | 54 | 00 | 463 | 21 8 |
| 1706 | 0 | 18 | 5 | 36 | 669 | 2 4 |
| 1707 | 0 | 8 | 17 | 12 | 875 | 208 4 |
| 1708 | 0 | 26 | 57 | 24 | 1082 | 190 0 |
| 1709 | 0 | 7 | 9 | 00 | 1288 | 170 6 |
| 1710 | 1 | 15 | 49 | 12 | 1495 | 152 3 |
| 1711 | 1 | 6 | 00 | 48 | 1701 | 132 9 |
| 1712 | 1 | 14 | 41 | 00 | 1908 | 114 5 |
| 1713 | 1 | 4 | 52 | 36 | 2114 | 95 1 |
| 1714 | 0 | 19 | 4 | 12 | 2320 | 75 8 |
| 1715 | 0 | 9 | 15 | 48 | 2527 | 56 4 |
| 1716 | 0 | 17 | 56 | 00 | 2734 | 38 0 |
| 1717 | 0 | 8 | 7 | 36 | 2941 | 18 6 |
| 1718 | 1 | 16 | 47 | 8 | 3148 | 0 3 |
| 1719 | 1 | 6 | 59 | 24 | 3355 | 206 3 |
| 1720 | 1 | 15 | 39 | 36 | 3562 | 187 9 |

Tabula Revolutionum Primi Satellitis Jovis in Anno.

| Jannarius. | | | | N.I. | N.II. | Martius. | | | | N.I. | N.II. |
|-------------|----|----|----|------|-------|----------|----|----|----|------|-------|
| d. | h. | ' | " | | | d. | h. | ' | " | | |
| 0 | 0 | 0 | 0 | 0 | 0 0 | 1 | 4 | 12 | 24 | 34 | 34 8 |
| 1 | 18 | 28 | 36 | 1 | 1 0 | 2 | 22 | 41 | 00 | 35 | 35 8 |
| 3 | 12 | 57 | 12 | 2 | 2 1 | 4 | 17 | 9 | 36 | 36 | 36 8 |
| 5 | 7 | 25 | 48 | 3 | 3 1 | 6 | 11 | 38 | 12 | 37 | 37 9 |
| 7 | 1 | 54 | 24 | 4 | 4 1 | 8 | 6 | 6 | 48 | 38 | 38 9 |
| 8 | 20 | 23 | 00 | 5 | 5 2 | 10 | 00 | 33 | 24 | 39 | 39 9 |
| 10 | 14 | 51 | 36 | 6 | 6 2 | 11 | 19 | 4 | 00 | 40 | 40 9 |
| 12 | 9 | 20 | 12 | 7 | 7 2 | 13 | 13 | 32 | 36 | 41 | 41 9 |
| 14 | 3 | 48 | 48 | 8 | 8 2 | 15 | 8 | 1 | 12 | 42 | 42 9 |
| 15 | 22 | 17 | 24 | 9 | 9 3 | 17 | 2 | 29 | 48 | 43 | 43 9 |
| 17 | 16 | 46 | 00 | 10 | 10 3 | 18 | 20 | 58 | 24 | 44 | 44 9 |
| 19 | 11 | 14 | 36 | 11 | 11 3 | 20 | 15 | 27 | 00 | 45 | 45 9 |
| 21 | 5 | 43 | 12 | 12 | 12 3 | 22 | 9 | 55 | 36 | 46 | 46 9 |
| 23 | 00 | 11 | 48 | 13 | 13 4 | 24 | 4 | 24 | 12 | 47 | 47 9 |
| 24 | 18 | 40 | 24 | 14 | 14 4 | 25 | 22 | 52 | 48 | 48 | 48 9 |
| 26 | 13 | 9 | 00 | 15 | 15 4 | 27 | 17 | 21 | 24 | 49 | 49 9 |
| 28 | 7 | 37 | 36 | 16 | 16 5 | 29 | 11 | 50 | 00 | 50 | 50 9 |
| 30 | 2 | 6 | 12 | 17 | 17 5 | 31 | 6 | 18 | 36 | 51 | 51 9 |
| 31 | 20 | 34 | 48 | 18 | 18 5 | | | | | | |
| Februarius. | | | | | | Aprilis. | | | | | |
| 0 | 20 | 34 | 48 | 18 | 18 5 | 0 | 6 | 18 | 36 | 51 | 51 9 |
| 2 | 15 | 3 | 24 | 19 | 19 6 | 2 | 00 | 47 | 12 | 52 | 52 9 |
| 4 | 19 | 32 | 00 | 20 | 20 6 | 3 | 19 | 15 | 48 | 53 | 53 9 |
| 6 | 4 | 00 | 36 | 21 | 21 6 | 5 | 13 | 44 | 24 | 54 | 54 9 |
| 7 | 22 | 29 | 12 | 22 | 22 6 | 7 | 8 | 13 | 00 | 55 | 55 9 |
| 9 | 16 | 57 | 48 | 23 | 23 7 | 9 | 2 | 41 | 36 | 56 | 56 9 |
| 11 | 11 | 26 | 24 | 24 | 24 7 | 10 | 21 | 10 | 12 | 57 | 57 9 |
| 13 | 5 | 55 | 00 | 25 | 25 7 | 12 | 15 | 38 | 48 | 58 | 58 9 |
| 15 | 00 | 23 | 36 | 26 | 26 7 | 14 | 10 | 7 | 24 | 59 | 59 9 |
| 16 | 18 | 52 | 12 | 27 | 27 7 | 16 | 4 | 36 | 00 | 60 | 60 8 |
| 18 | 13 | 20 | 48 | 28 | 28 7 | 17 | 23 | 4 | 36 | 61 | 61 8 |
| 20 | 7 | 49 | 24 | 29 | 29 7 | 19 | 17 | 33 | 12 | 62 | 62 8 |
| 22 | 2 | 18 | 00 | 30 | 30 8 | 21 | 12 | 1 | 48 | 63 | 63 8 |
| 23 | 20 | 46 | 36 | 31 | 31 8 | 23 | 6 | 30 | 24 | 64 | 64 8 |
| 25 | 15 | 15 | 12 | 32 | 32 8 | 25 | 00 | 59 | 00 | 65 | 65 7 |
| 27 | 9 | 43 | 48 | 33 | 33 8 | 26 | 19 | 27 | 36 | 66 | 66 7 |
| | | | | | | 28 | 13 | 56 | 12 | 67 | 67 7 |
| | | | | | | 30 | 8 | 24 | 48 | 68 | 68 6 |

Tabula Revolutionum Primi Satellitis Jovis in Anno.

| <i>Mains.</i> | | | | N.I. | N.II. | <i>Julius.</i> | | | | N.I. | N.II. |
|----------------|----|----|----|------|-------|------------------|----|----|----|------|-------|
| d. | h. | ' | " | | | d. | h. | ' | " | | |
| 0 | 8 | 24 | 48 | 68 | 68 6 | 1 | 7 | 5 | 48 | 103 | 102 5 |
| 2 | 2 | 53 | 24 | 69 | 69 6 | 3 | 1 | 34 | 24 | 104 | 103 5 |
| 3 | 21 | 22 | 00 | 70 | 70 6 | 4 | 20 | 3 | 00 | 105 | 104 4 |
| 5 | 15 | 50 | 36 | 71 | 71 6 | 6 | 14 | 31 | 36 | 106 | 105 4 |
| 7 | 10 | 19 | 12 | 72 | 72 5 | 8 | 9 | 00 | 12 | 107 | 106 5 |
| 9 | 4 | 47 | 48 | 73 | 73 5 | 10 | 3 | 28 | 48 | 108 | 107 3 |
| 10 | 23 | 16 | 24 | 74 | 74 5 | 11 | 31 | 57 | 24 | 109 | 108 3 |
| 12 | 17 | 45 | 00 | 75 | 75 5 | 13 | 16 | 26 | 00 | 110 | 109 3 |
| 14 | 12 | 13 | 36 | 76 | 76 4 | 15 | 10 | 54 | 36 | 111 | 110 2 |
| 16 | 6 | 42 | 12 | 77 | 77 4 | 17 | 5 | 23 | 12 | 112 | 111 2 |
| 18 | 1 | 10 | 48 | 78 | 78 4 | 18 | 23 | 51 | 48 | 113 | 112 2 |
| 19 | 19 | 39 | 24 | 79 | 79 3 | 20 | 18 | 20 | 24 | 114 | 113 1 |
| 21 | 14 | 8 | 00 | 80 | 80 3 | 22 | 12 | 49 | 00 | 115 | 114 1 |
| 23 | 8 | 36 | 36 | 81 | 81 3 | 24 | 7 | 17 | 36 | 116 | 115 1 |
| 25 | 3 | 5 | 12 | 82 | 82 3 | 26 | 1 | 46 | 12 | 117 | 116 0 |
| 26 | 21 | 33 | 48 | 83 | 83 3 | 27 | 20 | 14 | 48 | 118 | 117 0 |
| 28 | 16 | 2 | 24 | 84 | 84 2 | 29 | 14 | 43 | 24 | 119 | 118 0 |
| 30 | 10 | 31 | 00 | 85 | 85 2 | 31 | 9 | 12 | 00 | 120 | 119 0 |
| <i>Junius.</i> | | | | N.I. | N.II. | <i>Augustus.</i> | | | | N.I. | N.II. |
| 1 | 4 | 59 | 36 | 86 | 86 1 | 0 | 9 | 12 | 00 | 120 | 119 0 |
| 2 | 23 | 28 | 12 | 87 | 87 1 | 2 | 3 | 40 | 36 | 121 | 119 9 |
| 4 | 17 | 56 | 48 | 88 | 88 0 | 3 | 22 | 9 | 12 | 122 | 120 9 |
| 6 | 12 | 25 | 24 | 89 | 89 0 | 5 | 16 | 37 | 48 | 123 | 121 9 |
| 8 | 6 | 54 | 00 | 90 | 90 0 | 7 | 11 | 6 | 24 | 124 | 122 9 |
| 10 | 1 | 22 | 36 | 91 | 90 9 | 9 | 5 | 35 | 00 | 125 | 123 8 |
| 11 | 19 | 51 | 12 | 92 | 91 9 | 11 | 00 | 3 | 36 | 126 | 124 8 |
| 13 | 14 | 19 | 48 | 93 | 92 9 | 12 | 18 | 32 | 12 | 127 | 125 8 |
| 15 | 8 | 48 | 24 | 94 | 93 8 | 14 | 13 | 00 | 48 | 128 | 126 8 |
| 17 | 3 | 17 | 00 | 95 | 94 8 | 16 | 7 | 29 | 24 | 129 | 127 7 |
| 18 | 21 | 45 | 36 | 96 | 95 7 | 18 | 1 | 58 | 00 | 130 | 128 7 |
| 20 | 16 | 14 | 12 | 97 | 96 7 | 19 | 20 | 26 | 36 | 131 | 129 7 |
| 22 | 10 | 42 | 48 | 98 | 97 7 | 21 | 14 | 55 | 12 | 132 | 130 7 |
| 24 | 5 | 11 | 24 | 99 | 98 6 | 23 | 9 | 23 | 48 | 133 | 131 7 |
| 25 | 23 | 40 | 00 | 100 | 99 6 | 25 | 3 | 52 | 24 | 134 | 132 7 |
| 27 | 18 | 8 | 36 | 101 | 100 6 | 26 | 22 | 21 | 00 | 135 | 133 6 |
| 29 | 12 | 37 | 12 | 102 | 101 5 | 28 | 16 | 49 | 36 | 136 | 134 6 |
| | | | | | | 30 | 11 | 18 | 12 | 137 | 135 6 |

Tabula Revolutionum Primi Satellitis Jovis in Anno.

| September. | | | | N.I. | N.II. | November. | | | | N.I. | N.II. |
|------------|----|----|----|------|-------|-----------|----|----|----|------|-------|
| d. | h. | ' | " | | | d. | h. | ' | " | | |
| 1 | 5 | 46 | 48 | 138 | 136 6 | 0 | 9 | 59 | 12 | 172 | 170 7 |
| 3 | 00 | 15 | 24 | 139 | 137 6 | 2 | 4 | 27 | 48 | 173 | 171 8 |
| 4 | 18 | 44 | 00 | 140 | 138 6 | 3 | 22 | 56 | 24 | 174 | 172 8 |
| 6 | 13 | 12 | 36 | 141 | 139 6 | 5 | 17 | 25 | 00 | 175 | 173 8 |
| 8 | 7 | 41 | 12 | 142 | 140 6 | 7 | 11 | 53 | 36 | 176 | 174 8 |
| 10 | 2 | 9 | 48 | 143 | 141 3 | 9 | 6 | 22 | 12 | 177 | 175 9 |
| 11 | 20 | 38 | 24 | 144 | 142 3 | 11 | 00 | 50 | 48 | 178 | 176 9 |
| 13 | 15 | 7 | 00 | 145 | 143 3 | 12 | 19 | 19 | 24 | 179 | 177 9 |
| 15 | 9 | 35 | 36 | 146 | 144 3 | 14 | 13 | 48 | 00 | 180 | 178 9 |
| 17 | 4 | 4 | 12 | 147 | 145 3 | 16 | 8 | 16 | 36 | 181 | 180 0 |
| 18 | 22 | 32 | 48 | 148 | 146 3 | 18 | 2 | 45 | 12 | 182 | 181 0 |
| 20 | 17 | 1 | 24 | 149 | 147 3 | 19 | 21 | 13 | 48 | 183 | 182 0 |
| 22 | 11 | 30 | 00 | 150 | 148 3 | 21 | 15 | 42 | 24 | 184 | 183 0 |
| 24 | 5 | 58 | 36 | 151 | 149 3 | 23 | 10 | 11 | 00 | 185 | 184 0 |
| 26 | 00 | 27 | 12 | 152 | 150 3 | 25 | 4 | 39 | 36 | 186 | 185 1 |
| 27 | 18 | 55 | 48 | 153 | 151 3 | 26 | 23 | 8 | 12 | 187 | 186 1 |
| 29 | 13 | 24 | 24 | 154 | 152 3 | 28 | 17 | 36 | 48 | 188 | 187 2 |
| October. | | | | | | 30 | 12 | 5 | 24 | 189 | 188 2 |
| 1 | 7 | 53 | 00 | 155 | 153 3 | December. | | | | | |
| 3 | 2 | 21 | 36 | 156 | 154 3 | 0 | 12 | 5 | 24 | 189 | 188 2 |
| 4 | 20 | 50 | 12 | 157 | 155 3 | 2 | 6 | 34 | 00 | 190 | 189 2 |
| 6 | 15 | 18 | 48 | 158 | 156 3 | 4 | 1 | 2 | 36 | 191 | 190 3 |
| 8 | 9 | 47 | 24 | 159 | 157 3 | 5 | 19 | 31 | 12 | 192 | 191 3 |
| 10 | 4 | 16 | 00 | 160 | 158 3 | 7 | 13 | 59 | 48 | 193 | 192 3 |
| 11 | 22 | 44 | 36 | 161 | 159 3 | 9 | 8 | 28 | 24 | 194 | 193 4 |
| 13 | 17 | 13 | 12 | 162 | 160 3 | 11 | 2 | 57 | 00 | 195 | 194 4 |
| 15 | 11 | 41 | 48 | 163 | 161 6 | 12 | 21 | 25 | 36 | 196 | 195 5 |
| 17 | 6 | 10 | 24 | 164 | 162 6 | 14 | 15 | 54 | 12 | 197 | 196 5 |
| 19 | 00 | 39 | 00 | 165 | 163 6 | 16 | 10 | 22 | 48 | 198 | 197 6 |
| 20 | 19 | 7 | 36 | 166 | 164 6 | 18 | 4 | 51 | 24 | 199 | 198 6 |
| 22 | 13 | 36 | 12 | 167 | 165 6 | 19 | 23 | 20 | 00 | 200 | 199 7 |
| 24 | 8 | 4 | 48 | 168 | 166 6 | 21 | 17 | 48 | 36 | 201 | 200 7 |
| 26 | 2 | 33 | 24 | 169 | 167 7 | 23 | 12 | 17 | 12 | 202 | 201 8 |
| 27 | 21 | 2 | 00 | 170 | 168 7 | 25 | 6 | 45 | 48 | 203 | 202 8 |
| 29 | 15 | 30 | 36 | 171 | 169 7 | 27 | 1 | 14 | 24 | 204 | 203 9 |
| 31 | 9 | 59 | 12 | 172 | 170 7 | 28 | 19 | 43 | 00 | 205 | 204 9 |
| | | | | | | 30 | 14 | 11 | 36 | 206 | 206 0 |

Tabula Prima Aequationis Conjunctionum Primi Satellitis cum Jove.

| N. I. Aequat. | | | N. I. Aequat. | | | N. I. Aequat. | | |
|---------------|----|----|---------------|----|----|---------------|----|----|
| | I | II | | I | II | | I | II |
| 0 | 0 | 0 | 400 | 34 | 20 | 810 | 33 | 21 |
| 10 | 1 | 3 | 410 | 34 | 51 | 820 | 32 | 50 |
| 20 | 2 | 5 | 420 | 35 | 21 | 830 | 32 | 17 |
| 30 | 3 | 8 | 430 | 35 | 47 | 840 | 31 | 44 |
| 40 | 4 | 12 | 440 | 36 | 6 | 850 | 31 | 10 |
| 50 | 5 | 15 | 450 | 36 | 26 | 860 | 30 | 32 |
| 60 | 6 | 16 | 460 | 36 | 47 | 870 | 29 | 56 |
| 70 | 7 | 19 | 470 | 37 | 8 | 880 | 29 | 19 |
| 80 | 8 | 20 | 480 | 37 | 29 | 890 | 28 | 40 |
| 90 | 9 | 23 | 490 | 37 | 44 | 900 | 27 | 59 |
| 100 | 10 | 25 | 500 | 37 | 59 | 910 | 27 | 19 |
| 110 | 11 | 25 | 510 | 38 | 16 | 920 | 26 | 37 |
| 120 | 12 | 25 | 520 | 38 | 29 | 930 | 25 | 53 |
| 130 | 13 | 25 | 530 | 38 | 39 | 940 | 25 | 8 |
| 140 | 14 | 25 | 540 | 38 | 49 | 950 | 24 | 23 |
| 150 | 15 | 22 | 550 | 38 | 55 | 960 | 23 | 37 |
| 160 | 16 | 18 | 560 | 38 | 59 | 970 | 22 | 50 |
| 170 | 17 | 17 | 570 | 39 | 3 | 980 | 22 | 3 |
| 180 | 18 | 11 | 580 | 39 | 6 | 990 | 21 | 15 |
| 190 | 19 | 9 | 590 | 39 | 8 | 1000 | 20 | 26 |
| 200 | 20 | 5 | 600 | 39 | 7 | 1010 | 19 | 37 |
| 210 | 20 | 56 | 610 | 39 | 5 | 1020 | 18 | 47 |
| 220 | 21 | 49 | 620 | 39 | 03 | 1030 | 17 | 56 |
| 230 | 22 | 41 | 630 | 38 | 58 | 1040 | 17 | 5 |
| 240 | 23 | 32 | 640 | 38 | 51 | 1050 | 16 | 13 |
| 250 | 24 | 20 | 650 | 38 | 44 | 1060 | 15 | 19 |
| 260 | 25 | 7 | 660 | 38 | 34 | 1070 | 14 | 25 |
| 270 | 25 | 57 | 670 | 38 | 24 | 1080 | 13 | 32 |
| 280 | 26 | 43 | 680 | 38 | 10 | 1090 | 12 | 37 |
| 290 | 27 | 27 | 690 | 37 | 56 | 1100 | 11 | 42 |
| 300 | 28 | 9 | 700 | 37 | 40 | 1110 | 10 | 47 |
| 310 | 28 | 54 | 710 | 37 | 24 | 1120 | 9 | 52 |
| 320 | 29 | 35 | 720 | 37 | 5 | 1130 | 8 | 57 |
| 330 | 30 | 11 | 730 | 36 | 45 | 1140 | 8 | 00 |
| 340 | 30 | 45 | 740 | 36 | 25 | 1150 | 7 | 3 |
| 350 | 31 | 28 | 750 | 36 | 4 | 1160 | 6 | 7 |
| 360 | 32 | 10 | 760 | 35 | 40 | 1170 | 5 | 1 |
| 370 | 32 | 44 | 770 | 35 | 15 | 1180 | 4 | 13 |
| 380 | 33 | 15 | 780 | 34 | 49 | 1190 | 3 | 15 |
| 390 | 33 | 49 | 790 | 34 | 19 | 1200 | 2 | 19 |
| 400 | 34 | 20 | 800 | 34 | 49 | 1210 | 1 | 21 |
| | | | 810 | 33 | 21 | 1220 | 0 | 20 |
| | | | | | | 1224 | 0 | 0 |

Tabula Secunda Equationis Conjunctionum Primi Satellitis cum Jove.

| N.
II. | Equat.
add. | N.
II. | Equat.
add. | N.
II. | Equat.
add. | N.
II. | Equat.
add. |
|-----------|----------------|-----------|----------------|-----------|----------------|-----------|----------------|
| 0 | 0 0 | 28 | 2 4 | 56 | 7 0 | 84 | 12 0 |
| 1 | 0 0 | 29 | 2 13 | 57 | 7 12 | 85 | 12 9 |
| 2 | 0 1 | 30 | 2 21 | 58 | 7 24 | 86 | 12 16 |
| 3 | 0 2 | 31 | 2 30 | 59 | 7 36 | 87 | 12 24 |
| 4 | 0 3 | 32 | 2 39 | 60 | 7 47 | 88 | 12 32 |
| 5 | 0 4 | 33 | 2 48 | 61 | 7 59 | 89 | 12 40 |
| 6 | 0 6 | 34 | 2 58 | 62 | 8 11 | 90 | 12 47 |
| 7 | 0 8 | 35 | 3 8 | 63 | 8 22 | 91 | 12 53 |
| 8 | 0 10 | 36 | 3 17 | 64 | 8 34 | 92 | 13 00 |
| 9 | 0 14 | 37 | 3 27 | 65 | 8 46 | 93 | 13 6 |
| 10 | 0 17 | 38 | 3 37 | 66 | 8 57 | 94 | 13 13 |
| 11 | 0 20 | 39 | 3 48 | 67 | 9 8 | 95 | 13 19 |
| 12 | 0 23 | 40 | 3 59 | 68 | 9 20 | 96 | 13 24 |
| 13 | 0 27 | 41 | 4 9 | 69 | 9 32 | 97 | 13 30 |
| 14 | 0 32 | 42 | 4 20 | 70 | 9 44 | 98 | 13 35 |
| 15 | 0 37 | 43 | 4 31 | 71 | 9 54 | 99 | 13 39 |
| 16 | 0 42 | 44 | 4 41 | 72 | 10 3 | 100 | 13 45 |
| 17 | 0 47 | 45 | 4 53 | 73 | 10 14 | 101 | 13 48 |
| 18 | 0 53 | 46 | 5 4 | 74 | 10 25 | 102 | 13 51 |
| 19 | 0 58 | 47 | 5 15 | 75 | 10 35 | 103 | 13 54 |
| 20 | 1 4 | 48 | 5 27 | 76 | 10 45 | 104 | 13 57 |
| 21 | 1 11 | 49 | 5 39 | 77 | 10 55 | 105 | 14 00 |
| 22 | 1 18 | 50 | 5 50 | 78 | 11 5 | 106 | 14 3 |
| 23 | 1 25 | 51 | 6 2 | 79 | 11 15 | 107 | 14 5 |
| 24 | 1 32 | 52 | 6 14 | 80 | 11 25 | 108 | 14 7 |
| 25 | 1 40 | 53 | 6 25 | 81 | 11 34 | 109 | 14 8 |
| 26 | 1 47 | 54 | 6 37 | 82 | 11 43 | 110 | 14 9 |
| 27 | 1 56 | 55 | 6 49 | 83 | 11 52 | 111 | 14 10 |
| 28 | 2 4 | 56 | 7 00 | 84 | 12 00 | 112 | 14 10 |

Tabula Dimidia Mora Primi Satellitis in Umbra Jovis.

| N.I. | h. | ' | " | N.I. | h. | ' | " |
|------|----|---|----|------|----|---|----|
| 0 | 1 | 4 | 56 | 1200 | 1 | 5 | 6 |
| 40 | 1 | 4 | 33 | 1240 | 1 | 4 | 48 |
| 80 | 1 | 4 | 12 | 1280 | 1 | 4 | 26 |
| 120 | 1 | 3 | 59 | 1320 | 1 | 4 | 7 |
| 160 | 1 | 3 | 48 | 1360 | 1 | 3 | 54 |
| 200 | 1 | 3 | 39 | 1400 | 1 | 3 | 38 |
| 240 | 1 | 3 | 38 | 1440 | 1 | 3 | 38 |
| 280 | 1 | 3 | 48 | 1480 | 1 | 3 | 44 |
| 320 | 1 | 4 | 1 | 1520 | 1 | 3 | 52 |
| 360 | 1 | 4 | 16 | 1560 | 1 | 4 | 7 |
| 400 | 1 | 4 | 36 | 1600 | 1 | 4 | 24 |
| 440 | 1 | 4 | 56 | 1640 | 1 | 4 | 42 |
| 480 | 1 | 5 | 18 | 1680 | 1 | 5 | 00 |
| 520 | 1 | 5 | 41 | 1720 | 1 | 5 | 22 |
| 560 | 1 | 6 | 1 | 1760 | 1 | 5 | 46 |
| 600 | 1 | 6 | 21 | 1800 | 1 | 6 | 10 |
| 640 | 1 | 6 | 39 | 1840 | 1 | 6 | 28 |
| 680 | 1 | 6 | 53 | 1880 | 1 | 6 | 45 |
| 720 | 1 | 7 | 3 | 1920 | 1 | 6 | 57 |
| 760 | 1 | 7 | 11 | 1960 | 1 | 7 | 7 |
| 800 | 1 | 7 | 15 | 2000 | 1 | 7 | 13 |
| 840 | 1 | 7 | 13 | 2040 | 1 | 7 | 14 |
| 880 | 1 | 7 | 9 | 2080 | 1 | 7 | 15 |
| 920 | 1 | 7 | 2 | 2120 | 1 | 7 | 15 |
| 960 | 1 | 6 | 54 | 2160 | 1 | 7 | 10 |
| 1000 | 1 | 6 | 39 | 2200 | 1 | 6 | 49 |
| 1040 | 1 | 6 | 22 | 2240 | 1 | 6 | 32 |
| 1080 | 1 | 6 | 5 | 2280 | 1 | 6 | 15 |
| 1120 | 1 | 5 | 45 | 2320 | 1 | 5 | 58 |
| 1160 | 1 | 5 | 26 | 2360 | 1 | 5 | 38 |
| 1200 | 1 | 5 | 6 | 2400 | 1 | 5 | 18 |
| | | | | 2440 | 1 | 5 | 2 |

From these Tables, to any given Year, Month and Day, to find the next Eclipse of the First Satellite of Jupiter proceed thus.

1. In the Table of the *Epoche* find the Year of our Lord, and set down the Day, Hours, Minutes and Seconds, with *Num. I.* and *Num. II.* thereto annex'd; and in the Table of Revolutions, seek the Month, and Day of the Month, with the Hours and Minutes, and *Num. I.* and *Num. II.* affix'd, and add them together; and the respective Sums shall shew the Mean Time of the Middle of the *Eclipse* sought, with *Num. I.* and *Num. II.* required. But it must be Observed, that in *Jan.* and *Feb.* in the *Leap Year*, one Day is to be added to the Day thus found.

2. If *Num. I.* be found less than 1224, with *Num. I.* or if the greater than 2448, subtracting 2448 therefrom, with the Residue; enter the Table, and you will have the First *Æquation* to be Added to the Mean Time before found. But if *Num. I.* be less than 2448, but greater than 1224, subtract it from 2448, and entering the same Table with the remainder, you shall have the First *Æquation* to be Subtracted from the Mean Time. Then divide the Minutes of the said First *Æquation* by 11, or rather $\frac{3}{2}$, and the *Quote* shall be the *Æquation* of *Num. II.* (answering to the Eccentric Motion of *Jupiter*) to be Added thereto when the First *Æquation* Subtracts, and *contra* Subtracted when that Adds.

3. If *Num. II.* thus *Æquated* exceed 225,4, Subtract 225,4, therefrom; and if the Remainder or *Num. II.* be less than 113, with the said Remainder or Number; or if greater than 113, with the Complement thereof to 225,4, seek in the Table the second *Æquation*, which being Added to the Time before found, gives the True Time of the Middle of the *Eclipse*.

4. With *Num. I.* seek the Half Continuance of the Total *Eclipse*, which is to be Added for the *Emersion* when the *Æquated Num. II.* is less than 113, or if more than 225,4, it be less than 338. But if it exceed 113, or 338, then is the *Seminora* to be Subtracted for the *Immerson*.

5. Lastly, With the *Sun's True Place* take out the *Æquation* of Natural Days, which Added or Subtracted according to the Title, gives the Time of the *Immerson* or *Emersion* sought.

Now how few Figures serve for this Computation, will best appear by an Example.



An. 1677. Sept. 17 8^h 9' 40". at Greenwich, Mr. Flamsteed observed the First Satellite to begin to Emerge; that is 8^h 9' 26". at London.

| | d. | h. | ' | " | N. I. | N. II. |
|-------------|----|----|----|------|--------|-------------|
| 1677. | 0 | 3 | 14 | 36 | 2028 | 102, 5 |
| Sept. | 17 | 4 | 4 | 12 | 147 | 145, 5 |
| Sept. | 17 | 7 | 18 | 48 | 2175 | 248, 0 |
| Aquat. 1. | — | | 26 | 11 | 2448 | 2, 3 |
| Aquat. 2. | 17 | 6 | 52 | 37 | 273 | 250, 3 |
| | + | 0 | 1 | 39 | | 225, 4 |
| Seminora | + | 1 | 7 | 00 | | 24, 9 |
| Equal Time | 17 | 8 | 1 | 16 | 11 | 26, 2 (2, 3 |
| Equation | + | 0 | 9 | 25 | ⊙ in ♈ | 5°, 00 |
| Apparent T. | 17 | 8 | 10 | 41 | | |
| Observ. | 00 | 8 | 9 | 20 | | |
| Error | | | | 1 21 | | |

An Immersion of this Satellite being computed after the same manner according to these Tables, ought to have happened, An. 1683. Nov. 30. 16^h 52' 7". but I observed it at 16^h 48' 40". so that the Error was, — 3' 27".

Again, M. Cassini observed an Emerision at Paris, An. 1693. Jan. 14^d 10^h 40' 28". that is, at London 10^h 30' 48". Which these Tables give at 10^h 30' 39". and therefore the Error was no more than + 9".

After this manner I have compared these Tables with many good and certain Observations, and scarce ever find them Err above 3 or 4 Minutes of Time; which Errors are exceeding small in comparison of the short Time that the Satellites have been discovered.

In the Construction of the Table, which shews the Half Continuance of these Eclipses, the Semidiameter of the Shadow of Jupiter is made by Cassini just 10 Deg. and that of the Satellite 30'; and the Satellite's Ascending Node, being supposed in 15° of Aquarius, at the End of this Century (that is 55° 20' before the Perihelion of Jupiter) it will thence follow, that Num. I. being 116, or 2102, Jupiter passes the Nodes of the Satellites Orb, and consequently these Eclipses are Central, and of the Greatest Duration. But Num. I. being 215, or 1481, the Satellite passes the Shadow with the Greatest Obliquity, viz. 2° 55' from the Center, whence the Seminora becomes of all the shortest.

3. The Tables of the other 3 Satellites not being so perfect or exact as those of the other 3 of the First, are here given in another Form. The Periods of their Revolutions to Jupiter's Shade are as follows.

1b. p. 253.

Period.

| | d. | h. | ' | " | ''' | | |
|-----------------|----|----|----|----|-----|---------------------|-------------|
| Period. Secundi | 3 | 13 | 17 | 54 | 3 | five $2\frac{1}{3}$ | Rev. Primi. |
| Period. Tertii | 7 | 3 | 59 | 39 | 22 | five $4\frac{1}{3}$ | Rev. Primi. |
| Period. Quarti | 16 | 18 | 5 | 6 | 50 | five $9\frac{1}{3}$ | Rev. Primi. |

Whence the Table of the First *Æ*quation of the *First Satellite*, or M. *Cassini's* larger Table, may by an easy Reduction serve the other three; the *Æ*quation of the 2^d being $2\frac{1}{3}$, or twice the Minutes with half so many Seconds as there are Minutes in the *Æ*quation of the *First*, and the greatest *Æ*quation thereof $1^h 18' 35''$. *Æ*quation of the 3^d is $4\frac{1}{3}$ times greater than that of the *First*, and when greatest amounteth to $2^h 29''$. And the *Æ*quation of the 4th being $9\frac{1}{3}$ times that of the *First*, is had by subtracting $\frac{1}{3}$ and $\frac{1}{3}$ from 10 times the *Æ*quation of the *First*, whence the greatest becomes $6^h 10' 28''$. so that *Num. I.* and *Num. II.* as here collected for the *First*, may indifferently serve all the rest.

M. Romer's
*Æ*quation of
Light defended.
Ibid. p. 254.
Vid. Sup.

4. As to the Second *Æ*quation of the other *Satellites*, M. *Cassini* has, by his *Præcepta Calculi* (as is before mentioned) supposed the Minutes thereof to be increased in the same Proportion, as instead of $14' 10''$ in the *First* to be $28' 27''$ in the *Second*, $57' 22''$ in the *Third*, and no less than $2^h 14' 7''$ in the *Fourth*; whereas if this Second Inequality did proceed from the Successive Propagation of Light, this *Æ*quation ought to be the same in all of them, which M. *Cassini* says, was wanting to be shewn, to perfect M. *Romer's* Demonstration; wherefore he has rejected it as ill founded. But there is a good Cause to believe, that his Motive thereto, is what he has thought not proper to discover. And the following Observations do sufficiently supply the Defect complained of in the making out of that Hypothesis.

An. 1676, *Oct.* st. n. $6^h 10' 37''$. *App.* but $5^h 59' 37''$. *Equal Time.* M. *Cassini* at *Paris* observed the *Emersion* of the 3^d *Satellite* from *Jupiter's* Shadow. And again *Nov.* 14. following $6^h 20' 59''$. *App.* Time, but $6^h 51' 55''$. *Eq. Time.* he observed the like *Emersion* of the same *Satellite*. The observed Interval of Time between these *Emersions*, was $43^d 0^h 6' 18''$, which is $8' 22''$ more than 6 Mean Revolutions of this *Satellite*, of which $4' 27''$ arises from the Difference of the First Equation, and the greater Continuance of the Latter *Eclipse*; so that the other 4 Minutes is all that is left to answer for the Difference of the 2^d *Æ*quations; and *Num. II.* in that Time increasing from 48 to 72, gives $4' 36''$ for the Difference of the 2^d *Æ*quations of the *First Satellite*. So that here the 2^d *Æ*quation of the *Third* is found rather less than that of the *First*, but the Difference is so small, that it may rather be attributed to the Uncertainty of Observation. Whereas according to M. *Cassini's* Method of Calculating, instead of four Minutes it ought to be $18' 38''$. and the Interval of these two *Emersions* $43^d 0^h 21'$. exceeding the Time observed by a whole Quarter of an Hour; which that curious Observer could not be deceived in.

The like appears yet more evidently in the *Fourth Satellite*. By the Observations of Mr. *Flamsteed* at *Greenwich*, *An.* 1682. *Sept.* 24. 17^h 45'. *T. App.* but 17^h 32'. *T. Eq.* the *Fourth Satellite* was seen newly come out of the Shadow, so that about 17^h 30'. *T. Eq.* the first beginning of *Emerfion* was conjectured; and after 5 *Revolutions*, *viz.* *Decemb.* 17^d 11^h 16' or 11^h 18'. *T. Eq.* he again observed the first Appearance of the *Satellite* beginning to *Emerge*, that is, after an Interval of 83^d 17^h 48'; whereas this *Satellite* makes five mean *Revolutions* in 83^d 18^h 25'. Here we have 37' to be accounted for by the several *Inequalities*. Of this 21' is due to the first *Æquations*, which is reduced to 19' by the Greater Continuance of the latter *Eclipse*, *Jupiter* then Approaching to his *Descending Node*. So that there remains only 18' for the Difference of the 2^d *Æquations*, whilst the Earth approached *Jupiter*, by more than the *Radius* of its own Orb: and the Difference of the 2^d *Æquations* of the *First Satellite*, being according to *Cassini* 8' 30". the said Difference in the *Fourth* ought to be 1^h 20', instead of 18'; whence the Interval of these two *Emerfions* would be according to his Precepts, but 83^d 16^h 46', instead of 83^d 17^h 48' observed. And whereas 18' may seem too great a Difference; it must be noted, first, that M. *Romer* had stated the whole 2^d *Æquation* 22' 00". which M. *Cassini* has diminished to 14' 10". so that instead of 8'. M. *Romer* allows 13', and secondly, that in the first of these Observations, being about half an Hour before Sun-rise, the Brightness of the Morning might well hinder the seeing of this smallest and slowest *Satellite*, till such time as a good part thereof was *Emerged*.

XCIII. Having a great Desire to observe the Body of *Mars*, whilst *Acro-* The Phases and
mycal and *Retrograde* (having formerly with a Glas about 12 Foot long, ob- Revolution of
served some kind of Spots in the Phase of it) though it was not in the *Peri-* Mars about his
helium of its Orb, but nearer its *Aphelium*; yet I found that the Face of it, Hook.
when near its Opposition to the Sun (with a Charge, the 36 Foot-Glas, I n. 11. p. 198.
made use of, would well bear) appeared very near as big, as that of the Moon n. 14. p. 239.
to the naked Eye. Mar. An. 1666.

But such had been the ill Disposition of the Air for several Nights, that from more than 20 Observations of it which I had made since its being *Retrograde*, I could find nothing of Satisfaction, though I often imagined, I saw *Spots*, yet the Inflective Veins of the Air (if I may so call those parts, which being interspers'd up and down in it, have a greater or less Refractive Power, than the Air next adjoining, with which they are mix'd) did make it so confus'd and Glaring, that I could not conclude upon any thing.

On the 3^d of *Mar.* 0^h 30'. 1666 in the Morning though the Air was still bad enough, yet I could see now and then the Body of *Mars*, which I described by the Scheme B; as exactly representing what I saw through the Glas as I could.

Fig. 146.

An. 1666, *Mar.* 10. 0^h 20'. in the Morning finding the Air very bad, I made use of a very shallow Eye-Glas, as finding nothing distinct with the greater Charge, and saw the Appearance of it as in C, which I imagined might be the Representation of the former *Spots* by a lesser Charge. About 3 of the Clock

Fig. 147.

Fig. 148. Clock the same Morning, the Air being very bad (though to appearance exceeding clear, and causing all the Stars to twinkle, and the Minute Stars to appear very thick) the Body seemed like D; which I still supposed to be the Representation of the same *Spots* through a more confused and glaring Air.

Fig. 149. But observing *Mar. 21.* I was surprized to find the Air (though not so clear, as to the Appearance of small Stars) so exceeding transparent, and the Face of *Mars* so very well defined, and round and distinct, that I could manifestly see it of the Shape in E, about half an Hour after 9 at Night. The Triangular *Spot* on the Right-side (as it was inverted by the Telescope) according to the Appearances, through which all the preceding Figures are drawn) appeared very black and distinct, and the other towards the left more dim; but both of them sufficiently plain and defined. About a Quarter before 12 of the Clock the same Night, I observed it again with the same Glass, and found the Appearance exactly, as in F; which I imagined to shew me a Motion of the former Triangular *Spot*.

Fig. 151. *Mar. 22.* about half an Hour after 8 at Night, finding the same *Spots* in the same Posture (as in G) I concluded that the preceding Observation was only the Appearance of the same *Spots* at another Height and Thickness of the Air: and thought my self confirmed in this Opinion, by finding them in much the same Posture, *Mar. 23.* about half an Hour after 9 (as in H) though the Air was nothing so good as before.

Fig. 153. *Mar. 28.* about 3 of the Clock, the Air being light (in Weight) though moist and a little hazy; I plainly saw it, to have the Form represented in I. which is not reconcileable with the other Appearances, unless we allow a Turbinated Motion of *Mars* upon its Center: Which, if such there be, from the Observations made *Mar. 21, 22, and 23.* we may guess it to be once or twice in about 24 Hours, unless it may have some kind of Librating Motion; which seems not so likely.

The Parallax of Mars; by Mr. Flamsteed. n. 89. p. 5118. n. 96. p. 6100. *XCIV. An. 1672. Sept. Micrometro & Tubo 14 pedum, Martis Distantias à duabus Fixis eadem Nocte dimensus sum; unde didici Parallaxin ejus Acronici & Perigei nunquam majorem esse Scrupulis Secundis 25''; unde sequitur, Solis esse summum 10'', & Distantiam 21000 Terra Semidiametros.*

Places of Mars Observed at Darby; by Mr. Flamsteed. n. 86. p. 5039. *XCV. 1. An. 1672. Maii 14 mane, ibat Martis Sidus prope Stellam dictam, Quæ ad Clunes Aquarii; cujus Latitudo 2° 0' 0''. Locus tum mihi, 24° 12' 9''; Streetio, 24° 9' 00''. è qua notabam.*

| | | | |
|-----------------------|-------|--|---------|
| <i>Alia Fixa verè</i> | 9 40 | 2h 29' mane, <i>Martis</i> in eodem }
<i>Azymutho præcisè Distantiam</i> } | 0 1 11 |
| | 11 12 | eandem denuo <i>Distantiam</i> — | 0 24 17 |
| | 12 00 | 2h 51'. <i>Planeta</i> discesserat ab eodem <i>Azymutho</i> , eratque ad Ortum à <i>Linca, Azym. Differ.</i>) | 0 24 24 |
| | | | 0 00 55 |

| | | | | | | |
|---|---|---|-----------------------|----|----|----|
| 2 ^h 29'. mane <i>Angulus Parallaxicus</i> erat | — | — | — | 82 | 26 | 00 |
| Ergo <i>Mars</i> in <i>Consequentia Fixa</i> | — | — | — | 00 | 3 | 11 |
| cum majori Latitudine Australi | — | — | — | 00 | 24 | 4 |
| <i>Martis</i> propterea Latitudo | — | — | — | 2 | 24 | 4 |
| Locus verus | — | — | — | 24 | 12 | 17 |
| | | | { <i>Sireetio.</i> ~. | 24 | 15 | 40 |
| | | | { <i>Mibi.</i> ~. | | | |

2. An. 1683. Die 3. Maii 9^h 12'. *Mars* Conjunctionem celebrabat cum *As Dantick*; Penultima Stella *Ale Austrina Virginis*, quæ modo degit in 13° 49' 14". by *M. Hevelius*. *z.* & Latit. 1° 48' 33". Bor. sic ut in ipsa Conjunctione vix ultra 40". n. 151. p. 332. *Mars* infra dictam Stellam incederet.

XCVI. 1. *M. Burattini* hath signified from *Poland*, that he hath Observed Spots in *Venus*; Inequalities in *Venus* as in the *Moon*. by *M. Burattini*. n. 10. p. 173.

2. I have at last discovered towards the Middle of the Body of *Venus* a part clearer than the rest, by which one may Judge of the Motion, or the Rest of this Planet. The Rotation of *Venus*; by *M. Cassini*. n. 32.

The first time I saw it, was Octob. 14. 1666. 3^h 45'. p. m. and then this Bright part was very near the Center, on the North-side. And at the same time I observ'd Westward two obscure Spots, somewhat oblong; but I could not then see that Resplendent part long enough to conclude any thing from thence, nor was I able to see any thing well of those parts till April 28. 1667. on which Day a quarter of an Hour before Sun-rising, I saw again a Bright part, situated near the Section, and distant from the Southern Horn a little more than $\frac{1}{4}$ of its Diameter. And near the Eastern Ring I saw a Dark and somewhat Oblong Spot, which was nearer to the Northern than the Southern Horn. At the Rising of the Sun I perceived, that this Bright part was then no more so near the Southern Horn, but distant from it $\frac{1}{4}$ of its Diameter. This gave me great Satisfaction. But I was surpris'd at the same time to find, that the same Motion, which was made from South to North in the Inferiour part of the Disk was on the contrary made from North to South in the Superiour part; whence the Determination of the Motion may be better taken: For we have no Example of the like Motion, except it be in that of the Libration of the Moon. p. 615. n. 35. p. 687.

The next Day, at the Rising of the Sun, the said Bright part was not far from the Section, and distant from the Southern Horn $\frac{1}{3}$ of the Diameter. When the Sun was 4 Degrees high, the same was situated near the Section, remote from the Southern Horn $\frac{1}{4}$ of the Diameter. The Sun being high 6° 10'. it seem'd to have pass'd the Center, and that the Section of the Disk did cut the same. The Sun being 7° high, it appear'd yet more advanced Northward, together with two Obscure Spots seated between the Section and the Circumference, and equally distant from one another, and from each Horn on both Sides. And the Sky being very clear, I observ'd the Motion of the Bright part for 1^h. which then seem'd to be exactly made from South to North without any sensible Inclination Eastward or Westward.

Mean time I perceived in the Motion of the dark Spots so great a Variation, that it cannot be ascribed to any Reason in *Opticks*.

May 10. and 13. Before *Sun-Rising*, I saw still the Bright part near the Center Northward.

Lastly, June 5. and 6. Before the Rising of the *Sun*, I saw the same between the Northern Horn and the Center of this Planet, and I noted the same Irregular Variation in the Obscure Spots. But when *Venus* began to be farther removed from the *Earth*, it was more difficult to observe the *Phenomena*.

It is very difficult to determine any Thing of the Motion of these Spots; Yet this I can say (supposing that this Bright part of *Venus*, which I have observ'd, especially this Year 1667, hath always been the same) that in less than one Day it absolves its Motion, whether of Revolution or Libration, so as in near 23 Hours it returns to the same Situation in this Planet, which yet happens not without some Irregularity.

A Place of *Venus*, Observ'd at Dantzick; by M. Hevelius. n. 154. p. 419.

XCVII. An. 1583. Aug. 4. hora fere 2. m. *Venus* à Stella Fixa 3. Magnitud. à *Ventre* scil. *Pollucis* non nisi 16'. removebatur Austrum versus, id quod ex *Micrometro* accuratè comperi.

XCVIII. Mercury observ'd in the Sun, 1690. at Nuremberg; by M. Jo. Phil. Wurtzelbaur. u. 192. p. 483.

| Temp. Horol. Oscil. | Observationes. | Tempora supputata. |
|---------------------|---|--------------------|
| h. ' " | Die 30. Octob. p. m. A. 1690. | h. ' " |
| 6 32 00 | Culminat Os <i>Pegasi</i> | 6 28 45 |
| 9 00 00 | Culm. Cap. <i>Andromeda</i> | 8 52 17 |
| 9 4 30 | Culm. <i>Jupiter</i> | |
| 8 30 00 | Die 31. Octob. m. Sol à nubibus emerfit, <i>Mercurius</i> in disco ejus superne in Tab. observatoria, à Verticali ad dextram (revera ad <i>Lævam</i>) distans plus quam Dim. Dig. à Limbo exiturus apparuit. | |
| 8 36 00 | <i>Mercurius</i> postquam Undulanti Limbo <i>Solis</i> ad Min. Temporis adhæserat, exiit ad 14° à Zenith Septentrionem versus | |
| 8 49 00 | Altitudo <i>Solis</i> | h. ' " |
| 8 59 45 | | 8 38 38 |
| 9 7 10 | | 8 47 48 |
| 9 50 00 | | 8 56 24 |
| 11 1 30 | | 9 38 7 |
| | | 10 56 32 |

Ratio Diametrorum *Solis* & Nuclei *Mercurii*, dum Lucido *Solis* Disco immorabatur, quantum per auram haud satis defæcatam conjici poterat, erat ut

1000 ad 8 $\frac{1}{2}$. Postea quam ad Limbum *Solis* pervenerat, ejusque Limbo Undulanti ad Minutum ferè adhæsitâr, & genuinam Rotunditatem suam (quæ antea ex luce Disci *Solaris* formam quasi Ellipticam mentiebatur) recuperâr, erat ut 1000 ad 12 $\frac{1}{2}$.

XCIX. *An.* 1697. *Nov.* 3. *st. n.* 7h 25' cum *Sol* è nubibus Emerfisset di-Mercury ob-
recto ad ipsum Telescopio Differentia Ascensionis Rectæ Centri *Mercurii* Oc-served in the
cidental & Centri *Solis* observata per Horologium fuit Hor. *Sun.* Nov. 3. *st.*
n. 1697; by M.
Cassini. *n.* 245.

| | | | | |
|---|---|----|-----|-----|
| | — | 0° | 11' | 52" |
| Differentia Declinationis <i>Mercurii</i> Meridionalis fuit Grad. | — | 0 | 6 | 20 |
| 8h 3'. Differentia Ascensionis Rectæ Centrorum <i>Mercurii</i> } | | 0 | 15 | 30 |
| Occidental & <i>Solis</i> fuit Hor. | — | | | |
| Differentia Declinationis Grad. | — | 0 | 4 | 42 |

8h 8' 38". *Margo* Præcedens *Mercurii* pervenit ad *Solis* Marginem Præcedentem.

8h 10' 24". *Mercurius* totus Emerfit è *Solis* disco Telescopio pedum 18 observatus.

Ex his observationibus invicem comparatis, quantum ex hoc brevi intervallo inferri potuit, adventum *Mercurii* ad medium ipsius semitæ in *Solis* Disco Trigonometricè deduxi 6h 11' 18". post Meridiem.

Nodum vero *Ascendentem* *Mercurii* in $\odot$ 14° 42'. adhuc promotiorem quam per observationes *Anni* 1677.

Inclinationem autem *Orbitæ* *Mercurii* ad *Eclipticam* ex postremarum observationum comparatione inveni 6° 23', quam nihilominus ob breve harum observationum intervallum præferre non ausim ei quam ex *Sinemis* observationibus R. P. *Fontenay* longè majori intervallo distantibus deduxi. sc. 6° 40'. propius accedentem ad Tabulas Rudolphinas.

C. *Mercurii* *Venerisque* Sidera *Solis* Discum subintrare, ac instar Macula-
rum Nigricantium in Lucido ejus Orbe aliquando conspici, tam ex Verioris
Astronomia Principiis, quam ex indubitata Observantium Fide, dudum com-
pertum est. Qua vero Lege, quibusve Conditionibus, quantisque *Annorum*
Intervallis, hæc Phænomena nobis spectanda præbentur, nescio an aliquis ex
Astronomis Hodiernis ritè definiverit: Certe nihil hac de re inter Typis man-
data hucusque mihi visum est. Quapropter non Ingratum fore arbitratus,
huic Inquisitioni serio operam dedi, ac Dissertatione hanc rem maximè per-
plexam paucisque intellectam me plenius enucleaturum confido.

Has *Planetarum* Horum Phases semper in Retrogradientium cum *Sole* Con-
junctionibus fieri, cum scil. *Sol* *Nodis* eorum adeo vicinus sit ut *Planeta* *Soli*
juncti Latitudo Semidiametrum *Solis* non excedat, per se satis conspicuum est;
quò vero facilius Limites ac Conditiones harum Conjunctionum pervestigem,
cumque Calculi Elementa omninò diversi sint, uterque *Planeta* sigillatim tra-
ctandus est: à *Mercurio* itaque exordiamur.

Hujus *Planetæ* *Nodum* *Ascendentem*, juxta nuperas & accuratas Observa-
tiones, prope 15° *Tauri*, seu potius ad $\odot$ 15° 44'. à 11° * γ . hoc nostro
seculo reperiri, pro comperto habemus. *Descendentem* vero Oppositam ad

The Visible Con-
junctions of the
Inferior Pla-
nets with the
Sun; by Mr.
Halley. *n.* 193.
p. 511. Mar.
An. 1691.

6° 15' 44". à 12 * v. Angulum autem quo Planum Orbitæ Mercurialis ad *Eclipticam* Inclinator satis bene se habet apud *Keplerum*, viz. 6° 54'. Jam ex probatissimis Hypothesibus constat *Mercurii* in *Nodo Ascendente* constituti Distantiam à *Sole* esse partium 31365, quarum media *Solis* Distantia à *Terra* sit 100000; dum vero *Nodum* Alterum occupat, Distantia ista, in iisdem partibus mensurata, fit 45308. Sol vero *Nodo Ascendenti* Oppositus distat à *Terra* eidem Juncta partium istarum 98955, ad *Nodum* vero Alterum eadem Inter-capedo fit 101007. Atque idcirco *Mercurius* Soli Conjunctus ad *Nodum Ascend.* distat à *Terra* partibus 67591: ad *Nodum* vero *Descend.* partibus 55699. Quæ, cum inter se valde discrepent, separatim etiam considerandæ veniunt Conjunctiones illæ quæ ad diversos *Nodos* fiunt, Calculi Elementis Compendii gratia Synopticè expofitis.

Conjuncturæ Mercurius Retrogradus cum Sole Centraliter ad Nodum Ascendentem, Mense Octobri; ac ex dictis Hypothesibus habebitur.

| | | |
|---|-------|-------------|
| Longitudo Solis à Prima Stella Arietis | — | 6 15 44 00 |
| Longitudo Mercurii ex Sole visi | — | 0 15 44 00 |
| Distantia Mercurii à Sole | 31365 | |
| Distantia Mercurii à Terra | 67591 | |
| Angulus Inclinationis Orbis Mercurii | — | 0 6 54 00 |
| Motus 6 Horar. Mercurii ex Sole visi | — | 0 1 30 58 |
| Motus Solis in iisdem 6 Horis | — | 0 0 15 5 |
| Hinc Motus Mercurii à Sole, sex Horis | — | 0 1 15 53 |
| Et Motus ejus à Sole ex Terra viso, 6 Horis | — | 0 0 35 12 |
| Et Angulus Viæ Mercurii intra Solem visæ cum Ecliptica | — | 0 8 15 00 |
| Hinc Motus Mercurii in Orbis sua visibili 6 Horis | — | 0 0 35 40 |
| Deinde Motus Mercurii in Anno Siderio ultra quatuor Revolutiones | — | 1 24 45 8 |
| In Annis Tredecim itaque | — | 11 21 46 44 |
| Defunt itaque ad Revolutiones 54 Integra | — | 0 8 13 16 |
| | | d. h. |
| Quod spatium percurrit Mercurius in | — | 2 00 11 |
| Quibus promovetur Solis Locus; ac q̄ in <i>Nodo</i> Situs tantundem distat à Conjunctione Terra | — | 0 1 " |
| At Arcus iste ex Terra spectatus fit | — | 2 1 00 |
| Unde ex dato Angulo Viæ visæ 8° 15' provenit Basis, sive Distantia à Conjunctione Visibili | — | 0 55 34 |
| Qui Arcus percurritur à Mercurio juxta Horariam datam | — | h. |
| | | 9 21 |
| Excedunt vero 13. Anni Siderii totidem Julianos cum Inter-calculationibus tribus | — | 8 0 |

Itaque

| | |
|--|------------|
| Itaque <i>Mercurius</i> revertitur ad <i>Solem</i> post <i>Annos Julianos</i> | d. h. " |
| 13 atque insuper | 2 17 34 |
| Vel cum quatuor <i>Intercalationibus</i> , si præcedens <i>Annus</i> | |
| fit tertius à <i>Bissexto</i> | 1 17 34 |
| Ex arcu vero 56' 10". & Angulo dato, fit perpendicularis, sive proxima Distantia à <i>Sole</i> | 0 8 3 |
| Itaque 8 post 13 <i>Annos</i> intra <i>Solem</i> conspicuus, 8' 3". | |
| <i>Borealius</i> Incedit. | |
| _____ | _____ |
| Pari Argumento in 46. <i>Annis Sideriis</i> movetur 8 | 11 28 36 8 |
| Defunt itaque ad <i>Revoluciones</i> 191 Integras | 0 1 23 52 |
| | h. " |
| Hoc est in Tempore | 8 12 |
| | 0 1 " |
| Quo promovetur <i>Sol</i> | 0 20 41 |
| Hic arcus è <i>Terra</i> visus fit | 0 9 36 |
| <i>Basis</i> vero ei competens | 0 9 30 |
| | h. " |
| Tempus, vero quo <i>Mercurius</i> <i>Basin</i> percurrit, fit | 1 30 00 |
| Excedunt vero 46 <i>Annis Sideriis</i> totidem <i>Julianos</i> cum 11 | |
| <i>Intercalationibus</i> | 19 3 00 |
| Ac <i>Mercurius</i> revertitur ad <i>Solem</i> post 46 <i>Annos Julianos</i> | d. h. " |
| atque insuper | 1 4 51 |
| Vel cum duodecim <i>Intercalationibus</i> , ut fit cum <i>Annis</i> | |
| præcedens, fit Secundus vel Tertius à <i>Bissextili</i> | 0 4 51 |
| Perpendicularis vero, quo <i>Mercurius</i> in <i>Boream</i> provehitur, fit | 0 1 22 |
| | h. " |
| Periodus vero maxime Accurata <i>Mercurii</i> ad <i>Solem</i> absolvitur <i>Annis Sideriis</i> 263 atque insuper | 1 11 30 |
| Hi vero <i>Siderii</i> superant totidem <i>Julianos</i> cum 66 <i>Intercalationibus</i> | 10 20 0 |
| Unde post 263 <i>Annos Julianos</i> , <i>Mercurius</i> ad <i>Solem</i> revolvitur, tardius vero | 11 33 30 |
| | d. |
| Quod si præcedens <i>Annus Bissextilis</i> fuerit, addantur | 1 11 31 30 |
| Post hoc demum Intervallum <i>Borealius</i> Incedit | 0 0 0 10 |

Cæteræ vero Periodi Latiores ex jam Inventis facili negotio eruuntur, suntque vel 6 vel 7 Annorum.

Quæ Septem Annis absolvitur, *Mercurium* deprimit versus Austrum 22' 47", ac Septem Dies integros, minus 9 Minutis, citius provenit, si duæ fuerint *Intercalationes*. At cum una *Intercalatione*, cum scil. *Annus* prior *Bissextilis* sit, 6 dies subducendi sunt, additis tamen 9 Minutis, ut prius.

Rarius

Rarius vero post sex Annos in *Solis* disco conspicitur iterum Vagus ille *Planeta* qui exacta hoc Periodo 30' 50". Borealius transit; idque tardius, 9^d 17^h 25', si Annus præcedens sit Secundus vel Tertius à *Bissextili*, aliter 8^d 17^h 25' addendi sunt.

Pariter si fiat Conjunctio ad Nodum Descendentem Mense Aprili.

| | s. | ° | " | ''' |
|--|----|----|----|-------|
| Longitudo <i>Solis</i> à Prima Stella Arietis | — | — | — | — |
| <i>Mercurii</i> Longitudo ex Sole visi | — | — | — | — |
| Distantia Planetæ à Sole ut prius | — | — | — | 45308 |
| Distantia ejus à Terra | — | — | — | 55699 |
| Motus <i>Mercurii</i> à Sole visi Sex Horis | — | — | — | — |
| Motus <i>Solis</i> in eodem tempore | — | — | — | — |
| Motus <i>Mercurii</i> à Sole | — | — | — | — |
| Hinc Angulus Viæ visæ <i>Mercurii</i> intra <i>Solis</i> discum cum
Ecliptica fit | — | — | — | — |
| Motus vero visus à Terra in 6 Horis | — | — | — | — |
| | 8. | ° | " | ''' |
| | 0 | 15 | 44 | 00 |
| | 6 | 15 | 44 | 00 |
| | 0 | 0 | 14 | 29 |
| | 0 | 0 | 43 | 21 |
| | 0 | 0 | 28 | 52 |
| | 0 | 10 | 18 | 00 |
| | 0 | 0 | 23 | 52 |

Unde sequendo methodum calculi præcedentis, evincitur *Mercurium* post 13. Annos atque insuper 3^d 7^h 37' ad *Solis* Conjunctionem revolvī; quod si præcedens Annus fuerit Tertius à *Bissextili*, tunc addendi sunt 2^d 7^h 37'. tantum: ac tum *Mercurius* 16' 55". Australius incedere reperietur.

Post 46 vero Annos, cum 12 Intercalationibus addantur 0^d 7^h 14'. & habebitur *Mercurius Soli* Conjunctus in Tramite Australiore 2' 23". si vero 'Annus' Prior *Bissextilis* fuerit; vel ab eo Primus, addendus est 1^d 7^h 14'. ut habeatur accurate Synodus. Similiter post 263 Annos, quibus *Mercurius* in Austrum deflectitur 0' 22". Addendus est vel 1^d 11^h 49'. vel 11^h 49', juxta Legem in priori Casu præscriptam.

At Annis sex vel septem ob viciniam *Terra* ac *Planeta*, atque idcirco ob ampliatos Arcus ad hunc *Nodum*, non revertitur ad *Solem*, ut intra Discum appareat. Post Annos autem 33 *Solem* Transiit Viâ magis Boreali 14' 2". ac habetur momentum Conjunctionis subducendo à Prioris tempore 3^d 0^h 23'. si fuerit in Anno Tertio à *Bissextili*; aliter subduc 2^d 0^h 23'. tantum.

His Inventis facile erit continuare Calculum pro omnibus hisce Conjunctionibus *Mercurii* cum *Sole*, idque cum summa certitudine, ac sine ulla hæsitazione, an omnes possibiles habeantur necne: Sola Additione obtinentur momenta Conjunctionum ac Distantiæ Planetæ à Centro *Solis*; unde etiam ope Tabellæ depromuntur Durationes harum, ut ita dicam, *Eclipsium*, ut nihil sit quod in hac re desiderari videatur.

Epochas vero quod spectat, ex tutius Observatorum industria comparantur, quam calculi cujusvis Subtilitate: adeoque elegimus in primo Casu, notabilem illum Transiitum *Mercurii* quem ipse in Insula Sancta Helena perfectissime observavi, Oct. 28. An. 1677. St. Vet. & cujus Medium ex Initio & Fine determinavi in prædicta Insula quidem 0^h 4'. p. m. Londini vero 0^h 28'. p. m.

Semita

Semita vero qua incedere visus est Planeta $4' 40''$. Borealior erat Solis centro. In altero Casu, viz. cum *Mercurius Soli* conjungitur Mense *Aprili*, ex Cl. *Hevelii Mercurio in Sole viso*, p. 72. 75. Epocham defumere placuit; nempe quod *Apr. 23. An. 1662. St. Vet. 6h 8'. p. m. Gedani*, hoc est $4^h 52'$. *Londini*, *Mercurius Solis* centro proximus apparuit, utpote in medio Transitu, simulque distabat ab eodem Centro $4' 27''$ ad Boream. Hinc juxta præcepta præmissa, omnes ordine visibiles Conjunctiones *Mercurii* cum *Sole* simul exhibere, exigui Laboris opus erat: ac in Exemplum quod cuivis in posterum imitari licet, accipe hujus Seculi, ab invento Telescopio, quotquot usquam apparuere hujusmodi Phænomena, vel quæ etiam in sequentis Seculi posteris apparitura sunt.

Series Momentorum quibus Mercurius Soli Conjunctus intra Discum ejus conspicitur, per præsens & futurum Seculum, cum Distantiis Planeta à Solis centro.

A P R I L I.

| Ann. | Temp. Conj. | | | Dist. à Cent. ☉ | | |
|------|-------------|----|--------|-----------------|----|---|
| | d. | h. | ' | ' | '' | |
| 1615 | 22 | 21 | 38 * * | 7 | 20 | B |
| 1628 | 25 | 5 | 15 * | 9 | 35 | A |
| 1661 | 23 | 4 | 52 * | 4 | 27 | B |
| 1674 | 26 | 12 | 29 | 12 | 28 | A |
| 1707 | 24 | 12 | 6 | 1 | 34 | B |
| 1720 | 26 | 19 | 43 * | 15 | 21 | A |
| 1740 | 21 | 11 | 43 | 15 | 36 | B |
| 1753 | 24 | 19 | 20 * | 1 | 19 | A |
| 1786 | 22 | 18 | 57 * | 12 | 43 | B |
| 1799 | 26 | 2 | 34 * | 4 | 12 | A |

| Ann. | Temp. Conj. | | | Dist. à Cent. ☉. | | |
|------|-------------|----|--------|------------------|----|---|
| | d | h. | ' | ' | " | |
| 1605 | 22 | 8 | 29 | 12 | 48 | A |
| 1618 | 25 | 2 | 4 * | 4 | 45 | A |
| 1631 | 27 | 19 | 37 * | 3 | 18 | B |
| 1644 | 30 | 13 | 11 | 11 | 21 | B |
| 1651 | 23 | 13 | 20 | 11 | 26 | A |
| 1664 | 25 | 6 | 54 * | 3 | 23 | A |
| 1677 | 28 | 0 | 28 * * | 4 | 40 | B |
| 1690 | 30 | 18 | 2 * | 12 | 43 | B |
| 1697 | 23 | 18 | 11 * | 10 | 4 | A |
| 1710 | 26 | 11 | 45 | 2 | 1 | A |
| 1723 | 29 | 5 | 19 * | 6 | 2 | B |
| 1730 | 22 | 5 | 18 | 16 | 45 | A |
| 1736 | 30 | 22 | 53 * * | 13 | 5 | B |
| 1743 | 24 | 23 | 2 * * | 8 | 42 | A |
| 1756 | 26 | 16 | 36 | 0 | 39 | A |
| 1769 | 29 | 10 | 10 | 7 | 24 | B |
| 1776 | 22 | 10 | 19 | 15 | 23 | A |
| Nov. | | | | | | |
| 1782 | 1 | 3 | 44 * | 15 | 27 | B |
| Oët. | | | | | | |
| 1789 | 25 | 3 | 53 * | 7 | 20 | A |

Transitus qui signo * notantur, *Londini* ex parte Visibiles sunt, qui vero signo **, toti conspici possunt.

Notandum vero est *Solis* Diametrum ad *Nodum ♀ rii Ascendentem* mense *Octobri* occupare 32' 34'', atque adeo Maximam Durationem centralis Transitus esse 3^h 29', Mense vero *Aprili* Diameter *Solis* fit 31' 54''; unde ob tardiozem Planetæ Motum oritur Duratio Maxima 8^h 1': Quod si Oblique incidat *Mercurius*, Durationes hæ Breviores redduntur pro ratione Distantiæ à centro *Solis*: Quoque perfectior Calculus hic reddatur, sequentes Tabellæ adjunxi, quibus exhibentur dimidiata Durationes harum *Eclipsium* ad singula minuta Distantiæ Visæ à centro *Solis*; quæ Additæ ac Sublatæ à Conjunctionis momento in priori Tabula invento, *Initium* ac *Finem* totius Phænomeni designant.

OCTOBRI.

APRILI.

| Min.
Dist. | Semi-
durat. | Min.
Dist. | Semi-
durat. |
|--------------------------------|-------------------------------------|--------------------------------|----------------------------------|
| 0 | h. 2 44 ¹ / ₂ | 0 | 4 0 ¹ / ₂ |
| 1 | 2 44 | 1 | 4 0 |
| 2 | 2 43 | 2 | 3 58 ¹ / ₂ |
| 3 | 2 41 ¹ / ₂ | 3 | 3 56 |
| 4 | 2 39 ¹ / ₂ | 4 | 3 53 |
| 5 | 2 36 ¹ / ₂ | 5 | 3 48 ¹ / ₂ |
| 6 | 2 33 | 6 | 3 43 |
| 7 | 2 28 ¹ / ₂ | 7 | 3 36 |
| 8 | 2 23 | 8 | 3 28 |
| 9 | 2 17 | 9 | 3 18 ¹ / ₂ |
| 10 | 2 10 | 10 | 3 7 |
| 11 | 2 1 | 11 | 2 54 |
| 12 | 1 51 | 12 | 2 38 |
| 13 | 1 39 | 13 | 2 19 |
| 14 | 1 24 | 14 | 1 55 |
| 15 | 1 4 | 15 | 1 21 ¹ / ₂ |
| 15 ¹ / ₂ | 0 50 | 15 ¹ / ₂ | 0 56 |
| 16 | 0 30 | | |

Observationes omnes hucusque Habitus ritè representant hi Numeri, nec est quod dubitem de Futuris, cum ex omnibus Planetis *Mercurius*, *Soli* proximus, ejus Centro adeo vicinus sit, ut aliorum Centrorum interventu minimè cieatur, nec deviationibus illis quæ à cæterorum Systemate oriuntur, quibusque superiores, præsertim *Saturnus*, obnoxii sunt, quod sentiri possit interturbetur.

Parallaxes consultò omisi, ut perexiguas, quæque Locis diversis diversæ obvenientes Generaliori Calculo immisceri non debent; quodque etiam quantæ sint non satis adhuc constat, sed potius ex hujusmodi Observatis tutissime derivari possint: Diametri etiam *Mercurii* rationem non habui, quia supra fidem Parvus perpauca solum Minuta Limbo adherere videtur. Ex Observatione accuratissimâ deprehendi vix duo Minuta elapsa dum totus è *Sole* egrederetur Octob. 28, 1677. unde conclusi Diametrum ejus 0' 11", ac jux-

ta rationem Distantiarum à Terra ad Nodum Alterum $0' 13'' \frac{1}{2}$ fere; adeoque tunc $3 \frac{1}{3}$ Temporis Minuta ipsi sumi, dum Totus Planeta Solis Limbum Directe pervadit: Oblique vero transiens paulo diutius hæret, secundum ac Secantes Angulorum Incidentiæ augentur. Equationes etiam Temporis haud opus est ut æstimemus, quia per plures dies hinc inde in utroque Mense Constantes ac quasi Invariatae persistunt.

De Visibili Veneris cum Sole Conjunctione.

Venus, quamvis Syderum omnium speciosissima, more Sexus sui, sine mutato Cultu ac Splendore ascititio in Conspectum prodire veretur: Hoc etenim Spectaculum inter *Astronomica* longe nobilissimum, instar Ludorum Secularium, integri Seculi Mortalibus invident Motuum arctæ Leges. Unico vero hoc observato, summa cum certitudine Distantiam Solis à Terra determinari posse, quæ ob Parallaxin alias prorsus Insensibilem Vagis Terminis hucusque definita est, posthac declarabitur. Periodos vero quod attinet, illæ non adeo Accurate ac *Mercuriales* describi possunt, cum *Venus* semel tantum ab Orbe Condito, idque ab *Horroxio* nostro, intra Solis Discum deprehensa sit: Correctis autem Motibus, quantum per Rudiores Veterum observationes licet, accipe jam summam Calculi.

| | |
|---|-------------|
| Longitudo Nodi Ascendentis Veneris à prima Stella Arietis | s. 0' 11" |
| Sol itaque ei jungitur in puncto opposito; hoc est, per hæc Secula, circa finem Novembris | 1 15 16 00 |
| Distantia Veneris à Sole partium | 7 15 16 00 |
| Distantia Veneris à Terra | 71997 |
| Inclinatio Orbitæ Veneris ad Eclipticam | 26438 |
| Motus Veneris in 8 Annis Syderiis, supra 13 Revolutiones | 0 3 23 0 |
| Motus Veneris in 235 Annis Syderiis supra 381 Revolutiones | 0 1 30 28' |
| Motus Veneris in 243 Annis Syderiis supra 395 Revolutiones | 11 29 17 39 |
| | 0 00 48 8 |

Ex his Principiis, imo Calculo juxta Methodum in *Mercurio* expositam, proveniunt intervalla Temporum ac Distantiarum ut sequitur.

Post 18 Annos *Venus* revolvitur ad Solem sc. sublati à prioris Transitus momento $2^d 10^h 52^m \frac{1}{2}$. Incedit vero Planeta Semitâ $24' 41''$. priori magis Australi.

Post Annos 235, Additis $2^d 10^h 9'$, *Venus* iterum Solem ingredi potest, sed viâ, $11' 33''$. Borealiore: Quod si præcedens Annus Bissextilis fuerit $3^d 10^h 9'$. addendi sunt.

Post Annos 243 *Venus* etiam Solem transire potest, auferendo tantum $0^h 43'$. à prioris Tempore; Australius vero incedit $13' 8''$: Quod si præcedens Annus Bissextilis fuerit, adde $23^h 17'$.

Et

Et in omnibus his Appulsibus *Veneris* ad *Solem*, Mense *Novembri*, Angulus *Vizæ* *Vizæ Veneris* cum *Ecliptica* fit $9^{\circ} 5'$. ac *Motus* ejus *Horarius* intra *Solem* $4' 7''$; cumq; *Semidiameter Solis* fit $16' 21''$. provenit *Maxima Duratio* *Transitus* centri *Veneris* $7^h 56'$.

Deinde conjungantur *Sol* & *Venus* ad *Nodum Descendentem* Mense *Mai*; ac juxta *Numeros* eisdem supputantur intervalla eadem. Post 8 *Annos* auferendi sunt $2^d 6^h 55''$, ac *Venus* *Orbita* $19' 58''$. Borealiori pertransibit.

Post *Annos* 235 adde $2^d 8^h 18'$, vel si prior *Annus Bissextilis* fuerit $3^d 8^h 18'$. & habebis *Venerem* *Australiorem* $9' 21''$.

Denique post 243 *Annos*, adde $0^d 1^h 23'$. vel si prior *Annus Bissextilis* fuerit $1^d 1^h 23'$. & reperitur *Venus* iterum *Soli* *Conjuncta*, sed in tramite $10' 37''$. magis Boreali.

In omni ad hunc *Nodum* *Transitu* intra *Solem*, Angulus *Vizæ* *Vizæ Veneris* cum *Ecliptica* fit $8^{\circ} 28'$; ac *Horarius* ejus *Motus* $4' 00''$; ac *Solis* *Semidiameter* subtendente $15' 51''$; provenit *Duratio* *Maxima* *centralis* *Transitus* etiam $7^h 56'$; præcise eadem ac ad *Nodum Alterum*.

Quoad *Epochas*; Ex *Ingressu* quem solum vidit *Horroxius* in *Sole* jamjam occasuro concluditur, *Venerem* *Soli* *Junctam* fuisse *Londoni* 1639. *Nov.* 24. $6^h 37'$. sed versus *Austrum* incessisse $8' 30''$. Mense *Mai* vero à nemine *Mortalium* hucusque intra *Solem* visa est, sed ex *Numeris* meis quos non multum à *Cælo* ablufuros confido, constat *Venerem* proximâ vice *Solem* subituram *A.* 1761. *Maii* 25 $17^h 55'$. mediâ scil. *Eclipsi*, ac tunc *Distare* à *Centro* ejus versus *Austrum* $4' 15''$. Hinc & ex præmissis *Revolutionibus* facili negotio omnia hujus generis *Phænomena*, per *Millenium* integrum computavi, ut in sequenti *Tabella* exhibentur.

NOVEMBRI.

| Ann. | Temp. Conj. | | | Dist. à Cent. ☉ | | |
|------|-------------|----|----|-----------------|----|---|
| | d. | h. | ' | ' | " | |
| 918 | 20 | 21 | 33 | 6 | 12 | B |
| 1161 | 20 | 21 | 10 | 6 | 55 | A |
| 1396 | 23 | 7 | 20 | 4 | 38 | B |
| 1631 | 26 | 17 | 29 | 16 | 11 | B |
| 1839 | 24 | 6 | 37 | 8 | 30 | A |
| 1874 | 26 | 16 | 46 | 3 | 3 | B |
| 2109 | 29 | 2 | 56 | 14 | 36 | B |
| 2117 | 26 | 16 | 3 | 10 | 5 | A |

| Ann. | Temp. Conj. | | | Dist. à Cent. ☉ | | |
|------|-------------|----|----|-----------------|----|---|
| | d. | h. | ' | ' | " | |
| 1048 | 24 | 13 | 45 | 3 | 50 | B |
| 1283 | 23 | 8 | 14 | 5 | 31 | A |
| 1291 | 25 | 15 | 9 | 14 | 27 | B |
| 1518 | 25 | 16 | 32 | 14 | 52 | A |
| 1526 | 23 | 9 | 37 | 5 | 6 | B |
| 1761 | 25 | 17 | 55 | 4 | 15 | A |
| 1769 | 23 | 11 | 00 | 15 | 43 | B |
| 1996 | 28 | 2 | 13 | 13 | 36 | A |
| 2004 | 25 | 19 | 18 | 6 | 22 | B |

Durations harum *Veneriarum Eclipsium* quod attinet, respectu Centri eodem modo supputari possunt ac *Mercuriales*; sed cum Diameter *Veneris* satis ampla sit, cumque Parallaxes etiam Differentiam valde Notabilem quoad Tempus ingerere possunt, Calculus peculiaris pro Locis singulis necessariò subeandus est.

Veneris autem Diameter tanta est, ut dum Limbo *Solis* adhæret, ferme 20 Temporis Minuta præterfluunt, cum scil. *Solem* Directe aggreditur; Oblique vero Incidens, etiam diutius Limbo immoratur: Occupat autem Diameter ista, juxta *Horroxii* Observationem 1' 18", dum ad *Nodum Ascend. Soli* Jungitur, ac 1' 12" ad *Nodum Alterum*. Præcipuus autem harum Conjunctionum usus est, *Solis à Terra* Distantiam sive Parallaxin ejus accurate determinate, quam quidem frustra variis Methodis tantaverunt *Astronomi*; dum Instrumenta quantumvis Subtilia Angulorum quæditorum Minutiæ facile eludunt. At in observando *Veneris* in *Solem* Ingressu & ab eodem Egressu, spatium Temporis inter Momenta Contactuum Internorum, ad ipsum Temporis Minutum secundum, hoc est, ad $\frac{1}{4}$ Minuti Secundi five 4". Arcus observati, ope mediocris Telescopii & Horologii Oscillatorii, per 6 vel 8^h accuratè sibi constantis, obtineri potest. Ex duabus autem talibus Observationibus in Locis Idoneis debire institutis intra Quingentesimam partem certo concludi *Solis Distantiam* proximâ Occasione commonstrabo.

Fig. 154, 155.

Ne quid Obscure Lectoribus *Astronomicè* mihi Doctis videretur, Schemata pro utriusque *Planete* Transitu delineavi, quibus rem oculis subijcere conatus sum.

The Motion of
the Comet. A.
1664. Predicted;
by M. Auzout. n. 1. p. 3.
n. 2. p. 18.

CI. 1. M. Auzout, after he had seen the Comet (which was first observed in Holland, Decemb. 2. 1664.) 4 or 5 times, made the *Ephemerides* of its Motion upon an Hypothesis that it moveth justly enough in the Plain of a great Circle,

Circle, which inclineth to the *Equinoctial* about 30° , and to the *Ecliptick* 49° or $49^{\circ}\frac{1}{2}$, cutting the *Aequator* at about $45^{\circ}\frac{1}{2}$, and the *Ecliptick* at 28° of *Aries*, or a little more.

He takes Notice, that more *Comets* enter into our System by the Sign of *n. 2. p. 19.* *Libra* and about *Spica Virginis* than by all the other parts of the Heavens : For, both the present *Comet* and many others registred in *History* have entered that way, and consequently pass'd out of it by the Sign of *Aries* ; by which also many have entered.

2. Till the 6th of *Feb.* this *Comet* always advanced : But after that Day, *Observed; by*
I found that it returned in Augmenting always its Latitude. I left it *Mar. 8. M. Auzout. ib.*
at the 18. of the *Horn of Aries*, almost in the same Latitude ; and I am apt to believe it will be *Eclipsed* this Evening.

I shall only add, that on *Feb. 3.* we were surprized, to see the *Comet* again much Brighter than ordinary, and with a considerable *Train*. Some did believe, that it approached again to us. But having beheld it with a *Telescope*, I soon said, that it was joined with two small Stars, whereof one was pretty Bright, and that this *Conjunction* gave the *Comet* that Brightness. Hence it was, that I assur'd my Friends here, that we should no more see it so Bright.

M. *Auzout* also strongly conceives, that this *Comet* could not be *Feb. 18. n. 6. p. 107.*
st. n. where M. *Hevelius*, in his *Prodromus Cometicus*, hath placed it, viz. in *prima Arietis* ; unless it be said that it visited that Star of *Aries* on the 18. and returned thence the 19th, into its ordinary Course : For, according to his, and his several Correspondents Observations, the *Comet* on *Feb. 17.* was distant from that *First Star of Aries* at least $1^{\circ} 17'$; and on *Feb. 19.* (He having missed, as well as his other Friends, the Observation on *Feb. 18.*) was Advanced in its way $12'$ or $13'$, but yet Distant from the said Star some Minutes above a whole Degree, and consequently far from having then passed it. After which time M. *Auzout* affirms to have seen it as well as several others, for many Days, and that until *Mar. 17.* Observing, that about *Feb. 26* or *27*, when the *Comet* was nearest to the often mentioned *First of Aries*, it approached not nearer than $50'$.

3. Some Eminent *English Astronomers*, who have attentively observed the *By some English Astronomers. ib. p. 108. n. 9. p. 150.*
Position of this *Comet*, do jointly Conclude, that whatever that Appearance was, which was seen near the *First Star of Aries* by M. *Hevelius* (the truth of whose Relation concerning the same they do in no wise Question) the said *Comet* did not come near that Star in the *left Ear of Aries*, where the said M. *Hevelius* supposes it to have passed, but took its Course near the Bright Star in its *Left Horn*, according to *Bayer's Tables*.

4. I have easily found the Principle of M. *Auzout's Ephemerides* : and 'tis this, that this *Comet* moves about a Centre, in a straight Line drawn from the Earth thro' the *Great Dog*, in so great a Circle, that that Portion which is described, is exceeding small in respect of the whole Circumference thereof, and hardly distinguishable by us from a Straight Line. *The Principle of M. Auzout's Hypothesis; and the Motion of that Comet Observed; by M. Cassini. p. 2. p. 17.*

Concerning the *New Comet* you mention, I observ'd it *Feb. 11.* about the 24° of *Aries*, with a Northern Latitude of $24^{\circ} 40'$.

The Motion of
the Comet. A.
1665. Predict-
ed; by M. Au-
zout. n. 3. p. 36.

CH. I. M. Auzout, after 3 or 4 Observations, hath published another *Ephemerides* concerning the Motion of the Comet, which he first began to Observe Apr. 2. 1665. He affirms that the Line described by this Star resembles hitherto a Great Circle, as it is found in all other Comets in the midst of their Course. He finds the said Circle inclined to the *Ecliptick* about 26° . $30'$. and the *Nodes* where he cuts it, towards the Beginning of *Gemini* and *Sagittary*; that it declines from the *Aequator* about 26° . and cuts it towards the 11° . and consequently that its greatest Latitude hath been towards *Pisces*, and its greatest Declination towards the 25° . of the *Aequator*. He puts it in its *Perigee* about 15° . of *Pisces*, a little more Westerly than *Marchab*, or the *Wing of Pegasus*.

Observed; by
M. Auzout. ib.
P. 37.

2. He observes in General, that this *Second Comet* is contrary to the precedent, almost in all Particulars: Seeing that the former moved very swift, this pretty slow; that against the Order of the *Signs* from East to West, this, following them, from West to East; that, from South to North, this, from North to South, as far as it hath been hitherto, that we hear of, Observed; that, on the side opposite to the *Sun*, this, on the same side; that, having been in its *Perigee* at the time of its Opposition, this having been there, out of the time of its Conjunction. He taketh also notice, that this Comet differs in Brightness from the other, as well in its *Body*, which is far more Vivid and Distinct, as in its *Train*, whose Splendor is much greater, since it may be seen even with great Telescopes, which were useless in the former by reason of its Dimness.

A Comet. An.
1668. at Bono-
nia; by M.
Cassini. n. 35.
P. 683.

CIII. I. An. 1668. Mar. 10d. 1h. of the following Night (after the Italian way of counting) I observ'd a Path of Light extended from the *Whale* thro' *Eridanus*; which I judged to be the *Train* of a Comet both by the Figure and Colour, as also because that the Direction of it was to the part opposite to the *Sun*, like other Comets. By its extream Point it reached to that Star in *Eridanus*, which is called the 14 by *Bayerus*: But it issued out of the Horizontal Clouds, so that I thought the *Head* of the Comet was either veiled by them, or hid under the *Horizon*. Mar. 11. there was seen a Brightness in the *Whale*, amongst the thin Clouds, at least for half an Hour, which was very like the Splendor of *Venus*, likewise veiled with thin Clouds.

Mar. 12. When the *Great Dog* was in the *Mid-heaven*, the same *Tail* appeared again. It passed thro' the Star in *Eridanus*, which *Bayerus* calls the 15, and left to the Southward the 14, where it did terminate Mar. 10. Being by the Imagination drawn out to about 3° . and further, it tended to that Southern Star which precedes the *Ear* of *Lepus*. It was therefore more Northerly than the Day before Yesterday, and more Easterly. We were doubtful whether its head was hid by the Clouds or under the *Horizon*. But the Line from *Jupiter* to the Extremity of the said *Tail* in the Clouds was Perpendicular to that *Tail*; so that it was in the *Whale*, and the Apparent part of the *Train* reached out in Length about 32° .

3. Mar.

2. *Mar. 5. ft. n.* The *Comet* was first discovered : but for as much as it *At Lisbon; by* set few hours after the *Sun*, there could hitherto be taken no considerable P. 684.
Observations of it. The *Body* thereof is not seen, because it remains hid in the *Horizon*. Its *Train* is of a Stupendious Length, extended in Appearance over almost the 4th part of the Visible Heaven, from West to East; its apparent Breadth is of a good *Palm*, and its Splendour very great, but it Lasts but a few hours.

At *St. Salvador Mar. 5. (ft. n.)* at 7 a Clock at Night, *F. Estancel* began *In Brasil; by* to see this *Comet* a little above the *Horizon* from West to E. S. E. The Beginning of its *Tail*, was a little under the two Lucid Stars, the 15 and 16^h *P. Valentine* of the *Whale's Back*, over which it then passed, its Point being as 'twere at *Estancel.* the 8 and 9^h which are in the Bottom of the *Whale's Belly*; and thus the whole Length thereof was about 23°. The *Globe* or *Head* of it was so small and thin, that very few could discern it with the Naked Eye. *n. 150. p. 92.*

Mar. 7. The former Brightness was somewhat less, and become so Thin, that the Eye could easily see the Stars that were behind it, which by Conjecture were the 14 and 20^h.

The *Tail* was always directly opposite to the *Sun*; and when it appear'd the first time almost Horizontal, it was seen in the form of a Pillar, the *Head* standing a little under, and on the side of the Star of the *Whale*, which is in the Lat. of 15°. 46'. and the Long. of 12°. 42'. of *Aries*. And the Point did shew the 14^h, North of the three that are in the *Belly*, in the Lat. of 20°. 30'. and Long. of 15° 57'. of *Aries*.

This *Comet* was at first very Splendid, and cast it self with that Vividness upon the Sea, that the Rays thereof were reverberated unto the Shoar, where the Observers stood. But this Brightness lasted only for 3 Days, after which it did considerably Decay. But that which seemed somewhat strange was, that having lost so much of its Light, yet its Bulk was not diminished, but continued rather increasing until the *Comet* disappear'd. It pass'd more swiftly than *Venus*, whence he infers that it was under *Venus*; yet the Anticipation was not so great, that it could be believ'd to be under the *Moon*, as he would have it.

4. *P. Pietro Susarte*, Rector of *Macao*, in the *East-Indies*, well versed in *In Africa; by* Matters *Astronomical*, writes to have seen the same all along the Coast of *Bona* *P. Pietro Susarte. ib.* *Speranza*.

CIV. There hath been seen here a New *Comet* from the 2. of *Mar. ft. n.* *A Comet. An.* 1671. It is but little, having a *Train* not above a Degree or a Degree and an half long. It is now (*Mar. 9.*) about the Stars in the Right Arm of *Andromeda* on her Shoulder Blade. As far as I can collect from one or two Observations, it tends towards the *Lucida* of *Andromeda's Girdle*, and that with a Direct Diurnal Motion of about 2 Degrees in its Course. *1671. at Dantzick; by M. Heilius. n. 81. p. 4017.*

The 6 of *March* in the Evening 7^h. 40' it was in 7°. *γ*. and in 35° of Northern Latitude; as I guess'd by the hasty Inspection of a Globe.

Mar. 7. in the Morning 3^h 30' its Longitude was about 8° *γ*. with a somewhat lesser Latitude than before: in the Evening of the same day it's Longitude was 10° *γ*. and Lat. 34°. *fer.* *Mar.*

Mar. 8. in the Morning 4^h the Long. was 12° . γ . and the Lat. 33° . Which yet I would not have taken precisely, because I cannot yet reduce my Observations to a *Calculus*.

At by
Mr. Newton.
ib. p. 4018.

2. Mr. *Isaac Newton* about the 16 of *March* st. *V.* saw a dull Star South-West of *Perseus*, which he now takes to have been that *Comet*. It was very small, and had not any Visible *Tail*, which made him regard it no further.

At Paris; by
M. Cassini.
n. 81. p. 4018.
n. 82. p. 4042.

3. The *Mathematicians* of *La Fleche* perceived him from the 16 of *March*, st. *n.* and gave us here at *Paris* the first notice of it. Those of the *College* of *Clermont* being advertised of it, saw him the 25 of the same Month.

Mar. 26. 7^h 30' in the Evening, M. *Cassini* saw him between the *Head* of *Medusa* and the *Pleiades*; without a *Telescope* he appeared no otherwise than a Star of the 3^d Magnitude; His *Head*, seen with a *Telescope* of 17 Foot, appear'd almost Round; but it was well defined, and distinguish'd from the *Mistiness*, which formed a kind of *Chevelure*, wherewith it was encompassed; and even the middle was a little confused, and seem'd to have *Inequalities*, as are seen in *Clouds*.

The *Tail* was almost imperceptible; yet by the *Telescope* it was seen turned Opposite to the *Sun*, and it appear'd of the Length of two Diameters of the *Head* or thereabout: For it was not easie to measure it precisely, because being Thinner according as it was farther from the *Head*, its Extremity was insensibly lost. And to the whole *Comet*, *Head*, *Tail*, and *Chevelure*, taken altogether, took up no more than 3 or 4 Minutes of a Degree. At 7^h 48'. he was in a straight Line with the *Lucida* in the *Head* of *Medusa*, and with the most Occidental one of the *Pleiades*; and above the two Clearest Stars of the *Southern Foot* of *Perseus*; so that a straight Line drawn through these two Stars, did almost touch the Southern Extremity of his *Chevelure*. This Place of the *Comet*, transferred upon the Map of the fix'd Stars, fell precisely enough upon $23^{\circ} 25'$. of *Taurus*, in 15° Northern Latitude.

With a *Telescope* 3 Foot, we saw near the *Comet* two small Stars, distant one Diameter of the *Sun* from one another, which Stars are not in the Catalogues. The *Comet* was in a Straight Line, drawn from one of those two Stars to the other precisely at 9^h 15'. but a little nigher to that which was Westward; But 9^h 33'. he was equally Distant from them both. It was taken notice of, that from 8^h 5'. till 10^h 26'. He made, in respect of these two Stars, an oblique Motion sensible enough, going from North to South in the same time that he advanced from West to East.

Mar. 28. 7^h 42'. in the Evening the *Comet* was distant from the less bright Star of the *Southern Foot* of *Perseus*, no more than about 24' Westward. He had almost the same Latitude with this Star; so that he was precisely enough at $26^{\circ} 8'$. δ , and in the Lat. $12^{\circ} 8'$. At 8^h 14'. we took, as well as we could, the Distance of the *Comet* to the Star in the Eye of *Taurus*, called *Aldebaran*, $19^{\circ} 38'$. And 8^h 29'. the Distance of the *Comet* to the Star, called *Capella*, was found to be of $22^{\circ} 32'$.

Mar. 30. 9h 35'. at Night the *Comet*, seen without a Telescope, appeared no otherwise than a Star of the 4th Magnitude: thro' the Telescope he exceeded even those of the First; but he was very Dark, and in what manner soever we look'd upon him, we could Observe almost no Tail at all of him. He had pass'd one Degree and an half beneath the *Lucida* of the *Southern Foot* of *Perseus*; so that this Star was exactly in the midst of the *Comet* and the little Star of the *Leg* of *Perseus*, marked π by *Bayerus*, which then we saw not but by a Telescope. A Straight Line drawn from one of these Stars to the other, did almost touch the *Southern Limb* of the *Comet*, which being transferred upon the Map of the *Fix'd Stars*, fell upon $28^{\circ} 45'$ of *Taurus* in the Northern Latit. of $90^{\circ} 56'$. At $9^h 45'$. the Western Limb of the *Comet* touched a Straight Line, drawn thro' this less bright Star of *Perseus's Southern Foot*, and thro' the most Northern of the Head of *Taurus*; but that he was already got somewhat nearer to the Latter.

Mar. 31. 8^h in the Evening, the *Comet* was in a direct Line with the *Lucida* in the *Foot* of *Perseus*, and with the most Northern in the *Head* of *Taurus*; but he was more than twice as much remoter from the first than the other, and being transferred upon the Map of the *Fix'd Stars*, he was found at $15'$. from *Gemini*, in the Latit. of $80^{\circ} 49'$. During the whole time that we could observe him this Night (which was till 10 a Clock) he quitted not this Straight Line, which was almost parallel to the Horizon: notwithstanding that his own particular motion should raise him a little above it; as the Parallax, on the contrary, should sink him beneath it in approaching to the Horizon. It may be, there was a compensation made of these two contrary Motions: possibly also the effect of both was not sensible.

April 1. The *Comet* could not be seen without a Telescope, because the *Moon*, being very near it, hid him from our sight. But with a Telescope only of one Foot we discerned him easily enough, and found that he had pass'd $45'$. beyond the most Northern Star of the *Head* of *Taurus*, and that he must have touch'd it by his Southern Limb; as also that he was Distant $1^{\circ} 43'$, from the Star that was nearest to that toward the South; which is equally Bright, yet not marked by *Bayerus*. This place being transferred upon the Map of the *Fix'd Stars*, we found that he was at $1^{\circ} 30'$ of *Gemini*, in the Northern Latit. of $70^{\circ} 44'$.

April 2. 8^h in even, M. *Cassini*, having observed the *Comet* with a Telescope of one Foot, which discovered 5° found that he was two Deg. and half Distant from the most Northern Star of *Taurus*; and one Deg. from the Star of the *Ear* marked ϕ by *Bayerus*, and by *Tycho* called *Sequentis Laceris Borei*.

Two Lines drawn from the most Northern Star of *Taurus*, one to the *Comet*, the other to the Star that is wanting in *Bayerus*, made a Right Angle; and the Distance of the *Comet* to this Angle, was double to that which is between these two Stars. This Place transferred upon the Map of the *Fix'd Stars*, fell on $2^{\circ} 48'$ of *Gemini*, in the Northern Latit. of $6^{\circ} 40'$.

April 3. 9^h we saw him with the one Foot Telescope. He had pass'd over the Upper Star of the *Ear* of *Taurus*, and he made with this Star the *Basis* of an *Isosceles* Triangle, on the Top whereof was the Inferior Star of the

Ear. The two Sides of this Triangle were two times and an half bigger than the *Basis*; so that the *Comet* was 4° of *Gemini*, in the Northern Latit. of $5^{\circ} 38'$.

Apr. 5. 8^h at even, the *Comet* had passed the Northern *Ear* of *Taurus*, and was equally Distant from the Upper Star of the Northern *Ear* and from that which was on the *Front* of *Taurus*. He was also as Distant from the Inferior Star of the *Ear* of *Taurus*, as this Star is from the next Westward, by *Tycho* called *Inferior precedentis Lateris Quadrilateri*; and a Straight Line, drawn thro' the *Comet* and the Upper Star of the *Ear*, made an almost Right Angle with another Line, drawn from the *Comet* to the Inferior of the two Stars, that are above the *Eye* of *Taurus*. This Place being carried over to the Map of the *Fix'd Stars*, the *Comet* was found at $6^{\circ} 18'$ of *Gemini*, in the Northern Latit. of $3^{\circ} 41'$. He was so confused this Night, that even with the 17 Foot Telescope we could not exactly distinguish the *Head* from the *Chevelure* which environed him. The whole appeared a little bigger than the Disque of *Jupiter*, seen by the same Telescope.

Apr. 6. 8^h at even, a Straight Line drawn from the *Comet* to the Star that is in the *Front* of *Taurus*, made a Right Angle with another Straight Line drawn from this same Star to the Inferior of the two that are above the *Eye*: And the Distance of this Latter Star to that of the *Front* of *Taurus* was twice the Distance of the same Star of the *Front* of *Taurus* to the *Comet*. This Place being transferred upon the Map of the *Fix'd Stars*, the *Comet* was found at $7^{\circ} 25'$ of *Gemini*, in the Northern Latit. of $2^{\circ} 45'$. At $9^h 6'$ we saw on the side of the *Comet* a Star sufficiently clear, which was not farther Distant from him than a little more than the Diameter of the *Comet*, and that was at the same height of the Horizon.

Apr. 7. 9^h in the Evening, the *Comet* was equally Distant from the Inferior Star of the Northern *Ear* of *Taurus*, and from the Superior of the root of the Northern *Horn*. He was also as far Distant from this latter Star, as this Star is from that of the *Front*. This Place, being carried over to the Map of the *Fix'd Stars*, fell on $8^{\circ} 30'$ of *Gemini*, in the Northern Latit. of $1^{\circ} 56'$.

All the Places of the *Comet*, that we have Observed till now, fall into a Line little differing from an Arch of a great Circle, which cuts the *Ecliptick* in $10^{\circ} 45'$ of *Gemini*, and which consequently hath its greatest Latitude in $10^{\circ} 45'$ of *Pisces*; which Latitude is between 39° and 40° Northward. The same Circle cuts the *Aequator* at 101 degrees of the *Vernal Section* Eastward, and its greatest *Declination* from the *Aequator* Northward is of $38^{\circ} \frac{1}{2}$.

Having chosen two of our First Observations (because the latter are not so proper for this purpose) and having taken a Mean between the first Observations of the *Mathematicians* of *La Fleche*, we found, by our Method explained in the Theory of the *Comet* of 1665, that this *Comet* had been in his *Perigee* the 12 of *March* at 8 a Clock in the Morning: that in that time, which is that of his greatest apparent Celerity, he made about $2^{\circ} 32'$ a day in the Great Circle of his apparent Motion, and $\frac{444}{1000}$ of his *Perigee* Distance in the Line of his Equal Motion: that he was in his greatest *Declination*

Declination the 11th, and 12th of *March*; and that at that time, he passed thro' the Inferior Meridian at about two a Clock after Midnight.

If we have rightly determined his *Perigee*, and that the *Hypothesis* of the Equality of his Motion be Just for that time, he hath been visible since the Middle of *February*, at which time he was as far Distant from his *Perigee* by Approaching to the *Earth*, as he is at present by Receding from it. He must then have been at the extremity of the *Southern Wing* of the *Swan*, and arrived at the *Southern Foot* of *Pegasus* on the 23 of *Febr.* of the same bigness that he was seen to be of *Mar.* 28. He must have arrived at the Stars of the *Northern Arm* of *Andromeda* *Mar.* 9. at those of her *Girdle* 12. when he was in his *Perigee*, and in his greatest Declination; to her *Southern Leg*, *Mar.* 15; between her *Southern Leg* and the *Triangle* *Mar.* 18. very near as he was observed at *La Fleche*; and under the *Head* of *Medusa*, *Mar.* 25. The Days ensuing he must have arrived at the Places marked in our First Observations: But in the last he hath been swifter than this *Hypothesis* will bear. To represent these latter Observations, the Line of the Motion ought to have been made Curve, as we did for the end of the Apparent Motion of the *Comet* 1665. with this difference, that instead of that Lines being Convex in regard of the *Earth*, because the Motion was *Retrograde*, this was to be Concave towards the *Earth*, because that the Motion of this *Comet* is *Direct*.

It's a Thing worth Observing, that this *Comet* keeps his Course almost like that of the 2. *Comet* of 1665, and another of 1577. observed by *Tycho*. For they have passed thro' almost the same *Constellations*; tho' this be more inclined Northward, and cut the *Ecliptick* 5 or 6 Degrees more forward than that of 1665. So that it seems that in this Place of the Heavens there is, as 'twere, a *Zodiaque* for *Comets*.

CV. 1. *Cometam* novum D. *Romer* primum advertit 28 *Apr.* st. n. 1677. *A Comet. An.* 1677. at *Paris* & me statim de rei novitate admonito 4^h 6' 31". p. m. n. ejus *Altitudi-* by M. Cassini, n. 135. p. 858.
nem accepimus 12° 22' 10". Judicavi eum fuisse in Verticali declinante ab Ortu ad Septentrionem 33° circiter. Die 29 manè, momento per nubes à D. *Picardo* visus est, 3^h 9' 31". p. m. n. in *Altitudine* 4° 39'.

Die 2 *Maii* manè, *Ascensione Rectâ* Medii Cœli ex Fixis existente 267°, *Altitudo* *Cometæ* erat 4° 5'. *Distantia* Verticalis à Septentrione ad Ortum 42° 8' circiter. Die 4 manè 3^h 30'. p. m. n. *Altitudo* *Cometæ* fuit 5° 33'. *Distantia Azimuthalis* à Sept. ad Ortum 42° 32'. circiter.

Die 5. 3^h 32'. *Altitudo* *Cometæ* fuit 5° 10'. *Distantia Azimuthalis* à Septentr. ad Ortum 44° 10'. circiter.

Observationes quæ habitæ sunt, Initio *Cometam* reponunt in *Triangulo*, pectrem prope *Caput Medusæ*, ostenduntque *Cometam* procedere secundum Signorum Seriem per Lineam proximam, & fere Parallelam Illi quam descripsit *Cometa An.* 1590. Mense *Feb.* *Magnitudo Capitis*, visi *Telescopio*, videbatur ferè æqualis *Jovis* Disco, aut paulo minùs; nec perfectè Rotundum apparebat, sed Figuræ Ovalis, longiore *Diametro* *Horizonti* parallelo; quod *Refractioni* *Horizontali* videtur tribuendum.

Coma ejus, *Telescopio* visa, Latior, & fere Parabolica; Nudo autem Oculo Angusta, & parum Inflexa ad Occasum, videbatur.

At Dantzick;
by M. Hevelius
ib. p. 869.

2. Prodiit hiſce diebus *Sidus Crinum*, quod primâ vice hic *Gedani* die 27 *April* manè animadverſum fuit. Die 29 *Apr.* Oriebatur vel potius in Oculos incurrebat, 1^h 52', Meſaquilonem verſus (*h. e.* N. E. b. N.) *Capite* quidem haud eò Amplo, ſed tamen ſatis Splendido, ex unico Nucleo clariffimo compoſito, ad inſtar illius *An.* 1665. conſpecti. *Caudam* Lumine norabilem radiis divaricatis fuſſim verſus, duorum ferè graduum, exponibat. Linea Directionis continuata *Cauda* inter *Alamac*, Lucidum ſc. Pedem *Andromeda*, ejuſque *Cingulum* incedebat, & quaſi Diſtantiam harum Stellarum in duas æquales partes ſecabat. Verſabatur eo tempore ſupra *Caput Arietis* in *Triangulo*, inter *Apicem* & *Borealiorem* in ejus *Baſi*, nempe in 5° *Tauri*, & in Latit. 12° Bor. Diſtabat hoc tempore à *Sole* ſecundum Longitudinem tantummodo 5°. Suo Circulo verò Maximo 20°. Hincque cum adeò vicinus hic *Cometa* extiterit *Soli*, haud potuit Longiorem *Caudam*, utut meâ opinione reverà longè prolixiorẽ habuerit, oſtendere, imò ut puto proximis diebus aliquanto adhuc Breviorem oſtendet.

Apr. 30. Deprehenſus eſt in 90. °. & Lat. 18°. Bor. totidemque ferè à *Sole* exiſtente in 12 °; *Caudam* rurfus duorum grad. & aliquanto Longiorem, ad Borealiorem in *Baſi Trianguli* extenſam (quæ *Stella* planè in *Cuspide Caudæ* per *Tubos* optimè conſpecta) exhibebat.

Die 1. *Maii* 2^h 32'. m. in 110. °. repertus eſt, ſub Latitudine Bor. 18°, in ipſa propemodum Conjunctione *Solis*, totidem quoque gradibus à *Sole* diſtans. *Caudam* adhuc ſatis Lucidam referebat, ſed paulo Breviorem, utut Latiorẽ, quam ad *Lucidum Pedem Andromeda* exporrigebat.

A Die 29. *Apr.* quâ primùm à me Obſervatus, ad hunc uſque Diem 1. *Maii*, Motu Proprio propemodum 5° 30' abſolvit.

Quantum ex hiſce Obſervationibus conjicere poſſum, fertur Motu Directo ad *Siniſtrum Pedem Perſei*, ſupra *Taurum*, ad *Pedes Geminorum*, ſi eo uſque perdurabit. *Nodus* Descendens verſatur circa 20°. *Geminorum* (ſed ruditer id tantummodo refero) atque ſic ibidem *Eclipticam* pertranſibit, fietque tum Meridionalis ſub Inclinatione Orbitæ 27° ferè.

Die 2. *Maii* veſp. 8^h 45'. etiamſi eâ in parte *Cœli* nullæ adhuc *Stellæ* emicarent, intenſumque *Crepuſculum* exiſteret, nihilominus *Cometam* *Tubo Optico* protinus inveni. Paulo poſt, illum in Altitudine 3° 30'. deprehendi: *Caudam* referebat, ratione *Crepuſculi*, valde Tenuẽ, quam inter utrumque *Genu Caſſiopeæ*, proprius tamen *Siniſtro*, exporrigebat: occidebat eâ *Veſp.* 10^h *Circium* verſus (*h. e.* N. N. W.)

Die 3. *Maii* mane, *Cometa* oriebatur Boream verſus (*h. e.* N. N. E.) 1^h 23', quanquam *Cauda* paulò citius à nobis detecta, nempe 1^h 18'. Verſabatur in 14° °, cum *Sole* ferè in ipſa Conjunctione, Latitudinem habens 17° & tantam etiam Diſtantiam fere ab ipſo *Sole*. *Caudam* hâc die longè Prolixiorẽ & Accuratiorem ſatiquẽ ſplendidam, 2° vel 3° fere, oſtendebat. Hincque à me aliſque Spectatoribus viſu pollentibus nudo oculo ad 3^h 34' deprehenſus eſt, & *Teſcopio* ad 3^h 40'. in Altitudine 11° 30'. adeo ut *Sol* eo tempore tantummodo 6'. infra Horizontem lateret, imo diutius illum viſiſſemus, niſi nubeculæ illum nobis eripuiffent. Motus Diurnus decreſcere videbatur, quantum conjectura abſque omni calculo aſſequi potui: Nam inter

inter 29 & 30 April 20 45' fere extitit; inter 30 April & 1 Maii 20 15'; inter 1 & 2 Maii 10 55'; inter 2 & 3 Maii 10 40'; sed ipsæ Observationes calculusque id clarius ostendent. Die 4 Maii vesp. Aëre admodum sudo, 8^h 53', iterum *Cometa* detectus, sed Obscurior paulo extitit quam diebus præcedentibus, tum *Cauda* Brevior. Die 5 Maii mane 1^h 41'. *Caudam* Dextrum Genu *Cassiope*. versus exponens, versabatur in 17° 2', in 16° Latit. Bor. pariter in tanta Distantia à *Sole*. Motus Proprius à Die 3 ad 5 Maii fuit fere 20 40', decrescente Latitudine, ab ipso Initio scil. fere ad 3. sic ut à 29 April. Motus Proprius *Cometa* ad 5 Maii propemodum fuerit 120. Die 6 Maii mane commemorabatur in 18° 5', & Lat. Bor. 15° 30'. *Sole* existente in 17° 0'; Motus Diurnus erat 50' circit. Quoad *Caput*, quam *Caudam* multò Tenuior ac Debilior videbatur, ob *Solem* non nisi 160¹ à *Cometa* Remotum. Die 6 Maii vesp. visus quidem Tubo optico 8^h 35' *Cauda* adhuc Breviori & Dilutiori; sed cum in Decliviori Situ, atque in Crepusculo intenso existeret, nullo modo distinctè in Nudos incurrebat oculos.

Die 7 Maii deprehensus primùm 2^h 22'. in Alt. 3°, utut valde Tenuis videretur. Occupabat eo tempore 19° 0'; Motus ejus Proprius magis magisque decrescebat, quantum colligere absque calculo debatur. Die 8 Maii mane ab Hor. 1. sedulo nudis quæsitus est oculis, sed nusquam apparuit: Telescopio tamen 12. ped. inventus, *Caudam* quidem adhuc præ se ferens, sed Brevissimam, paulo à circulo Verticali sinistram versus extensam. Quantum conjectura assequi potui, versabatur in 20° 0', in Distantia à *Sole* 15°, qui tum 19° 0' possidebat; stabat fere hoc tempore in linea recta cum *Humero Dextro Persei* & *Algol Medusæ*. Diameter *Cometa*, ad *Jovis* Diametrum comparata, vix ad dimidiam partem accedebat. De reliquo, Tubi beneficio satis erat adhuc conspicuus, adeo ut eum ad 3^h 45' distinctè conspiciere potuerimus, in Altitudine scil. 9° ferè: unde colligere datur, Arcum Visionis vix 50° tum fuisse; *Sol* enim vix 5° sub Horizonte hærebat, quo tempore omnes jam *Stellæ*, excepto unico *Jove*, evanuerunt. Die 8 Maii vesp. *Cometam* nec nudis oculis, nec ullo Telescopio detegere amplius potuimus.

3. The first certain notice I had of this *Comet* was on April 21. The 22 of *Apr.* at about 2 a Clock after the Midnight following, I saw the *Tail* raised almost perpendicular to the Horizon; soon after the *Head* appeared thro' a thin Vapour, from which the *Tail* pointed, as near as I could guess, upon the * in the *Knee of Cassiopeia*, its Length being about 6 Deg. and Breadth at the Top about 7 or 8 Min. Viewing the *Head* with a Telescope of 16 Foot, I found it was not perfectly Round, but Indented, and not near one Min. Diameter. Afterwards I hastened to Measure its Distances from several Fixed Stars, which were as follow.

| h. | ' | " | | ° | ' | " |
|----|----|----|--|----|----|----|
| 2 | 44 | 00 | It's Head and the Foot of <i>Androm. Alamech</i> — | 11 | 26 | 0 |
| 2 | 47 | 15 | That Distance repeated — — — | 11 | 26 | 50 |
| 2 | 55 | 3 | Its Head from <i>Capella</i> — — — | 31 | 1 | 15 |
| 2 | 59 | 10 | — — — repeated | 31 | 1 | 24 |
| 3 | 12 | 2 | Its Head from <i>Algol</i> in <i>Medusa's</i> — — | 18 | 16 | 54 |
| 3 | 21 | 22 | — from <i>Mirach</i> — — — | 19 | 35 | 0 |
| 3 | 27 | 54 | — from <i>Alamech</i> again — — — | 11 | 33 | 30 |
| 3 | 36 | 20 | — from <i>Capella</i> again — — — | 30 | 59 | 45 |

At 4^h 21¹/₂ p. m. the height of the *Comet* was about 5⁰₂, therefore the Distance of the *Head* of the *Comet* from *Algol* corrected by refraction, 8⁰ 19'.
from *Mirach* — 19 37

And admitting with *Hevelius* the Place of *Mirach* now in γ . 21⁰ 40' 34" with North Latitude 25⁰ 57'. its Distance from *Algol* will be 23⁰ 42' 40". and the Place of the *Head* of the *Comet* in δ . 14⁰ 48¹/₂, with North Latitude 17⁰ 8'.

At 3^h 28'. I State the correct Distance of the *Comet's Head* from *Capella* 31⁰ 00'; from *Alamech* 11⁰ 40'; and therefore its true Place δ . 14⁰ 50¹/₂, with North Latitude 17⁰ 8' 25": agreeing very well with the Place derived from the former Distances from two other and different Stars.

The *Tail* was not, it seems, directly Opposite to the *Sun*: for the *Sun's* Place was now α . 13⁰ 7'; but the *Comet* being in 14⁰ 47'. of the same Sign, that is 1⁰ 40', in the Consequence of the *Sun*, the *Tail* ought, if it had been exactly opposite to the *Sun*, to have lain in Consequence of the *Head*; but the *Knee* of *Cassiopeia* is now in α . 13⁰ 24'. in Antecedence of the *Comet*, whose *Tail* lay not therefore in Consequence, but in Antecedence of the Line passing thro' its *Head* and the *Sun*, at about an Angle of 10⁰.

Next Night, being that following the 23 of *April*, about ¹/₄ of an Hour after two, its *Tail* appeared much shorter than last Morning: At 2^h 51'. its *Head* was from *Mirach* 21⁰ 9'. Hence and from a Course of Observation of it sent me by an ingenious Friend, I found its Motion was Direct, and its Latitude Decreasing.

A Comet. An. CVI. Nuperum *Cometam* Observavi primum m. ante *Solis* Ortum, à Die 2. 1680. at Dantzick; by M. Hevelius. *Ph.* ad 4 Dec. An. 1680. deinde vesp. à 24 Decemb. ad *Aequinoctium Vernum*. Mane versabatur in α , & η sub Latit. Australi. Vesperti vero in $\nu\beta$, α , κ , γ , & δ . sub Latit. Bor.

A Comet. An. CVII. Plurimas distantias à Fixis, tum Altitudines Nuperi *Cometae* Meridianas, Impetravi: quas autem omnes hic recensere nimis longum foret, nec vacat eas rigidiori Calculo subicere. Sufficiat hac vice dixisse *Cometam* hunc hic *Ge-dani* Die 25 Aug. st. n. 1682. primum detectum, atque à Die 26 Aug. ad 17. *Septemb.*

Septemb. debite à me observatum esse. Qua via autem, qua velocitate, sub quo Angulo Orbitæ & *Ecliptica* progressus fuerit, ex adjecta Tabella patet; quam tamen (quod scias velim) non ex accurato Calculo, sed ex Globo tantummodo Laxiori ratione concinnavi.

| <i>Mens.
dies.</i> | | <i>Long. Cometa</i> | <i>Lat. Cometa</i> | <i>Motus in Pro-
Orbit.</i> | <i>Motus Di-
urnus ali-
quanto Ac-
curatius.</i> |
|------------------------|-------------------|---------------------|--------------------|---------------------------------|--|
| | <i>h. ' "</i> | <i>o ' "</i> | <i>o ' "</i> | <i>o ' "</i> | <i>o ' "</i> |
| <i>Aug.</i> 26 | 3 0 <i>Mat.</i> | 23 30 $\odot$ | 21 0 <i>Bor.</i> | | |
| <i>Aug.</i> 27 | 11 0 <i>Vesp.</i> | 5 0 $\odot$ | 23 30 <i>Bor.</i> | 10 0 fere | 5 28 |
| <i>Aug.</i> 28 | — <i>Vesp.</i> | — | — | — | 5 35 |
| <i>Aug.</i> 29 | — <i>Vesp.</i> | — | — | — | 5 41 |
| <i>Aug.</i> 30 | 3 30 <i>Mat.</i> | 18 0 $\odot$ | 25 20 <i>Bor.</i> | 13 20 — | |
| <i>Aug.</i> 30 | 9 0 <i>Vesp.</i> | 22 0 $\odot$ | 25 40 <i>Bor.</i> | 3 30 — | 5 46 |
| <i>Aug.</i> 31 | 3 30 <i>Mat.</i> | 24 30 $\odot$ | 26 0 <i>Bor.</i> | 2 20 — | 5 50 |
| <i>Sept.</i> 1 | 3 30 <i>Mat.</i> | 1 0 $\odot$ | 26 0 fere | 5 45 | |
| <i>Sept.</i> 1 | 9 0 <i>Vesp.</i> | 6 0 fere $\odot$ | 25 40 <i>Bor.</i> | 4 45 — | 5 46 |
| <i>Sept.</i> 2 | — | — | — | — | 5 43 |
| <i>Sept.</i> 3 | 8 30 <i>Vesp.</i> | 20 0 fere $\odot$ | 24 30 <i>Bor.</i> | 11 30 — | 5 40 |
| <i>Sept.</i> 4 | — <i>Vesp.</i> | — | — | — | 5 34 |
| <i>Sept.</i> 5 | — <i>Vesp.</i> | — | — | — | 5 24 |
| <i>Sept.</i> 6 | 9 0 <i>Vesp.</i> | 5 0 $\odot$ | 20 30 <i>Bor.</i> | 15 0 — | 5 00 |
| <i>Sept.</i> 7 | — <i>Vesp.</i> | — | — | — | 4 30 |
| <i>Sept.</i> 8 | 8 0 <i>Vesp.</i> | 12 0 $\odot$ | 18 15 <i>Bor.</i> | 8 0 fere | 4 00 |
| <i>Sept.</i> 9 | 8 30 <i>Vesp.</i> | 15 30 $\odot$ | 17 15 <i>Bor.</i> | 3 30 — | 3 30 |
| <i>Sept.</i> 10 | 8 0 <i>Vesp.</i> | 18 30 $\odot$ | 15 45 <i>Bor.</i> | 3 0 fere | 3 00 |
| <i>Sept.</i> 11 | — <i>Vesp.</i> | — | — | — | 2 40 |
| <i>Sept.</i> 12 | 8 0 <i>Vesp.</i> | 23 0 $\odot$ | 14 0 <i>Bor.</i> | 5 0 — | 2 20 |
| <i>Sept.</i> 13 | 7 30 <i>Vesp.</i> | 25 0 $\odot$ | 13 30 <i>Bor.</i> | 2 0 — | 2 00 |

Sic ut *Notu Proprio* in suo *Orbita* confecerit Die 26 *Aug.* ad 13 *Sept.* 83° 27' ; & in *Ecliptica* 91° 30'. *Latitudo* vero *Borealis* creverit ad 26°. rursus decreverit ad 12° 30'.

Not. Nodus Boreus in 24°. $\odot$, & *Nodus Aust.* in 24°. $\odot$: *Limites* vero in 24°. $\odot$ & $\odot$ extiterunt. *Angulus Orbitæ & Eclipticæ* fuit 26°. fere. Utrum autem toto *Durationis Tempore* omnino constans cum *Nodis* extiterit? An vero & quousque sese variaverit? ut sæpius fieri solet, ex *Calculo* patebit.

Toto *Durationis Tempore*, *Lucidius* ac etiam aliquanto *Majus Caput*, quam iste *An. 1681.* è contrario multo *Breviorem Caudam* exhibuit. In ipso *Capite*, beneficio longioris *Telescopii*, non nisi unicum *Nucleum Figure Ovalis & Gibbosæ* constanter notavimus; nisi quod Die præsertim 8 *Sept.* ex dicto

Fig. 156.

dicto Nucleo clarissimus simul Radius, ex parte etiam incurvatus, in *Caudam* exiret; quod notari meretur, cum ejus generis faciem in nullo adhuc *Cometa* (quantum memini) observaverim. Præterea sciendum, quod nonnunquam, ut Die 30 *Aug.* mane, *Caudam* satis præcise in *Oppositum Solis* direxerit; sed sæpius etiam notabilem *Deviationem* (prout in plurimis *Cometis* sæpius fieri solet) exhibuerit. Longitudinem quoque *Coma* non semper eandem conservavit. Initio *Cauda* ferè 12° videbatur; deinde nonnunquam brevior, interdum etiam longior ad 15° & 16° extitit; circa finem vero quotidie diminuta est.

A Comet. An.
1683. at Dant-
zick; by M.
Hevelius.
n. 154. p. 416.

CVIII. Die 30. *Jul.* An. 1683. 11h 30' in Novo Nostro Sydere, *Tigride* vel *Lynce*, *Sydus Crinitum* hic *Gedani* deprehenderem, *Caudam* haud adeo Longam inter *Stellam Polarem*, & *Cassiopeiam* sursum cum aliqua *Inclinatione* exporrigens, constituerebat lineam Rectam cum *Suprema Capitis Aurigæ* & *Dextro Humero Persei*; non minus cum *Ventre Urse Majoris* & *Dextro Humero Aurigæ*; item cum *Media Cauda* & *Latere Urse Majoris*. Deinde Tubo 10 pedum arrepto, istud Phænomenum contemplatus sum, *Caput* erat quidem satis Amplum, sed *Materia* non admodum *Condensata*; sic ut nullus *Lucidus Nucleus* neque distincta *Corpuscula*, ut quidem alias in plurimis aliis deprehensum est, in eo apparerent. 12h ferè, *Altitudo* ejus erat 19° 57'.

Die 31 *Jul.* vesp. scil. 12h 30'. *Altus* cum esset 21° 28'. Rectam cum *Pede Aurigæ* & *Capella* constituerebat. *Cauda* erat diutissima, ac Rarior quam die hesternæ, sed paulo Longior.

Die 4 *Aug.* mane; Removebatur eo tempore tanto spatio à *Dextro Humero Aurigæ*, quanto alias distat dictus *Humerus* à *Capite Hædi*. Cæterum *Sinistra Tibia Persei*, *Capella*, & *Cometa*, Rectam referebant.

Die 16 *Aug.* vesp. hora ferè 11. *Cometa* inter quatuor *Stellulas* versabatur, quarum una à parte *Comete* superiori, in ipsa *Conjunctione*, non nisi 1'. distabat, adeo arcè limbo adhærebat: quo tempore simul *Diametrum Comete* *Micrometro* meo dimensus sum, nimirum 6' 5'' existere.

Die 20 *Aug.* vesp. Brevissimam ac Rarissimam *Comam* inter *Capellam* & *Caput Hædi* exporrigebat.

Die 20 *Aug.* vesp. Utroque *Hædo* erat vicinissimus, ita ut cum his *Triangulum* ferè æquilaterum constitueret, cujus *Latera* fere *Distantiam Hædorum* (quæ est 47'. circ.) æquabant. Ad hæc *Cometa* cum *Capella* & illa in *Planta Dextri Pedis Persei*, *Triangulum* *Æquilaterum*, cujus *Basis* erat *Distantia* distantiarum *Fixarum*, exhibebat.

Aug. 24. vesp. versabatur inter *Capellam* & *Pleiadas*, sic ut à *Capella* & *Pleiadibus* in eadem fere *Remotione* videretur. Deinde *Capella*, *Cometa*, & *Pleiades*; item *Almanac*, *Caput Medusæ*, & *Cometa*; nec non *Dexter Humerus Aurigæ*, *Cometa*, & *Sequens Sinistri Pedis Persei*, *Lineam* fere Rectam constituerebant, in hac tamen ultima constitutione, *Cometa* fere infra paulo Rectam jam incedebat.

Aug. 25. vesp. cum *Capella* & *Cap. Hædi* Rectam fere constituerebat.

Aug.

Aug. 29. 1h 5'. mane, plurimis minutissimis, & clarissimis Fixis stipatus erat, nempe *Stellis Subiescians*, atque à Cuspide Occidentali *Pleiadum* fursum versus, non nisi 42' 35'' removebatur.

Eadem die vesp. longe jam promotior contra S. S. spatio scil. 24 horarum, ad 4°. fere reperiebatur.

Aug. 30. vesp. *Cometa* à *Stellula* quadam bene conspicua non nisi 32' 41'' aberat; & cum *Musca* & in *Basi Trianguli*, deinde etiam cum præcedente in *Pede* & illa in *Genu Persei*, constituēbat Rectam.

Sept. 2. mane, *Cometa* inter *Pleidas* & *Nodum Lini* versabatur, constituens Lineam Rectam cum *Musca* & *Lucida Mandib. Ceti*. & cum infima in *Arme* & *Mandibula*, Triangulum fere æquicrurum, cujus vertex dicta erat *Mandibula*. Præterea quoque Lineam referebat Rectam cum duabus in *Fronte Ceti*, tum tanto fere spatio ab Occidentiori distabat, quam alias utraque ab invicem removentur.

Hac die iterum *Diametrum Comete Capitis* Micrometro diligenter dimensus sum, 9' 7''. die 16. Aug. eodem Micrometro obtenta tantummodo 6' 5''. sic ut notabiliter spatio 17. Dierum creverit. Non nemo diceret id factum esse, quod in ultima Observatione vicinior multo fuerit Terræ: Atque ideo Clarius & Lucidius *Caput* exhibere debebat, præsertim si Corpus esset æternum (ut quidam statuunt) quod rursus certo tempore absoluto suo circulo, nobis in conspectum redit. Sed è contrario *Caput* longe Obtusius, Rariusque ultimo extitit, sic ut distinctissimè notari potuerimus materiam *Capitis* sensim se dissolvere; id quod autem multo melius cum nostra convenit Hypothesi.

Sept. 4. mane, videbatur *Cometa* in Linea existere Recta cum illa in *Fronte Occid. Ceti* & *Lucida γ*; item cum illa in *Ore* & *Mandibula Ceti*; ad hæc fere Triangulum æquilaterum cum illa in *Ore* & ad *Genam Ceti* constituēbat. De cætero autem ex optato mihi extitit, quod *Altitudinem Meridianam* in *Austro exactissimo Quadrante*, *Hora, viz. Matutina 3h 40'*. ferè, nimirum 38°. 15'. esse impetraverim.



| <i>Mens. Dies.</i> | <i>Longitudo.</i> | <i>Latitudo.</i> | <i>Motus
Diurnus.</i> | <i>Declinatio.</i> | <i>Ascens.
Rect.</i> |
|--------------------|-------------------|------------------|---------------------------|--------------------|--------------------------|
| | ° / | ° / | ° / | ° / | ° / |
| <i>Jul.</i> 30 | 7 0 S | 29 15 B | | 51 30 B | 100 0 |
| 31 | 6 25 S | 29 0 B | 0 42 | | |
| <i>Aug.</i> 1 | 5 45 S | 28 45 B | 0 44 | | |
| 2 | 5 0 S | 28 30 B | 0 46 | | |
| 3 | 4 10 S | 28 15 B | 0 48 | | |
| 4 | 3 20 S | 28 0 B | 0 50 | 51 40 B | 96 0 |
| <i>Aug.</i> 5 | 2 20 S | 27 45 B | 0 52 | | |
| 6 | 1 20 S | 27 30 B | 0 54 | | |
| 7 | 0 20 S | 27 15 B | 0 56 | | |
| 8 | 29 20 II | 27 0 B | 0 58 | | |
| <i>Aug.</i> 9 | 28 20 II | 26 40 B | 1 0 | | |
| 10 | 27 20 II | 26 20 B | 1 2 | | |
| 11 | 26 20 II | 25 55 B | 1 4 | | |
| 12 | 25 20 II | 25 30 B | 1 6 | | |
| <i>Aug.</i> 13 | 24 20 II | 25 0 B | 1 8 | | |
| 14 | 23 20 II | 24 30 B | 1 10 | | |
| 15 | 22 20 II | 24 0 B | 1 12 | | |
| 16 | 21 10 II | 23 20 B | 1 14 | 46 0 B | 77 0 |
| <i>Aug.</i> 17 | 19 20 II | 22 30 B | 1 19 | | |
| 18 | 17 40 II | 21 30 B | 1 25 | 44 0 B | 73 30 |
| 19 | 16 0 II | 20 30 B | 1 35 | | |
| 20 | 14 20 II | 19 15 B | 1 45 | 41 0 B | 69 30 |
| <i>Aug.</i> 21 | 12 20 II | 18 0 B | 1 55 | | |
| 22 | 10 20 II | 16 45 B | 2 5 | | |
| 23 | 8 20 II | 15 30 B | 2 15 | | |
| 24 | 6 20 II | 14 15 B | 2 25 | 25 0 | 60 40 |
| <i>Aug.</i> 25 | 3 50 II | 12 45 B | 2 40 | | |
| 26 | 1 5 II | 11 0 B | 2 55 | | |
| 27 | 28 15 O | 9 0 B | 3 10 | | |
| 28 | 25 15 O | 6 30 B | 3 25 | 24 30 B | 51 0 |
| <i>Aug.</i> 29 | 22 15 O | 4 0 B | 3 40 | 21 30 B | 48 30 |
| 30 | 18 55 O | 1 30 B | 3 55 | 18 00 B | 45 40 |
| 31 | 16 25 O | 1 0 A | 4 5 | | |
| 1 | 12 15 O | 1 30 A | 4 20 | | |
| <i>Sept.</i> 2 | 9 55 O | 6 0 A | 4 40 | 10 30 B | 40 0 |
| 3 | 6 25 O | 9 40 A | 5 5 | | |
| 4 | 2 35 O | 11 20 A | 5 30 | 1 30 B | 34 0 |

Ex quibus nunc luculenter videre est, *Cometam* hunc continuo contra S. S. incessisse; sic ut in *Ecliptica* $63^{\circ} 55'$. in sua vero *Orbita* $74^{\circ} 35'$. peragraverit, sub Angulo, viz. *Orbita* & *Ecliptica* 39° fere, sub Angulo vero *Orbita* & *Aequatoria* 56° . *Latitudo* Initio $29^{\circ} 15'$. *Bor.* & ultimo $11^{\circ} 20'$. *Aust.* extitit; adeo ut ad 41° . fere eam variaverit.

De *Capite* hæc notandum habeo, quod Initio, quoad *Diametrum*, longe minus quam ultimo; è contrario Initio longe *Lucidius*, quam circa *Finem* extiterit; nullos tamen distinctos & fulgentes *Nucleos*, prout in plurimis videre nobis obtingit, exhibuerit, sed confusam materiam, & circa *Finem* multo *Tenuiorem*. Jure hic *Cometa* (cum plerumque absque omni *Cauda* visus) inter *Sidera Comata*, vel *Crimina*, sive inter *Barbata* & *Hircos* refertur. Nam non nisi ad 18 *Aug.* Brevissimam & Dilutissimam *Comam* sursum versus exporrigebat; quæ postmodum vero omnino Evanuit.

CIX. Novus *Cometes* nuper Cælo visus est à Lynceò oculo Abb. *Blanchini*, *A Comet. An.* Discipuli Cl. *Geminiani Montanarii*. *Cometes* parvus quidem, sed in sua *Orbita* 1684. at Rome; regularis apparuit, Lumine tenui, & tanquam *Stella* Subobscura; at Tubo by S. Ciampini. optico exceptus, Luminosior. n. 169. p. 920.

Jun. 30. st. n. *An.* 1684. *Cometa* primum mihi visus est, in grad. 9. cum aliquibus minutis *Libra*, *Latitudo* ejusdem *Borealis* fuit graduum 8. & aliquot minutorum.

| <i>Jul.</i>
<i>Die.</i> | <i>Long. Co-</i>
<i>meta.</i> | <i>Lat. Bor.</i> | <i>Jul.</i>
<i>Die.</i> | <i>Long. Co-</i>
<i>meta.</i> | <i>Lat. Bor.</i> |
|----------------------------|----------------------------------|------------------|----------------------------|----------------------------------|------------------|
| | 0 / | 0 / | | 0 / | 0 / |
| 1 | 11 18 ≈ | 13 12 | 10 | 24 42 ≈ | 39 40 |
| 2 | 13 16 ≈ | 17 57 | 11 | 25 44 ≈ | 40 49 |
| 3 | 14 58 ≈ | 22 12 | 12 | 26 38 ≈ | 41 58 |
| 4 | 16 45 ≈ | 25 40 | 13 | 27 23 ≈ | 43 5 |
| 5 | 18 30 ≈ | 28 50 | 14 | 28 32 ≈ | 44 9 |
| 6 | 19 50 ≈ | 31 34 | 15 | 29 49 ≈ | 45 12 |
| 7 | 21 7 ≈ | 33 54 | 16 | 00 50 m | 46 20 |
| 8 | 22 20 ≈ | 36 6 | 17 | 2 8 m | 47 40 |
| 9 | 23 32 ≈ | 38 00 | | | |

Quatuor ultimæ meliori indigent Calculo, nam aliquis fortasse Minutorum Error irrepsit. Observatio *Diei Primi Jul.* accuratissima est, *Cometa* enim Telescopio apparuit una cum *Stella Virginis*, quæ *Bayero* inscribitur Ω , & accidit sub Cingulo Septentrionalis partis primæ. Omnium certissima est Observatio *Diei Sextæ*, qua *Die Arcturus* in Tubo Optico simul cum *Cometa* conspiciebatur. *Die pariter 14. Cometa* & *Stella* χ . in *Colorrobo Bootis*, seu *Vénabulo*, subter *Humernum*, uno Intuitu detegebantur.

A Comet. An.
1686. at Leip-
sick; by M.
Kirk.
n. 186. p. 256.

CX. Sept. 8. ft. v. A. 1686. 4^h. mane, about Day-break, M. Kirk found this Comet in the Constellation of *Leo*, to the Right-hand of the *Lucida in Lumbis* ϵ , (as it is conceived, for the *Latin* Copy is Defective in this place) and resembling that Star in Colour and Magnitude, with a Thin and short Tail extended upright. Over the Comet in the same Vertical was the Star of θ ϵ of *Bayer*, or 21 *Tychoni*, distant therefrom, by the Micrometer, exactly a Degree; and a Line drawn from the *Lucida in Lumbis* ϵ to the Comet, passed much about half a Degree to the Right-hand of the θ *Leonis*. The Distance of the Comet from *Regulus* taken by a Radius was about 17° . The next Morning Sept. 9. at 3^h 58'. the Distance thereof from θ ϵ was found by the Micrometer $2^{\circ} 23' \frac{1}{2}$, and at 4^h 40', again $2^{\circ} 25' \frac{1}{2}$. To verify the Times, the Altitude of the *Lucida in Lumbis* ϵ was observed $11^{\circ} 10'$, at 4^h 8'. mane. A Right-line drawn by the Comet, and the said θ *Leonis* towards β *Leonis*, or the *Lucida Colli*, left that Star a little to the Right-hand.

This Comet was seen by a Countryman, who first gave Notice thereof, from the 6th to the 12th of Sept.

The Result of these Observations is, that the Comet was Direct in Motion, that it mov'd about $1 \frac{1}{2}$ Degree per Diem, and that it seem'd rather to Decrease in Latitude. On the 7th of Sept. it was about $24'$. distant from θ *Leonis*, but its bearing therefrom is not set down.

This Star, θ ϵ , was then in $9^{\circ} 2'$. of W , with North Lat. $9^{\circ} 41' \frac{1}{2}$. Whence at the time of the first Observation it may be Concluded, that the Comet was in $9^{\circ} 55'$. of W , with North Lat. $9^{\circ} 15'$. And at the 2^d Observation, the Longitude of the Comet will be found about $11^{\circ} 20'$. of W , with much the same North Latitude as before.

A Comet. An.
1693. at Paris.
by M. Cassini.
n. 250. p. 79.

CXI. Feb. 19. ft. n. An. 1699. in *Observatorio Regio Parisiensi*, videri cepit exiguus Cometa, instar Stellæ Nebulosæ tertiæ Magnitudinis; illi per similes quæ Mense Sept. 1698. fuit Observatus.

Situs erat inter Stellas Informes 6 Magnit. prope Circulum Polarem Arcticum supra Caput Aurigæ, æquali ferè Intervallo inter Cubitum Occidentalem Persei & Caput Majoris Urse; illas adscribit Tycho Informibus circa Ursum Minorem. Continuatis Observationibus, visus est, Proprio Motu, Iter suum dirigere Capellam, versus, cum exigua Deviatione ab ejus Circulo Declinationis. Ea erat ejus Velocitas ut unius Diei Spatio, Septem circiter Gradus Magni Circuli perficeret, quò Motu potuit ante Dies 4. ipsi Polo fermè adhærere, & Stellæ Polari sociari.

Hora 6. p. m. n. Comparavimus Cometam cum Stella 6. Mag. quam Tycho appellat Secundam earum quæ sunt in Linea Recta cum Polo; Cometa in Transitu per Circulum Horarium præcedebat hanc Stellam Min. Hor. $15' 53''$. quibus dabitur Differentia Ascensionis Rectæ $4^{\circ} 43'$. erat autem Septentrionalior eadem Stella 8'. Unde supposita hujus Stellæ Longitudine & Latitudine Tycho-nica ad hoc Tempus, Cometa refertur ad $15^{\circ} 51'$. Gem. cum Latitudine Sept. $37^{\circ} 25'$.

Movetur Cometa hic ad Cæli partes Oppositas illis ad quas tendebat Cometa An. præteriti, cum esset ferme in eadem Distantia à Polo in qua noster hic cum primum visus est, nec valde ab eodem loco remotus.

Cometa

Cometa: autem *Mensis Sept.* eandem profecutus est viam quam inter *Sidera* tenuerat *Cometa An. 1652.* à Nobis *Bononia* Observatus, cujus Occasione editis Literis ad Serenissimum *Fran. Estensem Martina* Ducem, eam viam per eadem *Sidera* quæ noster tenuit *An. 1698.* distinctè descripsimus. Ille *Mense Dec.* ab Australibus Cæli partibus per *Astra Leporis, Orionis & Tauri*, ubi *Eclipticam* secuit cum *Inclinatione 76°.* & per *Perseum & Cassiopeiam* pervenit, ubi videri desiit *Mense Jan. An. 1653.* Hic videri cœpit *Initio Mensis Sept.* in eadem *Cassiopeæ* parte ubi ille videri desierat, indeque pergens per *Humeros & Brachia Cephei*, ubi *Latitudinem Maximam* ab *Ecliptica* habuit *76°.* transit inter *Draconem & Cygnum*, per *Pellem Leonis* in *Hercule*, per *Ophiuncum*, usque ad *Constellationem Scorpii*, quam tenebat in ultimis Observationibus à *Die 24.* ad *28. Sept.* habitis. Ex his autem Observationibus collegimus *Cometam* hunc *Perigenum* obtinuisse *Die 7. Sept. Vesperis*, cum maxima *Velocitate* *Apparenti* fere *10°.* unius *Diei Spatio.*

CXII. Papers of less Use General Omitted.

1. The Constellation of *Cygnus*, with the *New Star in Pectore* in it, by *He-Cygnus*, n. 21. *veline*; together with the Names of the Stars in that *Constellation* by *Tycho*, p. 372. n. 65. and of those Added by himself. p. 2088, 2090.

2. Mr. *Flamsteed* having perused Mr. *Street's* Discourse, and considered the Contrivance of his *Moon-Wiser*, assures, that for the Motion of *Longitude* 'tis the very same, and for the Motion of *Latitude* not much better than Mr. *Horrox's*. *Mr. Horrox's Lunar System.* n. 110. p. 219. n. 116. p. 368.

But Mr. *Flamsteed* hath thought of another Contrivance that will shew the *Moon's* true Place to a Minute.

3. 1. The more Notable *Celestial Appearances* Calculated, by Mr. *Flamsteed*, for the Year 1670: *Celestial Phenomena Calculated.*

2. The same for the Year 1671. n. 55. p. 1099.

3. The same for the Year 1672. n. 66. p. 2029.

4. The same for the Year 1673. n. 77. p. 2297.

5. The same for the Year 1674. n. 79. p. 3061.

4. 1. The *Eclipses* of the *Satellites* of *Jupiter* Visible at *Uraniburg* the last four Months of the Year 1671. Calculated by M. *Cassini*. n. 86. p. 5040. n. 89. p. 5118. n. 99. p. 6162.

2. The *Eclipses* of the *Satellites* of *Jupiter* Visible at the *Observatory at Greenwich* in the three last Months of the Year 1683. Calculated by Mr. *Flamsteed*. *Satellite Eclipses* n. 77. p. 2238. n. 151. p. 322.

3. The *Satellite Eclipses* Calculated by Mr. *Flamsteed* for the Year 1684. n. 154. p. 404.

4. The same for the Year 1685. n. 165. p. 760.

5. The same, together with the *Parallaxes* of *Jupiter's Orb* and his *Geocentric Places*, for the Year 1686. n. 177. p. 1215.

6. The same for the Year 1687. n. 184. p. 196.

7. The *Satellites Eclipses* Calculated by Mr. *Halley*, for the Year 1688. n. 191. p. 435.

9. 1. An Account of the *Ephemerides* of the *Comet, A. 1665.* Calculated by M. *Auzout*; and the Principle of his *Hypothesis* discovered by M. *Cassini*. *Comets*, n. 1. p. 3. n. 2. p. 17. 18.

2. An Account of the *Ephemerides* of the *Comet, A. 1665.* Calculated by M. *Auzout*. n. 3. p. 36.

CXIII. Accounts of Books, and Emendations, Omitted.

- n. 102. p. 40. 1. A new Size of Globes about 15 Inches Diameter Rectified by R. Mor-
den and Will. Berry.
- Phil. Col. 2. A Representation of the Heavens in two large Hemispheres of 30 Inches
n. 1. p. 44. Diameter, Stereographically Projected upon the Plain of the Equinox; by
Mr. Fr. Lamb.
- n. 90. p. 5150. 3. Deux Machines propres à faire les Quadrans, avec tres grande facilité,
par le P. Ignace Gaston Pardies. S. J. à Paris. 1673. in 12°.
- n. 184. p. 213. 4. Sciotericum Telescopium, or a new Contrivance of adapting a Telescope
to an Horizontal Dial, for Observing the Moment of Time by Day or Night;
by Will. Molinæux R. S. S. Dublin. 1686. in 4to.
- n. 241. p. 240. 5. The Meridian Line of the Church of St. Petronio, Drawn and Fitted
for Astronomical Observations, in the Year 1655. Revised and Restored in
the Year 1695; by Jo. Dom. Cassini. At Bononia 1695. Fol.
- n. 66. p. 2028. 6. Joa. Hevelii Machinae Cælestis Pars prior. Organographiam Astronomicam
n. 99. p. 6171 plurimis Iconibus illustratam & exornatam exhibens, &c. Gedani 1673. in Fol.
- n. 109. p. 215. 7. Animadversions on the First Part of the Machina Cælestis of Joa. Hevelius,
together with an Explication of some Instruments made by R. Hook. P. of
n. 111. p. 243. Geometry in Gresh. Coll. and R. S. S. London 1674. in 4to. Dr. Wallis's Letter
to M. Hevelius, concerning Divisions by Diagonals there inserted, but faultily,
is here Reprinted more correctly.
- n. 175. p. 1162. 8. Joannis Hevelii Consulis Daniscani Annus Climactericus. Gedani 1685.
Fol. Wherein (among other Things) M. Hevelius vindicates the Justness of
his Celestial Observations against the Exceptions by some made to the Accuracy
of them. The Controversy between Him and Dr. Hook, about the Use of Te-
lescopic and Plain Sights, and Dr. Wallis's Calculation, for Dividing the Limb
n. 111. p. 244. of Instruments by Diagonals, are also here Abridged.
- n. 175. p. 1176. 9. Excerpta ex Literis Ill. & Clariss. Virorum ad Nob. Ampliss. & Con-
sultiss. D. Jo. Hevelium Cons. Gedanensem perscriptis, Judicia de Rebus Astro-
nomicis, ejusdemque Scriptis, exhibentia; Studio ac Operâ Joa. Eriici Olhoffii
Secretarii Gedani 1683. in 4to.
- n. 150. p. 308. 10. A Description of Helioscop, and some other Instruments, made by R.
Hook. R. S. S. Lond. 1675. in 4to.
- n. 118. p. 440. 11. The Sphere of M. Manilius made an English Poem, with Annotations,
and an Astronomical Appendix; by Ed. Sherburn, Esq; Lond. 1675. in Fol.
- n. 101. p. 233. 12. Albareni Observaciones Astronomicas, Quas ex Arabico in Latinum Tran-
stulit Plao Tiburtinus. Noribergæ 1537; & Bononia 1645. The Arabick Copy
n. 204. p. 913. of those Observations does not Appear, whereby that Translation might be Ex-
amined: But Mr. Halley, by Calculating Tables from the Principles there De-
livered, hath here Discovered and Corrected above 30 considerable Faults in a
few Pages.
- n. 43. p. 868. 13. Historia Cælestis; ex Libris & Commentariis MS. Observationum Vi-
cennialium Tychois Brahe, Dani. Augusta Vindelic. An. 1666. in Fol.
- n. 102. p. 27. 14. All the Manuscripts of the Famous Kepler, (both Published and Unpub-
lished) which are Purchased, and carefully preserved by M. Hevelius.

15. *Jeremie Horroccii Angli Opera Posthuma*: una cum *Guil. Crabrai Observationibus Caelestibus*; nec non *Jo. Flamstedii de Temporis Aequatione Diatriba*, Numerisq; *Lunaribus ad Novum Luna Systema Horroccii*. Lond. 1672. in 4to.

16. *Astronomia Reformata*. Auctore *Joan. Bapt. Riccioli S. J. Stephano de Angeli*, Conceiving the Arguments of this Author, against the Motion of the Earth, to be none of the strongest, taketh Occasion to let the World see, that they are not more esteem'd in *Italy*, than in other places: *Manfredi*, in behalf of *Riccioli*, endeavours to Answer the Objections of *Angeli*, and this latter replies to *Manfredi's* Answer. The Substance of which Controversy is here given by Mr. *Ja. Gregory*; with some Remarks and Explications of his own upon it.

17. An Attempt to prove the Motion of the Earth from Observations, made by *R. Hook*, *F. R. S.* Lond. 1674. in 4to. The Method of this Undertaking is Approved and Commended by *M. Chr. Hugen*, and *M. Cassini*.

18. *Nicolai Mercatoris* *Hollati*, è Soc. Regia, *Institutionum Astronomicarum* Libri duo. Lond. 1676. in Octavo.

19. *Annales Cæli & Temporum perpetui, sive Mysteria Astronomo-Chronologica à Seculo Abscondita, nunc per Dei Gratiam Detecta & evidenter Asserta*, Libris tribus. *Kiloni*. This Book is Preparing by Dr. *Wasmuth*.

20. A Catalogue of Fixed Stars with their Longitudes, Latitudes, and Magnitudes, according to the Observations of *Uleg Beig*. Oxford 1686.

21. *Catalogus Stellarum Australium, sive Supplementum Catalogi Tycho-nici*; exhibens Longitudines & Latitudines Stellarum Fixarum quæ prope Polum Antarcticum sitæ, in Horizonte Uraniburgico, Tycho-ni inconspicue fuere. Authore *Edm. Halleio*, è Col. Reg. Oxon. in 4to.

22. *Congietture Physico Astronomiche della Natura del Universo*; da *Pietro M. Cavina*. in Faenza 1669. in 4to.

23. *Prose de Signori Academici di Bologna*, in Bologna 1672. in 4to. *Montanari's* Discourse concerning the admirable Changes and other Noveltyes observed in the Heavens.

24. *Ismaelis Bullialdi ad Astronomos Monita duo*. Primum de *Stella Nova*, quæ in *Collo Ceti* ante An. aliquot visa est. Alterum de *Nebulosa in Andromeda* Cinguli parte Borea, ante Biennium iterum orta. Approv'd by *M. Hevelius*.

25. Three Letters of *Jo. Dominicus Cassinus*, concerning his Hypothesis of the Sun's Motion, and his Doctrine of Refractions. At Bononia. in 4to.

26. *Refractio Solis Inocidui*, in Septentrionalibus Oris circa Solstitium Æstivum, An. 1695. aliquot Observationibus Astronomicis detecta. *Holmie*. in 4to. Translated into English. Lond. in 8vo.

27. *Tabularum Astronomicarum Pars Prior*; de Motibus Solis & Lune, nec non de Positione Fixarum, ex ipsis Observationibus deductis: Authore *Ph. de la Hire*. Paris 1687. in 4to. Some Animadversions on it are here inserted.

28. 1. The Royal Almanack for the Year 1675; by *N. Stevenson*. in 12^o.

2. ——— For the Year 1676.

3. ——— For the Year 1677.

29. *Ephemeris, ad Annum 1686, exactissime supputata*. Lond. in 8vo.

30. The Celestial World discovered, or Conjectures concerning the Inhabitants, Plants, and Productions, of the Worlds in the Planets. Written in Latin by *M. Chr. Hugen*. in 8vo.

31. Rag-

- n. 1. p. 2. n. 45. 31. *Ragguaglio de Nuove Osservationi da S. Gioseppe Campani*. This Book was Answer'd by M. *Auzout*, who gives his Opinion of *Campani's* Glasses, and his new *Osservations* of *Saturn* and *Jupiter* made with them; To this *S. Campani* publishes a *Reply*, and M. *Auzout* his *Animadversions* thereon.
- 1b. p. 74. 32. 1. *Ephemerides Mediceorum Siderum, ex Hypothesibus & Tabulis Jo. Dom. Cassini. Bononia 1668. in Fol.* The like *Tables* have been formerly published by the Learned *J. Bapr. Hodierna* at *Rome*, about 1658.
- n. 35. p. 688. 2. The *Table* of the *Eclipses* of the *First Satellite* of *Jupiter* by M. *Cassini*, Publish'd at *Paris* in the *Recueil d' Observations faites en plusieurs Voyages pour perfectionner l' Astronomie & la Geographie*, being not printed with the usual Care of the *Imprimerie Royale*, Mr. *Halley* here amends some of the *Errata*.
- n. 44. p. 892. 33. *Martis, circa Axem proprium Revolutibilis, Observationes Bononia à Jo. Dominico Cassino habitæ 1666.* Here M. *Cassini* judges it Evident, that the *Period* of this *Planets Revolution* is not performed in the *Space* of $12^h 20'$, but in about $24^h 40'$; and that those, who affirm the former, must have been deceived by not well Distinguishing the two *Faces*.
- n. 214. p. 256. 34. *Mercurius in Sole visus; à Jo. Hevelio. Ged. 1662.*
- n. 14. p. 242. 35. *Prodromus Cometicus; by Hevelius.*
- n. 35. p. 687. 36. *Joannis Hevelii Descriptio Cometae, An. Æræ Christianæ 1665. exorti: una cum Mantissa Prodromi Cometici, Observationes omnes Prioris Cometae 1654, ex iisque Genuinum Motum accurate deductum, cum Notis & Animadversionibus, exhibens.*
- n. 134. p. 853. 37. *Jo. Hevelii Cometagraphia. Dantzick. in Fol.*
- n. 6. p. 104. 38. *Stanislai de Lubienietz Theatrum Cometicum. Amstelod. 1668. in Fol.*
- n. 17. p. 301. 39. *Del Movimento della Cometa, apparsa il mese di Decembri 1664. da Pietro Maria Mutoli. in Pisa. in 4to.*
- n. 40. p. 805. 40. *Erasmi Bartholini de Cometis, An. 1664, & 1665. Opusculum; ex Observationibus Hafniæ habitis adornatum. Hafnia. in 4to.*
- n. 35. p. 691. 41. *Joh. Wallisii, De Cometarum Distantiis investigandis. Lond. 1678.*
- n. 53. p. 1069. 42. *Lectures and Collections made by R. Hook, Sec. of the R. S. Lond. 1678. in 4to.*
- Ph. Col. n. 4. 43. *Observat. of the Comet of 1680. and 1681. made at the Col. of Clermont; by P. J. de Fontenay è S. J. Profess. of Mathematicks, Paris. 1681.*
- p. 106. 44. *A Treatise concerning the late Comet, Published at Turin. 1681. by Donato Rossetti S. T. D. Canon of Leghorn, and Tutor in Mathematicks to the Duke of Savoy.*
- 1b. p. 114. 45. *An Explication of the Comet which appeared at the End of 1680. and in the Beginning of 1681. upon the Observations of D. Anthelme, Carthusian of Dijon. at Dijon. 1681. in one single Sheet.*
- 1b. p. 116. 46. *A small Discourse about Comets, published in the High Dutch at Nuremberg 1681; by a Lover of Astronomy.*
- Rh. Col. n. 7. 47. *A new Introduction, shewing how the Motions of the Comets may be reduced to some certain and Geometrical Rules, so that their Appearance may be Predicted, in High Dutch; by Ja. Bernouly, at Bazil. An. 1681.*
- p. 196. 48. *Joannis Jacobi Zimmermanni Cometo-scopia. Or, Three Astronomical Relations concerning the Comets that have been seen in the Years 1680, 1681, 1682. Stutgard, 1682. in 4to.*
- 1b. p. 199. C. H. A. P.

C H A P. V.

MECHANICKS. ACOUSTICKS.

I. I.



I Agens ut A Efficit ut E ; Agens ut 2 A Efficiet ut 2 E, 3 A ut 3 E, &c. cæteris paribus : Et universaliter, m A ut m E ; cujuscunque rationis Exponens sit m .

The General Laws of Motion, by Dr. Wallis. n. 43. p. 864. Nov. An. 1668.

2. Ergo si Vis ut V moveat Pondus P ; Vis m V movebit m P, cæteris paribus, puta per eandem Longitudinem eodem Tempore, *h. e.* eadem Celeritate.

3. Item si Tempore T moveat illud per Longitudinem L ; Tempore n T movebit per Longitudinem n L.

4. Adeoque si Vis V, Tempore T, moveat Pondus P, per Longitudinem L ; Vis m V, Tempore n T, movebit m P, per Longitud. n L. Et Propterea, ut V T (Factum ex Viribus & Tempore) ad P L (Factum ex Pondere & Longitudine) sic m V T, ad m P L.

5. Quoniam Celeritatis gradus sunt Longitudinibus eodem Tempore transactis Proportionales, seu (quod eodem recidit) Reciprocè Proportionales Temporibus eidem Longitudini transigendæ impensis : erit $\frac{L}{T} : C :: \frac{m L}{n T} :$

$\frac{m}{n} C$. *h. e.* Gradus Celeritatum, in ratione composita ex directa Longitudinum & Reciproca Temporum.

6. Ergo propter V T : P L :: m V T : m P L : erit V : $\frac{P L}{T} :: m V : \frac{m P L}{n T}$: *h. e.* V : P C :: m V : m P C = m P x C = P x m C.

7. Hoc est, si Vis V movere potis sit Pondus P, Celeritate C : Vis m V movebit vel idem Pondus P, Celeritate m C ; vel eadem Celeritate, Pondus m P ; vel denique quodvis Pondus ea Celeritate, ut Factum ex Pondere & Celeritate sit m P C.

8. Atque hinc dependet omnium Machinarum (pro facilitandis Moribus) construendarum ratio : nempe ut qua ratione augetur Pondus, eadem minuat. Celeritas, quo fiat, ut Factum ex Celeritate & Pondere, eadem Vi movendo, idem sit : puta V : P C :: V : m P x $\frac{1}{m} C$ = P C.

9. Si Pondus P , Vi V , Celeritate C , latum in Pondus Quiescens (non impeditum) m P directe impingat; ferentur utraque Celeritate $\frac{1}{1+m} C$. Nam propter eandem Vim, majori Ponderi movendo adhibitam, eadem ratione minuetur aucti Celeritas; nempe $V : PC :: V : \frac{1+m}{1} P \times \frac{1}{1+m} C$, $= PC$. Adeoque alterius Impetus (intellige Factum ex Pondere & Celeritate) fiet $\frac{1}{1+m} PC$; reliqui $\frac{1}{1+m} m PC$.

10. Si in Pondus P (Vi V) Celeritate C latum, directe impingat aliud, eadem via, majori Celeritate insequens; puta Pondus m P , Celeritate n C , (adeoque Vi m n V latum; ferentur ambo Celeritate $\frac{1+m n}{1+m} C$. Nam $V : PC :: m n V : m n PC :: V + m n V = \frac{1+m n}{1} V : \frac{1+m n}{1} PC = \frac{1+m}{1} P \times \frac{1+m n}{1+m} C$. Adeoque Præcedentis impetus fiet $\frac{1+m n}{1+m} PC$, Subsequentis, $\frac{1+m n}{1+m} m PC$.

11. Si Pondera contrariis viis lata, sibi directe occurrant five impingant mutuo, puta, Pondus P , (Vi V) Celeritate C , dextrorsum; & Pondus m P , Celeritate n C (adeoque Vi m n V) sinistrorsum: utriusque Celeritas, impetus & directio, sic colliguntur, Pondus dextrorsum latum reliquo si quiesceret, inferret Celeritatem $\frac{1}{1+m} C$, adeoque Impetum $\frac{1}{1+m} m PC$, dextrorsum, sibi que retineret hanc eandem Celeritatem, adeoque Impetum $\frac{1}{1+m} PC$ dextrorsum (per Sect. 9.) Pondusque sinistrorsum latum (simili ratione) reliquo si quiesceret, inferret Celeritatem $\frac{m n}{1+m} C$, adeoque Impetum $\frac{m n}{1+m} m PC$ sinistrorsum; sibi que retineret hanc eandem Celeritatem, adeoque Impetum $\frac{m n}{1+m} PC$ sinistrorsum. Cum itaque Motus utrinque fiat; Impetus dextrorsum prius lati, jam aggregatus erit ex $\frac{1}{1+m} PC$ dextrorsum,

trorsum, & $\frac{m n}{1+m}$ P C sinistrorsum ; adeoque reapse vel dextrorsum vel sinistrorsum, prout hic vel ille major fuerit, eo Impetu quæ est duorum differentia : *h. e.* (posito + signo dextrorsum, & - sinistrorsum significante)

Impetus erit $+\frac{1}{1+m}$ P C $-\frac{m n}{1+m}$ P C $=\frac{1-m n}{1+m}$ P C ; Celeritas

$\frac{1-m n}{1+m}$ C ; adeoque dextrorsum vel sinistrorsum, prout 1 vel $m n$

major fuerit.) Et similiter Impetus prius lati, erit $+\frac{1}{1+m}$ m P C

$-\frac{m n}{1+m}$ m P C $=\frac{1-m n}{1+m}$ m P C ; Celeritas $\frac{1-m n}{1+m}$ C : adeoque

dextrorsum vel sinistrorsum, prout 1 vel $m n$, major fuerit.

12. Si vero Pondera nec eadem directe via procedant, nec directe contrario, sed oblique sibi mutuo impingant ; moderandus erit præcedens Calculus pro Obliquitatis mensura. Impetus autem Oblique impingentis, ad ejusdem Impetum qui esset si Directe impingeret (cæteris paribus) est in ea ratione qua Radius ad Secantem Anguli Obliquitatis ; (quod etiam intelligendum est, ubi perpendiculariter, sed Oblique cadit in percussi Superficiem, non minus quam ubi viæ Motuum se mutuo Oblique decussant :) quæ quidem consideratio, cum calculo priori debite adhibita, determinabit, quanam futura sint sic Oblique Impingentium Celeritas, Impetus & Directio, *h. e.* quo Impetu, qua Celeritate, & in quas Partes ab invicem resilient, quæ sic Impingunt. Eademque est ratio Gravitationis gravium Oblique descendentium, ad eorundem Perpendiculariter descendentium Gravitationem.

13. Si quæ sic Impingunt Corpora, intelligantur non absolute dura (prout hactenus supposuimus) sed ita Ictui cedentia, ut Elastica tamen Vi se valeant restituere, hinc fieri potuit ut à se mutuo resilient ea corpora, quæ secus essent simul processura ; (& quidem plus minusve, prout hæc Vis Restitutiva major minorve fuerit,) nempe si Impetus ex Vi Restitutiva sit Progressivo major.

In Motibus Acceleratis & Retardatis, Impetus pro singulis Momentis is reputandus est, qui gradui Celeritatis tum acquisito convenit. Ubi autem per Curvam fit Motus, ea reputanda est in singulis punctis Motus Directio, quæ est rectæ ibidem Tangentis. Et si quando Motus tum Acceleratus vel Retardatus sit, tum & per Curvam fiat (ut in Vibrationibus Penduli) Impetus æstimandus erit, pro singulis punctis, secundum tum gradum Accelerationis, tum Obliquitatem ibidem Tangentis.

2. Lex Naturæ de Collisione Corporum.

Velocitates Corporum propriæ & maximè naturales sunt ad Corpora reciproce proportionales.

Itaque Corpora R, S, habentia proprias Velocitates, etiam post Impulsum retinent proprias.

N n n 2

Et

By Sir Christ.
Wren,
1644. p. 867.

Et Corpora R, S, improprias Velocitates habentia ex Impulſu reſtituuntur ad *Æquilibrium*; hoc eſt, Quantum R ſuperat, & S deficit à propria Velocitate ante Impulſum, tantum ex Impulſu abſtrahitur ab R, & additur ipſi S, & è contra.

Quare Collifio Corporum proprias Velocitates habentium æquipollet Libræ Oscillanti ſuper bina Centra æqualiter hinc inde à Centro Gravitatis diſtancia: Libræ verò Jugum, ubi opus eſt producitur.

Itaque Corporum æqualium improprie Moventium tres ſunt Caſus. Corporum verò inæqualium improprie Moventium (ſive ad contrarias ſive ad eaſdem partes) decem ſunt omnino Caſus, quorum quinque oriuntur ex converſione.

Fig. 157.

R, S, Corpora æqualia; vel R, Corpus majus, S, Corpus minus.

a Centrum Gravitatis ſive Anſa Libræ. Z, Summa Velocitatum utriuſque Corporis.

$$\left. \begin{array}{l} \{ R \ c \} \text{ Veloc. } \{ R \} \text{ ante Impul-} \\ \{ S \ e \} \text{ Corp. } \{ S \} \text{ ſum data. } \end{array} \right\} \left. \begin{array}{l} \{ S \ o \} \text{ Veloc. } \{ S \} \text{ ante Impul-} \\ \{ R \ a \} \text{ Corp. } \{ R \} \text{ ſum data. } \end{array} \right\} \text{ vel } \left. \begin{array}{l} \{ o \ R \} \text{ Veloc. } \{ R \} \text{ poſt Impul.} \\ \{ o \ S \} \text{ Corp. } \{ S \} \text{ quæſita. } \end{array} \right\} \left. \begin{array}{l} \{ e \ S \} \text{ Veloc. } \{ S \} \text{ poſt Impul.} \\ \{ e \ R \} \text{ Corp. } \{ R \} \text{ quæſita. } \end{array} \right\}$$

Regula. R e, S e, faciunt o R, o S: R o, S o, faciunt e S, e R.

[Lege Syllabas (quamvis diſjunctas) R e, S e, o R, e S, vel R o, S o, o S, e R, in Linea cujuſlibet Caſus, & harum quæ ſcribitur in Schemate more *Hebraico*, ea indicat Motum contrarium Motui quem notat cujuſvis Syllabæ ſcriptio *Latina*. Syllaba conjuncta quietem Corporis denotat.]

$$\text{Calcul. } \begin{array}{l} R + S : S :: Z : R a \mid R e - 2 R a = o R \mid S o - 2 S a = e S. \\ R + S : R :: Z : S a \mid 2 S a + S e = o S \mid 2 R a + R o = e R. \end{array}$$

Natura obſervat. regulas Additionis & Subductionis *Specioſa*.

By M. Hugenſ.
n. 46. p. 927.
Apr. An. 1669.

3. *Regule de Motu Corporum ex mutuo Impulſu.*

1. Si Corpori Quieſcenti duro aliud æquale Corpus durum occurrat, poſt contactum hoc quidem Quieſcet, Quieſcenti verò acquireretur eadem quæ fuit in Impellente Celeritas.

2. At ſi alterum illud Corpus æquale etiam moveatur, feraturque in eadem Linea recta, poſt contactum permutatis invicem Celeritatibus ferentur.

3. Corpus quamlibet magnum à Corpore quamlibet exiguo & qualicunque Celeritate impactu movetur.

4. *Regula Generalis* determinandi Motum, quem corpora dura per occurſum ſuum directum acquirunt, hæc eſt:

Fig. 158.

Sint corpora A & B, quorum A moveatur Celeritate A D, B vero ipſi occurrat, vel in eandem partem moveatur Celeritate B D, vel denique Quieſcat, hoc eſt, cadit in hoc caſu punctum D in B. Diviſa Linea A B in C, (Centro Gravitatis corporum A B,) ſumatur C E æqualis C D. Dico E A habebit Celeritatem corporis A poſt Occurſum; E B verò, corporis B, & utrumque in eam partem, quam demonſtrat Ordo punctorum E A, E B. Quod ſi E incidat in punctum A vel B, ad Quietem redigentur corpora A vel B.

5. Quantitas

5. *Quantitas Motus duorum Corporum augeri minui potest per eorum occursum; at semper ibi remanet eadem quantitas versus eandem partem, ablatâ inde quantitate Motus contrarii.*

6. *Summa Productorum factorum à Mole cujuscunque corporis duri ducta in quadratum suæ Celeritatis, eadem semper est ante & post occursum eorum.*

7. *Corpus durum Quiescens, accipiet plus Motus ab alio corpore duro, se majori minorive, per alicujus Tertii, quod mediæ fuerit quantitatis, interpositionem, quam si percussum ab eo fuisset immediatè. Et si corpus illud interpositum, fuerit medium proportionale inter duo reliqua, fortissime omnium aget in Quiescens.*

Considerat *Author* in his omnibus (ut ipse ait) Corpora ejusdem materiæ, five id vult, ut eorum moles æstimetur ex pondere.

Cæterum subjungit, notâsse se miram quandam *Naturæ Legem*, quam Démonstrare se posse affirmat in corporibus Sphæricis, quæque Generalis ipsi videtur in reliquis omnibus five Duris five Mollibus, five Directe five Oblique sibi occurrentibus, viz. *Centrum Commune Gravitatis* duorum, trium, vel quotlibet Corporum, æqualiter semper promoveri versus eandem partem in linea recta, ante & post occursum.

4. Cum novissimis Mensibus nonnulli è *Societate Regia* in publico ejusdem concessu enixius urgerent, ut gravissimum illud de *Regulis Motus* Argumentum, non semel inter Ipsos ante hac agitatum, sed pluribus aliis intercurrentibus rebus, nunquam, uti par erat, discussum expensumve, tandem aliquando Examini Rigido subiectum conficeretur; visum equidem fuit Illustrissimo isti *Cæui* decernere, ut quotquot è *Sociis* suis indagandæ Motus Indoli præcæteris incubuissent, rogarentur, ut sua in rem illam Meditata & Inventa depromere, simul & ea, quæ ab aliis Viris Præcellentibus, *Gallileo* puta, *Cartesio*, *Honorato Fabri*, *Joachimo Jungio*, *Petro Borrelli*, aliisque, de Argumento isto fuerant excogitata, congerere & procurare vellent; eo scilicet fine, ut consultis hoc pacto collatisque omnium sententiis, illa dehinc Theoria, quæ cum Observationibus & Experimentis, debitâ curâ & fide crebro peractis, quammaximè congrueret, Civitate Philosophica suo jure donaretur.

Edito hoc Celeusmate, incitati protinus è dicta *Societate* fuerunt, imprimis *Christianus Hugenius*, *Johannes Wallisius*, *Christopherus Wrennus*, ut suas de *Motu Hypotheses & Regulas*, quibus condendis aliquamdiu insudassent, maturare atque expedire satagerent. Factum hinc, ut selectus ille Virorum præstantissimorum Trias, post paucarum septimanarum spatium, Theorias suas, eleganter compendifactas, tantum non certatim transmitterent, Regiæque Societatis super iis sententiam exquirerent. Primus omnium D. *Wallisius*, sua de *Motibus æstimandis Principia*, Literis, d. 15. Novemb. 1688. datis, ejusdemque Mensis die 26 traditis & prælectis, communicavit. Mox eum excepit D. *Christopherus Wren*, qui *Naturæ Legem de Collisione Corporum*, proximo Mense Decembri, ejusque die 17. eidem Societati publice exhiberi curavit: quæ in mandatis mox dedit (præhabito tamen utriusque hujus Authoris consensu) ut ad commodiorem horum scriptorum communicationem, discussionemque diffusio-rem, res tota Typis mandaretur.

Hæc

Hæc dum apud Nos geruntur, Ecce adfert Nobis Tabellarius d. 4. *Januarii* insequentis (*St. Ang.*) D. *Hugenii* literas ejusdem Mensis d. 5. (at *St. N.*) exaratas, ejusque scripti, de *Motu Corporum* ex mutuo *Impulsu*, priores Regulas quatuor, una cum Demonstrationibus, continentes. Habebam ego in promptu *Theoria Wrenniana* Apographum, idque actutum eodem planè die, sic favente Tabellione Publico, D. *Hugenio*, redhostienti vice, remittebam, dilata interim Literarum *Hugeniarum* (quibus tale quid includi, ob Molem, & antegressum Authoris promissum suspicabar) resignatione, donec ferret occasio Nobilissimum & Sapientissimum *Regiæ Societatis Præsidentem*, D. *Vice-Comitem Brouncker*, compellandi. Quo facto, amborumque Regulis in modo dicta *Societate* collatis, mirus confestim in utroque consensus effulsit; id quod insignem in nobis lubentiam pariebat, utrumque hoc scriptum prælo nostro committendi. Nihil hic Nobis deerat à parte *Hugenii*, quam ejus consensus; absque quo fas nequaquam judicabamus, ipsius Inventum, maximè cum illud haud integrum eo tempore nobis dedisset, in lucem emittere. Curæ interim nobis erat, scriptum ipsius publicis *Regiæ Societatis* Monumentis inferendi; simul & Authori d. 11. *Januarii* solennes pro Cordata illa Communicatione gratias reponendi, additâ dehinc (die *Scil.* 4. *Febr.*) sollicitâ commonefactione, ut suam hanc *Theoriam* vel *Parisiis* (quod proclive erat factu in *Eruditorum*, ut vocant, *Diario*) vel hic *Londini* in *Adversariis Philosophicis*, imprimendam curaret, vel saltem permetteret. Quibus expeditis Literis paulo post secundas accepimus ab *Hugenio*, scripti *Wrenniani* de hoc argumento recte traditi mentionem facientes, nihil tamen quicquam de suimet scripti Editione, vel *Parisiis* vel *Londini* paranda, commemorantes.

Unde liquere omnino autumem, ipsum sibi defuisse *Hugenium* in illa publicatione maturanda; quin imo occasionem dedisse procrastinando, ut Laudatus Dn. *Wren*, pro ingenii sui sagacitate Geminam omnino *Theoriam* eruens in Gloriæ, huic speculationi debitæ, partem jure veniret; cum extra omne sit dubium neutrum horum *Theoriæ* illius quicquam, priusquam scripta eorum simul compararent, rescivisse ab altero, sed utrumque propriâ Ingenii fecunditate, pulchellam hanc sobolem enixum fuisse.

Solvit equidem *Hugenius*, ante aliquot jam Annos, *Londini* cum ageret, illos de Motu Casus qui ipsi tunc proponebantur; luculento sane Argumento, eum jam tum exploratas habuisse Regulas, quarum id evidentia præstaret. At non affirmabit ipse, cuiquam se *Anglorum* suæ *Theoriæ* quicquam appertuisse; quin fateri tenentur, se ab eorum nonnullis ad communicationem ejus sollicitatum, nec tamen unquam, nisi nuperrime, ad id faciendum pertractum fuisse.

*The Synchronism
of the Vibrations
made in a Cy-
cloide; Demon-
strated by a Per-
son of Quality.
n. 94. p. 6032.
Fig. 159.
May An. 1673.*

II. Sint $a b$, $b c$, $c d$, $e f$, &c. omnes invicem æquales; & b 1, c 2, d 3, e 4, f 5, &c. æqualiter crescant ut, 1, 3, 5, 7, 9, &c.

Dico in hac Linea, Grave quodlibet, Cadens ex quovis ejus Puncto, attingere fundum in eodem Temporis spatio, quo eum attingeret si caderet ex quovis ejusdem Puncto alio.

Nam si ponas, $a = ab = bc = cd$, &c. & $b = b_1$, & x pro quolibet Numero alterutrorum; tunc si $x a$ ponatur pro $a f$, $x x b$ repræsentet oportet $f d$, proindeque Tempus Descensus necessario erit $\frac{x x b}{x x a a}$ seu $\frac{b}{a}$; atque idem in omnibus obtinet Casibus. Ergo, &c.

Dico insuper, Curvam hanc esse *Cycloidem*, quod Demonstratu est facile ex Constructione, atque ex eo quod jam innuo; nempe, Curvam hanc $abcd efz$ æquare duplum Ultimæ Rectarum, $h. e. z z a$, & $a a$ æqualem esse Semicircumferentiæ Circuli cujus $z a$ est Diameter; ac universim Triangulum $\gamma \delta \pi$ repræsentare Rectam $z a$; & Quadratum $\gamma \delta \pi \sigma$, Curvam $abcd efz$, & Quadrantem $\gamma \delta \sigma$ repræsentare Rectam $a a$; ac partes unius, partes alterius respicivè. Uti si $\gamma \pi$ repræsentat $f d$, tunc $\gamma \sigma$ repræsentat $a d$, & $\gamma \pi \sigma$ repræsentat $a f$. At non vacat fusiùs hæc prosequi.

Dico denique; Globulum suspensum è Funiculo (Justæ Longitudinis) & intra duas *Cycloides* vibrantem, moveri in *Cycloide*. Quare Vibrationes ejusmodi sunt *Synchrone*. Q. E. D.

III. 1. Probl.] *Determinare Lineam Curvam data duo Puncta, in diversis ab Horizonte Distantibus & non in eadem Rectâ Verticali posita, connectentem, super qua Mobile, Propriâ Gravitate decurrens, & à Superiori Puncto Moveri incipiens, Descendat ad Punctum Inferius.* A Problem concerning the Line of quickest Descent between two Points given; proposed by M. Jo. Bernoulli. n. 224. p. 384. Jan. An. 1697.

Sensus Problematis hic est, ex Infinitis Lineis quæ duo illa data Puncta conjungunt, vel ab uno ad alterum duci possunt, eligatur illa, juxta quam si incurvetur Lamina Tubi Canalisve Formam habens, ut ipsi impositus Globulus & libere dimissus iter suum ab uno Puncto ad alterum emetiatur Tempore Brevissimo.

2. Accepi hesterno die duo Problematum à Joanne Bernoulli Mathematico-Solv'd; by num. acutissimo propositorum Exemplaria, Groningæ edita, Cal. Jan. 1697. Ibid.

A Dato Puncto A, ducatur Recta Infinita A P C Z Horizonti parallela, & super eadem Recta describatur tum *Cyclois* quæcunque A Q P, Rectæ per alterum Datum Punctum B ductæ (& si opus est productæ) Occurrens in Puncto Q, tum *Cyclois* alia A B C cujus Basis & Altitudo sit ad prioris Basem & Altitudinem respective ut A B ad A Q; Et hæc *Cyclois* Novissima transibit per Punctum B, & erit Curva illa, linea in qua Grave à Puncto A ad Punctum B, Vi Gravitatis suæ, Cirrissimè perveniet. Q. E. I. Fig. 160.

3. Sit A P, Linea Horizontalis; P, Punctum à quo corpus Grave descendit, per Curvam Lineam quæsitam A D E, C & D, Puncta duo infinitè propinqua, per quæ Corpus decisurum sit, C D Recta, duo Puncta connectens, D C & s C, D F & S G, F S & G C vel s H, Momenta Curvæ Abscissæ, & Ordinatum applicatæ respective. Capiatur, D r = D s, & s C = B C. The Demonstration; by Mr. R. Sault. n. 246. p. 425. Nov. An. 1698. Fig. 161.

Quoniam

Quoniam in Lineolis Nascentibus, Tempus est ut Via percurſa directè & Velocitas (*i. e.* in hoc caſu, ut Radix Quadrata Altitudinis corporis deſcenſu) inverſe, per Hypoth. $\frac{D s}{\sqrt{QD}} + \frac{SC}{\sqrt{QF}} = \text{Tempori Minimo}$. Et quia Velocitas in Punctis æquialtis S & B per Curvam D s C & Rectam D B C eadem eſt, Tempus per D C, quod evidenter Minimum eſt, erit ut $\frac{BD}{\sqrt{QD}} + \frac{BC}{\sqrt{QF}}$; æquantur ergo hæc Tempora, & $\frac{D s}{\sqrt{QD}} + \frac{SC}{\sqrt{QF}} = \frac{BD}{\sqrt{QD}} + \frac{BC}{\sqrt{QF}}$, hoc eſt, $\frac{DB - D s}{\sqrt{QD}} = \frac{SC - BC}{\sqrt{QF}}$, vel $\frac{Br}{\sqrt{QD}} = \frac{ss}{\sqrt{QF}}$

Sed Triangula Evaneſcentia B r s, B t s, æquiangula ſunt Triangulis D s F, H s C; Ergo $\frac{B s}{D s} = \frac{Br}{sF}$, & $\frac{ts}{H s} = \frac{B s}{s t}$. Componantur hæc duæ rationes æqualitatis, & $\frac{Br}{D s \times H s} = \frac{ts}{sF \times s t}$. Ex æque $\frac{\sqrt{QD}}{sF \times s t} = \frac{\sqrt{QF}}{D s \times H s}$. Quandoquidem autem quidvis ex Elementis æqualiter Fluere ſupponatur, ponamus D s = S C, & evadet ſimpliciſſima Curvæ expreſſio $\frac{\sqrt{QD}}{sF} = \frac{\sqrt{QF}}{D s}$ ubique, *i. e.* in Puncto Flexuræ Curva ſemper erit in Ratione compoſita Velocitatis directè, & Momenti applicatim Ordinatæ inverſe. Sint x y & z Fluxiones Abſciſſæ, Ordinatum applicatæ, & Curvæ reſpectivè, $\frac{x^{\frac{1}{2}}}{y}$ conſtans eſt, ut ſupra. Ergo $\frac{x^{\frac{1}{2}}}{y} = 1$, ſed poſuimus z (= $\sqrt{x x + y y}$) conſtans. Ergo ut hæc unitas conſtans ſit & dimenſiones debitas retineat $\frac{x^{\frac{1}{2}}}{y} = \frac{a^{\frac{1}{2}}}{\sqrt{x x + y y}}$, & poſt reductionem $y = \frac{x^{\frac{1}{2}} x}{\sqrt{a - x}}$. Expreſſio Notiſſima Cycloidis PEL. Q.E.I.

IV. Theorema.] Si in Cycloide A V D, cujus Basis-A D est Horizonti *How much the* Parallela, Vertice V deorsum spectante, ex A ducatur utcumque Recta A B *Descent is* Cycloidi occurrens in B, ex quo ducatur Recta B C Curva Cycloidis B D *Quicker in the* in B Normalis, ad quam ex A demittatur Perpendicularis Recta A C. Dico *Cycloid than in* Tempus quo Grave è Quiete cadens ex A, Vi sua Gravitatis decurrit Rectam A B, *a straight Line,* esse ad Tempus quo percurrit Curvam A V B, sicut Recta A B ad Rectam *p. 424. Feb.* A C. *An. 1697.*

Per B ducatur B L, Parallela Cycloidis Axi V E; & B K, Basi A D Pa- *Fig. 161.* rallela, occurrens Axi in G, & Circulo super Diametrum E V descripto in F & H, Cycloidi denique in K. Ducatur Recta E F, quæ ex Cycloidis natura parallela est Rectæ B C. Unde B M est æqualis E F, & E M æqualis B F; quæ, propter Cycloidem, æquatur Arcui V F; & proinde A M est æqualis Arcui E H V F.

Per Prop. 25. Part II. Horologii Oscillatorii Hugonii, Tempus quo grave è Quiete cadens percurrit A V, est ad Tempus Casus per E V, ut Semicircumferentia ad Diametrum; & per dictæ Partis Prop. Ultimam, Tempus quo Grave percurrit V B, post decursam A V (nempe æquale Tempori quo Grave percurrit K V, post decursam A K) est ad Tempus Lapsus per A V, sicut Arcus V F, ad Semicircumferentiam; adeoque ad Tempus Casus per E V, sicut F V ad Diametrum. Quare Tempus quo Grave percurrit Curvam A V B, est ad Tempus Casus per E V, sicut Arcus E H V F, ad Diametrum E V. Sed Tempus Casus per E V, est ad Tempus Casus per L B, five E G, sicut E V ad E F: Igitur ex æquo, Tempus quo Grave percurrit A V B, est ad Tempus Casus per L B, sicut Arcus E H V F ad subtensum E F; hoc est, ut Recta A M, ad Rectam M B. Rursus, Tempus Casus per L B, est ad Tempus Lapsus per A B, ut L B ad A B: Ergo Ratio Temporis quo Grave percurrit A V B, ad Tempus quo percurrit A B, componitur ex Ratione A M ad M B, & Ratione L B ad B A; adeoque æqualis est Rationi $A M \times L B$ ad $M B \times B A$. Sed $A M \times L B$, est æquale $M B \times A C$, quia utrumque æquatur duplo Trianguli A B M: Et igitur Tempus quo Grave è Quiete cadens percurrit Curvam Cycloidis A V B, est ad Tempus quo percurrit Rectam A B, sicut $M B \times A C$ ad $M B \times B A$; id est sicut A C ad A B. Q. E. D. Similiterque procedet Demonstratio, si Punctum B, sit inter A & V.

V. 1. The upper Plate of the Watch is A B: The Circular Balance- *Watch Portable* Wheel C D, of which the Arbor is E F: The Spring turned Spirally, *Watches; by* GHM, fastned to the Arbor of the Balance-Wheel in M, and to the piece *M. Hugens.* that is fast to the Watch-Plate, in G, all the Spires or Windings of the *n. 112. p. 272.* Spring being free without touching any thing. NOPQ, is the Cock, in *Fig. 162.* which one of the Pivots of the Balance-Wheel turns; R S, is one of the *Mar. An. 1675.* Indented-Wheels of the Watch having a Ballancing Motion, which the Balance-Wheel gives to it. And this Wheel R S, catches in the Piston T, which holds on the Arbor of the Balance, of which by this means the Motion is entertained as much as is necessary. These Watches are exact for the Pocket,

and when made greater, will be useful to find the *Longitudes* both by Sea and Land.

By Dr. Goth.
Guil. Leibnitz.
n. 113. p. 285.
Apr. An. 1675.

2. The Principle I thought upon some Years ago for making Exact *Portable Watches*, is altogether different from that of M. *Hugens*; His depending upon a Physical Observation, but mine upon a meer Mechanical Reflection; which hath not been taken notice of for want of the *Art of Combination*, the use of which is far more general than that of *Algebra*. For, having considered with my self, that a *Spring* being Bent to the same Degree, will always Unbend it self in the same Time, provided it find the same freedom of Unbending it self suddenly; I inferred from thence, that there might be employed two such, one of which should play, whilst the *First Mover* of the *Watch* did Bend the other again.

Fig. 164.

These Thoughts I have executed in the following manner: Let A B, be one of the *Watch-Plates*, C and M, two indented Barrels wherein the small *Springs* are inclosed. The Teeth of the Barrels catch those of the Pinions *d d*, which carry the Ballances *e e*, and other Teeth of the said Barrels are caught by those of the Interrupted Wheel F G. Now let us imagine, that this Wheel F G, being moved towards H F, by the force of the *First Mover* of the *Watch*, and turning the Barrel C, Bends the *Spring* inclosed in it, and stops with the Barrel as soon as it hath Bent this *Spring*. This piece which serves to stop, is easie, and hath not been thought necessary to be marked here, to avoid embarrassing the Figure. But whilst one Indented part of the Interrupted Wheel F G, viz. F, turns the Barrel C, the empty part, opposed thereunto, which is G, answers to the other Barrel M, and gives Liberty to the *Spring*, it incloseth, to Unbend it self. Thus whilst the *Movement* of the *Watch* Bends the small *Spring* of the Barrel C, in the same Time the small *Spring* of the other Barrel M, Unbends of it self: I say, in the same Time, except the *Spring* C, shall have done Bending a little sooner than the *Spring* M shall have Unbent it self: So that the *Spring* C, being Bent, and the Wheel F G stopped; both of them stay in this Posture, till the *Spring* M, when it shall be quite Unbent, doth, at the end of its Motion, touch a piece which delivers it. And then the *Spring* C Unbends of it self in its turn; the Teeth of the Interrupted Wheel, which continues its Motion the same way as before, since 'tis delivered, not being any more able to hinder it therefrom, because the Barrel C, doth now meet with the Empty part H, of the said Wheel. But before it hath done with Unbending it self, the Indented part L, being opposite to the Empty part H, that turns the Barrel M, Bends its *Spring* again, and having done so, stops with it; whilst the *Spring* C, making an end of Unbending it self, delivers them by a Reciprocal good Office, and renders to the *Spring* M, the same Services which it had received from it, with an Expectation of receiving the like again.

Which being well considered, 'tis manifest, That the same *Alternative Motions* will continue always: That the Periods, taken from the very Moment that one *Spring* begins to Unbend, until the Moment it once Unbends it self again, will always be of *Equal Duration*, though the two small *Springs*

Springs be not equally strong: That the Ballance of such a *Watch* will be double, and may be Charged more or less, and receive Delay, by Advancing or Recoiling along the two Arms two equal Weights, Counter-ballancing one another, that so the Change of the Situation may not at all prejudice the Equality of the *Watch*. For the rest, we may in this kind of *Watches* spare the *Fusée*, and consequently the String or Chain. 'Tis also easie to judge, that such *Watches* as these may be of a Size sufficiently small; that they will make no more Noise than ordinary *Watches*; that they will be as exact as *Pendulums*, and cease not to go whilst they are Winding up. And though the Motion of the Watch Wheels may be altered by many Accidents, yet the *Periods* of the *small Springs* will not be concerned in all or any of them, provided the Motion of the Watch Wheels have always more Strength than it needs to Bend them again; which is in our Power.

The Objections that have been made against this Contrivance, if employed for *Finding Longitudes*, are these; that Tossing of Ships would shake the *Springs* as well other pieces; that Rust would spoil them, since the Salty Humidity of the Sea in remote Voyages, spares not the very Needles of Compasses though inclosed in Boxes; that the Changes of Seasons and Climates will sensibly alter the *Springs*, especially the great Heats or Rains within the Tropicks, which at length will somewhat Untemper the Steel; as is confirmed by the Experiments of the *Illustrious Academy of Florence*, shewing how easily that Heat and Cold do change *Slender Springs*: Besides that, the Air more or less condensed will also more or less resist the Motion of the Ballance. To which may be added, that *Springs* by working are weakened; And Lastly, That there will be always some little *Friction*, that will make the several pieces go more or less easily, and that even in length of time they will wear out.

But I answer, That all these Defects, that proceed from the Imperfection of the Matter, may be surmounted by a General Remedy, without Examining them here in particular: And that is, That for executing it in great, we make use of massy Springs, as are those of Cross-Bows, we being Masters of them, not wanting Force or Place in a Ship, to govern a great Weight that may serve to Bend them continually again. Now these *Massy Springs* may be so great, and their Restitution so speedy, by Augmenting their Number, that all the above-named Defects will have no considerable Proportion to this Strength, and the Aggregate of their Repetitions will not be sensible till after a very long time. And 'tis easie to *Demonstrate*, That by Augmenting the Bigness of the Engine, and the Force of the *Massy Springs*, we may make the Error as small as we will, provided we pass not the Bounds of Convenience, and content our selves with Exactness sufficient for their Chief End, viz. For finding the *Longitudes*.

VI. The Circle, FGH, being placed upon a Plain Inclined AB, is divided into two Unequal Parts by the Line GI. To restore to the Less Secture its *Equilibrium*, there is fastened to the Extremitie of the Radius DF, a Weight F, which is sufficiently heavy to Recover, what the lesser Secture loses.

n. 140. p. 1006 loses by its Situation. That a Wheel or Clock may thus stand not only in
 Fig. 165. *Equilibrium*, but also Ascend upward, there is placed in the middle of the
 July An. 1678. Clock a Drum, which encloses the Spring of the Pendulum; upon which Drum
 is fastned the Radius DF. For thus the Spring being Mounted, enforces the
 Drum to turn, and so to Raise the Weight, which it cannot Raise without
 its becoming more Heavy, in regard that coming to the Point E, it is farther
 from the Center, than when it was in F, and thus all the Wheel turns on that
 Side as the Spring gives way.

A Clock De-
 scendent on a
 Plain Inclined;
 by Mr. Mau-
 rice Wheeler.
 n. 161. p. 647.
 May An. 1684.

VII. Although the Marquis of Worcester is said to have contrived a Watch
 that should Move upon a Declivity, and M. de Gennes has given some Ac-
 count of a Clock Ascendent on a Plain Inclined; yet neither of them, nor any
 like them, was ever seen by me, and for ought I could ever learn, the Reason
 of their Motions remains to this Hour as great a Secret, as if they had never
 been. I shall therefore give an Account of a Movement, which I have de-
 sign'd to measure Time after a peculiar Manner.

Fig. 166.

1. The Exterior Structure of it is a Circular Body of $3\frac{1}{2}$ Inches Dia-
 meter, consisting of two Plates measured by the same Radius, and fixed in a
 Parallel Position to each other by the Hoop *b*, the Breadth of which is about
 an Inch. This Hoop and the two Plates Form the Case of the Movement;
 of which, that which appears in the Front, is towards the Verge thereof In-
 scribed with a Horary Circle, the Divisions whereof answer the Hours of a
 Natural Day. The Deep Shades within this Circle are intended to represent
 a Concave, of near half an Inch deep; and the Prominence *g*, in the Middle
 of this Concave, is a Hemisphere of Brass or Silver, riding loosely on a Pin,
 which lies hid, and is the Axis of the Movement. The Upper half of this
 Hemisphere is hollow, but the Nether filled with Lead; and the small Gen-
 tleman that sits thereon, does with an Erected Finger perform the Office of
 an Index. But this being only for Ornament, you may Substitute in the
 room thereof any other Index, provided the Axis, whereon it is supported,
 move freely in the Hole H, and the lower part thereof H L, so far prepon-
 derate to H P, as always to keep it Pendulous, with its Point to the Vertical
 Hour.

Fig. 169.

Fig. 166.

2. For the manner of its Motion, as far forth as it appears outwardly; it is
 thus: S E represents a Board or Shelf, of a Straight and Even Surface, about
 6 Foot long, and so Thick as not to be apt to Cast with change of Weather;
 nor to grow Camber under a small Weight; on this is the Movement placed,
 and here to perform its Course; and therefore I call it the Stage of the Move-
 ment. This Stage is Raised at the end S, about 10 Deg. above the Horizon
 or Line of Level H E; but this Angle of its Declivity D E H, is Variable.
 The two Plates which Form the Case of the Movement, are to be extant all
 round without the Hoop *b*, $\frac{1}{4}$ of an Inch, and the Edges of 'em lightly
 Indented; that while the Movement descends upon the Stage, it may Turn
 only, and not Slide. The Movement being placed as high as it may, near
 the point S, shall Move downward towards E, with that slowness, as to Finish
 one entire Revolution in 24 hours; and while it does so, the Divisions on
 the

John Flamsteed fecit
1685.

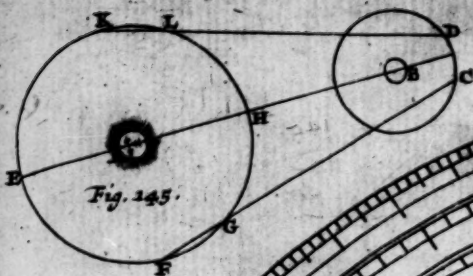


Fig. 145.



Fig. 146.

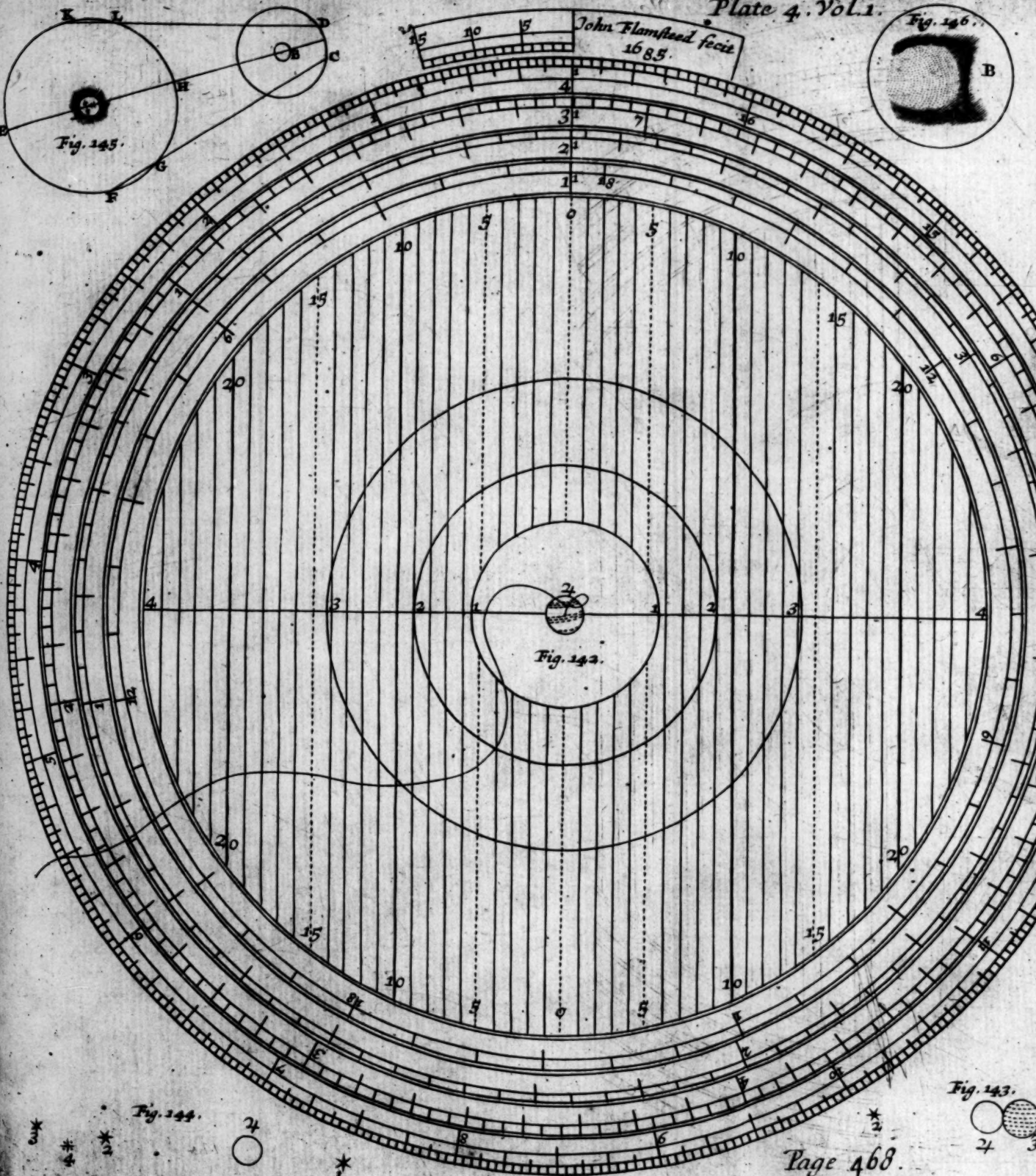


Fig. 142.

5
4
3
2
1

Fig. 144.

4

*

Fig. 143.

2
3

Fig. 147.



Fig. 149.



Fig. 151.



Fig. 153.



Fig. 148.



Fig. 150.



Fig. 152.

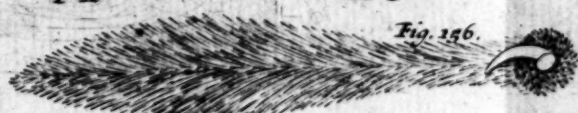
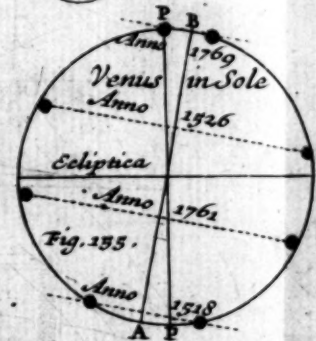
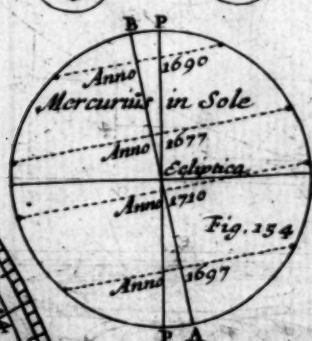
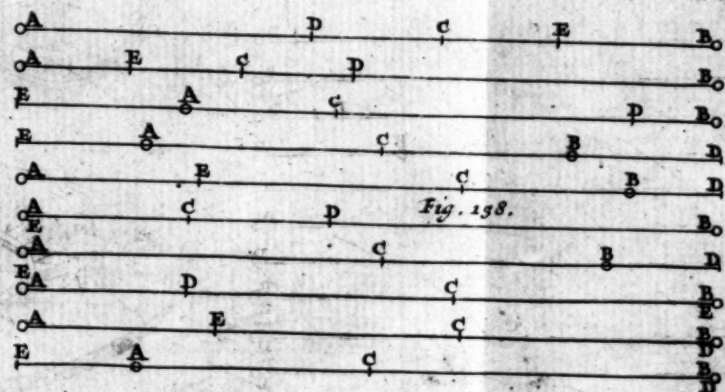
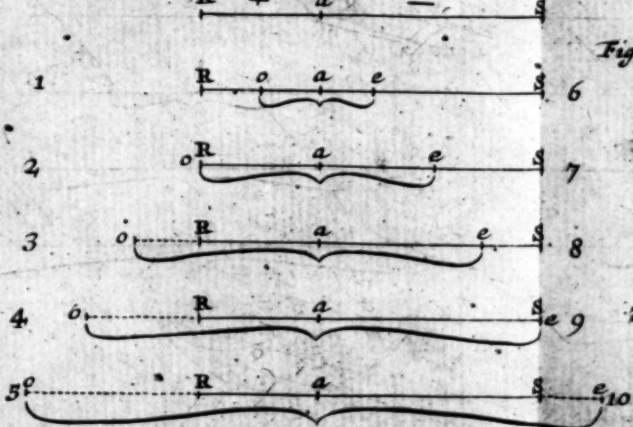


Fig. 156.



Inæqualia



Æqualia

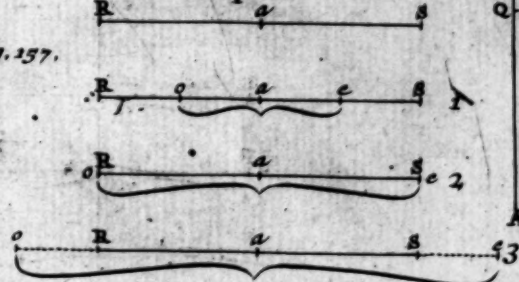


Fig. 160.



Fig. 161.



Fig. 165.

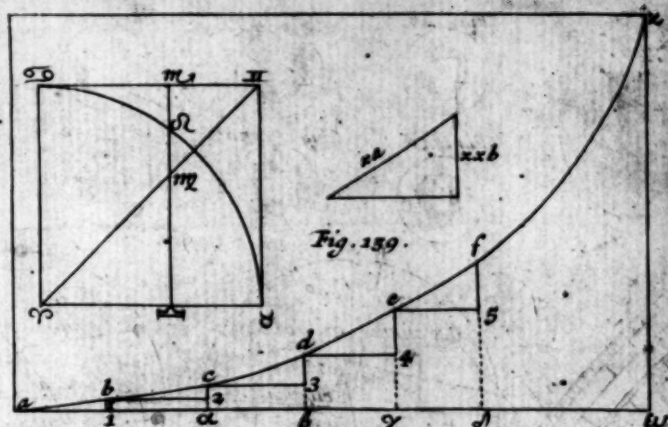


Fig. 159.



Fig. 163.



Fig. 164.

[illegible]

The first of these is the fact that the book is written in a very simple and direct style, which makes it accessible to a wide range of readers. The second is the fact that the book is written from a personal perspective, which gives it a unique voice and insight into the author's thoughts and feelings.

[illegible][illegible][illegible]

The only the number of the British and Indians are in
calculated, and the other numbers may correspond to the

The Commission has been informed that the Government of the United States has agreed to provide a loan of \$100,000,000 to the Government of the United Kingdom for the purpose of financing the reconstruction of the United Kingdom after the war. The loan is to be repaid over a period of 50 years, with interest at the rate of 4 per cent. The Government of the United States has also agreed to provide a loan of \$100,000,000 to the Government of the United Kingdom for the purpose of financing the reconstruction of the United Kingdom after the war. The loan is to be repaid over a period of 50 years, with interest at the rate of 4 per cent.

the *Horary Circle* (or Dial-Plate) successively Culminating over the Point of the Index (which is always to keep the same Position) will shew the Hours of the Day and Night. And when by several repeated Revolutions, it has measured the length of its *Stage*, it is to be replaced at S, as before, which may be done in less than half the time you are Winding up a Watch; and if the *Stage* be 6 Foot long, no oftner than once in a whole Week.

3. The way of Adjusting the Motion to the exact Measure of an Hour, and Rectifying its Errours, is thus: *viz.* By the turning a Skrew inserted at S, the *Stage* may be Elevated or Depress'd, and accordingly the *Movement* will go Faster or Slower: Faster, if Raised up, and Slower, if let Down; and by making the *Horary Circle* Moveable, and Inserting several small Bosses or Buttons, here and there upon the Verge thereof, it may with an easy touch of the Finger be moved to the right and left, as there shall be Occasion, till the just Time be brought to the Point of the *Suspended Index*.

The Reason of this *Movement* may be thus Explained: 1. Let the Circle LODN, represent any Circular Body, whose Centers both of Gravity and Magnitude are Coincident at M. Let this Circular Body be placed upon some Level Plain G G, and then 'tis Evident that the Angle of its Contact with that Plain at *a*, will also be the Point of its Libration, and consequently it must Rest there: *Quia Momentum & Impedimentum sunt equalia.*

2. Let DE, represent a Descending Plain, making an Angle of Contact with this Circular Body at *b*; and here 'tis manifest it cannot Rest; because the Line of Direction *ra*, which (while it Insisted upon a Level) divided the Circular Body by the Centers of Magnitude and Gravity into Parts *Æquiponderate*, is now removed to LD; which Line LD, falling without, or beside, the Center M, evidently destroys the *Æquipoise* of its Parts, and therefore must leave it to tumble down towards E. For here *Momentum Impedimento majus*. The Reason therefore of its Descent now, being the Over-balance of the Parts LND, to the remaining Section LDO, it must necessarily follow.

3. That if some Weight equal to the Excess of LND, above L O D, were affix'd to the Limb of the Quadrant O *a*, as at P; then the Circular Body would Rest as Quietly at *b*, as it did before at *a*. The Supposition cannot be denied, and the Consequence is unavoidable, because $LDO + P = LND$, *i. e.* *Impedimentum aequatur Momento.*

Let then the Numbers, 1, 2, 3, 4, represent a Train of Wheel-work, wherein there is no material difference from what is found in a common Watch; only the numbers of the Teeth on the Wheels and Pinions are to be so Calculated, that the Motion of the whole Train may correspond to the assigned Revolution of the Body of the *Movement*, which is to be once in 24 Hours: It would be Expedient also, That a *Spiral Spring* were applied to its *Balance*, as in later *Movements*, is usual; but of a *Fusee* here's no need, for the Turns of the Body of the *Movement*, as it descends upon the *Stage*, answer all the Intentions of a String or Chain; and the *Contravalance* of the Weight P, to the excess of LED, above LQD, serves instead of a Perpetual Spring; and the *Movement* wants only a *Perpetual Descent*, to make its *Motion*

Fig. 167.

Fig. 168.

tion so. And whereas the great Wheel in ordinary *Movements*, is placed as near the edge of the Framing Plate *f r*, as it may be; here it must (with its *Axis* or Arbor *M*) possess the Center of the *Movement*: Because this Wheel is to carry the Weight of Power *P*, by the *Vectis* *M P*, and that Weight *P*, must always keep an Equidistance from the Center of the *Movement*; that while the Body thereof (*i. e.* of the *Movement*) performs its Revolutions; the said Weight *P*, and the great Wheel (to which it is affixed) may, without any considerable Variation, continue in, or near the same Position, wherein they now are. Now suppose this Weight *P*, with its *Vectis* *M P*, to be taken quite out of the *Movement*, and the Body of the *Movement* to be placed on a Horizontal Plain *H H*, its Point of Contact in that Plain is *T*; where it should, but cannot, Rest; because the Weight of that part of the Train, marked with the Numbers, 2, 3, 4, Removes the Center of Gravity from *M*, and therefore on the opposite part of the *Movement* as about *C Q*, the Inside of the Hoop, which forms the Case, is to be loaded with a thin Lining of Lead, which may be a Counterpoise to that part of the Train; that so, the whole Body of the *Movement*, together with all its Furniture, within and without (excepting only *P*, with its *Vectis*) may on that Horizontal Plain, or while it Rides upon its own *Axis*, Rest indifferently in any Point. This reducing of the *Movement* to an Equilibration of all its Parts in the Center *M*, must be performed *Tentando, i. e.* by Rasping the Lead at *C Q*, as much and in such Places as is needful; which, to an Artificer of Ordinary Sagacity, will not be at all difficult.

Fig. 170.

Fig. 168.

The Center of Gravity being thus reduced to *M*, replace the Weight *P*, by the Hole *H*, on the Arbor of the Central-Wheel *M*. Then let the Body of the *Movement* be placed on the Declivity *D E*, and supposing $P + L Q D = L D E$, then the Body must needs Rest there: But because the Weight *P*, is not now fix'd to any part of the Quadrant *Q D*, but hangs upon the Train of Wheel-work 1, 2, 3, 4, it evidently follows, That if the Power thereof be Superiour to the Resistance of the Train, then the whole Body of the *Movement* must needs Descend, towards *E*. By this you see there are two Offices assigned to the Weight or Power *P*. The First is, to be a Counterpoise to the excess of the Weight of *L E D*, above *L Q D*. The Second is, that it be of Force sufficient to put the Train into a Motion so Adjusted, as may exactly comport with the Time assigned for the Revolution of the whole Body. So that if there be any Difficulty remaining, it consists in such an exact Stating of the Weight and Power of *P*, that it may Adequately serve both these Intentions. Now how easie this is, will be manifest from these Propositions following.

1. That whatever the Intrinsic Weight of *P* shall be (as suppose it 4 Ounces Troy;) yet the Power of that Weight will be Augmented or Diminished according to the different Degrees of its Elevation in the Quadrant *T Q*. Thus considering *P M*, as a *Vectis*, its *Hypomochlium* is *M*, the Point where it exerts its Power on the Train, is at *V*; I say then, whatever Power it has upon the Point *V*, in its present Elevation of 45 Deg. it will acquire a greater by being raised to 50, 55, &c. and the greatest of all in 90 Deg.

at

at Q: And on the contrary, let it Sink to $40^{\circ} . 35^{\circ} . \&c.$ its Power upon the Point V, will still be Diminished, infomuch that in T, it will be utterly extinguish'd. And therefore if P be of a competent Weight (*i. e.* not utterly too light) to Move the Train at all, it will certainly Move it in some Degree of Elevation or other in the Quadrant Q T.

2. If the Weight P, be considered as to its Office of being a Counterpoise to the Body of the *Movement*; as I need not to prove, that it will perform this no less, while it hangs by upon the *Vellis* M P, than if it were fast Rivetted in the same place to the Case of the *Movement*: so, in what Point of the Quadrant soever it will move the Train, it may be also a Counterpoise to the Body of the *Movement*. For,

1. At what Point soever of the Circle L E T Q, the Line of Declivity D E, makes an Angle of Contact; on the same Point will the Diameter S D, fall at Right Angles with D E.

2. The Line of Direction L D, will ever fall upon the Point of Contact D, making an Angle with the Diameter, as S D L.

3. The Angle S D L, will be always equal to D E H, *i. e.* As great as is the Elevation of the Line of Declivity D E, above the Horizontal E H, so great will the Angle of Distance be between the Diameter S D, and the Line of Direction L D.

Fig. 166.

4. The Greater the Angle of Declivity is, the Less will be the Section L Q D; and so on the contrary, the Less that Angle is, the Greater the Section. And therefore,

5. The Excess of the Weight of L E D, above L Q D, must be also Greater, by Raising up the *Stage* with the Skrew at S: and that Excess Less by Skrewing it down.

6. The Lighter that part of the Body is, which is represented by the Section L Q D, the more Heavy ought the Counterpoise P, to be; and that either in its own Intrinick Weight, (in Ounces and Parts of Ounces) or else in its Potential Weight, by being raised higher in the Quadrant Q T.

7. The Skrewing up the *Stage* of the *Movement* at S, will raise the Counterpoise Higher in the Quadrant Q T, by *Prop.* 3. and therefore Potentially Heavier. And from hence appears (I take it most clearly) both the Reason of the due Adjustment of the Motion of the Train to the exact Measure of an Hour, and what Weight is to be assign'd to P, that Moves it; and that we are not confined to Scruples and Grains, but are allowed such a considerable Latitude, as it is not easie to err therein.

Having therefore set the *Stage* (by the help of the Arched Skrew) at the Elevation of about 10 *Deg.* place the *Movement* thereon, and try what Weight, hanging at the end of the *Vellis* M P, will stir the Train, mean while holding the *Movement* with the Hand in such a Position, as the *Vellis* may make an Angle of about 30 *Deg.* with the Perpendicular M T: then let the *Movement* loose, to Undulate upon the *Stage*; and when the Vibration ceases, observe to what Degree of the Quadrant the *Vellis* Points, and at the same time mind the Pulses of the Ballance. If at this Observation, the Weight lies low, (as for instance, between 25 and 35 *Deg.* of the Quadrant) and the

the Beats of the Ballance are guessed to be not much different from their due time, the Weight P, is well enough proportioned. But if it chance to be much Heavier than is absolutely needful, that Excess will be moderated by Skrewing down the *Stage*; and if it be not absolutely too Light, its Defect will be compensated, by Skrewing the *Stage* Higher. Therefore of these two Extrems, choose the former; for the fewer Degrees that P arises in the Quadrant, beyond what is absolutely necessary, it will (for Reason very obvious) be so much the better.

The Effects of Gravity in the Descent of Heavy Bodies, and the Motion of Projects; by Mr. Halley.
n. 179. p. 3.
Jau. An. 1686.

VIII. *Des Cartes* his Notion, I must needs confess to be to me Incomprehensible, while he will have the Particles of his *Celestial Matter*, by being reflected on the Surface of the Earth, and so ascending therefrom, to drive Down into their Places those Terrestrial Bodies they find above them: This is as near as I can gather, the Scope of the 20, 21, 22, and 23 *Sections* of the last Book of his *Principia Philosophia*; yet neither he nor any of his Followers can shew, how a Body suspended in *Libero Aethere*, shall be carried downwards by a continual Impulse tending upwards, and acting upon all its Parts equally: And besides, the Obscurity wherewith he expresses himself, particularly, *Sect.* 23. does sufficiently argue, according to his own Rules, the *Confused Idea* he had of the thing he wrote.

Others, and among them, Dr. *Vossius*, asserts the Cause of the Descent of Heavy Bodies, to be the *Diurnal Rotation* of the *Earth* upon its *Axis*, without considering, that according to the Doctrine of Motion fortified with Demonstration, all Bodies moved in *Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all loose Bodies would be cast into the Air in a Tangent to the Parallel of Latitude. without the Intervention of some other Principle to keep them fast, such as is that of *Gravity*. Besides the Effect of this Principle is throughout the whole Surface of the Globe found nearly equal, and certain Experiment seems to argue it rather less near the Equinoctial, than towards the Poles, which could not be by any means, if the *Diurnal Rotation* of the *Earth* upon its *Axis* were the Cause of *Gravity*; for where the Motion was swiftest, the Effect would be most considerable.

Others assign the *Pressure* of the *Atmosphere* to be the Cause of this *Tendency* towards the *Center* of the *Earth*; but unhappily they have mistaken the Cause for the Effect, it being from undoubted Principles plain, that the *Atmosphere* has no other *Pressure*, but what it derives from it *Gravity*; and that the Weight of the upper parts of the *Air*, pressing on the lower parts thereof, do so far bend the Springs of that *Elastick Body*, as to give it a Force equal to the Weight that compressed it, having of it self no Force at all: And supposing it had, it will be very hard to Explain the *Modus*, how that *Pressure* should occasion the *Descent* of a Body circumscribed by it, and pressed equally above and below, without some other Force to *Draw* or *Thrust* it Downwards. But to Demonstrate the contrary of this Opinion, an Experiment was long since shewn before the *Royal Society*; whereby it appeared, that the *Atmosphere* was so far from being the Cause of *Gravity*, that the

Effects

Effects thereof were much more vigorous where the Pressure of the Atmosphere was taken off; for a long Glass-Receiver having a light Down-Feather included, being *Evacuated* of Air, the Feather which in the Air would hardly sink, did *in Vacuo* descend with nearly the same Velocity as if it had been a Stone.

Some think to Illustrate this Descent of *Heavy* Bodies, by comparing it with the Virtue of the *Loadstone*; but setting aside the Difference there is in the manner of their Attractions, the *Loadstone* drawing only in and about its Poles, and the Earth near equally in all Parts of its Surface, this Comparison avails no more than to explain *Ignotum per aquæ Ignotum*.

Others assign a certain *Sympathetical Attraction* between the Earth and its Parts, whereby they have, as it were, a Desire to be united, to be the Cause we enquire after: But this is so far from explaining the *Modus*, that it is little more, than to tell us in other terms, that *Heavy* Bodies Descend, because they Descend.

But though the *Efficient Cause* of Gravity be so obscure, yet the *Final Cause* thereof is clear enough; for it is by this Single Principle that the Earth and all the Celestial Bodies are kept from Dissolution: the least of their Particles not being suffered to recede far from their Surfaces, without being immediately brought down again by Virtue of this *Natural Tendency*, which, for their Preservation, the Infinite Wisdom of their Creator has ordained to be towards each of their Centers; nor can the Globes of the *Sun* and *Planets* otherwise be destroyed, but by taking from them this Power of keeping their Parts united.

The Affections or *Properties* of Gravity, and its manner of Acting upon Bodies Falling, have been in a great measure discovered, and most of them made out by Mathematical Demonstration in this our Century, by the accurate Diligence of *Galilæus*, *Torricellius*, *Hugenius*, and others; and now lately, by our worthy Countryman Mr. *Is. Newton*. Which *Properties* I shall here enumerate.

1. The First *Property* is, That by this *Principle* of Gravitation, all Bodies do Descend towards a *Point*, which either is, or else is very near to, the Center of *Magnitude* of the Earth and Sea, about which the Sea forms it self exactly into a Spherical Surface, and the Prominences of the Land, considering the Bulk of the whole, differ but insensibly therefrom.

2. That this *Point*, or Center of Gravitation, is *Fix'd* within the Earth, or at least has been so, ever since we have any Authentick History: For a Consequence of its Change, tho' never so little, would be the over-flowing of the Low-lands on that side of the Globe towards which it approached, and the leaving new Islands bare on the opposite side, from which it receded; but for this Two Thousand years it appears, that the Low Islands of the *Mediterranean* Sea (near to which the Ancientest Writers lived) have continued much at the same Height above the Water, as they now are found; and no Inundations or Recesses of the Sea arguing any such Change, are Recorded in History, excepting the *Universal Deluge*, which can no better way be accounted for, than by supposing this Center of Gravitation Removed for a time, towards the Middle of the then Inhabited Parts of the World; and a Change

of its place, but the two thousandth Part of the Radius of this Globe, were sufficient to bury the Tops of the Highest Hills under Water.

3. That in all Parts of the Surface of the Earth, or rather in all Points equidistant from its Center, the *Force of Gravity* is nearly *Equal*; so that the length of the Pendulum Vibrating Seconds of Time, is found in all Parts of the World to be very near the same. 'Tis true, at *St. Helena*, in the Latitude of 16 Deg. South, I found that the Pendulum of my Clock, which Vibrated Seconds, needed to be made shorter than it had been in *England*, by a very Sensible Space (but which at that time I neglected to observe accurately) before it would keep Time; and since the like Observations have been made by the *French Observers* near the Equinoctial: Yet I dare not affirm, that in mine it proceeded from any other Cause, than the great Height of my Place of Observation above the Surface of the Sea, whereby the *Gravity* being diminished, the Length of the Pendulum, Vibrating Seconds, is proportionably shortned.

4. That *Gravity* does *Equally Affect* all *Bodies*, without regard either to their Matter, Bulk, or Figure; so that the Impediment of the Medium being removed, the most Compact and most Loose, the Greatest and Smallest Bodies would Descend the same Spaces in Equal Times; the truth whereof will appear from the Experiment I before cited. In these two last Particulars, is shewn, the great Difference between *Gravity* and *Magnetism*, the one affecting only Iron, and that towards its Poles, the other all Bodies alike in every part. As a *Corollary*, from hence it will follow, That there is no such thing as *Positive Levity*, those things that appear *Light*, being only comparatively so; and whereas several things Rise and Swim in Fluids, tis because Bulk for Bulk, they are not so Heavy as those Fluids; nor is there any Reason, why *Cork*, for instance, should be said to be *Light* because it *Swims* on *Water*, any more than *Iron*, because it *Swims* on *Mercury*.

5. That this *Power Increases* as you *Descend* to, and *Decreases* as you *Ascend* from the *Center*, and that in the Proportion of the *Squares* of the *Distances* therefrom *Reciprocally*, so as at a double Distance to have but a quarter of the Force: This Property is the Principle on which Mr. *Newton* has made out all the Phænomena of the Celestial Motions, so easily and naturally, that its Truth is past dispute. Besides that, it is highly rational, that the *Attractive* or *Gravitating Power* should exert it self more vigorously in a Small Sphere, and weaker in a Greater, in proportion as it is Contracted or Expanded; and if so, seeing that the Surfaces of Spheres are as the Squares of their Radii, this Power at several Distances will be as the Squares of those Distances *Reciprocally*, and then its whole Action upon each Spherical Surface, be it great or small, will be always Equal. And this is evidently the Rule of *Gravitation* towards the Centers of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, and thence reasonably inferred, to be the General Principle observed by Nature in all the rest of the *Celestial Bodies*.

These are the Principal Affections of *Gravity*, from which the Rules of the *Fall* of *Bodies*, and the *Motion* of *Projects*, are Mathematically deducible. Mr. *J. Newton* hath shewed how to define the Spaces of the Descent of a Body,

Body, let fall from any given Height, down to the Center, supposing the *Gravitation* to Increase, as in the Fifth Property ; but considering the smallness of Height to which any *Project* can be made Ascend and over how little an Arch of the Globe it can be Cast by any of our Engines, we may well enough suppose the *Gravity* Equal throughout, and the *Descents* of *Projects* in *Parallel Lines*, which, in Truth, are towards the Center, the Difference being so small as by no means to be discovered in Practice.

Prop. I.] *The Velocities of Falling Bodies, are Proportionate to the Times from the Beginning of their Falls.*

This follows, for that the Action of Gravity being continual, in every space of Time, the Falling Body receives a new Impulse, Equal to what it had before in the same Space of Time, received from the same Power : For instance, in the First Second of Time, the Falling Body has acquired a Velocity, which in that Time would carry it to a certain Distance, suppose 32 Foot, and were there no new Force, would Descend at that Rate with an Equable Motion ; but in the next Second of Time, the same Power of Gravity continually Acting thereon, superadds a New Velocity equal to the former ; so that at the end of two Seconds, the Velocity is double to what it was at the end of the First ; and after the same manner may it be proved to be Triple, at the end of the Third Second, and so on. Wherefore the *Velocities of Falling Bodies, are proportionate to the Times of their Falls.* Q. E. D.

Propositions concerning the Descent of Heavy Bodies, and the Motion of Projects.
Ibid. p. 9.

Prop. II.] *The Spaces described by the Fall of a Body, are as the Squares of the Times from the Beginning of the Fall.*

Demonstration. Let AB represent the Time of the Fall of a Body, BC , perpendicular to AB , the Velocity acquired at the End of the Fall, and draw the Line AC ; then Divide the Line AB , representing the Time, into as many equal Parts as you please, as $b, b, b, b, \&c.$ and through these Points draw the Lines, $bc, bc, bc, bc, \&c.$ parallel to BC ; 'tis manifest that the several Lines, bc , represent the several Velocities of the Falling Body, in such parts of the Time, as Ab , is of AB , by the Former Proposition. It is evident likewise, that the Area, ABC , is the Sum of all the Lines bc , being taken according to the Method of Indivisibles infinitely many ; so that the Area ABC , represents the Sum of all the Velocities between none and BC , supposed infinitely many ; which Sum is the Space Descended in the Time represented by AB . And by the same Reason, the Area's Abc , will represent the Spaces Descended in the Times Ab ; so then the Spaces Descended in the Times AB, Ab , are as the Area's of the Triangle, ABC, Abc , which by the 20th of the 16th of Euclid are as the Squares of their Homologous Sides AB, Ab , that is to say, of the Times : Wherefore the *Descents of Falling Bodies, are as the Squares of the Times of their Fall.* Q. E. D.

Fig. 171.

Prop. III.] *The Velocity which a Falling Body acquires in any Space of Time, is double to that, wherewith it would have moved the Space Descended by an Equable Motion, in the same Time.*

Demonstration.] Draw the Line EC Parallel to AB , and AE Parallel to BC , and compleat the *Parallelogram* $ABCE$, it is evident that the *Area* thereof may represent the Space, a Body Moved *Equably* with the *Velocity* BC , would Describe in the Time AB ; and the *Triangle* ABC , represents the Space Described by the Fall of a Body, in the same Time AB , by the *Second Proposition*. Now the *Triangle* ABC , is Half of the *Parallelogram* $ABCE$, and consequently the Space described by the *Fall*, is Half what would have been described by an *Equable Motion* with the *Velocity* BC , in the same Time; wherefore the *Velocity* BC , at the End of the Fall, is *Double* to that *Velocity*, which in the Time AB , would have described the Space Fallen, represented by the *Triangle* ABC , with an *Equable Motion*. $Q.E.D.$

Prop. IV.] *All Bodies on or near the Surface of the Earth, in their Fall, Descend so, as at the End of the First Second of Time, they have described 16 Feet, one Inch, London Measure, and acquired the Velocity of 32 Feet, two Inches, in a Second.*

This is made out from the 25th, *Prop. Par. 2. Horol. Oscill. Hugen.* wherein he Demonstrates the Time of the least Vibrations of a *Pendulum*, to be to the Time of the Fall of a Body, from the Height of Half the Length of the *Pendulum*, as the Circumference of a Circle to its Diameter: Whence, as a *Corollary*, it follows, That as the Square of the Diameter to the Square of the Circumference, so half the Length of the *Pendulum* Vibrating Seconds, to the Space described by the Fall of a Body in a Second of Time: and the Length of the *Pendulum* Vibrating Seconds, being found 39, 125, or $\frac{1}{4}$ Inches, the Descent in a Second will be found, by the *forefaid Analogy*, 16 Foot and one Inch: And by the *Third Proposition*, the *Velocity* will be double thereto; and near to this it hath been found by several Experiments which by reason of the *Swiftnefs* of the Fall, cannot so exactly determine its Quantity.

From these Four Propositions, all Questions concerning the *Perpendicular Fall* of Bodies are easily solved, and either *Time*, *Height*, or *Velocity* being assigned, one may readily find the other two. From them likewise is the *Doctrine of Projects* deducible, assuming the two following *Axioms*; viz.

1. That a Body set a Moving, will Move on continually in a Right Line with an *Equable Motion*, unless some other Force or Impediment intervene, whereby it is Accelerated, or Retarded, or Deflected.

2. That a Body being agitated by two Motions at a Time, does by their Compounded Forces pass through the same Points, as it would do, were the two Motions divided and acted successively. As for instance.

Fig. 172.

Suppose a Body moved in the Line GF , from G to R , and there stopping, by another Impulse suppose it Moved in a Space of Time equal to the former from R towards K to V ; I say the Body shall pass through the point V , though these two several Forces Acted both in the same Time.

Prop. V.

Prop. V.] *The Motion of all Projects is in the Curve of a Parabola.*

Demonstration.] Let the Line GRF be the Line in which the Project is directed, and in which by the *First Axiom* it would Move Equal Spaces in Equal Times, were it not Deflected downwards by the Force of Gravity. Let GB be the Horizontal Line, and GC a Perpendicular thereto. Then the Line GRF, being divided into Equal Parts, answering to Equal Spaces of Time, let the Descents of the Project be laid down in Lines Parallel to GC, Proportioned as the Squares of the Lines, GS, GR, GL, GF, or as the Squares of the Times, from S to T, from R to V, from L to X, and from F to B, and draw the Lines TH, VD, XY, BC, Parallel to GF: I say, the Points T, V, X, B, are Points in the Curve described by the Project, and that that Curve is a Parabola. By the *Second Axiom* they are Points in the Curve; and the Parts of the Descent GH, GD, GY, GC, = to ST, RV, LX, FB, being as the Squares of the Times (by the *Second Prop.*) that is, as the Squares of the Ordinates HT, DV, YX, CB, Equal to GS, GR, GL, GF, the Spaces measured in those Times; and there being no other Curve but the Parabola, whose Parts of the Diameter are as the Squares of the Ordinates, it follows that the Curve described by a Project can be no other than a Parabola: And saying, as RV, the Descent in Time, to GR, or VD, the direct Motion in the same Time, so is VD, to a Third Proportional; that Third will be the Line called by all Writers of Conicks, the Parameter of the Parabola to the Diameter GC; which is always the same in Projects Cast with the same Velocity: and the Velocity being defined by the number of Feet, moved in a Second of Time, the Parameter will be found by dividing the Square of the Velocity by 16 Feet, 1 Inch, the Fall of a Body in the same Time.

Lemma.] *The Sine of the Double of any Arch, is equal to twice the Sine of that Arch into its Co-Sine, divided by Radius; and the Versed Sine of the Double of any Arch, is equal to the Square of the Sine thereof divided by Radius.*

Let the Arch BC be Double the Arch BF, and A the Center; Draw the Radii AB, AF, AC, and the Chord BDC, and let Fall BE, Perpendicular to AC, and the Angle EBC, will be equal to the Angle BAD, and the Triangle BCE, will be like the Triangle ABD; wherefore it will be as AB to AD; so BC, or twice BD, to BE; that is, as Radius to Co-Sine, so twice Sine to Sine of the Double Arch; and as AB to BD, so twice BD or BC, to EC; that is, as Radius to Sine, so twice that Sine to the Versed Sine of the Double Arch; which two Analogies resolved into Equations, are the Propositions contained in the Lemma to be proved.

Fig. 173:

Prop. VI.] *The Horizontal Distance of Projections made with the same Velocity, at several Elevations of the Line of Direction, are as the Sines of the doubled Angles of Elevation.*

Let

Fig. 172.

Let GB, the Horizontal Distance be $= z$, the Sine of the Angle of Elevation, FGB, be $= s$, its Co-Sine $= c$, Radius $= r$, and the Parameter $= p$. It will be as c to s , so z to $\frac{s z}{c} = FB = GC$, and by reason of the Parabola $\frac{p s z}{c} =$ to the Square of CB, or GF. Now as c to r , so is z to $\frac{z r}{c} = GF$, and its Square $\frac{z z r r}{c c}$ will be therefore $=$ to $\frac{p s z}{c}$: which Equation Reduced, will be $\frac{p s c}{r r} = z$. But by the former

Lemma $\frac{2 s c}{r}$ is equal to the Sine of the Double Angle, whereof s is the Sine: Wherefore 'twill be as Radius to Sine of Double the Angle FGB, so is Half the Parameter, to the Horizontal Range or Distance sought; and at the several Elevations, the Ranges are as the Sines of the Double Angles of Elevation. Q. E. D.

Coroll.] Hence it follows, That Half the Parameter is the greatest Range, and that that happens at the Elevation of 45° . The Sine of whose Double is Radius. Likewise, That the Ranges equally Distant above and below 45° . are equal, as are the Sines of all doubled Arches, to the Sines of their doubled Complements.

Prop. VII.] The Altitudes of Projections made with the same Velocity, at several Elevations, are as the Versed Sines of the doubled Angles of Elevation.

As c is to s , so is $\frac{p s c}{r r} = GB$, to $\frac{p s s}{r r} = BF$, and $VK = RV$

$= \frac{1}{4} BF$, the Altitude of the Projection $= \frac{p s s}{4 r r}$. Now by the foregoing

Lemma $\frac{2 s s}{r} =$ to the Versed Sine of the Double Angle, and therefore it will be as Radius to Versed Sine of Double the Angle FGB, so $\frac{1}{4}$ of Parameter to the Height of the Projection VK; and so those Heights at several Elevations are as the said Versed Sines. Q. E. D.

Coroll.] From hence it is plain, That the greatest Altitude of the Perpendicular Projection is a 4th of the Parameter, or half the greatest Horizontal Range: The Versed Sine of 180 Degrees being $= 2r$.

Prop. VIII.] The Lines GF, or Times of the Flight of a Project cast with the same Degree of Velocity at Different Elevations, are as the Sines of the Elevations.

As

As c is to r , so is $\frac{p s^2}{r r} = G B$ (by the 6 Prop.) to $\frac{p s^2}{r} = G F$, that is

as Radius to the Sine of Elevation, so the Parameter to the Line $G F$; so the Lines $G F$ are as the Sines of Elevation, and the Times are proportional to those Lines; wherefore the Times are as the Sines of Elevation: Ergo constat Propositio.

Prop. IX. Prob. 1.] A Projection being made, as you please, having the Distance and Altitude, or Descent of an Object, through which the Project passes, together with the Angle of Elevation of the Line of Direction, to find the Parameter and Velocity; that is, (having the Angle $F G B$), $G M$, and $M X$.

Solution.] As Radius to Secant of $F G B$, so $G M$ the Distance given, to $G L$; and as Radius to Tangent of $F G B$, so $G M$ to $L M$. Then $L M - M X$ in Heights, or $+ M X$ in Descents; or else $M X - M L$, if the Direction be Below the Horizontal-Line, is the Fall in the Time that the Direct Impulse given in G , would have carried the Project from G to $L = L X = G Y$; then by reason of the Parabola; as $L X$, or $G Y$, is to $G L$ or $Y X$; so is $G L$, to the Parameter sought. To find the Velocity of the Impulse, by Prop. 2. and 4. find the Time in Seconds that a Body would Fall the Space $L X$, and by that dividing the Line $G L$, the Quote will be the Velocity, or Space Moved in a Second Sought, which is always a mean Proportional between the Parameter and 16 Feet, 1 Inch.

Fig 172

Prop. X. Prob. 2.] Having the Parameter, Horizontal Distance, and Height or Descent of an Object, to find the Elevations of the Lines of Direction necessary to Hit the given Object; that is, having $G M$, $M X$, and the greatest Random equal to half the Parameter; to find the Angles $F G B$.

Let the Tangent of the Angle sought be $= t$, the Horizontal Distance $G M = b$, the Altitude of the Object $M X = h$, the Parameter $= p$, and

Radius $= r$, and it will be, as t to t , so b to $\frac{t b}{r} = M L$; and $\frac{t b}{r} + h$

$\left\{ \begin{array}{l} \text{in Ascents} \\ \text{in Descents} \end{array} \right\} = L X$, and $\frac{p t b}{r} + p b = G L q. = X Y q. \text{ ratione Pa-}$

rabola; but $b b + \frac{t t b b}{r r} = G L q. (47. 1. Euclid.)$ Wherefore $\frac{p t b}{r}$

$+ p b = b b + \frac{t t b b}{r r}$; which Equation transposed, is $\frac{t t b b}{r r} = \frac{p t b}{r}$

$+ p b - b b$; divided by $b b$, is $\frac{t t}{r r} = \frac{p t}{b r} + \frac{p b}{b b} - 1$. This Equati-

on shews the Question to have two Answers, and the Roots thereof are $\frac{t}{r}$

=

$$= \frac{p}{2b} + \frac{\sqrt{pp + 4ph}}{4bb} - 1; \text{ from which I derive the following Rule.}$$

Divide half the *Parameter* by the *Horizontal Distance*, and keep the *Quote*, viz. $\frac{p}{2b}$; then say, as Square of the *Distance* given to half the *Parameter*,

so half *Parameter* $\left\{ \begin{array}{c} - \\ + \end{array} \right\}$ double $\left\{ \begin{array}{c} \text{Height} \\ \text{Descent} \end{array} \right\}$ to the Square of a *Secant* =

$$\frac{pp + 4ph}{4bb}, \text{ the Tangent answering to that Secant will be } \frac{\sqrt{pp + 4ph}}{4bb} - 1,$$

or rr : so then the Sum and Difference of the afore-found *Quote* and this *Tangent*, will be the Root of the *Equation*, and the *Tangents* of the *Elevations* sought.

Note here, That in *Descents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb , the *Direction* of the Lower *Elevation* will be below the *Horizon*, and if $ph = bb$, it must be *Directed Horizontal*, and the

Tangent of the upper *Elevation* will be $\frac{pr}{b}$: Note likewise, That if $4bb + 4ph$ in *Ascents*, or $4bb - 4ph$ in *Descents*, be equal to pp , there

is but one *Elevation* that can Hit the *Object*, and its *Tangent* is $\frac{pr}{2b}$; and if

$4bb + 4ph$ in *Ascents*, or $4bb - 4ph$ in *Descents*, do exceed pp , the *Object* is without the *Reach* of a *Project* cast with that *Velocity*, and so the thing impossible.

From this *Equation* $4bb + 4ph = pp$, are determined the utmost *Limits* of the *Reach* of any *Project*, and the Figure assigned, wherein are all the *Heights* upon each *Horizontal Distance* beyond which it cannot pass;

for by Reduction of that *Equation*, b will be found $= \frac{1}{2}p - \frac{bb}{p}$

in *Heights*, and $\frac{bb}{p} - \frac{1}{2}p$ in *Descents*; from whence it follows, that all the

Points b are in the *Curve* of the *Parabola*, whose *Focus* is the Point from whence the *Project* is cast, and whose *Latus Rectum*, or *Parameter ad Axem* is $= p$. Likewise from the same *Equation* may the least *Parameter* or *Velocity*

be found capable to *Reach* the *Object* proposed; for $bb = \frac{1}{2}pp + ph$ being reduced, $\frac{1}{2}p$ will be $= \sqrt{bb + h} \left\{ \begin{array}{c} + b \text{ in } \textit{Ascents} \\ - b \text{ in } \textit{Descents} \end{array} \right\}$ which is the

Horizontal Range at 45° . that would just *Reach* the *Object*, and the *Elevation* requi-

require will be easily had; for dividing the so found *Semi-Parameter* by the *Horizontal Distance* given b , the Quote into *Radim* will be the *Tangent* of the *Elevation* sought.

But if a *Geometrical Construction* of this *Problem* be required; I think I have one, that is as easie as any can be expected, which I Deduce from the fore-

going *Analytical Solution*, viz. $\frac{r}{b} = \frac{p}{2b} + \frac{\sqrt{\frac{1}{4}pp + pb - b^2}}{b}$,

and 'tis this; Having made the Right Angle LDA, make DA, DF = p , or greatest *Range*, DG = b , the *Horizontal Distance*, and DB, DC = b , the *Perpendicular Height* of the Object; and draw GB, and make DE = thereto. Then with the Radius AC, and Center E, sweep an Arch, which if the thing be possible, will intersect the Line AD, in H; and the Line DH, being laid both ways from F, will give the Points K, and L, to which draw the Lines GL, GK; I say, the Angles LGD, KGD, are the *Elevations* required for *Hitting* the Object, B. But Note, That if B, be below the *Horizon*, its *Descent* DC = DB, must be laid upon A, so as to have AC = to AD + DC. Note likewise, That if in *Descents*, DH, be greater than FD, and so K, fall below D, the Angle KGD, shall be the *Depression* below the *Horizon*.

Fig. 174.

When I gave the preceding Solution of this Problem, viz. *To Hit an Object above or below the Horizontal Line, with the greatest Certainty and least Force*, I was not aware, that the *Elevation* there sought did constantly *Bisect* the Angle between the *Perpendicular* and the Object, as is Demonstrated from the Difference and Sum of the *Tangent* and *Secant* of any Arch, being always equal to the *Tangent* and *Co-Tangent* of the half Complement thereof to a Quadrant. But having discover'd this, I think nothing can be more compendious, or bid fairer to compleat the *Art of Gunnery*, it being as easie to Shoot with a *Mortar* at any Object on demand, as if it were on the Level; neither is there need of any Computation, but only simply laying the Gun, to pass in the middle Line between the *Zenith* and the Object, and giving it its due Charge. Nor is there any great need of Instruments for this purpose: For, if the *Muzzle* of the *Mortar* be turn'd truly Square to the Bore of the Piece, as it usually is, or ought to be, a piece of Looking-Glass Plate applied Parallel to the *Muzzle*, will, by its Reflection, give the true Position of the Piece; the *Bombardier* having no more to do, but to look Perpendicularly down on the Looking-Glass, along a small Thread with a Plumber, and to Raise or Depress the *Elevation* of the Piece, 'till the Object appear Reflected on the same Point of the *Speculum* on which the Plumber falls; for the Angle of Incidence and Reflection being Equal, in this case a Line at Right Angles to the *Speculum*, as is the *Axis* of the Chase of the Piece, will *Bisect* the Angle between the *Perpendicular* and the Object, according as our *Proposition* requires.

n. 216. p. 69.
Mar. An. 1695.

Prop. XI Prob. 3.] *A Shot being made on an Inclined Plain, having the Horizontal Distance of the Object it strikes, with the Elevation of the Piece, and the Angle at the Gun between the Object and the Perpendicular, to find the greatest Horizontal Range of that Piece, laden with the same Charge; that is, half the Latus Rectum of all the Parabolæ made with the same Impetus.*

Take half the Distance of the Object from the Nadir, and take the Difference of the given Elevation from that Half; the *Versed Sine* of twice that Difference Subtract from the *Versed Sine* of the Distance of the Object from the Zenith: Then shall the Difference of those *Versed Sines* be to the Sine of the Distance of the Object from the Zenith, as the Horizontal Distance of the Object struck to the greatest Horizontal Range at 45° .

Prop. XII. Prob. 4.] *Having the greatest Horizontal Range of a Gun, the Horizontal Distance and Angle of Inclination of an Object to the Perpendicular, to find the two Elevations necessary to strike that Object.*

Halve the Distance of the Object from the Nadir, this Half is always equal to the Half Sum of the two Elevations we seek. Then say; As the Greatest Horizontal Range, is to the Horizontal Distance of the Object; so is the Sine of the Angle of Inclination, or Distance of the Object from the Perpendicular, to a 4th Proportional; which 4th being Subtracted from the *Versed Sine* of the Distance of the Object from the Zenith, leaves the *Versed Sine* of the Difference of the Elevations sought; which Elevations are therefore had, by Adding and Subtracting that Half Difference to, and from, the aforesaid Half Sum.

n. 172. p. 18.

Prop. XIII.] *To determine the Force or Velocity of a Project, in every Point of the Curve it describes.*

To do this, we need no other *Præcognita*, but only the Third Proposition, viz. That the Velocity of Falling Bodies, is double to that which in the same Time would have described the Space fallen by an Equable Motion: For the Velocity of a Project is compounded of the constant equal Velocity of the impressed Motion, and the Velocity of the Fall, under a given Angle, viz. The Complement of the Elevation: For instance, In the Time wherein a Project would Move from G to L, it Descends from L to X, and by the Third Proposition has Acquired a Velocity, which in that Time would have carried it by an Equable Motion from L to Z, or twice the Descent L X; and drawing the Line G Z, I say the Velocity in the Point X, compounded of the Velocities G L, and L Z, under the Angle G L Z, is to the Velocity impressed in the Point G, as G Z is to G L; this follows from our Second Axiom; and by the 20th and 21st Prop. Lib. 1, Conic. Mordogii, X O, Parallel and Equal to G Z, shall touch the Parabola in the Point X. So that the Velocities in the several Points, are as the Lengths of the Tangents to the Parabola in those Points, intercepted between any two Diameters: And these again are as the Secants of the Angles, which those Tangents continued make with the Horizontal Line G B. From what is here laid down, may the comparative Force

Fig. 172.

of a Shot in any two Points of the Curve, be either Geometrically or Arithmetically discovered.

Coroll.) From hence it follows, That the force of a Shot is always least at V, or the *Vertex* of the *Parabola*, and that at equal Distances therefrom, as at T and X, G, and B, its force is always Equal, and that the least Force in V, is to that in G and B, as *Radius* to the *Secant* of the Angle of *Elevation*, FGB.

The *Tenth Proposition* contains a *Problem*, untouched by *Torricellius*, which is of the Greatest use in Gunnery, and for the sake of which this Discourse was principally intended. It was first Solved by Mr. *Anderson*, in his Book of the *Genuine Use and Effects of the Gun*, Printed in the Year 1674. but his *Solution* required so much Calculation, that it put me upon Search, whether it might not be done more easily; and thereupon in the Year 1678. I found out the *Rule* I now publish, and from it the Geometrical Construction: Since which time, there has a large Treatise of this Subject, Intitled, *L' Art de Jetter les Bombes*, been published in *France* by M. *Blondel*, wherein he gives the *Solutions* of this *Problem*, by Messieurs *Bout*, *Romer*, and *de la Hire*; but none of them are the same with mine, or in my Opinion more easie.

It was formerly the Opinion of those concerned in *Artillery*, That there n. 216. p. 63. was a certain Requisite of *Powder* for each *Gun*, and that in *Mortars*, where their Distance was to be varied, it must be done by giving a greater or lesser *Elevation* to the *Piece*. But now our Latter Experience has taught us, That the same thing may be more Certainly and Readily performed, by *Increasing and Diminishing the Quantity of Powder*, whether regard be had to the Execution to be done, or to the Charge of doing it. For when *Bombs* are Discharged with great *Elevations* of the *Mortar*, they fall too Perpendicular, and Bury themselves too deep in the Ground, to do all that Damage they might, if they came more Oblique, and Broke upon or near the Surface of the Earth; which is a thing acknowledged by the *Besieged* in all *Towns*, who Unpave their Streets to let the *Bombs* Bury themselves, and thereby stifle the force of their Splinters. A *Second Convenience* is, That at the Extream *Elevation*, the *Gunner* is not obliged to be so curious in the Direction of his *Piece*, but it will suffice to be within a Degree or two of the Truth; whereas in the other Method of *Shooting*, he ought to be very curious. But a *Third* and no less considerable *Advantage* is, in the saving of the *King's Powder*, which in so great and so numerous *Discharges*, as we have lately seen, must needs amount to a considerable Value. And for *Sea Mortars* it is scarce Practicable otherwise to use them, where the *Agitation* of the Sea continually Changes the *Direction* of the *Mortar*, and would render the Shot very uncertain, where it not that they are placed about 45°. *Elevation*, where several Degrees above or under makes very little Difference in the *Effect*.

It only remains by Good and Valid *Experiments* to be assured of the Force Ibid. p. 70. of *Gun-Powder*; How to make and conserve it equal; And to know the Effect thereof in each *Piece*; that is, How far differing *Charges* will cast the same Shot out of it; which may most conveniently be Engraven on the outside thereof, as a standing Direction to all *Gunners*, who shall from thence forward

forward have occasion to use that *Piece*: And where this Matter well ascertained, it might be worth the while to make all *Mortars* of the like *Diameter*, as near as may be alike in *Length* of *Chase*, *Weight*, *Chamber*, and all other *Circumstances*.

n. 179, p. 19.

Now the foregoing Rules would be Rigidly true, were it not for the *Opposition* of the *Medium*, whereby not only the Direct Impressed Motion is continually Retarded, but likewise the Increase of the Velocity of the Fall, so that the Spaces described thereby, are not exactly as the *Squares* of the *Times*: But what this *Opposition* of the *Air* is, against several *Velocities*, *Bulks*, and *Weights*, is not so easy to determine. 'Tis certain, That the *Weight* of the *Air* to that of *Water*, is nearly as 1 to 800, whence the *Weight* thereof, to that of any *Project* is given; 'tis very likely, that to the same *Velocity* and *Magnitude*, but of Different Matter, the *Opposition* should be reciprocally as the *Weights* of the *Shot*; as likewise that to *Shot* of the same *Velocity* and *Matter*, but of Different *Sizes*, it should be as the *Diameters* reciprocally: Whence generally the *Opposition* to *Shot* with the same *Velocity*, but of differing *Diameters*, and *Materials*, should be as their *Specifick Gravities* into their *Diameters* reciprocally; but whether the *Opposition*, to differing *Velocities* of the same *Shot*, be as the *Squares* of those *Velocities*, or as the *Velocities* themselves, or otherwise, is yet a harder Question. However it be, 'tis certain, That in large *Shot* of *Metal*, whose *Weight* many thousand times surpasses that of the *Air*, and whose *Force* is very great in Proportion to the *Surface* wherewith they press thereon, this *Opposition* is scarce *Discernible*: For by several *Experiments*, made with all Care and Circumspection with a *Mortar-Piece*, extraordinary well fixed to the *Earth* on purpose, which carried a Solid *Brass Shot* of $4\frac{1}{2}$ Inches *Diameter*, and of about 14 Pound weight, the *Ranges* above and below 45° . were found nearly equal; if there were any Difference, the under *Ranges* went rather the farthest, but those Differences were usually less than the Errors committed in Ordinary Practice, by the unequal goodness and dryness of the same Sort of *Powder*, by the Unfiness of the *Shot* to the *Bore*, and by the Looseness of the *Carriage*. In a smaller *Brass-Shot* of about an Inch and half *Diameter*, cast by a *Cross-Bow*, which Ranged it at most about 400 Foot, the *Force* being much more equal than the *Mortar-Piece*, this Difference was found more curiously, and constantly, and most evidently, the under *Ranges* outwent the upper. From which Tryals I conclude, That altho' in Small and Light *Shot*, the *Opposition* of the *Air*, ought and must be accounted for; yet in Shooting of Great and Weighty *Bombs*, there need be very little or no Allowance made; and so these Rules may be put in Practice to all Intents and Purposes, as if this *Impediment* were absolutely removed.

The Measure of
the Air's Resistance
to Bodies,
moved in it; by
Dr. Wallis.
n. 186. p. 269.
Jan. An. 1687.

IX. 1. In order to compute the *Resistance* of the *Air* to all *Projects*, I first Premise this *Lemma* (as the most Rational that doth occur, for my first footing.) That (supposing other things equal) the *Resistance* is proportional to the *Celerity*. For in a double *Celerity*, there is to be removed (in the same time) twice as much *Air* (which is a double *Impediment*;) in a treble, thrice as much; and so in other Proportions.

2. Sup.

2. Suppose we then the Force Impressed (and consequently the *Celerity*, if there were no *Resistance*) as 1; the *Resistance* as r (which must be less than the Force, or else the Force would not prevail over the Impediment, to create a *Motion*.) And therefore the Effective Force at a first *Moment*, is to be reputed as $1 - r$: That is, so much as the Force Impressed, is more than the *Impediment* or *Resistance*.

3. Be it as $1 - r$ to 1, so 1 to m (which m is therefore greater than 1.)

4. And therefore the effective Force (and consequently the *Celerity*) as to a first *Moment*, is to be $\frac{1}{m}$ of what it would be, had there been no *Resistance*.

5. This $\frac{1}{m}$ is also the remaining Force after such first *Moment*; and this remaining Force is (for the same Reason) to be proportionably abated as to a Second *Moment*: That is, we are to take $\frac{1}{m}$ thereof, that is, $\frac{1}{mm}$ of the Impressed Force. And for a third *Moment* (at equal Distance of Time) $\frac{1}{mmm}$; for a fourth $\frac{1}{m^4}$; and so onward infinitely.

6. Because the Length dispatched (in Equal Times) is proportional to the *Celerities*; the Lines of Motion (answering to those Equal Times) are to be as $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, $\frac{1}{m^4}$, &c. of what they would have been in the same Times, had there been no *Resistance*.

7. This therefore is a *Geometrical Progression*; and (because of m greater than 1) continually Decreasing.

8. This Decreasing Progression Infinitely continued (determining in the same Point of Rest, where the Motion is supposed to expire) is yet of a finite Magnitude, and equal to $\frac{1}{m-1}$ of what it would have been in so much time, if there had been no *Resistance*: As is demonstrated in my *Algebra*, Chap. 95. Prop. 8. For (as I have elsewhere Demonstrated) the

Sum or Aggregate of a *Geometrical Progression* is $\frac{VR - A}{R - 1}$, (supposing V the greatest Term, A the least, and R the Common Multiplier.) That is

$\frac{VR}{R-1} - \frac{A}{R-1}$. Now in the present Case (supposing the Progression Infinitely

Infinitely continued) the least Term A, becomes Infinitely Small, or = 0. And consequently $\frac{A}{R-1}$ doth also Vanish, and thereby the Aggregate becomes = $\frac{VR}{R-1}$. That is (as will appear by Dividing VR by $R-1$;

$V + \frac{V}{R} + \frac{V}{R^2} + \frac{V}{R^3} + \dots = \frac{VR}{R-1}$; (supposing the Progression to begin at $V = 1$.) That is, dividing all by R, that so the Progression may begin at $\frac{V}{R} = \frac{1}{m}$;) $\frac{V}{R-1} = \frac{V}{R} + \frac{V}{R^2} + \frac{V}{R^3}$

+ &c. That is in our present Case (because of $V = 1$, and $R = m$)

$\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3} + \dots = \frac{1}{m-1}$. That is (putting $n = m-1$)

$\frac{1}{n}$, of what it would have been, if there had been no Resistance.

Fig. 175.

9. This Infinite Progression is fitly expressed by an Ordinate in the Exterior Hyperbola, Parallel to one of the Asymptotes; and the several Members of that, by the several Members of this, put in Continual Proportion. As is there Demonstrated at Prop. 15. For let SH, be an Hyperbola between the Asymptotes, AB, AF: And let the Ordinate DH (in the Exterior Hyperbola, Parallel to AF) represent the Impressed Force undiminished; or the Line to be described in such Time, by a Celerity answerable to such Undiminished Force. And let BS (a like Ordinate) be $\frac{1}{m}$ thereof, which

therefore, being less than DH (as being equal to a part of it) will be further than it from AF. In A B (which I put = 1;) Let Bd, be such a part thereof, as is BS of DH. Now because (as is well known) all the Rectified Parallelograms, in the Exterior Hyperbola, A S, A H, &c. are Equal; and therefore their Sides Reciprocal: Therefore as $A d = 1 - \frac{1}{m}$ (supposing Bd, to be taken from B toward A,) to $AB = 1$, (or as $m-1$ to m ;) so is $BS = \frac{1}{m}$ DH to $d h$, which is therefore Equal to $\frac{1}{m}$

of DH; that is (as will appear by Dividing 1, by $m-1$;) to $\frac{1}{m-1}$ of DH. + $\frac{1}{m^2}$ + $\frac{1}{m^3}$ &c. of DH. Now in the present Case (supposing the Progression to begin at $V = 1$;) $\frac{V}{R-1} = \frac{V}{R} + \frac{V}{R^2} + \frac{V}{R^3} + \dots$

Or if B d be taken beyond B, then as $A d = m + \frac{1}{m}$ to $AB = 1$,

or as $m + 1$ to m , so is $\frac{1}{m}$ DH, to dh , which is therefore Equal to

$\frac{1}{m+1}$ DH; that is, (as will appear by like Dividing of 1 by $m+1$;))

= to $\frac{1}{m} - \frac{1}{mm} + \frac{1}{m^3} - \dots$, &c. of DH.

10. Let such Ordinate dh , or (Equal to it in the *Asymptote*) AF, be so Divided in L, M, N, &c. (by Perpendiculars cutting the *Hyperbola* in l, m, n , &c.) as that FL, LM, MN, be as $\frac{1}{m}, \frac{1}{mm}, \frac{1}{m^3}, \dots$, &c. That is, so

continually Decreasing, as that each Antecedent be to its Consequent, as 1 to

$\frac{1}{m}$, or as m to 1.

11. This is done by taking AF, AL, AN, &c. in such Proportion. For, of Continual Proportionals the Differences are also Continually Proportional, and in the same Proportion. For let A, B, C, D, &c. be such Proportionals; and their Differences, $a, b, c, \dots$, &c. That is, $A - B = a, B - C = b, C - D = c, \dots$, &c.

Then because, A, B, C, D, &c. are in Continual Proportion;

That is, $A : B :: B : C :: C : D :: \dots$, &c.

And Dividing $A - B : B :: B - C : C :: C - D : D :: \dots$, &c.

That is, $a : B :: b : C :: c : D :: \dots$, &c.

And Alternately; $a . b . c . \dots :: B . C . D . \dots :: A . B . C . \dots$

That is, In Continual Proportion, as A to B, or as m to 1;

12. This being done; the *Hyperbolic* Spaces FL, Lm, Mn, &c. are Equal, as is Demonstrated by Gregory San-Vincent; and as such is commonly admitted.

13. So that FL, Lm, Mn, &c. may fitly represent Equal Times in which are dispatched Unequal Lengths, represented by FL, LM, MN, &c.

14. And because they are in Number Infinite, (though Equal to a Finite Magnitude) the Duration is Infinite; and consequently the Impressed Force, and Motion thence Arising, never to be wholly Extinguished (without some further Impediment) but perpetually Approaching to A, in the Nature of *Asymptotes*.

15. The Spaces FL, Fm, Fn, &c. are therefore as *Logarithms* (in *Arithmetical Progression* Increasing) answering to the Lines, AF, AL, AM, &c. or to FL, LM, MN, &c. in *Geometrical Progression* Decreasing.

Fig. 176.

16. Because FL, LM, MN, &c. are as $\frac{1}{m}, \frac{1}{mm}, \frac{1}{mm^2}, \dots$ &c. Infinitely)

Terminated at A; therefore (by Prop. 8.) their Aggregate FA, or dh , is to DH, (so much Length as would have been dispatched in the same Time, by such impressed Force Undiminished) as 1 to $m - 1 = n$.

17. If therefore we take, as 1 to n , so AF to DH; this will represent the Length to be Dispatched, in the same Time, by such Undiminished Force.

18. And if such DH, be supposed to be Divided into Equal Parts Innumerable (and therefore Infinitely small;) these answer to those (as many) Parts Unequal in FA, or hd .

19. But, what is the Proportion of r to 1, or (which depends on it) of $1 - r$ to 1, or 1 to m ; remains to be enquired by Experiment.

20. If the Progression be not infinitely Continued; but End (suppose)

at N, and its least Term be $A = MN$: Then out of $\frac{V}{R - 1} + \frac{1}{m} +$

$\frac{1}{mm} = \frac{1}{m} + \dots$ is to be Subducted $\frac{A}{R - 1}$, (as at Prop. 8.) that is,

(as by Division will appear) $\frac{A}{R} + \frac{A}{R^2} + \frac{A}{R^3} + \dots$ That is, (in our

present Case) $\frac{a}{m} + \frac{a}{mm} + \frac{a}{mm^2} + \dots$ And so the Aggregate will be

$\frac{1 - a}{m} + \frac{1 - a}{mm} + \frac{1 - a}{mm^2} + \dots = \frac{1 - a}{n}$.

And thus as to the Line of Projection, in which (Secluding the Resistance) the Motion is reputed Uniform; dispatching Equal Lengths at Equal Times. Consider we next the Line of Descent.

21. In the Descent of Heavy Bodies, it is supposed, that to each Moment of Time, there is superadded a new Impulse of Gravity to what was before: And each of these, Secluding the Consideration of the Air's Resistance, to proceed equally (from their several beginnings) through the succeeding Moments As, (in the Erect Lines) I I I I &c. I I I &c. I I &c. I &c. and so continually, as in the Line of Projection.

22. Hence ariseth (in the Transverse Lines) for the first Moment 1, for the second $1 + 1$, for the third $1 + 1 + 1$, and so forth in Arithmetical Progression. As are the Ordinates in a Triangle at Equal Distance.

23. And such are the continual Increments of the Diameter, or of the Ordinates in the Exterior Parabola, answering to the interior Ordinates, or Segments of the Tangent, equally Increasing; As is known, and commonly admitted.

I
I I
I I I
I I I I
&c. &c. &c. &c.

24. If we take in the Consideration of the *Air's Resistance*; we are then for each of these Equal Progressions, to Substitute a Decreasing Progression Geometrical; in like manner (and for the same Reasons) as in the Line of Projection.

25. Hence ariseth for the first Moment $\frac{1}{m}$; for the second $\frac{1}{m^2}$ $\frac{1}{m}$
 $\frac{1}{m} + \frac{1}{m^2}$; for the third $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$; &c. And such is $\frac{1}{m^3}$ $\frac{1}{m^2}$ $\frac{1}{m}$
 therefore the Descent of a Heavy Body falling by its own Weight. The several Impulses of Gravity being supposed Equal. $\frac{1}{m^4}$ $\frac{1}{m^3}$ $\frac{1}{m^2}$ $\frac{1}{m}$

26. That is, as FL, FM, FN, &c. in the Line of Descent, answering to FL, LM, MN, &c. in the Line of Projection. *Fig. 176.*

27. But though the Progressions for the Line of Projection, are like to each of those many in the Line of Descent: It is not to be thence inferred, that therefore $\frac{1}{m}$ in the one is equal to $\frac{1}{m}$ in the other: But in the

Line of Projection (suppose) $\frac{1}{m} f$, (such a part of the Forced Impressed, and a Celerity answerable:) in the Line of Descent $\frac{1}{m} g$, (such a part

of the Impulse of Gravity.)

28. Those for the Line of Descent (of the same Body) are all equal each to other: Because g , (the new Impulse of Gravity) in each Moment is supposed to be the same.

29. But what is the Proportion of f to g (that is, of the Force Impressed, to the Impulse of Gravity, in each Body) remains to be enquired by Experiment.

30. This Proportion being found as to one Known Force; the same is thence known as to any other Force (whose Proportion to this is given) in the same Uniform Medium.

31. And this being Known as to one Medium, the same is thence Known as to any other Medium, the Proportion of whose Resistance to that of this is Known.

32. If a Heavy Body be Projected Downward in a Perpendicular Line, it Descends therefore at the rate $\frac{1}{m}$, $\frac{1}{m^2}$, $\frac{1}{m^3}$, &c. of f (the Impressed Force) Increased by $\frac{1}{m}$, $\frac{1}{m} + \frac{1}{m^2}$, $\frac{1}{m} + \frac{1}{m^2} + \frac{1}{m^3}$, &c. of g , the Impulse of Gravity: (by Prop. 5. and P. 25.) because both Forces are here United.

33. If in a Perpendicular Projection Upwards; it Ascends in the Rate of the former, Abated by that of the latter. Because here the Impulse of Gravity is contrary to the Force Impressed.

34. When therefore this latter (continually Increasing) becomes Equal to that former (continually Decreasing) it then ceaseth to Ascend; and doth thenceforth Descend at the rate wherein the latter continually Exceeds the former.

35. In an Horizon or Oblique Projection: If to a Tangent whose Increments are as FL, LM, MN, &c. that is, as $\frac{1}{m} f$, &c. be fitted

Ordinates (at a given Angle) whose Increments are as FL, FM, FN, &c.

that is, as $\frac{1}{m} g$, &c. The Curve answering to the Compound of these

Motions, is that wherein the Project is to move.

36. This Curve (being hitherto without a Name) may be called *Linea Projectorum*, the Line of Projects, or things Projected; which resembles a *Parabola Deformed*.

37. The *Celerity* and *Tendency*, as to each Point of the Line, is determined by a Tangent at that Point,

38. And that against which it makes the greatest Strock or Percussion, is that which (at that point) is at Right Angles to that Tangent.

39. If the Projection (at pag. 25) be not infinitely Continued, but Terminate (suppose) at N, so that the last Term in the first Column or Series Erect; be a ; and consequently in the second, ma ; in the third mma , &c. (each Series having one Term fewer than that before it :) then (for the same Reasons as at P. 20.) the *Aggregates* of the several Columns (or Erect Series) will be $\frac{1-a}{n}$, $\frac{1-ma}{n}$, $\frac{1-mma}{n}$, and so forth, till (the Multiple of a becoming $= 1$.) the Progression expire.

40. Now all the Abatements here, a , ma , mma , &c. are the same with the Terms of the first Column taken backward. For a is the last, ma the next before it; and so of the rest.

41. And the Aggregate of all the Numerators is so many times 1, as is the Number of Terms (suppose n .) wanting the first Column; that is,

$1 - \frac{1-a}{n}$, or $\frac{n-1+a}{n}$; and this again divided by the common De-

nominator n , becomes $\frac{n-1+a}{nn}$. And therefore, $\frac{n-1+a}{nn} g$, is the

Line of Descent by its own Gravity.

42. If therefore this be Added to a Projecting Force downward in a Perpendicular, or Subducted from such Projecting Force upward; that is, to or from $\frac{1-a}{n}f$: The Descent in the first Case, will be $\frac{1-a}{n}f + \frac{nt-1+a}{nn}g$; and the Ascent in the other case $\frac{1-a}{n}f - \frac{nt-1+a}{nn}g$.

And in this latter Case, when the Ablative part becomes equal to the Positive part, the Ascent is at the highest: And thenceforth (the Ablative part exceeding the Positive) it will Descend.

43. In an Horizontal or Oblique Projection having taken $\frac{1-a}{n}f$, in the Line of Projection, and thence (at the Angle given) $\frac{nt-1+a}{nn}g$, in the

Line of Descent; the Point in the Curve answering to these, is the place of the Project answering to that Moment.

44. I am aware of some Objections to be made, whether to some Points of the Process, or to some of the Suppositions. But I saw not well how to wave it, without making the Computation much more perplexed. And in a matter so Nice, and which must depend upon Physical Observations, 'twill be hard to attain such Accuracy, as not to stand in need of some Allowances.

45. Somewhat might have been further Added, to direct the *Experiments* suggested at P. 19. and 29. But that may be done at Leisure, after Deliberation had, which way to attempt the *Experiment*.

46. The like is to be said of the different Resistance which different Bodies may meet with in the Same *Medium*, according to their different Gravities, (extensively or intensively considered) and their different Figures and Positions in Motion. Whereof hitherto we have taken no account; but supposed them, as to all these, to be Alike and Equal.

47. The Computation (in P. 39, 40, 41) may, if that be also desired, be thus represented by Lines and Spaces. The Ablatives $a, m a, m m a$, &c. (being the same with the first Column taken backward) are fitly represented by the Segments of NF, (beginning at N.) and therefore by Parallelograms on these Bases, assuming the common Height of Fh, or NQ; The Aggregate of which is N h, or F Q, and so many times 1, by so many equal Spaces, on the same Bases; between the same Parallels terminated at the *Hyperbola*; The Aggregate of which is h F N Q. From whence if we Subduct the Aggregate of Ablatives F Q; the remaining Trilinear h Q n, represents the Descent.

48. If to this of Gravity, be joined a Projecting Force; which is to the Impulse of Gravity, as h K to h F, (be it greater, less, or equal) taken in the same Line; the same Parallels determine Proportional Parallelograms, whose Aggregate is K Q.

49 And therefore, if this be a perpendicular Projecting downwards; then $h K k m$ (the Sum of this with the former) represents the Descent.

50. If it be a Perpendicular Upwards; then the difference of these two represents the Motion: which so long as $K Q$ is the greater, is Ascendent: but Descendent, when $h Q n$ becomes greater; and it is then at the Higheft when they be equal.

51. If the Projection be not in the same Perpendicular, (but Horizontal or Oblique) then $K Q$ represents the Tangent of the Curve; and $h Q n$, the Ordinates to that Tangent, at the given Angle

52. But the Computation before given, I take to be of better use than this Representation in Figure. Because in such Mathematical Enquires, I chuse to separate (as much as may be) what purely concerns Proportions; and consider it abstractly from Lines or other matter wherewith it is incumbred.

As to the Question proposed; Whether the Resistance of the Medium does not always take off such a Proportional part of the Force, Moving through it, as is the Specifick Gravity of the Medium to that of the Body Moved in it: (for if so, it will save us the trouble of Observation.) I think this can by no means be admitted; For there be many other things of Consideration herein, beside the Intensive Gravity (or, as some call it, the Specifick Gravity) of the Medium.

A Vicious Medium shall more resist than one more Fluid, though of like Intensive Gravity.

And a sharp Arrow shall bore its way more easily through the Medium than a blunt headed Bolt, though of equal Weight and like Intensive Gravity.

And the same Pyramid with the Point, than with the Base forward.

And many other like Varieties, intended in my *P. 46.*

But this I think may be admitted, namely, That Different Mediums, equally Liquid, (and other Circumstances alike) do in such Proportion Resist, as is their Intensive Gravity. Because there is, in such Proportion, a Heavier Object to be Removed, by the same Force which is one of the things to which *P. 31.* refers.

And again: The Heavier Project once in Motion, (being equally swift, and all other Circumstances alike) Moves through the same Medium in such Proportion more strongly, as is its Intensive Gravity: For now the Force is in such Proportion greater, for the Removal of the same Resistance. And this is part of what my *P. 30.* insinuates.

But where there is a Complication of these Considerations one with another, and with many other Circumstances, whereof each is severally to be considered; there must be respect had to all of them.

Experiments, to
Determine the
Point Blank
Distance; the

X. 1. To know how far a Gun shoots Point Blank, (as they call it) that is, so near the Level of the Cylinder of the Piece, that the Difference is either not discernible, or not considerable.

On a fit *Platform* Place and Point the *Gun* at a Mark as large as the Bullet, some 50, 60, or more Yards distant, so as the underside of the Mark may be in the same Level or Line with the underside of the *Cylinder* of the *Piece*. Then between the *Gun* and the Mark at convenient Distances, place pieces of Canvas, Sheets of Paper pasted together, or the like, upon stakes fix'd in the ground, so as the underside, being Level with the Horizon, may just touch the Visual Line, that passeth from the Eye to the upper side of the Mark, when the Eye is placed in the Line that passeth from it to the upper side of the *Cylinder* of the *Gun*; the Canvas being so broad and long, that if the Bullet pass through it 2 or 3 Foot higher than the Level of the Mark, or of either hand, the Hole it makes may make it known, how much it Flieth higher than the Level of that place. If the *Bullet* falls Lower than the Mark, and touch not the Canvas, the *Gun* may be the next time raised a little, and so on till the *Bullet* Hit the Mark, or as High as it. If it fall as high as the Mark and Cut the Canvas, the Mark and Canvas may be brought nearer the *Gun*: But if it fall as high as the Mark and do not Cut the Canvas, the Mark may be removed to greater and greater Distances.

Charge of Powder; and best Size of Guns. Proposed; by Sir Rob. Moray. n. 26. p. 473. June An. 1667.

If this way of *Experiment* be made for further Distances and Raisings of the *Piece*, as High as conveniently may be above the Level, and the Distances measured, and then all *Randoms* above these likewise tried and measured, the Distance of an Object, to be Shot at, being known, and other necessary Cautions, beneath to be mentioned, carefully observed, good *Gunners* may with great Confidence undertake to Hit the Mark, be the Distance what it will, so it exceed not the reach of the *Gun*.

2. To know what Quantity of Powder is the just Charge of any *Piece*, so as it maketh the farthest Shot, and Fires totally.

1. Raise the *Gun* to a mean *Random*, as of 20° , or 25° , and Shoot with the ordinary Charge of Powder, in some convenient Ground where the Fall of the Bullet may be easily seen, and having made a Shot, measure the Distances with a Chain between the Hole made by the Bullet and the *Muzzle* of the *Gun*.

2. Then instead of a Full Charge of Powder used in the first Shot, take $\frac{1}{2}$ part less, or some such Proportion, for the next Trial, doing all things else as before.

3. For a third, fourth, or more Trials, diminish still the Quantity of Powder, by $\frac{1}{2}$ at a Time, till the Shot be considerably shorter than at first.

4. Then take $\frac{1}{2}$ more than the first Charge, and do all things else as before, and so continued more Trials, increasing still the Quantity of Powder in the same Proportion every new Trial, till you find the Increase of the Charge does not make the *Piece* Shoot further: Only Over-charge not so far as to endanger the *Gun*.

5. Three or more Shot are to be made with every different Charge, and at every several Trial, that the Certainty may the better appear.

6. The first Shot being Measured and Marked, the rest may all be Measured from it, or from one another to save Labour.

7. The *Gun* is to be Pointed, Placed and Ordered, every time in one and the same Place and Position, aiming still at the same Mark, or Pointing still at the same Line or Azimuth; that so all the Shot may fall in the same Line as near as is Possible.

8. The *Powder* (which ought to be all of the same Goodness) must be exactly Weighed every time the *Piece* is Charged, lest it having been Weighed long before, the Weight may be altered; though *Experiment* may be made with *Cartridges* and without.

9. The *Powder* and *Bullet* is to be Ramed home equally at every Shot; though the Looser the *Powder* lie, it *Fire* the better.

10. When the right *Charge* of a *Piece* is found, that makes the farthest Shot in the Ordinary and Plain way of Charging, *Mr. de Sons's* Contrivance of a *Wedge* may be tried, to make it Shoot farther; which is a piece of Board, so long, as being thrust home to the *Breech* of the *Piece* at one End, the other may reach farther out than the outside of the *Bullet* being Rammed up to its place; broad about an Inch, and thin so far as the *Wadd* before the *Bullet* reaches on the outside; there it is to have a Shoulder, from which forward to the end, it is to be cut a slope, like a *Wedge*, being of such thickness, as that at the place, where the Center of the *Bullet* is to be, it may make it stick so fast, that the *Powder* finding more Resistance, may at length Drive it out with the greater Violence.

11. Another of this Nature is a *Wooden Tampion*, like a piece of a Cylinder, big enough to fill the hollow Cylinder of the *Gun*, the length somewhat more than the Diameter of it, and Hollow'd towards the *Bullet*, so as to fit it; and either Flat or (which is better) Hollow likewise towards the *Powder*, and serving instead of *Wadd*. These, and such others, will probably render the Effect of the *Powder* greater, than otherwise it would be: But Care must be had that they do not endanger the *Piece*.

12. The Strength of the *Powder* must be examin'd by a *Powder Trier*, that raiseth a Weight, such an one as hath been contrived by *Mr. Hook*.

13. The same *Bullet* is to be made use of, if it can be had, till the Figure of it be Marred; otherwise another as near of the same Size, Shape and Weight, as is possible.

14. Observe the Strength and Position of the *Wind*, and at what Azimuth the Mark stands from the *Gun* at every time of Shooting: And take precise notice what effect it hath upon the *Bullet* in carrying it further, in hindring, or turning it aside.

15. Note the Figure, Dimensions, and Weight of the *Gun*, *Carriage*, and *Wheels*; and Record every thing exactly in a Book, as also every Accident and Observation.

16. After all other Experiments are made, every *Piece* may be Tried with the Right *Charge* of *Powder*, laying every time more and more Weight upon the *Carriage*; and at last Fixing the *Gun* so, as it may not Recoil at all, observing every time how far the *Bullet* goes, and how much less *Powder* than the full *Charge* will serve to Shoot the *Bullet*, when the *Piece* is Fixt, as far as the whole *Charge* does, when it Recoils freely.

17. The

17. The *Right Charge* found, the best *Random* is to be sought, by Trying all *Randoms*, by Degrees at a time.

3. To know what Gun shoots farthest.

1. A Gun to be prepared of *Culverine-Bore* (as being held the best for Shooting Far) but much Longer (double the ordinary Length may do well) and without any *Ring* about the *Muzzle*, is to be placed as in the former *Experiments*, and Charged with the Ordinary Charge of a *Culverine*, or rather with that Quantity, which by the former *Experiments* shall be found the best; and being Shot, the fall of the *Bullet* is to be Mark'd, and Distance Measured.

2. Then Try Less, and More *Powder* in her, as before

3. Then Cut off two Inches of the *Muzzle* with a Saw, and Place the Pieces so cut off in the *Carriage*, or their *Weight* of Lead in a convenient Figure, that the *Recoil* may still be the same; and try as before, doing every thing in the same manner: And so cut off still for new Trials, till the Shot begin to fall shorter than before.

4. The same may be done with *Guns* of Different *Bores*.

2. Mar. 18. 1651. At 200 Yards Distance from the Platform for great Ordinance at *Woolwich*, there were raised three *Butts*, one behind another: The Space between the first and second *Butt* was 14 Yards, between the second and the third, eight. The Thickness of each *Butt* was 19 Inches, whereof 13 was of Beams of *Massey Oak* fastened into the Ground, and set so close that they touched each other: On each side where Planks of Oak, 3 Inches a piece in Thickness and these were joyned close, and fastened on both Sides with Iron Bolts, and strong Pins of Wood; and on the Back, at the ends, and on the middle, there were 3 Braces of Elm, a Foot in Breadth, and 5 Inches in Thickness.

The first *Experiment* was with an *Iron Demy-Cannon*, having a *Cylinder Bore*, of 3500 *lib. Weight*, the *Bullet* 32 *lib.* of Iron, the *Powder* 10 *lib.* which Pierced through the two First *Butts*, and stuck in the third, so as the Ball was almost quite within, but the Timber not shivered (small) nor scarce Split. The *Butts* being touched by me, felt not warm; the like *Execution* was done, when it was Charged with 9 *lib.* as also when with 8 *lib.* of *Powder*.

The second *Experiment* was with an *Iron Demy-Cannon*, having a *Taper Bore*, and being 3600 *lib.* in *Weight*, and 4 Inches longer than the former, the *Iron Bullet* 32 *lib.* and the *Powder* 7 *lib.* which in three Trials seemed to have the same Force with the First. One of the *Shots* piercing through the second *Butt*, and Lighting near the Edge of the middle * *Butt* of Elm, Tore * *Brace*, it, but by the yielding of it, the *Bullet* glanced aside off the third *Butt*, and entered into the Earth.

The third *Experiment* was with a whole *Culverine* in *Brass*, of 5300 *lib.* in *Weight*, 11 Foot one Inch in *Length*, with a *Taper Bore*; the *Iron Bullet* was 18 *lib.* in *Weight*; the *Powder* in the first Trial 10 *lib.* in the second 9 *lib.* in the third 8 *lib.* which last Proportion did the best *Execution*, and passed through the two First *Butts*, Entering gently into the Third, which the former two did touch, but not Enter. The

Experiments for Trying the Force of great Guns; by Mr. Greaves. n. 173. p. 1090.

The Fourth Experiment was with a whole Culverine in Brass, made at Amsterdam for the French, with this Mark, 3580, being 10 Foot Long, and not very Thick in the Breech; the first Shot with 9 lib. of Powder, 18 lib. of Bullet-Iron, past through the three Butts and entered one Foot into the Ground; it passed by the Joints of the Timber, two Planks having been beat down before. The second Shot with 8 lib. Powder, passed through two Butts and Grazed between them. The third with 8 lib. past Two Butts, and 7 Inches into the Third, but the First Butt was much Battered before, where it Entered. The Fourth Shot passed, with 8 lib. of Powder, two Butts, and in both Butts through the midst of a Massey-strong Beam (below) that had not been Battered.

The Fifth Experiment was with an Iron Demy-Culverine, having 9 lib. Bullet in Iron, and 4 lib. Powder; this past one Butt (which was Torn before) and Entered the second.

This $\frac{1}{2}$ Culverine was Shot 8 Times, as fast as they could Charge it with Powder, and the Iron-Bullet, and yet was but scarce Lukewarm at the Breech, a little more in the Midst, most at the Muzzle, and this last scarce so Hot as my Hand, and yet the Gunners in Charging her, wet not at all the Scoop, or Spunge.

The Sixth Experiment was with a Brass Demy-Culverine, the Breech of her was 13 Inches $\frac{1}{2}$, the Mouth 9 $\frac{1}{2}$. The First Shot, with 4 lib. of Powder, 9 lib. Iron-Bullet, past two Butts: The Second Shot with 3 lib. of Powder, past almost two Butts: This proved to be the best Shot, because the Timbers were the Strongest.

Shooting by the
Rarefaction of
the Air; by
Dr. Papin.
n. 179. p. 21.

Fig. 178.
Jan. An. 1686.

XI. Whereas ordinary Wind Guns do their Effect by the Compression of the Air: Ottho Ghericke hath found a new sort that Shoots by Rarefaction; and he hath published that Device at large in his Book about Pneumatick Experiments. I have Contrived another which I take to be Better.

A A, is a Pipe, very Equal from one end to the other.

B B, a small Pipe Soder'd in a Hole near the end of the Pipe A A, and apply'd to the Plate of the Pneumatick Engine.

C C C C, Some kind of Stool, to bear up the Hinder part of the Pipe A A.

D, A Piece of Lead fitted to the Bore of the Pipe A A.

The Pipe A A, is to be Shut at both Ends by Valves ourwardly applied, and so the said Pipe A A, though never so Big, may be Exhausted of Air by means of the Pneumatick Engine: Which done, the Valve towards D must be suddenly opened, so that the whole Pressure of the Atmosphere Acting upon the Lead D, may drive it along the Pipe A A, with such a Swiftnes, that it will be able to carry it to a great Distance: And because such a Valve Shutting a great Hole, would prove very difficult to be Opened, when the Pipe A A, is of a great Bore, the Aperture towards D, may be left much smaller than the Pipe; the Swiftnes of the Air being so great, that even through a pretty small Aperture, it pressed the Lead D, as freely almost as if the whole Bore was quite Open.

Having

Having prepared a *Barrel* carrying a *Lead* of two Ounces, the *Experiment* was shewn before the *Royal Society*, and the Effect was found very Considerable, the Force being little less than that of the *Wind-Gun* by *Compression*; the same *Experiment* being afterwards repeated with a Longer *Barrel*, 'twas found that the Length in this way of *Shooting* was very little, if any Advantage.

XII. My Way of Computing the *Velocity* of the *Air* (which I think is better than the Trial made by the *Royal Academy* at *Paris*) is Grounded upon this *Hydrostatical Principle*, That *Liquors* have a Strength to Ascend as High as their Source is; and although the Resistance of the Medium does always hinder Jets of *Eau* in the Open *Air* from reaching quite so High, nevertheless the *Liquor* at its first Spouting out, hath the necessary *Swiftness* to come to that Height.

The Velocity
wherewith the
Air rushes into
an exhausted
Receiver;
by Dr. Papin.
n. 184. p. 103.
Oct. An. 1686.

Prop. I.] From this Principle may be easily deduced this Proposition, That, of two different *Liquors* driven by the same Pressure, that which is in Specie Lighter must ascend Higher than that which is heavier, and their Heights will be Reciprocally in the same Reason as their Specifick Gravities are.

Prop. II.] From the foregoing Proposition another may easily be Deduced, viz. That, of Differing *Liquors* bearing the same Pressure, those that are Lighter in Specie must acquire a greater Swiftness, and their Differing Velocities are to one another * as the Roots of the Specifick Gravities of the said *Liquors*.

For we have seen, Prop. I. That the Heights to be Attained are * in the same Reason as the Specifick Gravities; Now *Gallilæus*, *Hugenius*, and others, have Demonstrated, That the Velocities of Bodies are to one another, as the Square Roots of the Heights to which they may Ascend: And so in this Occasion they are also * as the Roots of the Specifick Gravities.

* Reciprocally.

* Reciprocally.

* Reciprocally.

If therefore we would know what is the *Velocity* of the *Air* being driven by any Degree of Pressure whatsoever, we ought but to find what would be the *Velocity* of *Water* under the same Pressure: And then take the Square Roots of the Specifick Gravities of these two *Liquors*; because as much as the Square Root of the Specifick Gravity of *Water* doth exceed the Square Root of the Specifick Gravity of the *Air*; so much in Proportion will the *Velocity* of *Air* exceed the *Velocity* of *Water*. For Example; when I would Compute what should be the Swiftness of a *Bullet* shot by my *Pneumatick Engine*, I should at first Compute what was the *Velocity* of the *Air* itself that drove the *Bullet*: I did therefore take notice, That in this Occasion the *Air* bears a Pressure much about the same as that of *Water* when its Spring is 32 Foot High. Now such *Water* would spout out with a sufficient *Velocity* to ascend 32 Foot Perpendicular, and therefore according to the Rules and Observations of *Gallilæus*, *Halley*, and others, such *Water* hath the *Velocity* of 45 Foot in a Second. It remains therefore but to know the Proportion of the Gravity of the *Air* to that of *Water*: And we have found it not to be always the same; because the Height, the Heat, and the Moisture, of the *Atmosphere*, are Variable: Nevertheless, we may say in general, That the Reason between the Specifick Gravities of *Water* and *Air* is much about 840 to 1. Taking then

their *Square Roots* as I have said above, which *Roots* are 29 and 1, we may conclude that the *Velocity* of *Air* must Exceed that of *Water*, by 29 Times: And so multiplying 45, the *Velocity* of *Water*, by 29, we shall find, that the *Velocity* of *Air* driven by the whole *Pressure* of the *Atmosphere*, is about 1305 Foot in a Second.

Wind produced
by the Fall of
Water; by
Dr. Wal. Pope.
n. 2. p. 25.
Fig. 179.
Apr. An. 1665.

XIII. In the *Brass-Works* at *Tivoli*, the *Waters* Blows the *Fire*, not by Moving the *Bellows*, but by affording the *Wind*. Thus: A, is the *River*. B, the *Fall* of it. C, the *Tube* into which it Falls. LG, a *Pipe*. G, the *Orifice* of the *Pipe*, or *Nose* of the *Bellows*. GK, the *Hearth*. E, a *Hole* in the *Pipe*. F, a *Stopper* to that *Hole*. D, a place *Under-ground*, by which the *Water* runs away. Stopping the *Hole* E, there is a perpetual strong *Wind*, issuing forth at G: and G being stopt, the *Wind* comes out so Vehemently at E, that it will, I believe, make a *Ball* play, like that at *Frescati*.

The Best Form of
Horizontal Sails
for a Mill; by
Dr. Rob. Hook,
Phil. Coll.
n. 3. p. 61.
Dec An. 1681.

XIV. Whatever Men may imagine concerning *Horizontal Sails*, I doubt there never will be found a better, and more Advantageous way, for receiving the Strength of the *Wind*, or Motion of the *Air*, than *Perpendicular Vanes* made of a True Form, so as every part thereof may draw alike. But because I find divers have of late attempted *Horizontal Vanes* for *Mills*, I shall explain a Way of making *Horizontal Vanes* capable of Performing the Most that is Possible with *Vanes* of equal *Extension*.

The *Invention* is founded upon the same Principle with that of the *Sailing* of *Ships*, and other *Vessels* upon the *Sea*; namely, Upon Disposing and Ordering of the *Vane* or *Sail* so, as to stand in the best *Posture* 'tis possible to Move the *Arms* of the *Mill*, or the *Body* of the *Ship*, in that Way it is to be Moved, by the Force of the *Wind* Blowing thus or thus against them.

The First Principle then common to both, is, that the *Vane* or *Sail* be as near as 'tis possible, a perfect Plain and Smooth *Superficies*, without any Bellying, Bunting, or Curvity in the *Superficies* thereof, upon which the Motion or Force of the *Wind* is Impressed.

Secondly, That the *Air* may have as many *Passages* between the parts of the *Vane* or *Sail* as may be, that the Moved *Air* may come to it as freely as may be, without being Intercepted by a *Stagnant Air* before it, to Impede or Divert its Force.

Thirdly, That the Plain of the *Vane* or *Sail* be put in the Middle *Inclination*, between the Way of the *Wind* and the Way of the *Arm*, or that of the *Body* of the *Ship*.

The Contrivance itself is This.

Fig. 180.

Let A B, signifie the *Stream* or *Current* of the *Air* or *Wind*, Moving from A to B, and let C represent the Center of the *Axis* or *Spindle*, standing Perpendicular to the *Horizon*, upon which, at the Top, is fixed at Right Angles, the piece D H, making the two *Arms* C D, and C H, upon the Ends of which the *Vanes* M N, are Moved on *Spindles*; so as that the Plain of the *Vane* doth always pass through the Point D: I say, these *Vanes* so ordered, shall be always placed in the most Advantageous *Posture* for Moving the

the *Arms* round upon the said *Spindle*, whose Center is C, in the Order of DEF GHI KLD.

First, For the *Vanes* placed at D and H, I say, They are set in the most *Advantageous Posture* possible, in those two Points : For First, the *Vane* M N at D being to move Directly against the *Wind*, the most *Advantageous Posture* is to turn its edge directly against the *Wind*, and thereby to give the least *Resistance* possible that being the only Point in which the *Vane* supposed only a *Superficies*, Draws not. And Secondly, For the *Vane* M N placed at H, it standeth the most *Advantageously*, because its Motion being directly from or before the *Wind*, it standeth full *Cross*, or opposed to the Motion thereof

Secondly, The *Vanes* at E, F, G, and I, K, L, stand the most *Advantageously*, because they Divide the *Angle*, between the *Way* of the *Wind*, and that of the *Arms* in those Points into two equal Parts and consequently the *Wind* Impresseth the Greatest Force in the most Direct Way : For it is easy to be Demonstrated, That the Force Impressed on the *Vane* by the *Wind*, is Perpendicular to the Surface, and consequently that the Obliquity of the Force to the *Way* of the *Arms*, Increased by the *Vanes* standing more full against the *Wind*, will have a less Proportion of Power to promote the Motion thereof, than in the Posture here Set. And supposing the *Vanes* set Sharper to the *Wind*, the Diminution of the Force Impressed by the *Wind* on its Surface, will be greater than the Augmentation of its Power, by being moved more Directly to the *Way* of the *Arms*. This is easy enough to be Geometrically Demonstrated.

The *Vane* may be so ordered, as always to Stand in this Posture by a great many ways : I shall only Instance in One, not the best for Practice, but the most Easy to be understood and Demonstrated.

Let the *Vane* be equally Expanded on each side of its *Axis*, by which the Pressure on the extreams of it are always Counterpoised, then Fasten upon the Lower end thereof a Wheel, which may be in Diameter about $\frac{1}{2}$ of the Length of the *Arms* from Hole to Hole; then Fix a Wheel upon the Frame in which the *Spindle* of the *Arms* do Move, that shall be of half the Diameter with the former, and to contain half the number of Teeth, Then by a third small Wheel, Fixed under the *Arms*, of a convenient Bigness. Communicate the Motion of the One to the Other ; for by this means each *Vane* being so provided, they will, being once set Right, always continue to be moved and Disposed in the true Posture desired.

This Contrivance will not only be Useful for all manner of common *Wind-Mills*, but also for *Water-Mills* in *Rivers*, where there can no *Dam* be made, as may also the *Perpendicular Vanes* of other *Mills*, neither of which has been so much, as Hinted by any Person whatsoever that I have hitherto heard of.

Fig. 181.

An Account of
Flying ; by
Dr. Hork.
Phil. coll.
n. 1. p. 14.

XV. 1. The Art of Flying hath been in all Ages Attempted by many, particularly in the Times of our famous Fryar Roger Bacon, who lived about 500 Years since. He was Believed a Magician or Conjuror, and to have performed what was related of him, by the help of Diabolical Magick, but from the perusal of several of his excellent Works yet Extant, I esteem him no such Person,

I rather find him to have been a good *Mathematician*, a knowing *Mechanick*, a rare *Chymist*, and a most Accomplished *Experimental Philosopher*, which was a Miracle for that Dark Age. This Man affirms the Art of *Flying* Possible, and that He himself knew how to make an *Engine*, in which a Man Sitting, might be able to carry himself through the *Air* like a *Bird*: And affirms, that there was then another Person who had actually Tried it with good Success. We have not wanted Later Instances in *England*, of several Ingenious Men, who have employed their Wits and Time about this *Design*. Particularly, I have been credibly informed, That one Mr. *Gascoigne* did about 40 Years since Try it with good Effect; tho' he since Dying, the Thing also Dyed with him. And even now, there are nor Wanting some in *England*, who affirm themselves able to do it, and that they have proved as much by *Experiment*. We have little or no account of the Ways they have taken to Effect their Designs; But we may conclude them Defective in somewhat or other, since we do not find them brought into Common Use.

The Art of Flying;
by S. Besnier.
Ibid. p. 15.
Fig. 182.

2. The *Sieur Besnier*, a Smith of *Sable* in the County of *Maine*, hath Invented an *Engine* for *Flying*. It consists of two *Poles* or *Rods*, which have at Each End of them an oblong *Chassie* of Taffety, which *Chassie* Folds from above Downwards, as the frame of a folding *Window Chassie*. He fits these *Poles* upon his Shoulders, so that two of the *Chassies* may be before him, and the other two behind him. The Order of Moving them is thus: When the *Right Hand* Strikes down the *Right Wing* before, A, the *Left Leg* by means of the String E, Pulls downwards the *Left Wing* behind, B; then immediately after, the *Left Hand* Moves or Strikes downwards the *Left Wing* before, C, and at the same time the *Right Foot*, by the String F, Moves or Pulls down the *Right Wing* behind D; and so successively, or Alternately, the Diagonally Opposite *Wings* always Moving downwards, or Striking the Air together.

A Flying Chariot;
by Fr. Lana.
Ibid. p. 18.

3. 1. P. *Francesco Lana* in his *Prodromo*, finding by an *Experiment*, That the *Weight* of the *Air* is $\frac{1}{14}$ part of the *Weight* of a like quantity of *Water*, he concludes certainly, That if we could make a Vessel of *Glass* or other Matter that might *Weigh* less than the *Air* that is in it, and should draw out all it's *Air*, this Vessel would be *Lighter in Specie* than *Air* itself, and therefore would Swim in it and Ascend on High. This He supposes may be done, by making a Round Vessel of Thin Plate *Brass*, (Weighing 3 Ounces in a Square Foot) of the *Diameter* of 14 Foot. For the *Surface* of the Vessel will be 616 Square Feet, and the *Brass* will *Weigh* no more than 1848 Ounces; whereas the *Content* will be $1437\frac{1}{2}$ Cub. Feet, and that Quantity of *Air* will *Weigh* $2155\frac{1}{2}$ Ounces: So that that *Air* being evacuated, the Vessel will be $307\frac{1}{2}$ Ounces *Lighter* than *Air*, and therefore will not only Ascend into the *Air*, but also carry up with it a *Weight* of $307\frac{1}{2}$ Ounces. And thus by Encreasing the Bulk of the Vessel, without encreasing the Thickness of the Plates of *Brass*, he supposes a kind of Ship may be made, to Swim in the *Air*, and to carry two or three Men in it.

2. The Fallacy of the *Author's* Reasoning lies in this; He supposes Copper of 3 Ounces in a Foot Square to be of sufficient Thickness to Resist the Pressure of the Air in a Globe of 14 Foot Diameter, nay of any Dimension. But in this we can nowise Assent to him: For the Pressure from without Inwards, tho' it be always the same upon Equal Surfaces, yet upon Unequal Surfaces the Case is quite otherways, for there the Pressure will be found, not the same, but to Encrease always in the same Proportion with the Surface, and thence consequently the Thickness of his Copper, or any Metal or Material, which he shall make use of, must Increase in the same Proportion, with the Diameter of the Sphere, and consequently the Weight of his Copper must always Increase in the same Proportion at least to the Solidity of his Sphere; so that by his Augmenting the Quantity of his Sphere, he has no manner of Advantage of making it proportionably Lighter than the Air, and proportionably Strong, but the contrary: For it is manifest, That a Bigger Sphere so made of any Matter we yet know, has less Power of Resisting the same Pressure of the Air than a Less, because of Finite Resistance of Matter to Pressure, there being some degree of Pressure that will Crush every Body.

Shown Impracticable; by Dr. Hook. Ibid. p. 27.

XVI. This Engine is composed of four Principal Parts; the *Serpent* A A, two Foot-Steps or *Treddles* B B, one *Clapper* C, and two *Arms* D D, D D.

An Engine to make Linen cloth; by M. de Gennev. n. 140. p. 1007. Fig. 183. July An. 1678.

The *Serpent* or Iron Bar A A, has two Elbows E E, where to the Ends of the Ropes are fix'd that Raise and put Down the *Foot-steps* B B, F F are two Fourths of a Circle, that successively Rest upon two Arches or Bows of Iron, G G, which are above the *Clapper* C, to Raise it. H H are two Teeth of Iron, added to the *Serpent* making an Angle of 25 Deg. with F F, and K K: which serve to put Down a *Bascule*, or *Sweep*, which is in the Arm that carries the *Shuttle*. The *Foot-steps* or *Treddles* differ in nothing from those which are usually made use of, only the Cords that hold them Pendent from the Ground are fix'd in the Elbows of the *Serpent*, which in turning Raises and puts them Down by the help of two little Pullies, upon which the Ropes turn.

The *Clapper* is supported between two Pillars, with a Rope double twisted, which occasions it to make a kind of Spring, and causes it naturally to give forwards to Beat the Cloth.

L M, is one of the *Arms* which Passes freely into the Canal or Pipe N N, supported by four Pillars of Wood O O O O. The Motion of it proceeds from the following Parts. P Q, is a *Bascule*, which, though unequally divided by its Supporter R, is yet in *Æquilibrio*, the End P R being made to weigh exactly as much as R Q.

At the Extremity of this *Bascule* is ty'd a Cord which passes through the Pulley S, and terminates at the Extremity of the Arm, where it is fastened, to a little Bow M. At the other Extremity of the same Arm, that is to say, towards L, is also fastened underneath, a Cord which passes through the Pulley T, and which carries the Weight V.

At the same end of the Arm is added a little *Niche* Z, about the bigness of half the *Shuttle*: Then over a little Bar X Y, which passes a-thwart the Arm there

there are two other little Pieces of Wood, having at the end of them two Teeth, which enter into the *Niche Z*, through two Holes which are there, of the one side and t'other.

To the Ends of these little pieces of Wood, there is a little *Bow* of Whalebone or Steel, which keeps the two Ends asunder, and forces the Teeth, which are at the other end, to enter into the *Niche*, before the said pieces can themselves. At the points 1 1, are two Ropes that pass through the Pullies 2 2, fastened to the Pillars, 0 3 0 4, and have each of them a little Weight at the end big enough to keep it from passing through a little Bowl which is under each Pulley.

This *Arm* thus disposed, goes and comes in the Hole *NN*, in the following manner. One Tooth of the *Serpent*, already described, strikes upon the Extremity of the *Bascule P Q*, and so causes the End *Q* to Rise up, which drawing the Cord fastened to the Point *Q*, makes the *Arm LM*, to Advance forward. But when afterwards the Tooth of the *Serpent* is come forth again, then the Weight *V*, tied to the other End of the same *Arm*, by a Cord that passes through the Pulley *T*, forces the said *Arm* by its own Weight to Return again.

When the *Arm LM* is in its ordinary place, the two little pieces of Wood into which enters the Bar *XY*, enclose the *Shuttle* by means of the Whalebone Spring. But when the said *Arm* approaches the other opposite *Arm*, then the Cords tied to the Points 1 1, being a little too short, and the Weight which is at the end of them not being able to pass through, the Spring gives way a little, and so the *Shuttle* is no longer enclosed by the *Arm* which carries it, but is wholly received and grasp'd by the other; which likewise in its Turn, delivers it back again in the same manner.

The Motion of the whole *Machine* is made at the rate as you Move the Handle of the *Serpent*, for then the *Arms* cause the Threads to open, and immediately one of the *Arms* begins to slide in towards the opposite *Arm*, to which it carries the *Shuttle* and Retires, immediately: At the same time, one of the Quarters of a Circle, which held the *Clapper*, Elevated, forsakes it, and leaves it for to flap, and then the opposite Quarter of a Circle Elevating itself, the other Elbow changes the Threads, and the other *Arm* Retires, and so successively.

The Advantages of this *Engine* are these. 1. One *Mill* will set 10 or 12 of these *Looms* at Work. 2. You may make the *Cloth* of what Breadth you please. 3. There will be fewer Knots in the *Cloth*, since the Threads will not break so fast as in other *Looms*, because the *Shuttle* that breaks the greatest Part, can never touch them. In short, The *Work* will be carried on Quicker, and at less Charge, in regard that instead of several Work-Folks which are required in making very large Cloths, One Boy will serve to tie the Threads at the several *Looms* as fast as they break and to order the *Quills* about the *Shuttle*.

XVII. I Order'd a Model of a part of a *Waggon* to be made consisting of four *Wheels*, two *Axes*, and a *Board* nailed upon the *Axes*. The *Lesser Wheels* were $4\frac{1}{2}$ Inches high, and the *Bigger Wheels* $5\frac{2}{3}$ Inches high, viz. $\frac{1}{3}$ of the Ordinary height of the *Wheels* of a *Waggon*: The Weight of the *Model* was almost $1\frac{1}{2}$ *lib*. I had also two other *Wheels* made $5\frac{2}{3}$ Inches high to be put on instead of the *Lesser*. The Middle of the two *Axes* were $6\frac{1}{4}$ Inches asunder. All the *Wheels* Turned very easily upon the *Axes*.

Advantages of High Wheels Experimented; by a Member of the Oxford Society.
n. 167. p. 856.
Jan. An. 1685.

A piece of *Lead* $50\frac{3}{4}$ *lib*. *Averdupoise*, was laid upon the *Model*, so forward, that the *Lesser Wheels* seemed to bear above $\frac{2}{3}$ parts of the *Weight*. Then the *Model* was drawn with a *String* laid over a *Pulley*, the Top whereof was $\frac{1}{4}$ of an Inch higher than the Top of the *Hinder Axes*, and the Middle of this *Pulley* was $7\frac{1}{2}$ Inches from the Middle of the *Fore Axis*.

The *Lesser Wheels* being put on, and the *String* being tied to the Top of their *Axis*.

1. *Three Pound* drew the *Model* on the smooth *Level Table*.
2. *Twenty Pound* drew the *Lesser Wheels* over a *Squared Rod* $\frac{1}{4}$ of an Inch thick.
3. *Thirty Pound* drew them over a *Round Rod* a little more than $\frac{1}{2}$ an Inch thick.
4. *Thirty One Pound* drew them over a *Square Rod* half an Inch thick.
5. *Twelve Pound* drew the *Hinder Wheels* over the bigger *Square Rod*.
The *String* being laid under the *Axis*, viz. $\frac{1}{4}$ of an Inch lower than before.
6. *Twenty nine Pound* drew the *Lesser Wheels* over the *Bigger Square Rod*.
Then the two *Bigger Wheels* being put on instead of the *Lesser*, and the *String* lying over the *Axis*.
7. *Three Pound* drew the *Model* on the *Table*.
8. *Twenty five Pound* drew the *Fore Wheels* over the *Round Rod*.
9. *Twenty five Pound* drew them over the *Bigger Square Rod*.
10. The *String* lying under the *Axis*, *16 Pound* drew them over the least *Rod*.
11. *Twenty three Pound* drew them over the *Round Rod*.
12. *Twenty three Pound* drew them over the *Bigger Square Rod*.
13. *Thirteen Pound* drew the *Hinder Wheels* over the *Bigger Square Rod*.

In all these *Experiments*, the *Lead* was laid exactly upon the same part of the *Board*, but yet when the *Lesser Wheels* were taken off, the *Lead* did not lean so much forward, so that the *Hinder Wheels* were somewhat more pressed than they were before.

By comparing the second, third, and fourth *Experiments*, with the tenth, eleventh, and twelfth, it appears how much more easily a *Waggon*, &c. might be drawn in *Rough Ways*, if the *Fore Wheels* were as high as the *Hinder Wheels*, and if the *Thills* were fix'd under the *Axis*. Such a *Waggon* as this, would likewise be Drawn more easily, where the *Wheels* cut in *Clay*, or *Sand*, or any *Soft Ground*. And moreover, *High Wheels* would not cut so deep as *Low Wheels*.

Low Wheels indeed are better for Turning in a narrow Compass than high Ones: But it seems probable that *Waggons* with four *High Wheels*, might be so contrived, that there should be no great Inconvenience in that Respect; at least, such *Waggons* as seldom have occasion to turn short, as *Carriers Waggon*s, and such like.

The Difference which you may observe in the eighth and eleventh *Experiments*, is agreeable to what is said by *S. Stevinus*, and *Dr. Wallis*, viz. That if a *Coach*, &c. must be Drawn over Rough, Uneven Places, it is best to fix the *Traces* to the Coach Lower than the Height of the Horses Shoulders.

14. A Table $2\frac{1}{2}$ Foot long, was set with one End $8\frac{1}{2}$ Inches Higher than the other End, and the *Model* being loaded as before, less Weight by 6 Ounces drew it up the Table, where the four *Bigger Wheels* were on, that when two *Bigger* and two *less* were on. Because in the first Case there was almost the same *Direction* of the Motion of the *Model* and of the String that drew it; but not in the second Case, when the *Fore Axis* was so much Lower than the Top of the Pulley.

A new sort of
Calesh described
by Sir R. B.
D. 172. p. 1028.
June An. 1685.

XVIII. This *Calesh* goes on two *Wheels*; Carries one Person, is Light enough; Tho' it hangs not on *Braces*, yet it is easier than the Common *Coach*; A Common *Coach* will Overturn, if one *Wheel* go on a *Superficies* a Foot and a half Higher than that of the other, but this will admit of the Difference of $3\frac{1}{2}$ Foot in Height of the *Superficies*, without danger of Overturning: We Chose all the irregular Banks, and sides of *Ditches* to run over; and I have this Day seen it at five several Times turn over and over, and the Horse not at all disordered. If the Horse should be in the least unruly, with the help of one Pin, you Disengage him from the *Calesh* without any Inconvenience. I my self have been once Overturned, and knew it not till I look'd up, and saw the *Wheel* flat over my Head; and if a Man went with his Eyes shut, he should imagine himself in the most smooth way, tho' at the same time there be three Foot Difference in the Height of the Ground of each *Wheel*.

The Contrivance
of a Perpetual
Motion; by M.
Explained; by
Dr. Papin.
N. 177. p. 1240.
Fig. 184.
Dec. An. 1685.

XIX. Let DEF, be a pair of *Bellows* 40 Inches long, that may be opened by removing the part F, from E: Let them be exactly shut every where, but at the Aperture E; and let a Pipe EG, 20 or 22 Inches long, be Sodered to the said Aperture E, having its other end in a Vessel G, full of *Mercury*, and placed near the Middle of the *Bellows*.

A, is an *Axis* for the *Bellows* to Turn upon.

B, A Counterpoise fastened to the lower end of the *Bellows*.

C, A Weight with a Clasp to keep the *Bellows* upright.

Now if we suppose the *Bellows* opened only to $\frac{1}{2}$, or $\frac{1}{3}$, standing Upright, and full of *Mercury*, it is plain that the said *Mercury* being 40 Inches High, must Fall, as in the *Torricellian Experiment*, to the Height of about 27 Inches, and consequently the *Bellows* must open before F, and leave a *Vacuity* there. This *Vacuity* must be filled with *Mercury* Ascending from G through the

Pipe

Pipe G E, the said Pipe being but 22 Inches long: By this means the *Bellows* must be opened more and more till the *Mercury* continuing to Ascend, makes the Upper part of the *Bellows* so heavy, that the Lower part must get loose from the Clasp C, and the *Bellows* should turn quite upside down; but the Vessel G, being set in a convenient place, keeps them Horizontal, and the part F, Engageth there in another Clasp C; then the *Mercury* by its Weight runs out from the *Bellows* into the Vessel C, through the Pipe E G, and the *Bellows* must shut closer and closer until the part E F comes to be so Light, that the Counterpoise B is able to make the part F, get loose from the Clasp C; then the *Bellows* comes to be Upright again as before; the *Mercury* left in them falls again to the Height of 27 Inches, and consequently all the other Effects will follow, as we have already seen, and the *Motion* will Continue for ever.

Fig. 185.

Upon this, it is to be observ'd, That the *Bellows* can never be opened by the Internal Pressure, unless the said Pressure be stronger than the External. Now in the Case before us, it is plain, That altho' the Lowermost part of the *Bellows* be Pressed Outward by 40 Inches of *Mercury*, yet the Upper part having no *Mercury* above it, bears none at all; the parts that lie in the Middle near the Axis of the *Bellows* bear but 20 Inches, and so all the rest must bear more or less, according as they lie Higher or Lower: It is evident therefore, That there are as many parts that bear less than 20 Inches, as there are that bear more, and the Increase of Pressure following an *Arithmetical Progression*, it is undeniable, that all these Pressures added together, will do more than one Uniform Pressure, that would be equal to 20 Inches every where. It is also plain, that the Weight of the *Atmosphere*, cannot come at the Inward Part of the *Bellows*, but through the Pipe G E, which containing 22 Perpendicular Inches, of *Mercury*, doth Counterpoise so much of the Weight of the *Atmosphere*; so that this being supposed to be 27 Inches of *Mercury*, it cannot press the Inward part of the *Bellows*, but with a Weight equivalent to 5 Perpendicular Inches of *Mercury*. So that we find, the Inward Pressure both of the *Mercury* and the *Atmosphere*; is equivalent but to 25 Inches of *Mercury* in all. Whereas the Pressure of the *Atmosphere* upon the Outside is every where equal to 27 Inches; from whence it appears, That the Pressure without, is stronger than the Pressure within. From this we may conclude, that the *Bellows* standing Upright will rather Shut than Open.

And shown Insufficient by him. *ib. p. 1241.* n. 182. p. 138.

I shall say nothing to the Alterations this *Author* may make in his *Engine*, n. 186. p. 267. resolving to leave it to others to shew him, that upon the Principle all he can do signifies nothing. And I doubt not, but if he pleases to consult *M. Perault, de la Hire*, or any other at *Paris*, he will find them of the same Opinion with *Mr. Boyle*, and *Mr. Hook*, and others *Here*.

XX. This *Reflecting Trumpet*, consists of two Parts. The Utmost B b, is a large Concave Pyramid, about a Yard long, (or may be of any manageable Length) Open at the Base B, and Closed not with a Flat, but a Concave Head, at the Cone B. Within this is fastened a Bended Tube A a. Vol. I. T t t This

The Speaking Trumpet Improved; by Mr. J. Conyers. n. 141. p. 1077. Sept. An. 1678.

Fig. 186.

This *Trumpet* did at a Meeting of the *Royal Society* at *Arundel House*, distinctly deliver some Words, cross the Garden and the River *Thames*, and that against the Wind which was then strong; and the Words were written down by one, that was sent over for that purpose: Whereby it appeared, That a *Reflecting Trumpet*, after this, or some other like manner, of Wood, Tin, Pewter, Stone, or Earth, or which may be best of Bell-Metal, will carry the Voice as far, if not farther, than the Long one Invented by Sir *Samuel Moreland*. Besides that, it seems to take off from the Astonishing Noise near at hand, which happens in use of the said Long *Trumpet*: By Sir *Sam. Moreland's Trumpet* Angularly Arched in the Middle, the delivery of Sound to any Distant Place was much shortened; and by another with three large Angular Arches, reaching almost from one End to the Other, the Sound was almost wholly Obstructed.

Fig. 187.

Fig. 188.

The Smifness of
Sounds and
their Reflections
or Echoes; by
Mr. Walker.
n. 247. p. 433.
Dec. An. 1698.

XXI. I provided a *Pendulum*, of small Virginal Wire, with a Pistol Bullet at the end of it, which had two Vibrations in one Second of Time. I took this *Pendulum*, and standing over forgainst a High Wall, I clapt two pieces of small Boards together, and observed how long it was ere the *Eccho* returned; and I Removed my Station till I found the place whither the *Eccho* Returned in about half a Second. But that I might Distinguish the Time more nicely, I clapt every Second of Time, 10 or 15 times together; so that by this means, I could the better Discover whether the Distances betwixt the Claps and the *Ecchoes*, and the following Claps, were Equal. And though it be very difficult to be Exact, yet I could come within some few Yards of the place I sought for, thus: I observed the two Places, where I could but just discover that I was too near, and where I was too far off; and from the midway betwixt them I measured to the Wall, which Measure doubled, was the Space that the Sound Moved in half a Second.

Here follow the Number of *Englisb Feet* which a Sound Moved in one Second of Time at several Trials.

| <i>Trials.</i> | <i>Feet.</i> | <i>Trials</i> | <i>Feet.</i> | <i>Trials</i> | <i>Feet.</i> |
|----------------|--------------|---------------|--------------|---------------|--------------|
| 1 | 1256 | 5 | 1292 | 9 | 1278 |
| 2 | 1507 | 6 | 1378 | 10 | 1290 |
| 3 | 1526 | 7 | 1292 | 11 | 1200 |
| 4 | 1150 | 8 | 1185 | | |

Mersennus mentions an *Experiment* wherein he found the Motion of the Sound to be 1474 Feet in a Second. The *Academy del Cimento* caused 6 *Harquebusses*, and 6 Chambers to be fired one after another at the Distance of 5739 *Englisb Feet*, and from the Flash to the Arrival of the Report each was 5: And repeating the *Experiment* at the Midway, the Motion was exactly in half the

the time; and Mr. Boyle observed. That the *Motion of Sound* passes above 400 Yards in a Second.

When the *First Trial* was made, there was some Wind stirring, tho' not much; the 2d, 3d and 6th were made in a Calm Morning. In the 8th, the *Eccho* was returned from a Wall at 395 Yards Distance in two Seconds. and in the 9th and 10th, at 213 and 215 Yards Distance, in one Second. The 4th was made at one end of St. John's Cloister, in Oxford, which is 104 Feet 7 Inches long, where the Sound was Reflected 11 times in two Seconds: And the 5th, on the North side of New College Cloister (which is 160 Feet 8 Inches long) where there are about $7\frac{1}{2}$ *Ecchoes* in two Seconds.

By some of those Experiments that I tried, I am inclined to think, That the Sound Moved Quicker when it was Calm, than in a Wind, even when the Sound Moved half way with the Wind, and that it Moves swifter at first, than afterwards.

There is seldom any *Eccho*, where there is not some Wall, Wood, Bank, or such like, directly Opposite, that may Reflect the Sound to the Person that makes it; but in St. John's Grove, if you stand near the Gate leading from the College to the Grove, and Clap, the *Eccho* will return to you from the Ball Court, though a Line drawn from you to the Ball Court be not Perpendicular to the Wall there, but as much Oblique as the Line AB, is to the Line BC; where A represents the Gate, BC the Ball Court Wall, and BD another Wall. Or, if you stand at E, the Corner of the Grove next to Trinity, and Clap, the *Eccho* will Return to you, from the Ball-Court.

Fig. 180.

In the same Grove, I stood about 20 Yards from the same Gate, and the Gate being shut, Clapt, and at other times Stamped, and the *Eccho* Returned from the Gate as loud, if not louder than the Clap or Stamp.

An *Eccho* Reflected from a Gate or Door, has usually a baser and duller Sound than that which is returned from a Wall, this being much brisker.

As I have been walking towards a Wall, I have Clapped my Hands together several times, and I could distinguish the *Eccho* from the Clap, 'till I came within 7 or 8 Yards of the Wall.

In the Cloisters, where, as we said before, the *Eccho* was Repeated several times, the first Repetition seemed to be slower than the second or third; but of all the Repetitions, besides the first, the subsequent seemed slower than the precedent.

I have observed the Tossing of a Sound forward and back again, in very many Places where there are Parallel Walls; and where the Distance of the Walls is less, there the *Ecchoes* follow one another quicker.

Wheresoever a Sound was thus tossed betwixt two Walls, if I stood about the Middle, I could hear the Sound twice as quick, that is twice as often Repeated in one Second, as if I stood near one Wall: The Sound being Reflected to me from both ends, when I stood in the middle.

In Trinity Ball Court, when I stood and Clapt at B, three or four Yards from the End of the Wall C, or at A, which is oppositeto B, the Sound was tossed betwixt the Opposite Walls, but not half so long as when I stood betwixt the Walls. In Places where there are Parallel Walls, not above six or

Fig. 150.

eight Yards asunder, as in *Trinity Ball Court*, and at the Entrance into *St. John's Grove*, &c. I have heard the *Ecchoes* of a Clap following one another distinctly enough: But there the *Ecchoes* of a *Musical Note*, which was longer than a Clap, were so confused, that they seemed one continued *Long Sound*: which makes me think, that the *Eccho* in some Vaults, is nothing else but the *Sound* tossed betwixt the Side Walls, and betwixt the Top and Bottom. This also makes me conjecture, That the Reason why *stringed Musical Instruments* give a greater and longer *Sound* to the *Strings* than if the *Strings* were fixt to a single Board, may be this; because the *Sound* is tossed from side to side in the *Belly* of the *Instrument*.

*The Doctrine of
Sounds; by
Narcissus,
Bishop of Ferns
and Leighlin.
p. 156. p. 472.
Nov. An. 1683.*

XXII. I cannot better Explain the *Usefulness* of this *Theory* of *Sounds*, than by making a Comparison 'twixt the Faculty of *Seeing* and *Hearing* as to their Improvements. In order to which, I Observe, That *Vision* is threefold, *Direct*, *Refracted*, and *Reflex'd*, answerable whereunto we have *Opticks*, *Dioptricks*, and *Catoptricks*.

In like manner *Hearing* may be Divided into *Direct*, *Refracted* and *Reflex'd*; where to answer three Parts of our *Doctrine* of *Acousticks*, which are yet nameless, unless we call them *Acousticks*, *Diaousticks*, and *Cataousticks* (or in another Sense, but to as good Purpose) *Phonicks*, *Diaphonicks*, and *Cataphonicks*.

Direct Vision has been Improved two ways.

1. *Ex parte Objecti*, by the Arts of *Producing*, *Conserving*, and *Imitating*, and duly *Applying*, *Light* and *Colours*.

2. *Ex parte Organi vel Medii*, by making use of *Tubes* without *Glasses*, or, a Man's Closed Hand to look through. So likewise *Direct Hearing*, partly has, and partly may further receive great and notable Improvements, both *ex parte Objecti*, and *ex parte Organi vel Medii*.

1. As to the Object of *Hearing*, which is *Sound* Improvement has been and may be made, both as to the *Begetting*; and as to the *Conveying* and *Propagating* (which is a kind of *Conserving*) of *Sounds*.

1. As to the *Begetting* of *Sounds*. The Art of *Imitating* any *Sound* whether by *Speaking* (that is *Pronouncing*) any kind of *Language* (which really is an *Art*; and the *Art* of *Speaking* perhaps one of the greatest) or by *Whistling*, or by *Singing* (which are allowed *Arts*) or by *Hollowing*, or *Luring*, (which the *Huntsman* and *Faulkner* would have to be an *Art* also) or by *Imitating* with the *Mouth* (or otherwise) the *Voice* of any *Animal*; as of *Quails*, *Cats*, and the like, or by representing any *Sound* begotten by the *Collision* of *Solid Bodies*, or after any other manner; these are all *Improvements* of *Direct Hearing*, and may be *Improved*.

Moreover, the Skill to make all sorts of *Musical Instruments*, both *Ancient*, and *Modern*, whether *Wind Instruments* or *String'd*, or of any other Sort, whereof there are very many (as *Drums*, *Bells*, the *Systrum* of the *Egyptians* or the like) that *Beget* (and not only *Propagate*) *Sounds*: The Skill of Making these, I say, is an *Art*, that has much *Improv'd Direct Hearing*, and an *Harmonious Sound* exceeds a *Single* and *Rude* one, that is an *Immusical Tune*: which *Art* is yet capable of farther Improvement. And I hope, That by the Rules,

Rules, which may happily be laid down, concerning the *Nature, Propagation, and Proportion or Adapting of Sounds*; a way may be found out, both to Improve *Musical Instruments* already in use, and to Invent New Ones, that shall be more Sweet and Lushious than any yet known. Besides, that by the same means *Instruments* may be made, that shall Imitate any *Sound* in Nature, that is not *Articulate*; be it of Bird, Beast, or what thing else soever.

2. The *Conveying and Propagating* (which is a kind of *Conserving*) of *Sounds*, is much helped by duly *Placing* the *Sonorous Body*, and also by the *Medium*.

For if the *Medium* be *Thin* and *Quiescent*, and the *Sounding Body Placed* conveniently, the *Sound* will be easily and regularly *Propagated* and mightily *Conserved*.

1. The *Medium* must be *Thin* and *Quiescent*; Hence in a still Evening, or the Dead of the Night (when the Wind ceases) a *Sound* is better sent out, and to a greater Distance than otherwise.

2. The *Sonorous Body* must be *Placed* conveniently, viz. Near a *Smooth Wall*, either *Plain* or *Arched* (*Cycloidically* or *Elliptically*, rather than otherwise; though a *Circular* or any *Arch* will do; but not so well.) Hence in a *Church*, the nearer the *Preacher* stands to the *Wall* (and certainly it's much the best way to place *Pulpits* near the *Wall*) the better is he heard, especially by those who stand near the *Wall*, also, though at a greater Distance from the *Pulpit*, those at the Remotest End of the *Church*, by laying their Ears somewhat close to the *Wall*, may hear him easier than those in the *Middle*.

Hence also do arise *Whispering Places*. For the *Voice* being applied to one End of an *Arch*, easily Rowls to the other. And indeed were the *Motion and Propagation of Sounds* but rightly understood, 'twould be no hard matter to contrive *Whispering Places* of infinite variety and use. And perhaps there could be no better or more pleasant Hearing a *Consort of Music*, than at such a Place as this; where the *Sounds* Rowling long together before they come to the Ear, must needs Consolidate and Imbody in one; which becomes a true *Composition of Sounds*, and is the very Life and Soul of *Consort*.

2. If the *Sonorous Body* be placed near *Water*, the *Sound* will easily be *Convey'd*, yet mollify'd; as Experience teacheth us from a *Ring of Bells* near a *River*, and a great *Gun* shot off at *Sea*; which differ much in the *Strength*, and yet softness and continuance, or *Propagation of their Sounds* from the same at *Land*; where the *Sound* is more *Harsh* and more *Perishing*, or much sooner *Decays*.

3. In a *Plain* a *Voice* may be heard at a far greater Distance, than in *Uneven Ground*. The Reason of all which, last nam'd *Phanomina* is the same, because the *Sonorous Air* meeting with little or no *Resistance* upon a *Plane* (much less upon an *Arch'd*) *smooth Superficies*, easily Rowls along it, without being let or hindred in its *Motion*, and consequently without having its *Parts* *Disfigured*, and put into another kind of *Revolution*, than what they had at the first *Begetting of the Sound*, which is the true Cause of its *Preservation* or *Progression*; and fails much when the *Air* passes over an *Uneven Surface*, according to the *Degrees of its Inequality*; and somewhat also, when it passes over the *Plane Superficies* of a *Body*, that is hard and resisting.

Where-

Wherefore the *smooth Top* of the *Water* (by reason of its yielding to the *Arched Air*, and gently rising again with a kind of *Refurge*, like to *Elasticity* though it be not so; by which *Refurge* is *Quicken*s and *Hasten*s the *Motion* of the *Air* Rowling over it, and by its yielding preserves it in its *Arched Cycloidical* or *Elliptical Figure*) the *smooth Top* of the *Water*, I say, for these *Reasons*, and by these means, *Conveys* a *Sound* more entire, and to a greater *Distance*, than the *Plane Surface* of a piece of *Ground*, a *Wall*, or any other *Solid Body* whatever, can do.

2. The *Organ*, which is the *Ear*, is help'd much by *Placing* it near a *Wall*, (especially at one end of an *Arch*, the *Sound* being *Begotten* at the other) or near the *Surface* of *Water*, or of the *Earth*; along which the *Sounds* are most easily and naturally *Conveyed*; as was before declared. And 'tis Incredible, how far a *Sound* made upon *Earth* (by the *Trampling* of a *Troop* of *Horses*, for Example) may be heard in a still *Night*, if a *Man* lays his *Ear* close to the *Ground* in a large *Plane*.

Acousticks here come in for helping the *Ear*; which may be so contrived (by a right understanding the *Progression* of *Sounds*, which is the Principal Thing to be known for the due regulating all such kinds of *Instruments*) as that the *Sound* might enter the *Ear* without any *Refraction*.

2. *Refracted Vision* (which is always made *ex parte Medii*) arises from the different *Density*, *Figure*, and *Magnitude* of the *Medium*; which is somewhat altered by the divers *Incidence* of the *Visible Rays*, and so it is in *Refracted Hearing*, all these *Causes* concur to its *Production*; and some others to be hereafter considered.

Now as any *Object* (a *Man* for Example) seen through a *Thickened Air*, by *Refraction* appears greater than really he is: So likewise a *Sound* heard through the same *Thickened* part of the *Atmosphere*, will be considerably vary'd from what it would seem to be, if heard through a *Thinner Medium*. And this I call a *Refracted Sound*.

Improvements of *Refracted Vision* have been made, by *Grinding* or *Blowing* *Glasses* into a certain *Figure*, and placing them at due *Distances*; whereby the *Object* may be (as 'twas) enabled to send forth its *Rays* more *Vigorously*, and the *Visive Faculty* Impowered the better to receive them. Thus,

1. A fine *Glass Bubble*, filled with *Clear Water*, and *Placed* before a *Burning Candle* or *Lamp*, does help it to dart forth its *Rays* to a *Prodigious Length* and *Brightness*.

2. The *Visive Faculty* is much *Helped*.

1. By *Spectacles* and other *Glasses* which are made to *Help* the *Purblind* and *Weak Eyes*, to see, at any competent *Distance*.

2. By *Perspective Glasses* and *Telescopes*, which *Help* the *Eye* to *See* *Objects* at a very great *Distance*, which otherwise would not be discernible.

3. By *Microscopes* or *Magnifying Glasses*, which help the *Eye* to see *Near Objects*, that by reason of their smallness were *Invisible* before.

4. By *Polyscopes* or *Multiplying Glasses*, whereby one thing is represented to the *Eye* as many, whether in the same or different *Shapes*.

After the same manner, Instruments may be contrived for assisting both the *Sonorous Body* to send forth its *Sound* more strongly, and the *Audible Faculty* to receive and discern it more easily and distinctly. And thus;

1. An Instrument may be Invented, that applied to the *Mouth*, (or any *Sonorous Body*) shall send forth the *Voice* Distinctly as to a prodigious *Distance* and *Loudness*. For if the *Stemato-phomon* (which is but a Rude and Inartificial Instrument,) does such great Feats; what might be done with One composed according to the Rules of Art? whose *Make* should comply with the Laws of *Sonorous Motion* which that does not.

2. There are some *Instruments*, and more such may be Invented to help the *Ear*: As,

1. *Oraucousticks* (and better may be made) to help *Weak Ears* to hear at a reasonable *Distance* also. Which would be as great a help to the *Infirmity* of *Old Age*, as the other Invention of *Spectacles* is, and perhaps greater; for as much as the *Hearing* what's spoken is of more daily use and concern to such Men, than to be able to *Read Books*, or to view *Pictures*.

2. A sort of *Oraucousticks* may be so contrived, as that they shall Receive in *Sounds* made at a very great *Distance*, which otherwise would have been *Inaudible*: And these *Oraucousticks*, in some *Respects*, would be of greater use than *Perspectives*.

1. In Time of *War* for discovering the *Enemy* at a good *Distance*, when he *Marches* or *Lies Incamp'd* behind a *Mountain* or *Wood*, or any such *Place* of *Shelter*, which hinder the *Sight* from reaching very far.

2. At *Sea*, when in dark *Foxy Weather* the *Air* is too thick, or in *Stormy Tempestuous Weather*, the *Waves* Rise too *High*, for the *Perspectives* to be made use of.

3. In *Dark Nights*, when *Perspectives* become almost *Insignificant*, and yet at such times, generally *Soldiers* take their *March*, when they would surprise their *Enemies*.

4. *Microphones*, or *Micraucousticks*, that is, *Magnifying Ear Instruments*, which may be Contrived after that manner, that they shall render the most *Minute Sound* in *Nature* distinctly *Audible*, by *Magnifying* it to an unconceivable *Loudness*. By the help whereof we may hear the different *Cries* and *Tones* of the smallest *Animals*.

4. A *Polyphone*, or *Polyucoustick*, so ordered that One *Sound* may be heard, either of the Same, or a Different *Note*. Inasmuch that who uses this Instrument, he shall at the Sound of a *Single Viol* seem to hear a whole *Consort*, and all *True Harmony*. By which means this *Instrument* has much the Advantage of the *Polyscope*.

I have called it *Refracted Hearing*, because made through a *Medium*, viz. Thick *Air*, or an *Instrument*, through which the *Sound* passing is broken or *Refracted*.

3. *Reflected Vision* (which is always made *ex parte Objecti*) hath been Improv'd by the Invention of *Looking Glasses* and *Polish'd Mirah*, whether *Plane*, *Concave* or *Convex*, of several *Figures*, and Placed at *Determinate Distances*.

In like manner, *Reflex'd Audition* (which is only made *ex parte Corporis Op-positi*) may be Improved by Contriving several sorts of *Artificial Echoes*. For (speaking in general) any *Sound* falling *Directly* or *Obliquely* upon any *Dense Body*, of a smooth (whether *Plane* or *Arch'd*) *Superficies*, is beat back again and *Reflected*, or does *Echo* more or less.

I say, (1.) *Falling Directly or Obliquely*; because, if the *Sound* be sent out and *Propagated Parallel* to the *Surface* of the *Dense Body*, there will be no *Reflection of Sound*, no *Echo*.

I say, (2.) Upon a *Body* of a *Smooth Superficies*; because if the *Surface* of the *Corpus Obstant* be *Uneven*, the *Air* by *Reverberation* will be put out of its *Regular Motion*, and the *Sound* thereby broken and extinguish'd: So that, tho' in this case also the *Air* be beaten back again, yet *Sound* is not *Reflected*, nor is there any *Echo*.

I say, (3.) It does *Echo More or Less*, to shew, that when all things are, as is before describ'd, there is still an *Echoing*, though it be not always *Heard*, either because the *Direct Sound* is too *Weak* to be beaten quite back again to him that made it; or that it does *Return* home to him, but so weak, that without the help of a good *Otacoustick* it cannot be discerned; or that he stands in a *wrong Place* to receive the *Reflected Sound*, which passes over his *Head*, under his *Feet*, or to one side of him; which therefore may be *Heard* by a *Man* standing in that place, where the *Reflected Sound*, will come, provided no *interpos'd Body*, does *intercept* it; but not by him that first made it.

These *Echoes* (like *Reflected Vision*) may be several ways *Produced*, as;
1. A *Plane Corpus Obstant* *Reflects* the *Sound* back in its due *Tone* and *Loudness*; if allowance be made for the proportionable *Decrease* of the *Sound* according to its *Distance*.

2. A *Convex Corpus Obstant* *Repels* the *Sound* (insensibly) *Smaller*; but somewhat quicker (though weaker) than otherwise it would be.

3. A *Concave Corpus Obstant* *Echoes* back the *Sound* (insensibly) *Bigger*, *Slower*, (though *Stronger*) and also *Inverted*; but never according to the order of *Words*. Nor do I think it possible for the *Art* of *Man* to *Contrive* a single *Echo*, that shall *Invert* the *Sound* and *Repeat* backwards; because then the *Words* last spoken, that is, which do last occur to the *Corpus Obstant*, must first be *Repell'd*; which cannot be. For where in the mean time should the first *Words* hang and be conceal'd or lie dormant? Or how, after such a *Pause* be *Reviv'd* and *Animated* again into *Motion*? Yet in *Complicated* or *Compound Echoes*, where many *Receive* from one another, I know not whether something that way may not be done.

From this *Determinate Concavity* or *Archedness* of these *Reflecting Bodies* it comes to pass, that some of them from a certain *Distance* or *Positure*, will *Echo* back but one *Determinate Note*, and from no other *Place* will they *Reverberate* any; because of the undue *Position* of the *Sounding Body*. Such an one (as I remember) is the *Vault* in *Merton Colledge* in *Oxford*.

4. *The Echoing Body*, being Removed farther off, *Reflects* more of the *Sound*, than when nearer. And this is the Reason, why some *Echoes* Repeat but one Syllable, some one Word, and some many.

5. *Echoing Bodies* may be so contriv'd and Plac'd, as that *Reflecting* the *Sound* from one to the other, either *Directly* and *Mutually*, or *Obliquely* and by Succession, out of one *Sound* shall many *Echoes* be begotten; which in the first Case will be altogether and somewhat Involv'd or Swallow'd up of each other; and thereby Confused (as a Face in Looking-Glasses obverted;) in the other they will be Distinct, Separate and Succeeding one another, as most *Multiple Echoes* do.

Moreover, a *Multiple Echo* may be made, by so Placing the *Eechoing Bodies*, at Unequal Distances, that they *Reflect* all one way, and not One on the Other; by which means a *Manifold Successive Sound* will be heard (not without astonishment;) One Clap of the Hands like many; One *Ha* like a Laughter; One single Word like many of the same *Tone* and *Accent*; and so one *Viol* like many of the same kind, Imitating each other.

Furthermore, *Echoing Bodies* may be so ordered, that from any one *Sound* given, they shall produce many *Echoes* different both as to their *Tone* and *Intention*. By this means a *Musical Room* may be so Contrived, that not only One *Instrument*, played on in it shall seem many of the same Sort and Size; but even a *Consort* of (somewhat) Different ones; only by Placing certain *Echoing Bodies* so, as that any *Note* (played) shall be Returned by them in *ds*, *5th*, and *8ths*, which is possible to be done otherwise than was mentioned before in *Refracted Audition*.

I have been thus large, that I might give you a little Prospect into the Excellency and Usefulness of *Acousticks*, and that thereby I might excite others to bend their Thoughts, towards the making of *Experiments* for the Compleating this (yet very Imperfect though Noble) Science; a *Specimen* whereof I will give you in these three *Problems*.

Prob. I.] *To make the least Sound (by the help of Instruments) as Loud as the Greatest; a Whisper to become as loud as the Shot of a Cannon.*

By the help of this *Problem* the most minute Sounds in Nature may be Clearly and Distinctly heard.

Prop. II.] *To Propagate any (the least) Sound to the greatest Distance.*

By the help hereof any *Sound* may be Conveyed to any, and therefore heard at any *Distance*, (I must add, within a certain, though very large Sphere.)

Moreover by this means a *Weather-Cock* may be so contrived, as that with an Ordinary Blast of Wind it shall Cry (or Whistle) Loud enough to be heard many Leagues. Which happily may be found of some Use, not only for Pilots in mighty Tempestuous Weather, when *Light Houses* are rendred almost uselefs: But also for the Measuring the Strength of Winds, if allowance be made for their Different Moisture. For I conceive, That the more Dry any Wind is, the Louder it will Whistle, *ceteris paribus*; I say, *ceteris paribus*, because, besides the Strength and Dryness of *Winds* or *Breath*, there

are a great many other things (hereafter to be consider'd) that Concur to the Increase of *Magnifying of Sounds*, begotten by them in an Instrument exposed to their Violence, or Blown into.

Prob. III.] That a Sound may be convey'd from one Extream to the other (or from one Distant Place to another) so as not to be heard in the Middle.

By the help of this Problem a Man may talk to his Friend at a very considerable Distance, so that those in the Middle Space shall hear nothing of what passed betwixt them.

Fig. 197.

I shall here Add a *Semiplane* of an *Acoustick* or *Phonical Sphere*, as an Attempt to explicate the great Principle in this Science, which is the *Progression of Sounds*.

You are to conceive this (Rude) *Semiplane* as *Parallel* to the *Horizon*; for, if it be *Perpendicular* thereunto, I suppose the upper *Extremity* will be no longer *Circular*, but *Hyperbolical*, and the lower part of it suited to a greater *Circle* of the *Earth*. So that the whole *Phonical Sphere* (if I may so call it) will be a solid *Hyperbola*, standing upon a *Concave Spherical Base*. I speak this concerning *Sounds* made (as usually they are) nigh the *Earth*, and whose *Sonorous Medium* has a free passage every way. For if they are Generated *High* in the *Air*, or *Directed* one way, the Case will be different; which is partly Designed in the *Inequality* of the *Draught*.

XXIII. A Paper, of less General Use, Omitted, viz.

Carriages.

n. 161. p. 666.

Experiments to be made, relating to Carriages; proposed by Sir William Petty.

XXIV. Accounts of Books and Additions, Omitted.

- n. 32. p. 626. I. **D**E vi Percussionis, Joh. Alphonsi. Borelli: Bononia, 1667. in 4to.
 n. 73. p. 2210. 2. De Motionibus à Graviate dependentibus Liber, Jo. Alphonsi Borelli, in Academia Pisana Matheseos Professoris, Regio Julio 1670. in 4to.
 n. 67. p. 2057. 3. Dialogi Physici, quorum Primus de Lumine; Secundus & Tertius de Vi Percussionis & Motu; Quartus de Hemoris Elevatione per Canaliculum; Quintus & Sextus de Variis Selectis. Auth. Honor. Fabry, S. Jesu. Lugduni Galliarum, 1669. in 8vo.
 n. 54. p. 1086. 4. Mechanica, sive de Motu, Tractatus Geometricus; Auth. Joa. Wallis, S. S. Th. D. Londini, 1670. 1671. in 4to. The Author here makes some Additions to Prop. I. Chap. XV. p. 753. concerning the Center of Gravity of the Hyperbola.
 n. 61. p. 2008. 5. Exercitationes Mechanicae, Alexandri Marchetti. Pisis, 1669. in 4to.
 n. 82. p. 4050. 6. De Resistentia Solidorum, Alexandri Marchetti in Pisana Academia Phil. Prof. Florentiae. 1665. in 4to.
 n. 73. p. 2213. 7. Hypothesis, Physica nova, sive Theoria Motus Concreti, una cum Theoria Motus Abstracti. Auth. Gothfredo Gulielmo Leibnitio. J. V. D. Lond. 1621 in 12. Of this Book Dr. Wallis here gives his Opinion.
 n. 74. p. 2227.

8. *La Statique, ou la Science, de Forces Mouvantes* par le P. Ignace Gaston n. 94. p. 6042
 Pardies. S. J. à Paris 1673. in 12°. *The First part being of Local Motion.*
 Printed at Paris, 1670. was Englished and Printed at London, the same Year, n. 63. p. 2010
 in 12°.

9. *Christiani Hugonii Zulichemii Horologium Oscillatorium.* Parifiis, 1673 n. 95. p. 6068.
 in Fol.

10. *A Discourse made before the Royal Society concerning the Use of Dupli- n. 109. p. 209.*
 cate Proportion in sundry Important Particulars; together with a New Hypo-
 thesis of Elasticque or Springy Bodies: By Sir William Petty.

11. *Traité de la Percussion ou Choq des Corps, &c. par M. Mariotte, de n. 134. p. 859*
 l'Academie Royal des Sciences. A Paris, 1673. in 12°.

12. *Philosophiæ Naturalis Principia Mathematica.* Authore Is. Newton n. 186. p. 291.
 Lond. in 4to. n. 226. p. 445.

13. *Traité de Mouvement des Eaux & des autres Corps Fluids par feu; n. 181. p. 191*
 M. Mariotte. A Paris. 1686, in 8vo.

14. *Mechanick Exercises; or, The Doctrine of Handy Works.* By Mr. Jos. n. 138. p. 967.
 Moxon. Lond. 1677. in 4to. n. 139. p. 987.

15. *The Speaking-Trumpet, as it hath been Controverted, and Published, by Sir n. 79. p. 3093*
 Samuel Moreland; together with its Uses both at Sea and Land. Lond. 1671.

CHAPTER VI.

Hydrostaticks. Hydraulicks.

1. **T**AKE a *Viol* with a very *Narrow Body*, and, when it is almost full, *To Weigh Water,*
 the Water is to be Dropt into it, drop by drop, till it can hold no *or other Fluids;*
 more. Then *Weigh* it exactly, and deduct the *Weight* of the *Empty Viol*.

2. A Is a *Glass Bottle* like a little *Mattacium*, of which the *Neck B C* *A New Armo-*
 is so small that a Drop of Water therein takes up the Space of 5 or 6 Lines, *meter; by*
 near that *Neck* is a little *Capiller Tube D*, about 6 Lines long, and Parallel *M. Comberg.*
 to the *Neck B C*; The *Opening B* is a little dilated, in the Fashion of a *n. 262. p. 330.*
Tunnel, for pouring more easily the *Liquors* into the *Bottle*, and the little *Fig. 192.*
Tube D is for giving a way to the *Air* contained in that *Vessel* to go out,
 when the *Liquor* is poured in at *B*; the *Point C* is a little *Mark* at the
 same height, as the end of the little *Tube D*.

When we fill the *Vessel*, we pour the *Liquors* into it, by the *Opening B*,
 until it goes out by the little *Tube D*, and, if the *Height* of the *Liquor* is
 even to the *Mark C*, 'tis well; if it is *Lower*, we must fill more to that
Point; if it is *Higher*, we must strike softly upon the *Opening B*, till the
Overplus of the *Liquor* be even to the *Point C* in the *Neck* of the *Bottle*.
 By that means we have always exactly the same *Volume* of *Liquor*, and we
 can know how the same *Volume* of the several *Liquors* weighs more or less than

another precisely. But we must consider the Variation of the Weather, when we compare the Weight of a Liquor which we weigh in Summer time, with the Weight of another which we have weighed in the Winter; for the same Liquor, being more Rarified in the Hot time, and condensed in the Cold, the same Volume of it will be more Weighty in Cold Weather. than in Warm.

*A New Essay
Instrument;
by M. Boyle.
n. 24. P. 447.
n. 115. P. 329.
Fig. 193.*

II. 1. Many Years ago I made use of a little *Glass Instrument*, consisting of a *Bubble*, and furnished with a *Long and Slender Stem* to compare the *Specifick Gravities* of *Different Liquors* by its more or less *Sinking in Them*: And I have since employed it to discover the *Specifick Gravities* of *Solid*, several appended, by its being more or less depressed by them in the same Liquor. For 'tis clearly deducible from the Grounds of *Hydrostaticks*, that any Solid Body Heavier than Water, loses in the Water as much of the Weight it had in Air, as Water of equal Bulk to the Immersed Solid would weigh in the Air; and consequently, since Gold is by far the most Ponderous of Metals, a piece of Gold and one of Equal Weight of Copper, Brass, or any other Metal, being proposed, the Gold must be less in bulk, than the Copper or Brass. And by this means, if both of them be weighed in the Water, the Gold must lose in that Liquor less of its former Weight than the Brass or Copper; because the baser Metal, as well as the Gold, grows lighter by the weight of a Bulk of Water equal to it; and the baser Metal being the more Voluminous, the correspondent Water must weigh more than that which is correspondent to the Gold. Whence I concluded, that the *Floating Instrument* abovementioned would be made to sink deeper by an Ounce, for Instance of Gold, hanging at it under Water, than by an Ounce of Brass, or any other Metal, which, by reason of its greater Bulk than Gold, losing more of its weight by the Immersion, must needs retain less, and so have less power to Depress the Instrument 'twas fastened to. Which Conclusion will also hold (though the Disparity be not so great and Conspicuous) in reference to other Metals, as Lead and Tin, that differ in *Specifick Gravity*.

This *Instrument* may be of *Glass*, *Copper*, *Silver*, or almost any other Solid Body, that is, or may be made, fit to Float on the Water, with a *Guinea*, &c. Hanging at it, and of a Texture close enough to keep out the Water. It consists of three Parts: The *Ball* or Globulous part; the *Stem* or Pipe; and that which Holds the *Coin*.

Fig. 194.

The *Ball* or Round part of BCDE (if of *Metal*) consists of two thin Concave Plates, exactly Sodered together in the middle; and at the distantest parts from the Commissure, there ought to be left two opposite *Holes*, one in each Plate, for the two other parts of the Instrument. This middle part, though for Brevities sake we name it the *Ball*, should not be exactly Round; but of any Shape that shall be found fit to make the *Instrument* keep to its Erect posture steadily in the Water. It must contain as much Air as may serve to keep the whole *Instrument* when loaded, from Sinking beneath the top of the *Stem*.

The *Stem* A B is to be Soldered on to the *Ball* at the uppermost of the two mention'd *Holes*. It may be either Hollow or Solid: But it ought to be made

made very slender, that the different Depressions of the Instrument in the Water may be the more Notable. And, for the same reason, it ought not to be too Short, especially if it be to be applied to other Uses than the Examining of *Guineys*.

At the Undermost of the two *Holes* in the *Ball*, is Inserted and Solder'd the undermost part of the *Instrument*, which I call the *Screws*, or the *Stirrup*. The *Screw F* is a very short Piece of *Brass* with a broad Slit in it, capable of receiving the edge of the *Guinea*, which with one turn or two of a small and slight lateral *Screw* may be kept fast in it, and readily, the Operation being ended, taken out again. The *Stirrup G* is made of a piece of *Wire*, that, a little beneath the bottom of the *Ball*, is bent round, so as to stand Horizontally, that the *Guinea* may be laid on it.

Fig. 193

It would be convenient, that the undermost *Stem* and the *Screw* be made by it self, that it may be at pleasure thrust upon the *Stem*, and taken off again. For, by this means, if the *Ball* of the *Instrument* be made large enough, you may have room to put on for *Ballast*, as occasion shall require, one, two, or three flat and round pieces of *Copper*, *Lead*, &c. with each of them a *Hole* in the middle fitted to the Size of the *Stem*, so that they may be put on as near the Lower part of the *Ball* as you think fit, and then the *Screw* may be thrust on after them, not only to take hold of the *Coin* or *Metalline* mixture to be *Examined*, but to support the thin *Plates*.

Fig. 194

To adjust this *Instrument* for the use of examining *Guineas*, which are by far the most usual *Gold Coins* that pass in *England*, you must by the help of the *Stirrup* or *Screw*, Hang, at the bottom of it, a piece of that *Coin* which you know to be *Genuine* and (having carefully stoppt the Orifice of the *Stem* (if it be a *Pipe*) that no *Water* may get in at it, Immerse the *Instrument* leisurely and perpendicularly into a *Vessel* full of clean *Water*, 'till it be Deprest almost to the top of the *Stem*, and then; letting it alone, if, being Settled, it continue in the same *Station* and *Posture*, your work is done, if it *Emerge*, you must add a little weight to it, either by putting into the *Stem*, if it be *Hollow*, some *Dust Shot*, *Filings* of *Lead*, or some other *Minute* and *Heavy* *Body*, or else by putting on the short *Stem* abovementioned, that comes out beneath the *Ball*, a flat, round and perforated piece of *Lead*, of *Weight* sufficient to enable the *Guinea* to Depress the weight as low as 'tis desired: But, if it sink quite under *Water*, you must lighten it either with a *File*, or by scraping or grating off a little of the *Ballast Plate* abovementioned; or, if you have put any *Weight* into the *Cavity* to poize it, by taking out some of that, till you have made it *Light* enough: This being done. a *Mark, H* is to be made just at the place where the *Surface* of the *Water* touches the *Stem*, and then taking out your *Instrument*, substitute, in the place of your *Guinea*, a little round *Plate* of *Brass*, of the same *Weight*, or a *Grain* or two heavier, in the *Air*; and, putting the *Instrument* into the *Water*, as before, suffer it to *Settle*, and make another *Mark I*, at the *Interfection* of the *Stem* and the *Horizontal Surface* of the *Water*.

Fig. 193.
Fig. 194.

There may (though 'tis like there very seldom will) happen a *Case*, wherein, though the *Principle*, our *Instrument* is framed on, will hold good, yet the

the Practical Application may be Unsecure. For, if a *Falsifier of Money* have the Skill, by *Washing* or otherwise, to take off much of the Quantity or Substance of the *Guinea*, without altering or impairing either the *Figure* or *Stamp*, the piece of Coin will not be able to Depress our Instrument to the Usual Mark, and may thereby make it to be judged *Counterfeit*, when 'tis indeed but too *Light*. But it presently shews, that the proposed *Guinea*, if it be not *Counterfeit*, is otherwise *Abused*; and, though it does not clearly determine, whether that likewise proceed from the want of *Specific Gravity* in the Metal, or from the Coins having been *Washed* or otherwise fraudulently *Lessened*; yet it probably resolves the doubt, because, if the want of Weight appear by the *Instrument* to be very great, as it usually does, where the piece has been Robbed of some of its Substance, 'tis a strong Presumption, that 'tis rather *Washed*, &c. than *Counterfeited*. However, it will be sure to prompt him that uses it, to employ the *Ballance*, which will presently assist him to resolve his Doubt. For if the Suspected Coin have in the Air its due Weight, 'twill argue that the great Lightness of it in the Water proceeds from its not being of the requisite fineness; and, if it want much of its due Weight in the Air, 'tis very probable, that 'tis *Washed*, &c. rather than of another Metal than *Gold*.

Any other kind of *Gold Coin*, that is near about the Weight of a *Guinea*, may be Examined by our *Instrument* after the manner above deliver'd. If the Coin be Heavier than a *Guinea*, as is a twenty Shilling piece of *Broad Gold*, the *Ballast*, whether internal or external, of the *Instrument*, must be taken off, that so Heavy a Coin may not quite sink it. But, if it be Lighter than a *Guinea*, one may add as much *Gold* (of the same *Alloy*) beaten into thin Plates, as with the Coin proposed, will make up in the Air the Weight of a *Guinea*. For then this Aggregate, being examined as if it were a *Guinea*, will discover in the Water, whether the Coin be *Right* or *Counterfeit*.

This *Instrument* may be also made to serve to examine some sorts of *White Money*, less Heavy than *Half-Crowns*. And, because it may be useful to know in General, what Coins may, and what may not, be Examined by this or that particular *Instrument* proposed, I shall here add a general way that is not difficult for finding this out; namely, first by Weighing the piece of *Gold* or *Silver* in the Air, and afterwards in the Water, and Subtracting the latter from the former, to obtain the Difference of the two Weights: And next by Weighing also in the Air and in the Water a piece of *Copper* or *Brass*, if this be the likeliest to be employed in *Counterfeiting* the Coin, and observing likewise the Difference between those Weights. For, the lesser of these Differences being Subtracted from the greater, the remains will shew, how much the true piece of Coin will out-weigh the other in the Water, and consequently, if so many Grains, as this residue amounts to, being Added to the Weight of the Lighter Metal, do make a sufficiently manifest Depression of it below the Mark it would stay at without that Addition, one may probably Conclude, that the Difference between a True and *Counterfeit* piece of Coin proposed will be discoverable by the *Instrument*. But it may be Expedient, for those that have frequent Occasions to Examine Various sorts of Coin,

to have a several *Instrument* adjusted for each of them, to save themselves some Pains and Trouble.

With this *Instrument*, Pure *Tin* may be certainly Distinguish'd from such as is Adulterated. For, as *Gold*, being the Heaviest of Metals, cannot be Allay'd by any other that will not Depress our *Instrument* less than *Gold* can do; so *Tin*, being the Lightest of Metals, cannot be mixed with any other that will not Sink it Lower than unmix'd *Tin*, (still supposing the Weight to be the same in the Air.)

After the same manner may *Pewter* be Compared and Examined. For, having once observ'd how much the *Instrument* is Depress'd by a piece of two, three, or four Drams, or even an Ounce Weight of *Pewter*, which is known to be good, and to contain such a proportion of Lead in reference to the *Tin*, if you load the *Instrument* with an equally Heavy piece of any other Mass of *Pewter* propounded, if the *Instrument* Sink deeper, 'twill be a sign that the former Proportion of Lead may be very probably argued to exceed in the mixture; I say probably, because perhaps 'tis possible to Embase *Pewter* by Mixing not only Lead, but other Mineral Substances, whose *Specifick Gravity* is not well known: But yet I say very probably, because the Addition of too much Lead is the most Gainful way of Adulterating *Pewter*.

This *Instrument* may also assist us, to make such an Estimate as will not much Deceive us of the *Fineness* of *Gold* and its differing *Allays* with *Silver*, or some other determinate Metal.

In order to this, the *Instrument* may be fitted to sink to the tip of the Pipe with some determinate weight of the *Finest Gold*, as of 24 Carats, as they call that which is most Pure and Fine. But 'twill be convenient, that this Metal, in the Air, be just an Ounce, or half an Ounce, or some such Determinate Weight, that is commodiously Divisible into many aliquot Parts. Then you may make a Mixture that contains a known proportion of the Metal wherewith you Allay the *Gold*; as if it hold 19 or 15 parts of *Gold*, and one of *Silver*; and, letting the *Instrument* settle in the Water, Mark the place where the Surface of the Water cuts the *Stem* or *Pipe*. And then putting in another Mixture, wherein the *Silver* has a new and greater Proportion to the *Gold*; as if the former be an 18th or a 14th part of the Latter, you may Observe, how much less than before this Depresses the *Instrument*, and so you may proceed with as many Mixtures or Degrees of *Allays* as you think fit, or can be Distinguish'd conveniently on the *Stem*; being always careful, that, whatever be the Proportion of the two Ingredients, the Weight of the Mass in the Air be just the same with that of the *Pure Gold*, which we may have lately supposed to be an Ounce, or half an Ounce.

By the same Method may be Examined the differing *Allays* of *Pure Silver*, upon the Admixture of such and such determinate Proportions of *Copper*, or any other Metal Lighter in Specie than *Silver*; and by the same way, with a slight Variation, 'twill not be difficult to Estimate, how much divers *Coins*, whether of *Silver* or *Gold*, are more or less Embas'd by the known Ignobler Metal that is mix'd in the piece propos'd. These Estimates (which may be made without much Trouble) will come nearer the Truth, not only than the

the Estimates wont to be made by the *Touch-Stone*, but perhaps too, than some of those that divers make with Trouble, Inconvenience, and Charge.

It may be also Employ'd to Examine other *Mixtures* besides *Alloy'd Coins*, and that if the *Instrument* be adjusted to an Ounce, for instance, of Pure Copper, it may help Men to make an Estimate of the *Alloy* of *Tin*, or the Quantity of it that is often times added to *Copper*, to make different Sorts of *Bell Metal*, and of those *Metalline Specula*, whether Plain or Concave, that are call'd Steel Glasses, as also of Solders consisting of certain Proportions of Silver and Brass, or Copper; in all which, and divers others, the Discovery of the Proportion of the Ingredients may, on some Occasions, be Useful to Tradesmen, as well as desirable to *Virtuosi*. And though I have Observed, that by Mixture, Tin and Copper acquire a *Specifick Gravity* somewhat differing from what their Ingredients promise; yet, since the *Instrument* is to be fitted for such Estimates, not by Calculation, but by Trials, the Estimates may be made near enough to the Truth.

Further Considered; by . . .
n. 116. p. 553.

2. Long since I took Notice, how Light and Silver-like the Pewter was which descended to us; but, as soon as, to follow the Fashion, we Changed it, the Weight and the very Colour was altered; and is in every Change more and more Embased. And, if our *Silver-Smiths* hold on their Degrading Mixtures, I shall Question, whether our Silver-Plate may not shortly come down to approach our Fore-Fathers Pewter: I mean, in the Country, where 'tis never or seldom Tried.

The Weight of
Water in Water.
by Mr. Boyle.
n. 50. p. 6001.

III. A Glas Bubble, of about the Bigness of a Puller's Egg, was purposely blown at the Flame of a Lamp, with a somewhat long Stem turn'd up at the End, that it might the more conveniently be broken off. This Bubble being very well heated to Rarifie the Air, and thereby drive out a good part of it, was nimble Sealed at the End, and, by the help of the Figure of the Stem, was by a convenient Weight of Lead depressed under Water, the Lead and Glas being tied by a String to one Scale of a good Ballance, in whose other there was put so much Weight, as sufficed to Counterpoise the Bubble, as it hung freely in the midst of the Water. Then with a long Iron Forceps I carefully broke off the Seal'd End of the Bubble under Water, so as no Bubble of Air appear'd to Emerge or Escape through the Water, but the Liquor by the Weight of the Atmosphere sprung into the un replenish'd part of the Glas Bubble, and fill'd the whole Cavity about half full and presently; as I foretold, the Bubble subided, and made the Scale it was fastened to Preponderate so much, that there needed 4 Drachms and 38 Grains to reduce the Ballance to an *Æquilibrium*. Then, taking out the Bubble with the Water in't, we did, by the help of the Flame of a Candle, warily applied, drive out the Water (which otherwise is not easily excluded at a very narrow Stem) into a Glas Counterpoised before; and we found it, as we expected, to Weigh about 4 Drachms and 30 Grains, besides some little that remained in the Egg, and some small matter that may have been Rarify'd into Vapours, which added to the Piece of Glas that was broken off under Water and lost there, might very well amount to 7 or 8 Grains. By which it appears not only, that Water
hath

hath some Weight in Water, but then it Weighs very near or altogether as much in Water, as the self same Portion of a Liquor would Weigh in the Air. We Repeated the Experiment with another Seal'd Bubble as big as a great Hen Egg, with like Success.

IV. Apr. 7. 1680. Being off of *Pantalara* near *Sicily* in a Calm, I let down a *Bottle* 70 *Fathom*, stopp'd with an Excellent good tender Cork, well Fitted, and the Cork came up in the Bottle $\frac{1}{4}$ full of Salt Water. The Bottle was again fitted with an Excellent good Cork, but of a Woodiness or Hardness as some Corks are, with the which, being let down in like manner, the Cork continued in its Place, but as it were Bruised, and the Bottle as before, about $\frac{3}{4}$ full of Salt Water: Whereupon I took a good Ox Bladder, and bound it four fold over the Mouth of the Bottle without any Cork at all, only I put a piece of Leather to keep the Glass from cutting the Bladder; and so ordered, it was let down as before, but taken up without any Water, or the least Moisture in it.

The pressure of Water in great Depths; by a Person of Honour.
n. 193. P. 504.

May 18. 1680. Being in a Stark Calm some Leagues distant from the Coast of *South Spain*, off the great Hills of *Granada*, we took a Bottle and clapt a Leather on the mouth of it, tying over that a single part of the Bladder, the which we let down 75 *Fathom*, but it came up again Entire; We then made a Hole in the Leather, about the Bigness of a large Pea, and let the same down again 75 *Fathom*, but it came up perforated in the Vacant place where the Leather had the Hole in it, and almost full of Water; we then bound over another part of the Bladder single, and let it down but 30 *Fathom*; but it came up whole and entire; whereupon immediately we let it down 50 *Fathom*, but it came up broke and full of Water. Then, we again fitted the Bottle with the said perforated piece of Leather and a Double Bladder; and let it down 50 *Fathom*, but it again came up Entire: So again, immediately we let it down 75 *Fathom*, but then it came up broken and full of Water:

June 24. 1680. Being in $39\frac{1}{4}$ Degrees of Latitude, and by the Ships Account 150 Leagues Westward of *Portugal*, I caused a *Florence Flask* to be well stopped with a Bladder over the Mouth of it, and Lower'd it down 30 *Fathom*, but it was taken up broken. Whereupon, imagining that the roughness of the Leads Halling so tender a Body so violently through the Water might be the breaking thereof, I caused another *Flask* in like manner to be fitted, and close by it I tied likewise another *Flask*, so as to be born with the Mouth downwards, as were the other, but which was not Stopp'd; and these I caused to be taken up when they had been but 10 *Fathom* under Water, and found them both Entire; but the Open *Flask* almost full of Water; the which being emptied, were both let down again and taken up at 20 *Fathom*, when the Open *Flask* was entire, tho' full of Water, but the Other broken to pieces.

Jun. 8. 1693. In the Bay of *Biscay*, when we had 100 *Fathom* of Water, we took a Quart Glass Bottle stop't with a large Cork: And, Fastening it to our Plumbing-Rope with a Lead at the End; we sunk it to the Bottom of the Sea,

By Dr. Oliver.
n. 204. P. 508.

which as soon as we perceived, we drew it up again, and found the Cork, quite pressed through the Neck of the Bottle into its Cavity, and the Bottle full of Salt Sea-Water. We Repeated our *Experiment* with another Bottle and Cork in the same manner as before, but, the Cork being not found, the Sea-Water soaked through it, and the Bottle was half full of Water; so the Cork remained in the Mouth of the Bottle, not pressed down at all. We repeated our *Experiment* a third time in 90 *Fathoms* of Water, with a very found Cork, and much larger than the Mouth of the Bottle. We beat it in with a Hammer as far as it would go, leaving about an Inch of the Cork above the Mouth of the Bottle. The Cork at this *Trial* was pressed down only into the Neck, and became Level with the Mouth of the Bottle: But I really believe, had we had 10 or 20 *Fathoms* of Water more, it would have succeeded as at our first *Trial*.

The Weight of
divers Bodies
try'd by the Di-
rection of the
Phil. Society
at Oxford.
n. 169. p. 926.
Mar. An. 1685.

V. 1. The following Bodies were poured gently into a Vessel of well seasoned Oak, whose Concave was an exact *Cubick Foot*. Those in the Twelve first *Experiments* were Weighed in *Scales* turning with two Ounces, but the last Seven were Weighed in *Scales* turning with one Ounce. The Pounds and Ounces here mentioned are *Averdupois*.

| | | lb. | oz. |
|----|---|-----|-----|
| 1 | A Foot of <i>Wheat</i> (worth 6 s. a Bushel.) | 47 | 8 |
| 2 | <i>Wheat</i> of the best sort (worth 6 s. 4 d. a Bushel.) | 48 | 4 |
| 3 | The same sort of <i>Wheat</i> measured the second time. | 48 | 2 |
| | Both sorts were <i>Red Lammas Wheat</i> of the last Year | | |
| 4 | <i>White Oats</i> of the last Year. | 29 | 8 |
| | The best sort of <i>Oats</i> were 2d. in a Bushel better than these. | | |
| 5 | <i>Blue Pease</i> (of the last Year,) and much Worm-eaten. | 49 | 12 |
| 6 | <i>White Pease</i> of the last Year but one. | 50 | 8 |
| 7 | { <i>Barley</i> of the last Year: (the best sort sells for 1 s. 6 d.)
in a Quarter more than this; } | 41 | 2 |
| 8 | <i>Malt</i> of the last Year's <i>Barley</i> , made two Months before. | 30 | 4 |
| 9 | <i>Field Beans</i> of the last Year but one. | 50 | 8 |
| 10 | <i>Wheaten Meal</i> (unsifted.) | 31 | 0 |
| 11 | <i>Rye Meal</i> (unsifted.) | 28 | 4 |
| 12 | <i>Pump Water</i> . | 62 | 8 |
| 13 | <i>Bay Salt</i> . | 54 | 1 |
| 14 | <i>White Sea Salt</i> . | 43 | 12 |
| 15 | <i>Sand</i> . | 85 | 4 |
| 16 | <i>Newcastle Coal</i> . | 67 | 12 |
| 17 | { <i>Pit Coal</i> from <i>Wednesbury</i> 63, but this is very uncertain
in the filling the Interstices between the greater pieces. } | 63 | 0 |
| 18 | <i>Gravel</i> . | 109 | 5 |
| 19 | <i>Wood-Asbes</i> . | 58 | 5 |

| | |
|--|-------|
| Pump-Water | 1000 |
| Fir Dry | 546 |
| Elm Dry. | 600 |
| Cedar Dry. | 613 |
| Walnut-Tree Dry. | 631 |
| Crab-Tree meanly Dry | 765 |
| Ash meanly Dry, and of the Out-side lax part of the Tree. | 734 |
| Ash more Dry, about the Heart. | 845 |
| Maple Dry. | 755 |
| Yew of a Knot or Root 16 Years Old. | 760 |
| Beech meanly Dry. | 854 |
| Oak very Dry almost Worm-eaten. | 753 |
| Oak of the Out-side sappy part Fell'd a Year since. | 870 |
| Oak Dry; but of a very sound close Texture. | 929 |
| The Same tryed another Time. | 932 |
| Logwood. | 913 |
| Claret. | 993 |
| Moil Cyder, not Clear. | 1017 |
| Sea-Water settled Clear. | 1028 |
| College Plain Ale the same. | 1028 |
| Urine. | 1030 |
| Milk. | 1031 |
| Box the same. | 1031 |
| Redwood the same. | 1031 |
| Sack. | 1033 |
| Beer Vinegar. | 1034 |
| Pitch. | 1150 |
| Pit-Coal of Staffordshire. | 1240 |
| Speckled Wood of Virginia | 1313 |
| Lignum Vita. | 1327 |
| Stone Bottle. | 1777 |
| Ivory. | 1826 |
| Alabaster. | 1872 |
| Brick. | 1979 |
| Hedington Stone, the Soft Lax kind. | 2029 |
| Barford Stone, an old Dry Piece. | 2049 |
| Paving Stone, a hard Sort from about Blaidon. | 2460 |
| Flint. | 2542 |
| Glass of a Quart Bottle. | 2666 |
| Black Italian Marble. | 2704 |
| White Italian Marble tryed twice. | 2707 |
| White Italian Marble of another sort, of a visible Closer Texture. | 2718 |
| Block Tin. | 7321 |
| Copper. | 8843 |
| Lead. | 11345 |
| Quick Silver. | 14019 |
| Quick Silver. | 13593 |

2
The Specifick
Gravities of
several Bodies,
by the Direction
of the Phil.
Society at Ox-
ford.
Ib. p. 927.

The last *Experiment* was tried with another quantity of *Quick-Silver*, which had been used in *Water* in the preceeding *Experiment* : However, I rather trust the last, for that I found a small mistake (though herein the Calculation allowed for) in the Weight of the *Glass* containing the *Quick-Silver*, in the Trial before.

The *Solids* here mentioned, were Examined *Hydrostatically* by Weighing them in *Air* and *Water* ; but the *Fluids*, by Weighing an equal Portion of each in a *Glass* holding about a *Quart*. The Numbers shew the Proportion of *Gravity* of equal Portions of these Bodies : But if of these Bodies we take Portions equally Heavy, their *Magnitudes* will be reciprocally proportional to their correspondent Numbers : e. g. a Cubic Foot of *Water* is to a Cubic Foot of *Alabaſter* in *Gravity* as 1000 to 1872 ; but a Pound Weight of *Water*, is to a pound Weight of *Alabaſter* in *Magnitude* as, 1872 to 1000. So that, knowing by the former Table, the Weight of a Cubic Foot of *Water*, and by this proportion in *Gravity* betwixt *Water* and *Alabaſter* ; we may by the Rule of Three find the Weight of a Cubic Foot of *Alabaſter*, and so of any other of these Bodies ; or we may know their *Magnitude* by knowing their *Gravity*. So that, an irregular piece or quantity of these Bodies being offered, 'tis but Weighing them, and we may know their just *Magnitude* without farther trouble.

By Mr. J. C.
n. 199. p 904.
Apr. An. 1693

| | |
|------------------------------------|------|
| Pump-Water | 1000 |
| Cork. | 237 |
| Sassafras Wood. | 482 |
| Juniper Wood (Dry.) | 556 |
| Plum-Tree (Dry.) | 663 |
| Mastic. | 849 |
| Santalum Citrinum. | 809 |
| Santalum Album. | 1041 |
| Santalum Rubrum. | 1128 |
| Ebony. | 1177 |
| Lignum Rhodium. | 1125 |
| Lignum Asphaltum. | 1179 |
| Aloes. | 1177 |
| Succinum Pellucidum. | 1065 |
| Succinum Pingue | 1087 |
| Jet. | 1238 |
| The top part of a Rhinoceros Horn. | 1242 |
| The Top part of an Ox Horn. | 1840 |
| The (Blade) Bone of an Ox. | 1656 |
| An Humane Calculus. | 1240 |
| Another Calculus Humani. | 1433 |
| Another Calculus. | 1664 |
| Brimstone, such as commonly Sold. | 1811 |
| Borax | 1720 |

| | |
|--|-------|
| A Spotted Facitious Marble. | 1822 |
| A Gally-Pot. | 1928 |
| Oyster Shell. | 2092 |
| Murex Shell. | 2590 |
| Lapis Manati. | 2270 |
| Selenitis. | 2322 |
| Wood Petrified in Lough Neagh. | 2341 |
| Onyx Stone. | 2510 |
| Turcois-Stone. | 2508 |
| English Agat. | 2512 |
| Grammatias Lapis. | 2515 |
| A Cornelian. | 2568 |
| Corallachates. | 2605 |
| Talo. | 2657 |
| Coral. | 2689 |
| Hyacinth (Spurious.) | 2631 |
| Jasper (Spurious.) | 2666 |
| A Pellucide Pebble. | 2641 |
| Rock Crystal. | 2659 |
| Crystallum Disdiacasticum. | 2704 |
| A Red Paste. | 2842 |
| Lapis Nephriticus. | 2894 |
| Lapis Amiantus from Wales. | 2913 |
| Lapis Lazuli. | 3054 |
| An Hone. | 3288 |
| Sardachutes. | 3598 |
| A Granat. | 3978 |
| A Golden Marcasite. | 4589 |
| A Blue-Slate with shining Particles. | 3500 |
| A Mineral Stone, yielding 1 part in 160 Metal. | 2650 |
| The Metal thence Extracted. | 8500 |
| The (reputed) Silver Ore of Wales. | 7464 |
| The Metal thence Extracted. | 11087 |
| Bismuth. | 9859 |
| Spelter. | 7065 |
| Spelter Solder. | 8362 |
| Iron of a Key. | 7643 |
| Steel. | 7852 |
| Cast Brass. | 8100 |
| Wrought Brass. | 8280 |
| Hammer'd Brass. | 8349 |
| A False Guinea. | 9075 |
| A True Guinea. | 18888 |
| Sterling Silver. | 10535 |
| A Brass Half Crown. | 9468 |

| | |
|---|-------|
| <i>Electrum a British Coin.</i> | 12971 |
| <i>A Gold Coin of Barbary.</i> | 17548 |
| <i>A Gold Medal from Morocco.</i> | 18420 |
| <i>A Mint Gold Ducat.</i> | 18261 |
| <i>A Gold Coin of Alexander's.</i> | 18893 |
| <i>A Gold Medal of Queen Mary's.</i> | 19100 |
| <i>A Gold Medal of Queen Elizabeth's.</i> | 19125 |
| <i>A Medal esteemed to be near Fine Gold.</i> | 19636 |

The Different
Weight of several
Liquors in
Winter and
Summer; by
M. Homberg.
n. 262. p. 530.
Vid. Sup.
S. I. 2.
Mar. An. 1700.

VI. M. Homberg, has given us the following Table of the *Various Weights* of some more Usual Liquors in the *Coldest* Time, and in the *Hottest*.

| | In Summer. | | | In Winter. | | |
|---|------------|---|-----|------------|---|-----|
| | 3 | 3 | gr. | 3 | 3 | gr. |
| The <i>Araometer</i> full of <i>Mercury</i> . | 14 | 0 | 6 | 11 | 0 | 32 |
| <i>Oil of Tartar.</i> | 1 | 3 | 8 | 1 | 3 | 31 |
| <i>Spirit of Urine.</i> | 1 | 0 | 32 | 1 | 0 | 43 |
| <i>Oil of Vitirol.</i> | 1 | 3 | 58 | 1 | 4 | 4 |
| <i>Spirit of Nitre.</i> | 1 | 1 | 40 | 1 | 1 | 70 |
| <i>Spirit of Salt.</i> | 1 | 0 | 39 | 1 | 0 | 47 |
| <i>Aqua Fortis.</i> | 1 | 1 | 38 | 1 | 1 | 55 |
| <i>Vinegar.</i> | 0 | 7 | 55 | 0 | 7 | 60 |
| <i>Spirit of Wine.</i> | 0 | 6 | 47 | 0 | 6 | 61 |
| <i>River Water.</i> | 0 | 7 | 53 | 0 | 7 | 57 |
| <i>Distilled Water.</i> | 0 | 7 | 50 | 0 | 7 | 54 |
| This Empty <i>Araometer</i> Weighs | | | | 0 | 1 | 28 |

Experiments about the
Superficial Figures of
Fluids, especially
Liquors Contiguous to
other Liquors.
and their Reflexive
Powers; by Mr. Boyle.
n. 131. p. 775.
Jan. An. 1675.

VII 1. Having poured a strongly *Alcalizat Menstruum* (I used that made of *Fix'd Nitre*, dissolved by the moisture of a Cellar) into a Pipe of Glass, sealed at one end, and not full a quarter of an Inch in Bore; that the Cavity, which in a greater depth would seem less deep, might be the more conspicuous: We gently poured on it some highly *Dephlegm'd Spirit of Wine*, which we knew would not Mix with it, but swim above it, and presently, as we had guess'd, we found the *Figure* of the *Surface* of the Lower Liquor Changed, and the Cavity quite destroyed; the *Surface* that seemed as it were, common to the two *Contiguous Liquors*, appearing Flat or Horizontal. And such a Level *Superficies* we had, by putting these two *Liquors* together in a much Wider Glass.

2. We found also, that by Employing *Oyl of Turpentine*, instead of the *Spirit of Wine*, the Liquor did almost totally Lose its Cavity.

3. But

3. But, if, instead of *Deliquated Tartar*, we put common Water into the Pipe, we found this Liquor to retain its Concave Surface, though we put to it some Oil of *Turpentine*, and left it to rest upon the Water a good while.

4. Having provided some pure Oil of the *Gum of Guaiacum*, and poured a little of it into a slender Pipe, we found the Upper Superficies of it to be Concave, almost, if not altogether, like that which Water would have had in the same Pipe. But, when I put a little Water upon this Oil, it presently changed the Figure of its Surface, which became visibly, though not very much, Protuberant or Convex.

5. Having put some Oil of *Tartar* into the slender Pipe, and put some Drops of the Oil of *Guaiacum* to it, we found, that this Liquor did not manifestly alter the Concave Figure of the Surface of the Liquor *Alkali*, as the Oil of *Turpentine* had done: And, having for Curiosities sake warily poured a little Water upon the Oil of *Guaiacum*, I found, as I had reason to suspect, that the upper Superficies of it Changed presently from a Concave Figure to a Convex, so that this Oil in the midst of the other two Liquors appeared like a little red Cylinder; which, instead of having Circular Bases, was Protuberant at both ends; but more at that which touched the Oil of *Tartar*.

6. I put some *Essential Oil* (as *Chymists* call it) of *Cloves* into a new slender Pipe, and, having observed it to be somewhat Concave at the Top, where it was Contiguous to the Air, we caused a little Common Water (perhaps a quarter of a Spoonful or less) to be put to it, and found, as we expected, the Surface of this Oil also to be Tumid. And in regard this Liquor, as well as the forementioned Oil of *Guaiacum*, though it were so heavy as to sink into Water, would not do so in *Deliquated Salt* of *Tartar*, we did, into another slender Pipe, put first some of this last named Liquor, then some of the *Aromatic Oil*, and lastly, a little Common Water; by which means we found, that the Little Cylinder of Oil, did, like that of the Oil of *Guaiacum*, appear Convex at both ends, but, was unlike it in one Circumstance, that the Oil of *Cloves* appear'd more Convex at the upper end where 'twas Contiguous to the Water, than at the Lower, that leaned upon the Surface of the Oil of *Tartar*.

7. Having taken a little slender Glass, that was much longer, but of the like Bore with the former, we put into it a small quantity of *Quick Silver*, and, having taken notice how the Upper Superficies swelled in the middle above the Level of the Parts where it touched the Glass, we poured some Water upon it, and found a Manifest and Considerable Depression of the Surface, though the Protuberance were not quite Suppressed.

8. This *Phænomenon*, having been for greater security several times Repeated, sometimes it seem'd, that when the *Aqueous Cylinder* was much longer, the Depression of the *Mercurial* Surface was somewhat greater. But this did not so constantly happen; But we often observed, that, though a very little Water sufficed by its Contact to make, in the Judgment of the Eye, a manifest Abatement of the Protuberance of the *Quick-Silver*; yet it had not the same effect on that Ponderous Fluid, that it had, when being Increased almost as high as the length of the Pipe would permit, a greater Weight of it was Incumbent

Incumbent on the *Mercury*, for then I manifestly perceived, and shewed to others, that the Surface of the *Quick Silver* being Depressed almost to a Level in those Parts of it that were near the inside of the Glass, there was about the Middle of the Surface an Elevation of *Mercurial Matter*, that appeared to be rather more than a half Globe, and was to the Height of its full Semidiameter, raised above the rest of the *Mercurial Surface*, and in that State it continued, as long as I thought fit to let it do so. And, lest this Tryal should Impose upon me, I caused it to be more than once Repeated; and, the better to confirm it, I afterwards caused the Incumbent *Water* to be Little by little suck'd up, and found as I expected, that, when the Incumbent *Water* began to be too much shorten'd, the little Teat or Segment of a Sphere, lately mentioned, began to be somewhat Flatned, and Subsided more and more as the *Water* was further taken off.

9. Having conveyed into one of our *Pneumatical Receivers*, a Couple of such slender Pipes as have been already described, one of them Furnished with *Common Water*, and the other with *Quick Silver*, we caused the Common Air to be diligently Pump'd out, without observing any Sensible Change in the Concave Figure of the *Water*: But, as for the *Quick Silver*, I knew not what to conclude about it. For, having Repeated the Tryal twice or thrice, the *Mercury* sometimes seem'd manifestly to Swell, to be more Protuberant upon the Exhaustation of the *Receiver*, than when it was put in, especially when its Figure was attentively Viewed, and the External Air, that was Pump'd out but slowly, was suffer'd to Re-enter with all convenient Celerity. But that which yet kept me doubtful was, that I observed, That, upon the diligent withdrawing of the Air's Pressure upon the *Quick Silver*, there disclosed themselves some little Bubbles, which, I fear'd, we had not been able to free it altogether from, and which might be suspected to have some Interest in the *Phenomenon*. We also convey'd into our *Receiver*, a clear *Cymical Oil* that was heavier than *Water*, and, whilst it was Contiguous to it, had not a Concave but a Convex Surface, and, having placed the Pipe furnish'd with both Liquors in the *Pneumatical Receiver*, we Pump'd out the Air, without finding that the *Oil* sensibly altered its Protuberant Surface, as neither did the *Water* lose the Concave Figure of it's upper Surface.

10. I took *Fix'd Nitre*, (or which is Analogous to it, *Salt of Tartar*) resolved *per Deliquium* into a Transparent Liquor, and having filled a clear Viol half full with this, I poured on it a convenient quantity of *Vinous Spirit* exactly *Rectified*, that there might be no Phlegm to occasion an Union betwixt the two Liquors, which ought, as ours did, to retain Distinct Surfaces, and speedily regain them, though the Glass were well shaken. Then, having found by a Tryal formerly mentioned, that Common *Oil of Turpentine*, if employed in a Competent Quantity, will not totally (and much less will readily) Dissolve in *Spirit of Wine*, and also having Observed (what may seem somewhat strange) that if this *Spirit of Wine* be exquisitely Dephlegm'd, the *Oil*, though a *Chymical One*, will not Swim on it, but Sink in it; I warily let fall some drops of *Oil* into the *Spirit*, and had the pleasure to see, as I expected, that they fell towards the bottom of the Glass, till their Descent was stop'd by the

the Horizontal (for it was not Concave) Surface of the *Alcalizat Liquor* of *Fix'd Nitre*. And, because my design was chiefly to observe the Superficial Figure of a Fluid Encompassed by other Fluids without touching any Solid Body, I shall here take notice of the chief *Phænomena* that were produced of that kind, without Staying to Enquire into the Causes or the Consequences of them.

1. If the *O'y Drops* were but small, they seem'd to the Eye exactly enough *Spherical*. For, the *Oil* differing but very little in the *Specifick Gravity* from the *Spirit of Wine*, the Drops did but just touch the Surface of the subjacent *Alkali*; and, the same Drops being but small, their own Weight was not great enough visibly to Depress them, and hinder that Roundness which the Pressure of the Ambient *Spirit*, or their own Viscosity, endeavoured to give them.

2. If an Aggregate of Drops were considerably bigger than those newly mentioned, as if it had about a third Part of an Inch in Diameter, it would then manifestly lean upon the *Alcalizat Liquor* as upon a Floor, and appear somewhat *Elliptical*, (for some little part of the bottom was a Plain;) the Weight of the upper parts Depressing the Drops, and making the Horizontal Diameter somewhat longer than the Transverse.

3. If a yet greater Portion of *Oil* were let fall upon the Heavy Liquor, it would for a pretty while appear in the form of a somewhat Imperfect *Hemisphere*, or some other large Section of a *Sphere*, the lower part being cut off; (as if a Globe were divided by a Plain) by the Horizontal Surface of the *Deliquated Salt*.

4. But, if the Quantity of *Oil* were not too great, 'twas pretty to observe, that, though at first putting in, it did perhaps spread itself over the Subjacent Liquor, and lie as it were flat upon it; yet, by little and little, (for 'twas but slowly) it would by the Action of the Ambient, concurring with it's own Tenacity, be raised above the Surface of the Fluid *Nitre*, and be Reduced to the Figure, either of half a Globe, or of a greater Segment of a Globe, or even of an Imperfect Ellipsis, according to the Bulk or Weight of the *Oil*.

5. Though these Globules, or Portions of *Oil*, did oftentimes readily mingle, when they touched one another, yet divers times also we observed, that, having warily approach'd them, we were able to make them touch without Mingling, insomuch that we have with pleasure made them so far bear against one another's Surfaces, as manifestly to press them inwards, though being parted they would presently Resume their former Figure. But, in case any of these *Oily Portions* came by a more Pressing contact to be United, they would then alter the Figures they had whilst separate, and take another suitable to the Bulk of the Aggregate.

6. When a large Portion of *Oil* rested upon the *Saline Liquors*, if then the Ambient *Spirit* was moderately and warily Agitated, 'twas not unpleasant to observe the various Figurations, which the Convex and Protuberant part of the mutilated Globe would be put into by the Shakes, without any Visible Solution of Continuity, or considerable Motion of the whole Body, which would very quickly recover its former Figure. Though, if the Agitation were too

strong, some Portions would be quite brok en off, and presently turned in-
to Little Globes.

11. I tried to produce another *Phænomenon*, that would not have been un-
pleasant, by putting together in a somewhat large Vessel, with other Liquors,
two *Oils*, (whereof one if I mistake not, was from *Turpentine*), which first by
reason of the Oleaginous Nature wherein they agreed, might exactly mingle
and make a Compounded Liquor; and then, by reason of there being one
Heavier, and the other Lighter in *Specie* than Water, might by this Liquor
be again separated, and Include betwixt them the Liquor that had Divided
them. But I found that the *Oils* being once united would not be easily par-
ted, but according to the Prevalency of the Lighter or Heavier Ingredient,
in the mixture, the Compounded *Oil*, would almost totally either Emerge to
the top of the Water, or Lie beneath at the Bottom of it; I say, almost total-
ly, because some Part of the *Oil*, which was not perhaps all uniformly Mix'd,
did not keep in a Body with the rest; but either was separated from the Mass
in the form of Globules, or else sticking to the side of the Glass, had the
other part of its Superficies, which was Contiguous to the Water, very
Variouly Figured, according as the Bulk and Degree of Gravity of the ad-
hering *Oil*, and other Circumstances happen'd to Determine.

These are some of the *Phænomena* I observed in *Oil of Turpentine*, when
'twas invironed only with Fluids; but, if it were permitted to be Contigu-
ous to the Inside of the Glass, and so to fasten part of its Surface to a Solid,
the greater part of the Surface, which remained exposed to one or both of
the Contiguous Liquors, would partly by their Action, and partly by the
Gravity of the *Oil* itself, be put into *Figures* so *Variom*, and sometimes so
Extravagant, that 'twas much more pleasant to behold them, than it would
be easy to describe them.

12. *Confining Fluids* may have Distinct Surfaces, without having, at least
in many Positions, *Refractions* differing enough, or *Reflexions* strong enough,
to make the Plain that Disterminates them, obvious to the Eye. Thus, when
the *Oil of Tartar*, or *Nitrom Alkali*, that I employed, happen'd to be very
Clear and Colourless, I have more than once made highly *Rectified Spirit of*
Wine Float upon it so, that in most Positions the Vial seem'd to have in it
but one uniform Liquor; the Plain that divided the two Fluids being unapt
to be discerned, but in a Position, wherein the Rays of Light passing thence
to the Eye, fell very Obliquely on it; and indeed, when there was no little
Dust or other Feculency, swimming upon the Surface of the *Oil of Tartar*;
I had sometimes much ado to convince Ordinary Spectators, that the Vial,
in two distinct Regions of it, contained two *Unsociable Liquors*.

13. We took a *Deliquated Alkali*, made of *Nitre* and *Tartar*, and deeply
Tinged with *Cochineel*; and, that the Liquors might not only be Heteroge-
neous, but as differing in *Gravity* and *Density*, as we could make them, we
poured on it a peculiar kind of *Oil* lighter than *Spirit of Wine*, and holding
the Plain where the two Liquors were Contiguous in a convenient Position,
in respect of the Light and the Eye, I observed it to make a strangely vivid
Reflexion of the Incident Beams of Light: So that this Physical Surface which
was

was flat, look'd almost, for 'twas not so *Specular*, like that of *Quick-Silver*; and, when I kept it 'till Night, and considered it by the Light of a Candle, the bright Figure of the Flame was strongly Reflected almost as from a Close *Specular Body*; which tempted me to suspect, that there might be something else than the bare smoothness of the Surface of the *Alcalizat Liquor* to produce so brisk a *Reflection*; and the rather, because I did not Observe, that the remains of the same Tinged *Alkali*, which I kept in another Glass, nor a Portion of the same *Oil*, which I had also by me in a separate Vial, did either of them afford so Vivid a *Reflection* from its Surface; tho' I did theless wonder at this, because of the great Disposition to *Reflect Light*, which I had formerly the Curiosity to Observe in the foremention'd *Oil*, when I joyned it with other Liquors. I shall add, that looking on this Liquor, as a Body which, though it have all the necessary Quantities of an *Oil*, does in regard of its Origin, and some Properties I have found in it, differ from common *Chymical Oils*; I was invited the more to observe its *Phenomena* in reference to *Reflection*, and I found among other Things (not pertinent to this Place,) First, That the Confining Plain often mentioned between the Tinged *Alkali*, and this *Liquor*, did not appear Red itself, nor communicate that Colour to the Image of the Flame of a Candle Reflected from it. Secondly, That when I warily shook the Vial, which contained the two Liquors, the uppermost would be reduced into a seeming Froth, consisting of a great number of Imperfectly Globular Bodies, which after a while would make a kind of a Rude Physical Plain; which, though neither very Horizontal, nor sensibly Smooth, would at its upper Superficies, send back the *Incident Light* with more Briskness than one would expect; and, when the seeming Froth consisted of smaller Particles, these, when they were of a certain Size, and conveniently Placed, in reference to the Flame of a Candle and the Eye, would (as more than one Trial informed me) Reflect the *Incident Light* so many ways, and so visibly, that they seemed, for multitude and splendor, like little Sparkling Corpuscles of Polished *Silver*; or almost like these Glistering ones, that appear when a clean Plate of *Copper* is first Immersed into a much allayed Solution of good *Silver*, made in *Aqua Fortis*. Thirdly, That though pure *Spirit of Wine* be so thin a Lipuor, and our *Oil* is nevertheless, so Light as to Swim upon it, yet I found the Confining Surface very strongly *Reflexive*.

I have also found, that some other *Essential Oils* (as *Chymists* call those that are Distilled with Water in *Limbecks*) and particularly an Unsophisticated *Oil of Lemons*, did with our Tinged *Alkali* afford most of the same *Phænomena*; but not so Brisk a *Reflection*: I say, most, chiefly because with *Spirit of Wine*, their *Subtile Oils*, as I formerly noted, will readily be confounded: Though our Anomalous *Oil* be unfociable with it.

14. In Cold Weather we took *Essential Oil of Anniseeds*, whose Property it is to Coagulate in such Weather, and, having in a gentle warmth brought it to be Fluid, we poured it into a slender Vial more than half filled with *Common Water*, that had been also a little warmed, that the *Oil* might not be too hastily reduced to its former State. This *Oil* being Lighter than so much *Water*, and, being poured on in a convenient Quantity, had its Upper Surface

somewhat Concave, as that of the Water was; but the Lower Surface, Surrounded by the Water, was very Convex, appearing almost (for it was not perfectly) of the Figure of a great Portion of a *Sphere*. This being done, the Vial was stoppt, and suffered to rest for some time in a Cold place, by which means the *Water* continuing Fluid as before, the *Oil* of *Anniseeds* was, as I expected, found *Coagulated* in a Form approaching to that it had whilst in a Fluid State; I say approaching, because it was not easy to discern the exact Figure in the Vial I was fain to make use of: And I suspected, that the *Oil* grown Consistent was become less Convex than before. But 'twas worth Observing, how great a Difference there was between the dull *Reflection* it made when it was *Coagulated*, and the fine *Reflection* it had made whilst 'twas a Liquor. The later of which *Reflections* brought into my mind, how Vivid the *Reflective Power* of some *Fluids* is in Comparison of that of the Generality of *Solid Bodies*.

15. Having observed, That *Quick-Silver*, and Rectified *Oleum Petra*, are, the Former of them the Heaviest, and the Latter the Lightest, of all the Visible Fluids that are yet known to me; I put some (Distill'd) *Quick Silver* into a small Vial, and held it in such a posture, That the *Incident Light* was strongly remitted to my Eye, I then slowly put to it some *Petroleum*, that being well Rectified was very clear, and observed, that, as this Liquor covered the *Quick Silver*, there was at the Imaginary Plain, where they both confined, a Brisker *Reflection* than the *Quick Silver* alone had given before. On this occasion it will not be amiss to take notice, that either the Surface of the Air itself, as thin and yielding a Fluid as it is, or the Surface of a Solid contiguous to included Air, or some interposed *Subtile Matter*, may *Reflect* the *Incident Beams* of *Light* more strongly than most Men would expect. To this purpose, I remember, that a Curious Person having one Day brought me a couple of Rarities, which he told me were two pieces of a Solid, but transparent Body, that he had casually found; in one of which there was a *Pearl*, Large, Round, and Orient, and in the other a less Perfect One: One of them was opened, and that which had appeared a *Pearl* was found to be but a Cavity, that contained no grosser Substance than Air. And I have by me, a well shap'd piece of Glass of a good Thickness, with an Aerial Bubble in the middle, which by some Qualities particularly its Pear-like shape and Vivid *Reflection*, does not ill resemble a fair, though not *Orient Pearl*. But, in suchlike Observations, the Position of the Eye, and that wherein the Body receives the Beams of Light, may be very considerable. For I have by me a Small Stone (with which I have puzzled a Skilful Jeweller to determine what kind of *Gem* it was) that being laid flat upon ones Hand, or a piece of Paper, and look'd on Directly downwards, looks almost like a piece of Common Glass, and is transparent: But, if the Eye be so Placed, that the *Incident Beams* of *Light*, by whose *Reflection* it's seen, fall with a convenient degree of Obliquity upon the Stone, it makes an exceeding pretty shew, sometimes appearing like a fine *Opal*, and sometimes not very unlike an *Orient Pearl*.

16. We made a competent Quantity of a *Resinous* or *Gummosus Substance*, that looked like High Coloured *Amber*, but was easy to melt. This we put into a deep round Glass with a wide Mouth, and held it by the Fire-side in a moderate Warmth, 'till it was brought into a Fluid State; then we Transferr'd it into one of our *Pneumatical Receivers*, where we presumed, that this Temporary Liquor, would, as well as Liquors that are constantly such, disclose *Aerial Bubbles*, when the Pressure of the Air was withdrawn from it; and accordingly having caused the Air to be Pumped out by degrees, we found, that store of Bubbles appeared at the Top of the Liquor, and made there a copious Froth, many of them being by reason of the Viscosity of the Fluid, very large, and divers of them, because of the Nature and Texture of it, and the Thinness of the Films, being adorned with the Colours of the Rainbow, whose Vividness made them pleasant to behold, and suggested to us some *Optical Considerations*. But, notwithstanding this Froth, I caused the Pumping to be continued, that those Bubbles that had most of Common Air in them, and which therefore are wont to Rise first, might get to the Top, and the Subsequent Bubbles might meet with more resistance from the Liquor still tending to grow Cold, and so might be the more Expanded, and yet kept from Emerging by the Concretion of the Resinous Substance; and answerable to this we found, That, when this Substance had resumed its consistent form, there were intercepted, between the Upper and the Lower Surfaces of it, some Bubbles that were not small, which yet had a considerable *Reflection*, notwithstanding the small Quantity of the grosser Particles of the Air, that may be supposed to be contained in Bubbles so very much Expanded.

17 'Tis taken for granted, That the *Falling Drops* of *Rain* are *Spherical*, yet their Descent is so swift, that I fear 'tis rather *Supposed* than *Observed* that their Figure is *Spherical*; which will be the more questionable, if it be true, which is vulgarly thought, That *Hail* is but *Rain Frozen* in its passage through the *Air*. For 'tis evident, That the Grains of *Hail* are frequently of other Figures than truly Orbicular. But the *Surface* of *Water* may have Differing Figures, according as 'tis totally Encompass'd with Heterogeneous Fluids, or as 'tis only in some places Contiguous to one or more of them. In the former case we found it not so easy to make an Observation, because, we know not of any two Liquors (setting *Mercury* aside) that will not Mingle either with One Another, or with *Water*. We therefore cautiously convey'd into some *Chymical Oil* of *Cleves* some Portions of *Common Water* of differing Bignesses, taking care, as far as we could, that they might not touch one another; by which means the *Oil* being Transparent, and yet somewhat coloured, 'twas easy to observe, that the smaller Portions of *Water* were so near totally Invironed with the *Oil*, that they were reduced into almost Perfect *Globes*; Those Portions, that were somewhat bigger (as about twice the bigness of a Pea,) would be of a Figure somewhat approaching to that of an *Ellipsis* (for 'twas not the same) and those Portions that were yet somewhat Larger, tho' they seemed to be sunk almost totally beneath the *Oil*; yet, they held to it by a small Portion of themselves, whose Surface was easily enough distinguishable from that of the *Oil*. These larger Portions of *Immersed Water* being almost

almost wholly Inviron'd with the other Liquor, were by it reduced into a Round Figure, which was ordinarily somewhat *Elliptical*, but more Depressed in the Middle than that Figure requires.

18. Having into a Slender Pipe, of that sort that has been described before, put a little *Oil of Cloves*, and upon this some *Oil of Turpentine*, that so the *Water* might both above and beneath be touched by Heterogeneous Liquors, I observed not the *Oil of Cloves* to be very manifestly Tumid at the top, nor the Lower Surface of the *Oil of Turpentine* (for the Upper was Concave) to be very Convex: For somewhat Convex it was Downwards. And, from this 'twill be easy to conclude, the Figure of the Cylindrical Portion of *Water* intercepted between these two Oils.

19. I took *Oil of Anniseeds*, thaw'd by a gentle warmth, and *Common Water*, and, having put them together in a convenient shap'd Glass, they were suffered to stand in a cold Place, 'till the *Oil* was coagulated; which done, it was Parted from the *Water*, and by the Roughness of its Superficies manifested, as I expected, that, when its Parts were no longer agitated and kept easily Displaceable, by the Subtile permeating matter, or whatever other Agent or Cause it were to which it owed its *Fluidity*, when the Contiguous *Water* grew unable to inflect, or otherwise Place them after the manner requisite to constitute a *Smooth Surface*. And, what happen'd to that part of the *Oil's* Surface that was touched by the *Water*, happen'd also to that which was contiguous to the *Air*, save that the *Asperity* of the last named Surface was differing from the other, which, whether it were an accidental or constant *Phænomenon*, farther Trial must determine. But I have often observed, That the Upper Surface of *Oil of Anniseeds*, when this Liquor comes to be Coagulated by the Cold Air, was far enough from being *Smooth*, being variously Asperated by many Flaky Particles, some of which lay with their broad, and others with their edged Parts upwards.

20. An Inequality and Ruggedness of Superficies I have also observed in *Water*, when, having covered it with *Chymical Oil of Juniper*, and exposed it in very cold Weather, though the *Oil* continued *Fluid*, yet the *Water* being Frozen had no longer a *Smooth Superficies*, as whilst in its Liquid state 'twas contiguous to the *Oil*. And the like Inequality, and rather a greater, we observed in the Surface of *Water* Frozen, which had *Chymical Oil of Turpentine* Swimming over it, yet a no less, if not a much greater, Roughness may be oftentimes observed in the Surfaces of divers Liquors that abound with *Water*, when, those Liquors being Frozen, their Surfaces have an immediate Contact with the Air. I shall here add, that, having purposely caused a Strong and Blood-red Decoction of the Soot of Wood to be exposed in a large Glass in a very Cold Night, I was more pleased than surprized, to find in the Morning a Cake of Ice, that was curiously Figured, being full of large Flakes shap'd almost like the broad Blades of Daggers, but neatly Fringed at the Edges. But that which I chiefly mention these Figures for, is, that they seem to be as it were Imboss, being both to the Eye and the Touch Raised above the Horizontal Plain or Level of the other Ice.

21. I have sometimes observed the like *Phænomenon* in one and the same Liquor, and particularly not long since in Frosty Weather, on a Vial where I had long kept *Oil of Vitriol*, I perceived that the Cold had reduced far the greatest part of the *Menstruum* into a Consistent Mass, whose upper Surface was very Rugged and oddly Figur'd, though it lay covered all over with a pretty deal of high coloured Liquor, that was not Frozen or Coagulated, nor seem'd to be disposed to be so, at least, in that Degree of Cold.

22. This may be also observ'd in the best sort of what the *Chymists* call *Regulus Martis Stellatus*, where the Figure of a Star, or a Figure somewhat like that of the Decoction of the Soot lately mentioned, will frequently appear Imboss'd upon the Upper Superficies of the *Regulus*; and such a Raised Figure I have seen on a Mass of *Regulus* made of *Antimony* without *Mars*. But, if to those two Bodies *Copper* be also Skillfully added, the Superficies will be often times adorned with new Figures according to the Circumstances; tho' the most usual I took notice of was that of a Net, that seem'd to cover the Surface of the Compounded *Regulus*. But this is not so constant, but that I have by me a Mass of the Conical Figure, consisting of two very Contiguous, but easily superable Parts, whereof the Lowermost, which abounds more in *Metal*, hath its upper Surface covered with Round Protuberances, in Shape and Bigness not unlike to small Pease cut in two; and these are so really Imboss'd and Elevated above the rest of the Superficies, that the other part of the Cone, which is of a more Scorious Nature, has in its Lower Surface, which exactly fits the Upper of the *Regulus*, Cavities, for Number, Shape and Bigness, answering to the Protuberances lately mentioned, which Argues that the *Regulus* cooled first with that Inequality of Surface we have described, and that the Lighter and more Recrementitious Substance, continuing longer Fluid, had thereby Opportunity to accommodate itself to the Superficial Figure of the *Regulus*, on which it first leaned, and afterwards Coagulated.

VIII. 1. My Brother, Mr. *Tho. Molyneux* (in the *Nouvelles de la Republique de Lettres*) has given this Reason for the *Phænomenon*, viz. That the Internal Motion of the Parts of the Liquor does keep up the Particles of the Dissolved Solid, for they, being so very minute, are moveable by the least Force Imaginable, and the Action of the Particles of the *Menstruum* is sufficient to Drive the Atomes of the Dissolved Solid Body from place to place; and consequently, notwithstanding their Gravity, they do not Sink in the Liquor Lighter than themselves.

Why Bodies Dissolved Swim in Menstrua Specifically Lighter than themselves: by Mr. Will. Molyneux. n. 181. p. 88. May An. 1686.

But I conceive another Account may be given of this Appearance, and that the Prime Law of *Hydrostaticks* is a little deficient. 'Tis true indeed, if we consider only the *Specifick Gravity* of a Liquor, and the *Specifick Gravity* of a Solid Particle Floating therein, the forementioned Rule is exact; but in Sinking there is requisite a Separation of the Parts of the Liquor by the Sinking Body; and there being a Natural Inclination in the Parts of all Liquors to Union arising from an Agreement or Congruity of their Parts, there is a Resistance therein to any thing that Separates this Conjunction: Now, unless a Body have Weight enough to overcome this Congruity or Union of Parts, such a Body will Float in a Liquor Specifically Lighter than itself. But that a Heavy Body,

Body, as *Mercury* or *Iron*, may have its Parts reduced to that Minuteness that their *Gravity* or Tendency Downwards, is not strong enough to Separate the *Cohesion* or Union of Parts of a Liquor, will be manifest, if we consider, that the Resistance made by the Medium to a *Falling* Body, is according to the Superficies of the Body: But, as the Body Decreases in Bulk, its Superficies does not proportionably Decrease; thus a Sphere of an Inch Diameter has not *eight* times less Superficies than a Sphere of two Inches Diameter, tho' it have *eight* times less bulk, and consequently passing through a Medium, as suppose Air or Water, the Sphere of an Inch Diameter, is, proportionably to its Bulk, more *Resisted*, than a Sphere of two Inches Diameter in proportion to its Bulk, and hence it will come to pass, that at last a Body may be reduced to that Minuteness, that its *Gravity Pressing* Downwards (which is according to its Bulk) may be less than the *Resistance* of the Medium, which operates on the Surface of the Body; seeing, as I said before, the Surfaces of Bodies do not Decrease so fast as their Bulks, these Decreasing in a Triplicate, but those in a Duplicate Ratio of the Bodies Diameters.

But, because I have said that the forementioned Law of *Hydrostaticks* is a little Defective, I desire to explain myself a little further in that point. In Weights *Falling* through the Air, were Gravity only considered, the Proportions of their Descents would be exactly as *Galileo* has Demonstrated; but it is allowed by all, that the *Resistance* of the Air, not being considered in those Demonstrations, they are not *Mathematically True* in Practice, but that really there is something of that Proportion hindered by the Air's *Resistance*. Now, what is this less than to say, that the *Resistance* of the Air takes off some of the Operation of Gravity, or is able to withstand or oppose part of its Action? And if so, what shall we say, were an *Iron* Sphere let through a Medium of Water? Surely, the Proportions of its Descents would be much more Disturbed herein, as Water is much more solid and difficult to be Separated or passed through than Air, and consequently we must needs grant, that more of the Operation of Gravity is taken off or resisted by this Opposition of the Water, than that of the Air. And if so, surely there may be a certain Degree of Gravity, that may be quite taken off by the *Resistance* of the Water: Were a Pistol Bullet let fall through the Air, it would Descend imperceptibly nigh the Proportions that *Galileo* has assigned, but were a single Grain of Sand so let fall, it would be much hindered in its Course, and half of this Grain would be more obstructed; what shall we then say of the Ten Thousandth part, or of a part of the Ten Thousandth Millionth of this, and again of the Infinite Subdivisions of that, 'till at last we come to a Part that would be wholly *Resisted*, or kept up; such as I conceive the Minute Particles of a Body *Dissolved* in a *Menstruum*.

On this Account, 'tis, I say, that the forementioned Principle of *Hydrostaticks* is a little Defective; for it considers not the Natural *Congruity* of the Parts of a Liquor, whereby they desire as 'twere to Unite and Keep together, just as we see two drops of Water on a dry Board being brought together, do jump and coalesce, and therefore Liquors have an Innate Power of *Resisting* a certain Degree of Force that would *Separate* them; such as I suppose

pose the Degree of Gravity, in the most minute Particles of a Body Dissolved in a *Menstruum*.

The forementioned Rule holds true to the most nice Sense in great Bodies, but in those that are by many Millions of Divisions smaller, it seems to Fail.

I would not however be thought wholly to Reject my *Brother's Solution* of this *Problem*; for certainly that *Motion* (whatsoever it is) in a *Menstruum*, which is able to dissolve such a Solid Body as Iron, that is, which is able to disturb the Close and strong Cohesion of the Parts of Iron, may very well be supposed sufficient to disturb or keep up these Parts from Resting in the bottom of the Vessel, wherein the *Solution* was made; and certainly no better account can possibly be given of such *Solutions*, than by supposing such an *Internal Motion* in the parts of the *Menstruum* Insinuating themselves into the Solid Body, and loosening its parts. But I leave to others to consider what Kind of *Motion* and Peculiar Conformation of Parts is requisite both in the *Menstruum* and in the Dissolved Body, that a *Solution* may result from their Commixture.

2. Tho' Liquors Consist of Parts United, and tho' this Union be easily Destroyed, yet of Necessity it requires some degree of Force for Effecting it; Yet this Property ought not to be rely'd on as the Sole Cause of this Appearance; For, in this *Solution* of the *Problem*, We first Suppose the Minute Particles of a Heavy Body Rais'd, and then give the Reason of their not Sinking; Whereas 'tis not to be questioned, but that that Force which Rais'd them, is the same that Keeps them from Falling to the bottom.

Consider'd; by
Mr. Tho. Mo-
lyneux.
Ibid. p. 93.

IX. Sir Sam Moreland undertakes to Demonstrate, (contrary to the Common and Received Opinion through England and all Europe.)

1. That he will Force Water 60 Foot high with treble the Weight that shall Raise it 20 Foot, and so proportionably, in infinitum.

2. That by how much Wider the Barrel is, in which the Forcer Works, than the Pipe through which the Water is forced up, by so much is the Engine Pressed with Unnecessary Weight.

An Undertaking
for Raising of
Water; by Sir
Sam. Moreland.
p. 102. p. 25.
Apr. An. 1674.

X. 1. Elapsa nuper Æstate, Ann. 1684. in manus incidit Tractatus quidam inscriptus *Sipho Wurtembergicus*, sive *Sipho Inversus Cruribus æque-altis Fluens & Refluens hætenus Inauditus*: de hac *Machina* magnifica prædicat Author, sed Lectorem orat ut ignoscat Patrono ejus Serenissimo quod Mysterium Structuræ ejus sibi servet. Hæc dum Legerem in mentem venit quo modo instrui possit *Sipho*, ut quæ de *Wurtembergico* illo narrantur præstaret. Hæbens ergo in manibus *Siphonem* quendam Vitreum, erexi eum supra duo Vascula quantum potui perpendiculariter; dumque in eo situ sisteretur, affudi in unum ex Vascularibus Aquam, donec Orificium *Siphonis* paululum superaret, & statim in alterum Vas ut expectabam effluebat Aqua: tunc Evacuato illo Vase in quod primum Aquam Infuderam in alterum Effudi, & immediate Aqua ista Refluebat in prius Vas. Licet non ausim Artificium hujus *Siphonis* mei cum illo *Wurtembergico* comparare tamen, si Utilitatem spectemus, cum

A Siphon performing the same things with the *Sipho Wurtembergicus*; by Mr. J. Davis. p. 167. p. 346. Nov. An. 1684.

eo certare posse (præsertim si addatur ei Instrumentum quoddam, modo quodam peculiari à me excogitato,) non multum dubito.

By Dr. Papia.
Ibid. p. 847.

2. In Tractatu de *Siphone Wurtembergico*, qui *Sturgardia* Authore D^o. *Salomone Reiselio* nuper Editus est, magna quædam atque inaudita, si & Utilitatem & Raritatem & Artificium spectes, de novo illo *Siphone* prædicantur: ipsius autem Proprietates Characteristicæ proponuntur his Verbis.

1. Ut Orificia Crurum Duorum Siphonis sui horizontaliter sita Labris inserantur; cum in Veterum inventis Crus longius infra Labrum seu *Æquilibrium* descendat semper.

2. Ut Orificiis vel partim vel dimidium Aqua repletis, Effluat tamen Aqua super Montem ducta; cum in reliquis Siphonibus totum Orificium Aqua adimpleri seu immergi Aquæ debeat.

3. Ut in Siccitate diuturna quiescens Machina tamen effectum producat Affluente iterum Aqua.

4. Ut Lumine seu Orificio alterutro Aperto, altero vero post Horas demum aut Diem, seu per Epistomium, seu Conum recluso, Effluat tamen Aqua; cum in aliis utraque Simul Lumina Aperiri debeant.

5. Ut in Horizontali Linea positis Orificiis, & equalibus quoad Altitudinem Cruribus exundet Liquor; cum in Portæ Aliorumve Machinis in equalia debeant Crura esse, & Perpendicularum majus.

6. Ut ab utroque Labro in utrumque Labrum infusa Ascendat & Effluat Aqua; cum in Veteribus non nisi per unicum nempe Longius, Crus Effluat, nunquam Refluat.

Hæc sunt ipsius verba Authoris, qua vero ratione, quibusve auxiliis tanti effectus produci possint, ne verbum quidem: Me itaque iussit Regia Societas Machinam construere, quæ eadem illa in Libro descripta Phænomena exhiberet. Rem quidem tribus variis modis exequutus sum; ne vero tædiosus nimium videar, sequentem methodum utpote facillimam instar omnium fore arbitror.

Fig. 197.

A A Sunt duo Vasa Metallica quibus duæ Siphonis extremitates inferuntur. BCDEDCB est Siphon, cujus Lumina B B, In eadem Horizontali Linea disponenda sunt. F est Tubulus foramini in superiori Siphonis parte adferuminatus, diligenterque obturandus, postquam Siphon Aqua exacte Repletus fuerit: Jam manifestum est Aquam in partibus C D, contentam, Aeri Externo ingressum prohibere ne ad superiorem Siphonis partem E penetret: Siphon igitur Aqua semper plenus (modo debitam Altitudinem non excedat,) effectum suum certissime producet statim atque Aqua in Vasibus A A, contenta alterutrius Orificii B, partem aliquam Replebit; quumque Ambobus Orificiis Aqua partim Repletis, in utroque Vase A ad eandem Horizontalem Lineam Superficies Aquæ pervenerit, si alterutri Vasi vel tantillum Aquæ infuderis, pars ejus per Siphonem statim in alterum Vas deferetur, eademque ratione cætera in Libro descripta Phænomena exhiberi poterunt.

3. Ne



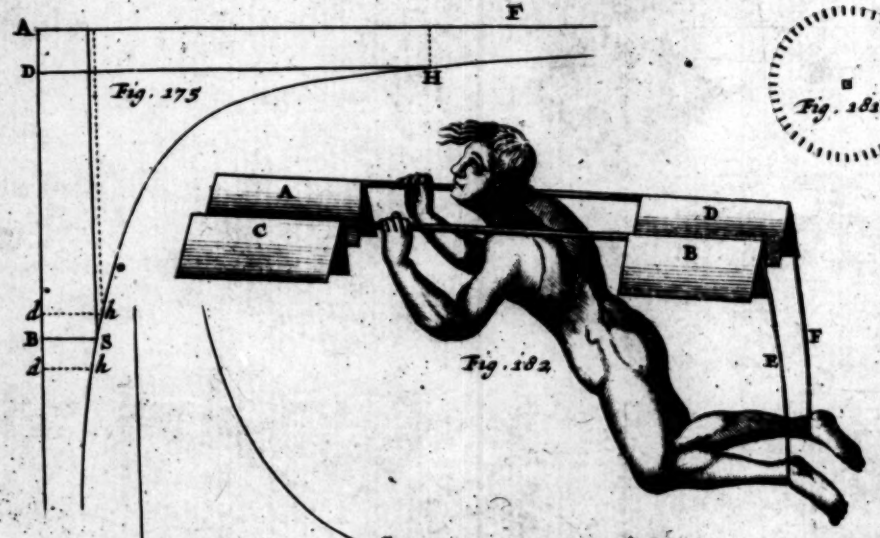
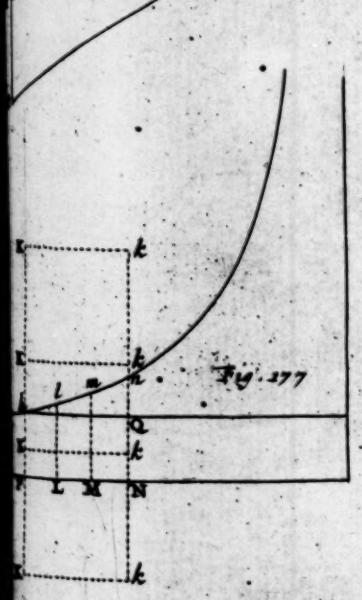
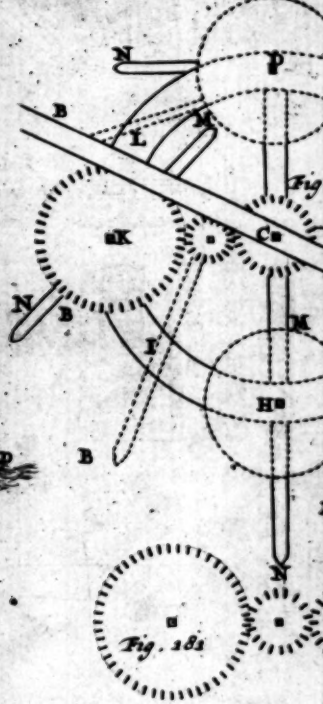
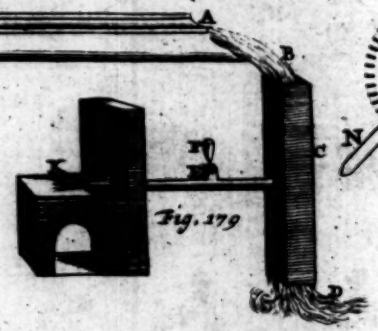
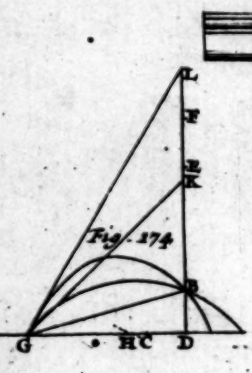
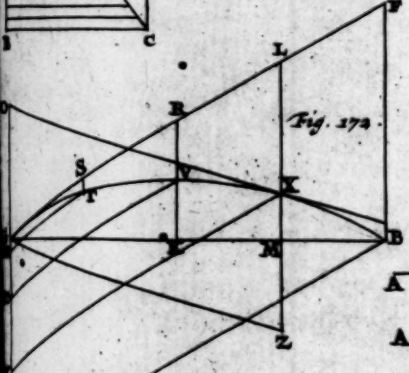
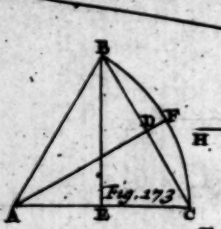
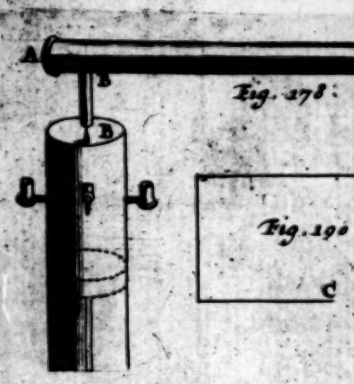
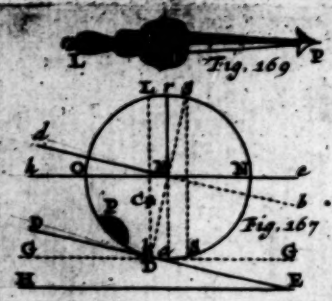
By D. Salomon
Reiseliu bim-
N, self.
n. 178. p. 1272.
Dec. An. 1685.

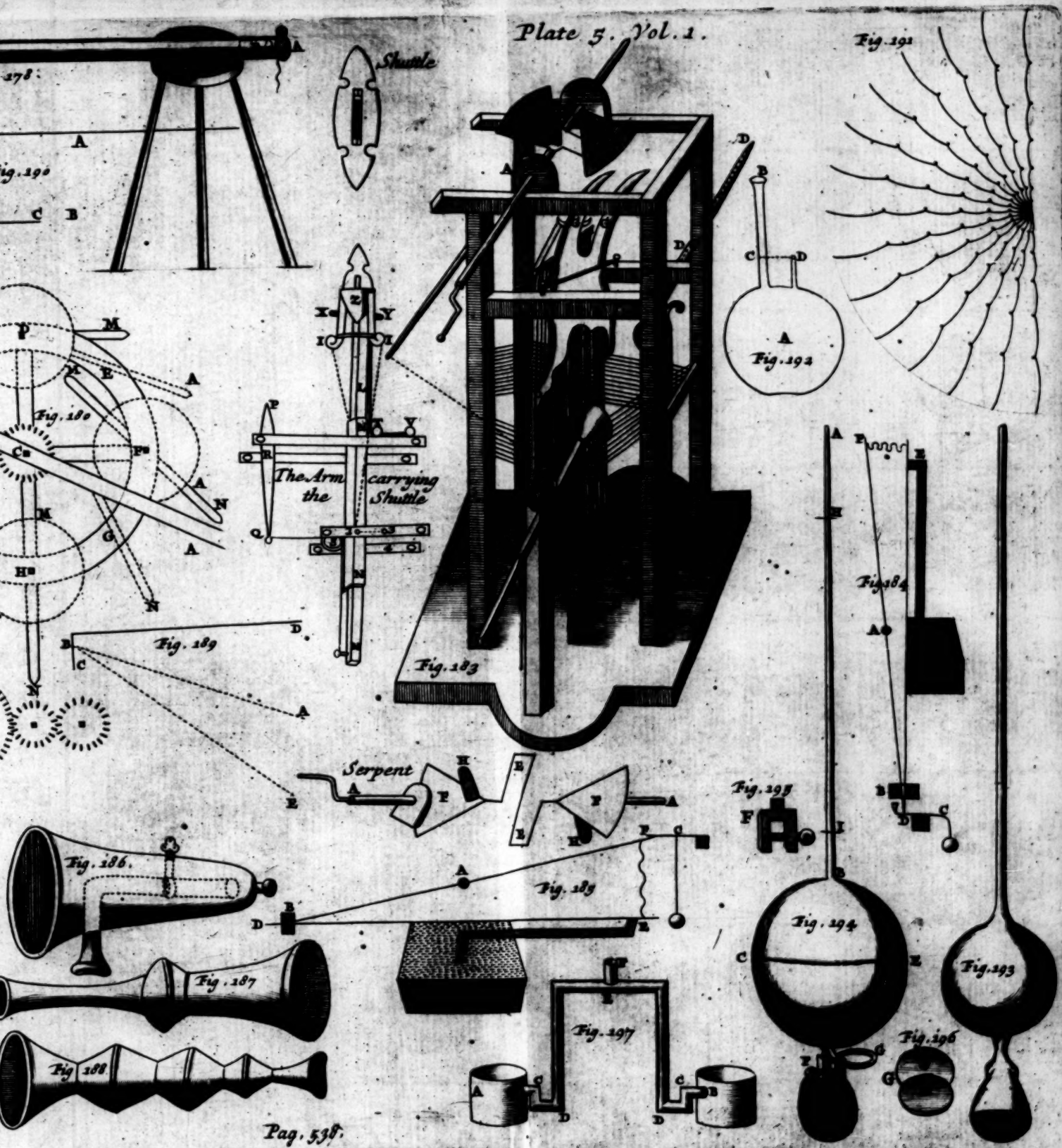
A New way of
Raising Water,
Enigmatically
proposed; by
Dr. Papin.
n. 173. p. 1093.
July An. 1685.
Fig. 198.

Solv'd : by
Dr. Nath.
Vincent.
n. 177. p. 1238.
Dec. An. 1686.

By Mr. R. A.
Ib. p. 1239.

In a Letter Sub-
scribed, W. Te-
non.
n. 178. p. 1254.
Fig. 199.
Dec. An. 1685.





By Dr. Papin.
Ibid. p. 847.

Fig. 197.

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3. Ne diutius *Serenissima Societatis* desiderium frustraer, aut morarer, fa-
 teor Excellentissimi Dⁿⁱ. D^{ris}. Papin Siphonem ipsissimum esse *Wurtembergicum*,
 etiam cum Recurvatione Pedum factum; neque aliud esse Mysterium, ut
Inventor ipse scripsit, in Summitate, quam quod per Infundibulum debeat
 Impleri, sine qua Impletione non potest transfluere. Quod proximè Typis
 amplius confirmabitur, quia longum nimis esset & tædiosum hic omnia
 scribere.

By D. Salomon
 Reiselius bim-
 self.
 n. 178. p. 1272.
 Dec. An. 1685.

Ast, ut in præsentī ostendam me hætenus nonnihil laborasse circa *Siphonis*
 effectus; ecce inter Experimentandum, hoc quoque mihi occurrit; ut didice-
 rim, quomodo in *Vertice* vel ad *Latèra* possit Effluere; quod hætenus multi
 promissere, vix quisquam efficit.

XI. 1. A A, Is a great Glass made like a Tumbler, but much bigger,
 and laid upon the Chimney Board, B B.

A New way of
 Raising Water,
 Enigmatically
 propos'd; by
 Dr. Papin.
 n. 173. p. 1093.
 July An. 1685.
 Fig. 198.

C C, Is the *Engine* like a small *Rock*, that doth constantly Spout out Water
 by the two Holes D D: This *Rock* is kept at a distance from the bottom of
 the Glass A A; so that it may plainly be seen that it cannot Receive any
 Water by Subterranean Tubes.

E E, Is a factitious *Coral*, reaching from the Center of the *Rock* C C, to
 the Center of the *Crown* F F.

F F, Is a *Crown* bearing upon the Aperture of the Glass A A, and holding
 the *Rock* C C, Suspended at a considerable Distance from the Bottom.

G G, A Glass Open at both ends, apply'd to the *Rock* C C, to keep the
 Water upon it from falling down.

The Water in this *Engine* runs constantly H H, Two *Shells* to receive the
 Water from the *Jetto's*.

Solv'd: by
 Dr. Nath.
 Vincent.
 n. 177. p. 1238.
 Dec. An. 1686.

2. Within the *Rock* C C, there may be a Vessel placed, which shall be
 made like the Body of a pair of *Bellows*, or those *Puffs* heretofore used by *Bar-*
bers, which being filled with Water, a piece of *Clockwork* put under it may
 produce the *Jetto's*; the Water being received into the *Shell* H H, and running
 thence into the hollow of the *Coral* E E; may be thereby conveyed into the
 Follicular Cavity in the same quantity it is Ejected from the two Emerging
 Tubes; and it will Circulate according to the going of the *Clock-work*.

3. I conceive that the Air is forced into the *Outer Glass* at the bottom thereof.
 That it then passes up between the two *Glasses*.

By Mr. R. A.
 Ib. p. 1239.

That the *Outer Glass* or Case being close Luted at the Head or *Crown* to
 which the *Inner Glass* is hung by the *Coral*, the Air is forced into the Mouth
 of the *Inner Glass*.

That the Air so forced pressing on the Surface of the Water that covers
 the *Rock*, forces the Water to rise through those two extream Parts that
 are not at all Clogg'd, or covered with Water.

4. ABDE, signifies a Cylindrical Vessel, Closed on every side, and Di-
 vided into two Rooms by the Floor E F.

In a Letter Sub-
 scribed, W. Te-
 non.

G L M H, Is another Cylindrical Vessel within that Upper Room, Ce-
 mented with its Mouth downwards to the Floor, and full of Water up to
 the Surface I K; the upper part thereof G I K H, being full of Air.

n. 178. p. 1254.
 Fig. 199.
 Dec. An. 1685.

QO, RP, Two Pipes, open above and below, and let through the Upper Room into this Vessel, and reaching almost down to the Floor EF.

VW, a Pipe open above and below, and let into the Upper Room. These Pipes must be close joyned round about them to the Floors C D, and GH.

XY, Two little Hemispherical Bladders prepared with Oil or some Oily Substance (as Butter and Turpentine) against Water, and Cemented with their Mouths upward to the Floor EF, underneath.

α , β , Two Valves Opening out of the Upper Room in the Bladders.

γ , δ , Two other Valves Opening out of the Bladders into the Inner Vessel above.

NZ, A *Pendulum* playing upon the Centre N, and having two Battle-door Arms a , b , to squeeze alternately the Bladders which rest upon them.

Let the Upper Room be filled with Water at the Pipe VW, and if the *Pendulum* be made to play by *Clock-work*, the Bladders will perpetually Pump it thence into the Inner Vessel, and the Comprest Air GIKH in the Upper part of that Vessel pressing upon the Surface of the Water IK, will force it thence into the Pipes OQ, PR, out of which Spouting with a perpetual even Stream into the Spoons ST, it will run down by the Pipe WV, into the Upper Room again, the *Pendulum* will play most easily when the Upper Room is filled to the Top of the Pipe WV. Instead of the Bladders may be other Contrivances, as of Suckers or little Organ Bellows, playing alternately with two Leaves about an Axis in the middle.

5. AA, Is the *Great Tumbler*, that must have some little hole in the bottom as I.

ILL, A slender Pipe hidden by the Chimney Board BB, whereby the Tumbler AA, hath Communication with the *Pump* or *Bellows* MM.

MM, Some kind of *Pump* or *Bellows* well shut, and having no other Aperture, but through the Pipe ILL. These are put in some secret Place where a Body may Play the same, and not be seen.

NN, A slender Pipe, that makes a Communication between the *Glass* AA, and the *Crown* FF; this Pipe reacheth near to the cover of the *Crown*, that the Water contained in it may not run down by that Aperture.

EE, the *Facitious Coral*, Hollow within, Shut at the Bottom and Open at the Top.

DD, DD, Two crooked Pipes Soldered to the Sides of the *Coral* EE, so that the Water running down the *Coral* may Spout out at the Holes DD.

OO, A Pipe hidden in the *Coral* EE, passing through the bottom of the same where it must be well Solder'd, and reaching near to the bottom of the *Rock* CC.

PP, A Pipe to convey the Water from the *Glass* GG, into the *Rock* CC; this Pipe is well Solder'd to the *Cover* of the said *Rock*.

Q, A *Valve* working by a Spring at the bottom of the Pipe PP, to keep the Water, that gets in that Way, from returning back.

R, Another *Valve* at the top of the Pipe OO, that the Water getting up that way, may not fall through the sam.

Now

Now it is plain, that the *Rock CC*, being filled partly with *Water*, partly with *Air*; if we open the *Bellows MM*, the *Air* from the *Crown FF*, must run through the *Pipe NN*, into the *Tumbler AA*: And thence through the *Pipe ILL*, into *MM* to fill the *Vacuity* madetherein: The *Air* in the *Crown FF*, being thus *Rarified*, gives liberty to the *Air* in the *Rock CC*, to *Rarify* too, by *Driving* the *Water* through the *Pipe OO*. The *Water* being got up into the *Crown FF*, runs down the *Coral EE*, and through the *Crooked Pipes DD, DD*, *Spouts* out at their upper *Apertures*, and from the *Shells HH*, falls upon the *Rock CC*: If we come afterwards to shut the *Bellows MM*, the *Air* got into their *Vacuity*, must run back into the *Tumbler AA*, and press upon the *Water* at the *Top* of the *Rock CC*: But the *Air* in the said *Rock* having been *Rarified*, its *Spring* is not sufficient to *Resist* this *Pressure*, and so the *Water* is forced into the said *Rock* through the *Pipe PP*: And by thus opening and shutting the *Bellows MM*, the *Water* must constantly circulate by the *Ways* aforesaid.

As for the *Uses* this *Way* for *Raising Water* may be applied to, this I do conceive: The *Glasses*, being merely to conceal the *Secret*, must be left out; and there may be made several *Receptacles* above one another to receive the *Raised Water*, so as doth the *Crown FF*: And there should be as many *Bellows* to *Communicate* every one with one *Receptacle*: These *Bellows* should be moved by an *Axis*, so that, when the *First* is open, the *Second* should be shut: the *Third* open, the *Fourth* shut; and so forth, alternatively; which may be easily done: By this means, the *First* or *Lowest Receptacle* would give the necessary *Supply* of *Water* to the *Second*, the *Second* to the *Third*, and the *Third* to the *Fourth*, &c. 'till the *Water* would be *Raised* to the intended *Height*; such *Receptacles* might easily be set at 12 or 15 *Foot* above one another, and so but few of them might *Raise Water* to a considerable *Height*, as well as ordinary *Pumps* do: But this new *Way* would have this advantage, that in the *Ordinary Pumps* the strength to be applied lieth near the *Water* to be *Raised*, but by this *Contrivance* the *Stream* of a *River* may be applied to draw *Water* out of a *Mine* far distant from it. By the same way the *Stream* of the *Thames* might keep constant *Water-Works* in *Windsor-Castle*, as easy almost as in the *Lowest Fields*: The *River Seine* might do the same at *St. Germain*, and perhaps at *Versailles* too, notwithstanding the great *Distance*. For it is to be observed, That the *Pipes* of *Communication* between the *Bellows* and the *Engine*, being merely for the *Conveying* of the *Air*, which moves very swiftly, they may be slender enough, and so contain but a small *Quantity* of *Air* to be *Rarified*; and besides, they will not be subject to *Burst* or *Leak*, since the *Pressure* they bear, being all external to the *Pipe*, will rather strengthen than break the same. From whence it follows, That the said *Pipes* need not be strong, but may be made at very small *Charges*. It is also to be observed, That those *Bellows* which are open, have the *Air* in them very much *Rarified*, so that the outward *Air* lieth heavy upon (to shut) them, by which means the *Motion* of the *Engine* must be help'd in *Lifting* up the opposite *Bellows*, that are to be opened: And this *Observation* may answer

The Use of this
Contrivance.

answer the greatest Difficulty that might be objected against this Contrivance: So that I don't question but this way for *Raising Water*, may on several Occasions be of great Advantage.

Further Explained.
Fig. 201.

A B, A B, Are several *Receptacles* set above one another, which must be well Shut and Soldered every where.

CDD, CDD, Are two slender Pipes, whereby the First and Third *Receptacles* have a Communication with the *Pump* HH.

EFF, EFF, Two other slender Pipes, whereby the Second and Fourth *Receptacles* have a Communication with the *Pump* II.

HH, II, Two *Pumps* whose Plugs are so moved by the Axis LL, that when one goeth down the other goeth up.

MM, A Wheel fastened to the Axis LL, that it may be moved by the Stream of a River.

NO, PQ, NO, PQ, Are big Pipes for the Water to go up, from a Lower into a Higher *Receptacle*.

O, Q, O, Q, Are *Valves* fitted to the Top of the aforesaid Pipes that the Water may not go down through the same.

Now it is plain, that, when the *Plug* in the *Pump* HH, is going up, the Air comes in through the Pipes CDD, and so it is Rarified in the First and Third *Receptacles* marked A, A: And by that means the Water may be Driven up into the said *Receptacles* through the Pipes NO, because at the same time the *Plug* in the *Pump* II, going down, causeth the Air to return to its Ordinary Pressure in the Second and Fourth *Receptacles*, that it may be able to drive up the Water through the said Pipes NO, and the Lowest Pipe draws the Water that lies open to the Air. By the same reason when the *Plug* in the *Pump* II, goeth up, the Air must come in through the Pipes EFF: And so it is Rarified in the Second and Fourth *Receptacles* marked B, B, and by that means the Water may be driven up into the said *Receptacles* through the Pipes PQ, PQ, because at the same time the *Plug* in the *Pump* HH, going down, causes the Air to return to its Ordinary Pressure in the First and Third *Receptacles*, so that it is able to Drive up the Water through the said Pipes PQ.

Several Objections, made by M. Nuis, Answered; by Dr. Papin. n. 186. p. 263. Jan. An. 1687.

6. 1. To keep the *Receptacles* from being fill'd too much, the Water may be let out by Inserting into each a Crooked Pipe, reaching a pretty way downwards, and having its Lower Aperture shut up with a Valve, whereby the Water may run out when the *Receptacle* shall be fill'd to a certain Height: And I may add, to prevent new Difficulties, that, lest the *Pumps* should be fill'd too much, a Valve may be made that shall open as soon as the Air in the *Pump* should be more Comprest than the Outward Air: So the Air getting in through any Pores would be constantly let out.

2. I have not positively promised a good Success, but for *Windsor* and *St. Germain*; but when I spoke of *Versailles* I used the Word *perhaps*, thereby shewing, that, before any one should go about such great Undertaking, he should Reflect upon it more than I would then do, not having Occasion for such Work. But I now make the following Computation.

Let the Distance of *Versailles*, as *M. Nuis* supposeth, be 12000 Foot, and the Capacity of each *Receptacle* be about one half of a Cubic Foot: I might make the Wheel with the Axis to make their *Revolution* in one Minute of Time, and so Order all Things that the Air under the Ascending *Plugs* might come to be Rarified to such a degree, that by its Elasticity it might not Counterpoise more than 7 Foot of Water; but at the same time the Air in the *Receptacles* A, A, B, B, would, even in its greatest Dilatation, be able to Counterpoise 17 Foot: So it is plain, that the Air will be driven from the *Receptacles* into the *Pumps* by a Strength equivalent to 10 Foot of Water: Now, if we compute the * *Velocity* of Air driven by such a Pressure: We shall find that the said *Velocity* will be about 740 Foot in a Second: so that in half a Minute, during which the Plug goeth up, this Air might pass above 22000 Foot, although it were not Rarified at all; but being Rarified, as we do suppose it to be, it might go a great deal further.

*Vid. Sup. Cap. V.
S. XII.*

I must now take notice, that according to the Honourable Mr. *Boyle's* Experiments, the *Rarefaction* of the Air is much lesser than Mr. *Nuis* takes it to be: For the Water contained in the Pipe N O, is so far from causing the Air to fill up a space four times bigger, that it will not Extend itself to a Space once bigger than before; Considering therefore the *Velocity* of the Air, and the small Dilatation it doth suffer, if any one will take the trouble to Compute, he will find, that if the *Pumps* have in Diameter the Diagonal of a Square Foot, and the same Height: And if the small Tubes of Communication be made of $\frac{1}{2}$ part of an Inch in Diameter; so that being 12000 Foot long, they may contain about one Cubic Foot of Air; that would be more than sufficient to make the necessary *Rarefaction* in the *Receptacles*.

But for the good Success of the *Engine* it is not enough to make the Air pass from the *Receptacles* into the *Pumps*, it must also Return from the *Pumps* into the *Receptacles*: Now for this Intent it would be necessary to set the *Receptacles* but five Foot above one another; so, to drive the Water up the Pipe N O, it would be enough that the Air in the *Receptacle* B, should Press with a strength Equivalent to 23 Foot of Water: For it is plain, that five Foot in the Pipe N O, together with a Pressure Equivalent to 17 Foot, which I have supposed to be in the Upper *Receptacle* A, will make but 22 Foot in all; and therefore 23 Foot Pressing in the *Receptacle* B, must prevail, and cause the Water to Ascend: Now the Pressure in the *Receptacle* being but 23 Foot, and the Air in the *Pump* Returning to it's Ordinary Pressure, which is about 33 Foot, it is plain that the Air going back to the *Receptacle*, will be Driven by Strength Equivalent to 10 Foot, as well as it had been in coming from the *Receptacle* towards the *Pump*: And so the Bigness assigned for the *Communication Pipes* will also prove more than sufficient to this Effect.

From what I have been saying, it is plain, That in great Distances there should be made as many *Pumps* as *Receptacles*, as hath been already propounded: And for to raise Water but 60 Foot High, there should be required 13 or 14 *Receptacles*, and as many *Pumps* of the Bigness aforesaid. Some People may take this for a great Difficulty. But I Answer, That in this

this *Engine*, this is not so much as it seems at First ; because, the Pressure being all from without, there is no need of any great strength to resist it, and so the Metal for the *Pump* will Cost but little : There may also be found Occasions where to make so good use of them, that such an *Engine* as I have Described would in a Years time save Labour enough to pay for many *Pumps*, since it might every hour Raise about 1800 Pounds of Water to the height of 60 Foot : Mean while I dont pretend to have given here the best Proportion for the Bigness of every part of the *Engine* ; but it may be, by Altering the Capacity of the *Pumps*, of the *Pipes*, or of the *Receptacles*, a much more considerable Effect might be Produced.

3. The Water doth a not any time Ascend higher than from a Lower *Receptacle* into the next Upper *Receptacle* ; which Height is but 12 Foot : So that it is plain enough, that the Pressure of the Air may be sufficient to Drive it up. It is Indifferent, whether it be by Rarefaction or otherwise that the Water comes into the *Receptacle* A ; it is enough that the Water is there, and that the Air Presses upon it with such a Strength as will prevail against all that opposeth it.

4. Though the Use of the *Pipes* be meerly for Conveying of Air : They may nevertheless be easily fill'd with Water when need Requires, and so the Defects in them may as well be found out as in *Pipes* that are used for the Conveying of Water.

An Engine for
Raising Water,
by the Help of
Fire ; by Mr.
Tho. Savery.
n. 253. p. 228.
Fig. 202.
June An. 1699.

XII. A, The Furnace,
B, The *Boyle*.

CC, Two *Cocks*, which convey the *Steam* by turns to the *Vessels* DD.

DD, The *Vessels* which receive the Water from the Bottom, in order to Discharge it again at the Top.

EEE E, *Valves*.

FF, *Cocks* which keep up the Water, while the *Valves* on Occasion are Cleans'd

G, The *Force Pipe*.

H, The *Sucking Pipe*.

I, The *Water*

An Hydraulic
Engine ; by
.....
n. 128. p. 679.
Fig. 203.
An. 1675.

XIII. This *Engine* is a *Chest* of Copper A, pierced with many Holes above BB, and holds within it the Body of a *Pump* E F M, whose *Sucker* DE, is Raised and Abased by two *Levers* C, O ; These *Levers* having each of them two Arms, and each Arm being fitted to be laid hold on by both Hands of a Man. Each *Lever* is pierced in the middle by a *Mortaise* a a, in which an-Iron Nail, which passes through the Handle of the *Sucker*, turns when the *Sucker* is raised or lower'd. Near the Body of the *Pump* there is a Copper Pot I H K, joined to it by the Tube G, and having another Tube K N L, which in N may be Turned every way.

To make this *Engine* Play, Water is pour'd upon the *Chest* to enter in at the Holes that are in the cover thereof. This Water is drawn into the Body of

of the *Pump* at the *Hole F*, at the time when the *Sucker* is Raised; and when the same is let down, the *Valve* of the same *Hole F* shuts, and forces the *Water* to pass through the *Hole M* into the *Tube G*, of which the *Valve H* being lifted up, the *Water* enters into the *Pot*, and filling the *Bottom*, it enters through the *Hole K*, into the *Tube K N L*, in such a manner, that when the *Water* is Higher than the *Tube K N L*, and the *Hole* of the *Tube G* is shut by the *Valve H*, the *Air* Inclosed in the *Pot* hath no issue, and it comes to pass, that, when you Continue to make the *Water* Enter into the *Pot* by the *Tube G*, which is much thicker than the Aperture of the end *L*, at which it must issue, it must needs be, that the Surplus of the *Water* that enters into the *Pot*, and exceeds that which at the same time issues thro' the small end of the *Jet*, compresses the *Air* to find place in the *Pot*: which makes that, whilst the *Sucker* is Raised again, to make new *Water* to Enter into the *Body* of the *Pump*; the *Air* which has been Compress'd in the *Pot*, Drives the Surplus of the *Water* by the Force of its *Spring*, mean time that a new Compression of the *Sucker* makes new *Water* to enter, and causes also a new Compression of *Air*. And thus the Course of the *Water*, which issues by the *Jet*, is always entertained in the same State.

XIV. A A, The Body of a Square Taper *Pump*, made of Oak, Elm, or Deal Planks; with a *Valve* at bottom a a.

BB, The *Bucket*, in the midst of which there is a *Valve b*, not visible in the Figure, being concealed by the Sides of the *Leather b b*.

CCC, The *Iron* to raise the *Bucket*.

DD, The *Wood* at the bottom of the *Bucket* containing the *Valve*.

EE, The *Handle* for Raising the *Bucket*, to be managed by fewer Hands than Ordinary *Pumps* are; which may be Altered so as to employ a Horse, or Mill, or other such like way, more Advantageous than that of this *Handle* managed by the Strength of Men.

FF, a Square Taper-Box with Holes in the Sides, and open at the Bottom; into the narrower part of which is enclosed the narrower End of the *Body* of the *Pump*.

GG, An Additional *Bucket* of a larger Dimension to be placed on the *Iron Work* of the *Pump* about H, when it shall be needful to Lengthen the Taper of of your *Pump*, and thereby to Raise the *Water* more forcibly to a greater Height.

II, The *Spout* of the *Pump*, to cast out the *Water* of the same breadth with the side of the *Pump*.

KK, The *Iron* or *Wooden Work* set off, or bent back (if need be) and placed at the back of this *Pump* for the easier and more capacious Motion of the *Pump Handle*, in which it moves.

This *Pump* was by me Contrived in 1673. when the New Canal of Fleet-River in London was Enlarged: It was found to Raise at least twice as much *Water* proportionably as those of the same, or rather bigger Bore, that were first made use of and cast by. It was 8½ Foot long, and 1 Foot 8 Inches

broad at the top, and about 8 Inches broad at the Bottom, where it is inserted in the *Box*, and did cast out 8 Gallons at a *Stroke*, and 21 *Strokes* being made in one *Minute*, there were Delivered about 169 Gallons in a *Minutes* time; whence it is easy to Compute, what Quantity is thrown out in an *Hour*. This kind of *Pump* may by the same Contrivance, be made of a Tree Bored through with a Taper-Bore; and a Basket may be used at the bottom of the *Pump* instead of the *Box-Colender*.

Phil. Coll.
n. 2. p. 35.

XV. Papers of less General Use; (Extracted from a Book of Jo. Alph. Borellius de Motu Animalium,) Omitted,

1. **A** Way how a Man may Swim under Water, and Breath by the Help of a Bag about his Head.
2. Another Way of Breathing under Water by the help of a Leathern Pipe kept open by Wreathed Wires, and extended by the Swimmer's Head to the top of the Water.
3. A way to make a Submarine Vessel accommodated with Ways to Row it, and to make it Rise and Sink in the Water.

XVI. Accounts of Books Omitted.

n. 8. p. 145.
n. 10. p. 173.
n. 226. p. 481.

1. **H**ydrostatical Paradoxes, made out by New Experiments, (for the most part Physical and Easy) by the Honourable R. Boyle Esq;
2. Recueil de diverses Pieces touchant quelques Nouvelles Machines, &c. Par. D. Papin. M. D. A Cassel. 1695. in 8vo.

C H A P. IX.

Geography. Navigation.

A new Place for the first Meridian propos'd; by a Professor of Math. at Seville.
n. 118. p. 425.
Oct. An. 1675.

- I. **L**ongitudo Terrestris est Arcus *Æquinoctialis* ab uno ad alium Meridianum interceptus sive Temporis Spatium, quod per *Æquinoctialem* numeratur, inter duo Loca; quare consonum fuerit, *Longitudinis Principium* in ipso *Æquinoctiali* constituere. Insuper cum Circulus ille *Æquinoctialis Globum* in Borealem & Australem partem dividat, si detur hujus *Primarii Meridiani* Fixatio in eo, erit inter Boreales & Australes *Æquitas* atque Conformitas. Deinceps, oportet, ad Præcisionem, Locum hujus *Primarii Meridiani* esse parvum, ut *Longitudinis Numeratio* exactius exprimatur: non ut aliqui,

aliqui, qui omnes *Insulas Fortunatas* pro hujus *Principio* assumebant, & *Distantiam* duorum *Graduum*, inter earum aliquas, non notabant; quod certe absurdum nimis erat. Iterum, *Præcautio* alia est habenda, ut *Primarius Meridianus* non confundatur cum *Terris* & *Locorum Imaginibus* in *Globo* vel *Mappa* exaratis, quod fiet si per *Medias Terras* Transeat: Et si præcipuas *Terræ Partes* dividat, ut *Americam*, *Africam*, & *Europam*, per *Maria* transiens, erit eoaptior & convenientior in *Globi Tarraquei* repræsentatione. Quas omnes memoratas causas considerans, Inveni, quòd *Natura* (nihil frustra suppeditans) posuit sub ipso *Æquinoctiali Circulo* *Insulam* quandam prope *Brasiliam*, olim *Abroxos*, nominatam, quæ *Insula* distat à *Teneriffæ Pico* Grad. 9. Occidentales, & ab *Uraniburgo* Grad. 42. Occidentales; in qua inveniuntur omnia ad *Primarium Meridianum* conducenda, ut à me æstimatur.

II. 1. Those that intend to make use of *Pendulum Watches* at *Sea*, must have two of them at least; that, if one of them should by mishap or neglect come to stop, or (being by length of time become foul) need to be made clean, there may likely always remain one in Motion.

2. The *Watches* on *Shipboard* are to be Hung in a Close Place, where they may be freest from Moisture or Dust, and out of Danger of being disordered by Knocking or Touching.

3. Before the *Watches* be brought on *Shipboard*, 'tis convenient they be adjusted to a *Middle* or *Mean Day*, the Use of them being then most easy.

4. Here take Notice, That the *Sun* passeth the 12 Signs, or makes one Entire Revolution in the *Ecliptick* in 365 Days, 5^h. 48'. or thereabout: And that those Days, reckoned from Noon to Noon, are of different Lengths; as is known to all that are versed in *Astronomy*. Now between the Longest and the Shortest of those Days, a Day may be taken of such a Length, as 365 such Days, 5^h. 49'. &c. make up, or are Equal to that Revolution, And this is called the *Equal* or *Mean Day*, according to which the *Watches* are to be Set; and therefore the Hour or Minute shewed by the *Watches*, though they be perfectly Just and Equal, must needs differ almost continually from those that are shew'd by the *Sun*, or are Reckoned according to it's Motion. But this Difference is Regular, and is otherwise called the *Æquation*; which is accounted from the first of *February* in the following Table.

M. Hugen's
Instructions for
finding the Longi-
tude with Pen-
dulum Watches;
Enlarged by . . .
n. 47. p. 937.
May An. 1669.

To adjust the
Watches.

| January | | | February | | March. | | April. | | May. | | June. | |
|---------|---|----|----------|----|--------|----|--------|----|------|----|-------|----|
| | ' | " | | " | | " | | " | | " | | " |
| 1 | 6 | 10 | 0 | 0 | 4 | 46 | 14 | 23 | 19 | 25 | 16 | 24 |
| 2 | 5 | 47 | 0 | 2 | 5 | 03 | 14 | 39 | 19 | 28 | 16 | 13 |
| 3 | 5 | 24 | 0 | 4 | 5 | 21 | 14 | 55 | 19 | 29 | 16 | 01 |
| 4 | 5 | 02 | 0 | 8 | 5 | 39 | 15 | 10 | 19 | 29 | 15 | 49 |
| 5 | 4 | 41 | 0 | 12 | 5 | 57 | 15 | 25 | 19 | 29 | 15 | 37 |
| 6 | 4 | 21 | 0 | 16 | 6 | 15 | 15 | 39 | 19 | 28 | 15 | 24 |
| 7 | 4 | 02 | 0 | 21 | 6 | 33 | 15 | 53 | 19 | 26 | 15 | 11 |
| 8 | 3 | 44 | 0 | 26 | 6 | 51 | 16 | 07 | 19 | 24 | 14 | 58 |
| 9 | 2 | 27 | 0 | 32 | 7 | 09 | 16 | 21 | 19 | 21 | 14 | 45 |
| 10 | 3 | 11 | 0 | 40 | 7 | 27 | 16 | 34 | 19 | 18 | 14 | 32 |
| 11 | 2 | 55 | 0 | 48 | 7 | 45 | 16 | 47 | 19 | 15 | 14 | 19 |
| 12 | 2 | 39 | 0 | 57 | 8 | 03 | 16 | 59 | 19 | 11 | 14 | 06 |
| 13 | 2 | 23 | 1 | 06 | 8 | 22 | 17 | 11 | 19 | 07 | 13 | 53 |
| 14 | 2 | 07 | 1 | 16 | 8 | 41 | 17 | 22 | 19 | 02 | 13 | 40 |
| 15 | 1 | 52 | 1 | 26 | 9 | 01 | 17 | 33 | 18 | 57 | 13 | 27 |
| 16 | 1 | 38 | 1 | 37 | 9 | 21 | 17 | 43 | 18 | 51 | 13 | 15 |
| 17 | 1 | 25 | 1 | 49 | 9 | 41 | 17 | 53 | 18 | 45 | 13 | 03 |
| 18 | 1 | 13 | 2 | 02 | 10 | 01 | 18 | 03 | 18 | 39 | 12 | 52 |
| 19 | 1 | 02 | 2 | 15 | 10 | 21 | 18 | 13 | 18 | 33 | 12 | 41 |
| 20 | 0 | 51 | 2 | 28 | 10 | 40 | 18 | 23 | 18 | 26 | 12 | 30 |
| 21 | 0 | 41 | 2 | 42 | 10 | 59 | 18 | 32 | 18 | 18 | 12 | 19 |
| 22 | 0 | 32 | 2 | 56 | 11 | 18 | 18 | 39 | 18 | 10 | 12 | 08 |
| 23 | 0 | 24 | 3 | 11 | 11 | 37 | 18 | 46 | 18 | 01 | 11 | 58 |
| 24 | 0 | 18 | 3 | 26 | 11 | 56 | 18 | 53 | 17 | 51 | 11 | 48 |
| 25 | 0 | 13 | 3 | 41 | 12 | 15 | 18 | 59 | 17 | 41 | 11 | 38 |
| 26 | 0 | 9 | 3 | 56 | 12 | 34 | 19 | 04 | 17 | 30 | 11 | 28 |
| 27 | 0 | 6 | 4 | 13 | 12 | 53 | 19 | 09 | 17 | 19 | 11 | 18 |
| 28 | 0 | 3 | 4 | 29 | 13 | 12 | 19 | 14 | 17 | 08 | 11 | 09 |
| 29 | 0 | 1 | | | 13 | 31 | 19 | 18 | 16 | 57 | 11 | 00 |
| 30 | 0 | 0 | | | 13 | 49 | 19 | 22 | 16 | 46 | 10 | 52 |
| 31 | 0 | 0 | | | 14 | 06 | | | 16 | 35 | | |

| | July. | | August. | | Septemb | | October. | | Novemb | | Decemb. | |
|----|-------|----|---------|----|---------|----|----------|----|--------|----|---------|----|
| | ' | " | ' | " | ' | " | ' | " | ' | " | ' | " |
| 1 | 10 | 45 | 11 | 07 | 19 | 41 | 29 | 16 | 31 | 13 | 21 | 14 |
| 2 | 10 | 38 | 11 | 16 | 20 | 01 | 29 | 30 | 31 | 05 | 20 | 44 |
| 3 | 10 | 31 | 11 | 25 | 20 | 22 | 29 | 43 | 30 | 53 | 20 | 14 |
| 4 | 10 | 25 | 11 | 36 | 20 | 43 | 29 | 56 | 30 | 43 | 19 | 44 |
| 5 | 10 | 19 | 11 | 48 | 21 | 04 | 30 | 09 | 30 | 32 | 19 | 14 |
| 6 | 10 | 13 | 12 | 01 | 21 | 25 | 30 | 22 | 30 | 20 | 18 | 44 |
| 7 | 10 | 07 | 12 | 14 | 21 | 47 | 30 | 34 | 30 | 08 | 18 | 14 |
| 8 | 10 | 02 | 12 | 28 | 22 | 09 | 30 | 45 | 29 | 55 | 17 | 44 |
| 9 | 9 | 58 | 12 | 42 | 22 | 31 | 30 | 55 | 29 | 40 | 17 | 14 |
| 10 | 9 | 54 | 12 | 57 | 23 | 52 | 31 | 04 | 29 | 23 | 16 | 44 |
| 11 | 9 | 51 | 13 | 12 | 23 | 13 | 31 | 12 | 29 | 06 | 16 | 14 |
| 12 | 9 | 49 | 13 | 27 | 23 | 33 | 31 | 19 | 28 | 48 | 15 | 44 |
| 13 | 9 | 47 | 13 | 43 | 23 | 53 | 31 | 26 | 28 | 30 | 15 | 14 |
| 14 | 9 | 46 | 13 | 59 | 24 | 13 | 31 | 32 | 28 | 11 | 14 | 43 |
| 15 | 9 | 46 | 14 | 16 | 24 | 33 | 31 | 38 | 27 | 51 | 14 | 12 |
| 16 | 9 | 46 | 14 | 33 | 24 | 53 | 31 | 43 | 27 | 30 | 13 | 41 |
| 17 | 9 | 47 | 14 | 50 | 25 | 13 | 31 | 47 | 27 | 08 | 13 | 10 |
| 18 | 9 | 49 | 15 | 08 | 25 | 33 | 31 | 50 | 26 | 45 | 12 | 40 |
| 19 | 9 | 52 | 15 | 26 | 25 | 52 | 31 | 53 | 26 | 22 | 12 | 10 |
| 20 | 9 | 56 | 15 | 45 | 26 | 11 | 31 | 55 | 25 | 58 | 11 | 40 |
| 21 | 10 | 00 | 16 | 04 | 26 | 30 | 31 | 55 | 25 | 34 | 11 | 10 |
| 22 | 10 | 04 | 16 | 23 | 26 | 49 | 31 | 55 | 25 | 10 | 10 | 40 |
| 23 | 10 | 08 | 16 | 42 | 27 | 08 | 31 | 55 | 24 | 45 | 10 | 10 |
| 24 | 10 | 13 | 17 | 01 | 27 | 26 | 31 | 54 | 24 | 20 | 9 | 41 |
| 25 | 10 | 18 | 17 | 21 | 27 | 43 | 31 | 52 | 23 | 55 | 9 | 13 |
| 26 | 10 | 23 | 17 | 41 | 28 | 00 | 31 | 50 | 23 | 30 | 8 | 45 |
| 27 | 10 | 28 | 18 | 01 | 28 | 16 | 31 | 47 | 23 | 04 | 8 | 17 |
| 28 | 10 | 34 | 18 | 21 | 28 | 32 | 31 | 43 | 22 | 38 | 7 | 50 |
| 29 | 10 | 41 | 18 | 41 | 28 | 47 | 31 | 37 | 22 | 11 | 7 | 23 |
| 30 | 10 | 49 | 19 | 01 | 29 | 02 | 31 | 30 | 21 | 43 | 6 | 58 |
| 31 | 10 | 58 | 19 | 21 | | | 31 | 22 | | | 6 | 34 |

By

By the help of the foregoing *Table* you will always know what a Clock it is by the *Sun* precisely, and consequently, whether the *Watches* have been set to the right Measure of the *Mean Day*, or no; using the *Table* as follows.

When you first Set your *Watch* by the *Sun*, you are to Subduct from the Time observed by the *Sun*, the *Æquation* adjoyned to that Day of the Month in the *Table*, and to Set the *Watches* to the remaining Hours, Minutes and Seconds; that is, the *Watches* are to be Set so much Slower than the Time of the *Sun*, as (in the *Table*) is the *Æquation* of that Day; so that the *Æquation* of the Day Added to the Time of the Clock, is the True Time by the *Sun*. And when after some Days, you desire to know by the *Watch* the Time by the *Sun*, you are to Add to the Timeshewed by the *Watch*, the *Æquation* of that Day; and the Aggregate shall be the Time by the *Sun*, if the *Watch* hath been perfectly well Adjusted after the Measure of the *Mean Days*; for the doing of which, this will be a convenient Way;

Draw a *Meridian Line* upon a Floor, and then hang two Plummets, each by a small Thread or Wire, directly over the said *Meridian*, at the distance of some two Foot or more one from the other, as the smallness of the Thread will admit. When the Middle of the *Sun* (the Eye being placed so, as to bring both the Threads into one Line) appears to be in the same Line exactly (for the better and more secure discerning whereof, you must be furnish'd with a Glass of a Dark Colour, or somewhat Black'd with the Smoak of a Candle,) you are then immediately to Set the *Watch*, not precisely to the Hour of 12, but by so much less as is the *Æquation* of that Day; e. g. If it were the 12th of *March*, the *Æquation* of that Day being by the *Table*, 8'. 3". These are to be Subducted from 12 Hours, and the Remainder will be 11^h. 51'. 57". to which Hours, Minutes, and Seconds, you are to Set the *Index* of the *Watch* respectively: Then after some Days you are to observe again in the same manner, and likewise to note the Hour, Minute, and Second of the *Watch*; to which you are to add the *Æquation* of these Days, taken out of the *Table*; and if the Aggregate do just make 12 Hours, the *Watch* is Adjusted to the Right Measure; but if it differ, you are to Divide the Minutes and Seconds of that Difference by the Number of the Days between both the Observations to get the Daily Difference.

Let us suppose this Second Observation to have been made the 20th of *March*, viz. Eight Days after the first, and finding that the Middle of the *Sun*, being seen in the *Meridian* in the same Line with the two Threads, as before, The *Watch* Points, ———— 11^h. 51' 07".

The *Æquation* of the 20th of *March*, by the *Table*, is ———— 00 10 40
Which being Added to the Time, shew'd by the *Watch*, gives — 12 01 47

If this had been just 12 Hours, the *Watch* would have been well adjusted, but being 1'. 47". more than 12, it hath gone so much too fast in eight Days. And these 1'. 47". that is 107". being divided by 8, there come 13 $\frac{1}{4}$ Seconds for the Difference of every 24 Hours; which Difference being known,

known, if you want time, or have no mind to take the Pains to Adjust the *Watch* to its Right Measure, (this being not necessary, since you may bring it thus on *Ship-Board*) note only the Daily Difference, and regulate yourself accordingly. But if you will Adjust it better, you must Remove the less Weight of the *Pendulum* a little downwards, which will make it go Slower, and then you must begin a-new to observe by the *Sun*, as before. If it had gone too Slow, you must have Removed the mentioned Weight somewhat upwards. And this is of that Importance in the finding out of *Longitudes*, that if it be not Observed, you may sometimes in the space of Three Months misreckon 7 Degrees, and more yet, (without any fault in the *Watches*;) which under the *Tropicks* will amount to above 400 *English Miles*.

The *Watch* may also be Adjusted on *Board*, when a Ship Rides at Anchor, thus: In the Morning, when the *Sun* is just half above the *Horizon*, Note what Hour, Minute, and Second, the *Watch* points at, if it be going; if not, set it a going, and put the *Indexes* at what Hour, Minute, and Second you please. Let them go till *Sun Set*, and when the Body of the *Sun* is just half under the *Horizon*, see what Hour, Minute, and Second, the *Indexes* of the *Watch* point at, and note them too; and reckon, how many Hours, &c. are passed by the *Watch* between the one and the other. Then take the half of that Number, and add it to the Hours, &c. of the Morning Observation, and you shall have the Hours, &c. which the *Watch* did show, when the *Sun* was in the *South*; whereunto Add the *Æquation* in the Table belonging to that Day, and note the Sum. Then, some Days being pass'd, (the more the better,) you are to do just the same: And if the Hour of this last Day be the same that was noted before, your *Watch* is well adjusted; but if it be more or less; the Difference divided by the Number elapsed between the two Observations, will give the Daily Difference. And if you will, you may let it Rest there or otherwise, Removing the Lesser Weight of the *Pendulum*, you may Adjust it better. You may also, instead of the *Sun's Rising* and *Setting*, take two Equal Altitudes of the *Sun*, before, and after Noon, and having noted the time given by the *Watches* at the time of both the Observations, proceed with it in the same manner, as was just now directed for Observing the *Sun* in the *Horizon*. In either of which ways there may be some Error, caused by the *Sun's Refraction*, which is considerable, and therefore needs not to be taken notice of.

5. Give to each of the *Watches* a Name, or a Mark, as A, B, C; and before you set *Sail*, set them to the Time observed by the *Sun* in the place where you are, and whence you are departing, allowing for the *Æquation* of the Day whereon you make your Observation; which Day you are to note, if the *Watches* be not well adjusted; otherwise it is not necessary.

Then afterwards being at *Sea*, and desiring to know the *Longitude* of the Place where you are; that is, How many Degrees the Meridian of that Place is more Easterly or Westerly, than the Meridian of that Place where you did set the *Watches*; you must observe by the *Sun* or *Stars*, what Time of the Day it is, as precisely as is possible, and Note at the same time, to what Hour, Minutes,

To find by them
the Longitude
at Sea.

Minutes and Seconds the *Watches* do point, (which Time, if the *Watches* be not set to the Right Measure, is by the known Daily Difference to be Adjusted,) adding thereunto the *Æquation* of the present Day, which gives you the time of the Day, shewed by the Sun, at the Place where the *Watches* were set : And, if this time of the Day be the same with that observed where you are, then you are under the same Meridian with the Place where the *Watches* were set by the Sun ; but, if the time of the Day, observed where you are, be greater than that shewed by the *Watches*, you may be assured, that you are come under a more Easterly Meridian ; and if less you are come under a more Westerly. And Counting for every Hour of Difference of Time, 15 Degrees of *Longitude*, and for every Minute, 15 Minutes, or $\frac{1}{4}$ of a Degree, you shall then know, how many Degrees, Minutes, &c. the said *Meridians* do Differ from one another. E. g. Suppose the *Watches* A, B, C, were Set at the Place, whence you parted, on the 20th of February, to the time of Day observed by the Sun, abating the *Æquation* of the 20th of February, (viz. 2^h 28^m.) and suppose that the Watch A, be set to its Right Measure, but that B goes every Day 7^m. too Slow, and C every Day 12^m to Fast. Some Days after, suppose the 5th of May, desiring to know the *Longitude* of the Place where you are at Sea ;

| | |
|---|-------------------------|
| You Observe the Time of the Day there to be | 05 ^h 18' 10" |
| And you find the Watch A to point at | 02 06 00 |
| But the Watch B to point at | 1 57 22 |
| Going too Slow by 7 ^m . every Day, which makes in 74 Days, | } 00 08 38 |
| (viz. From the 20th of Febr. to the 5th of May,) ----- | |
| Which being added to its own Time, gives the same with | } 02 06 00 |
| that of the Watch A, viz. ----- | |
| You find also the Watch C to point at | 02 20 48 |
| Going 12 ^m . too Fast every Day, which makes in 74 Days | 00 14 48 |
| Which being Subducted from its own Time, gives again | 02 06 00 |
| The Time of the Day therefore by the <i>Watches</i> being | 02 06 00 |
| Add thereunto the <i>Æquation</i> of the 5th of May | 00 19 29 |
| And so you have for the Time of Day at the Place where the | } 02 25 29 |
| <i>Watches</i> were set ----- | |
| But the Time Observed being | 05 18 10 |
| Exceeds this by | 02 52 41 |
| Wherefore the <i>Meridian</i> of the Place, where you are May 5. } | } 02 52 41 |
| is more Easterly, than the Place where the <i>Watches</i> were Set by | |
| Which being reduced to Degrees, reckoning 15 Degrees } | } 43° 10' 15" |
| for an Hour comes to ----- | |

'Tis true, that from the same Reckoning it may be concluded, that you are 180 Degrees more Easterly, which happens, because the *Hour Index* goes round in the space of 12 Hours in the *Watches* ; but the Difference is so great, that one cannot be deceived in it ; else the *Watch* might be so made, that the *Index* shall go round about once in 24 Hours.

6. Since that for finding the *Longitude*, the Time of the Day, at the Place where you are, must be known, (as hath been said above) you must have a care

care to Observe that Time as precisely as possible. For every Minute of Time, that you misreckon, makes $\frac{1}{2}$ Degree in *Longitude*, which amounts, near the *Æquator*, to above 15 *English Miles*, but less elsewhere. Wherefore to find the Time of the Day with certainty, the best way is to Observe the *Sun's Altitude* when it is in the East or West, (the nearer the better :) for, being there, its *Altitude* Changes in a short Time more sensibly than before or after; and thus from the Height of the Pole and the Declination of the Sun the Hour may be Calculated.

7. At the *Rising* and *Setting* of the *Sun*, when it is half above the *Horizon*, mark the Time of the Day, which the *Watches* then shew; and, though ye have in the mean time sailed on, it is not Considerable. Then Reckon by the *Watches* what Time is elapsed between them, and Add the half thereof to the Time of the *Rising*, and you shall have the Time by the *Watches*, when the *Sun* was at *South*, to which is to be added the *Æquation* of the present Day by the Table. And, if this together makes 12 Hours, then was the Ship at Noon under the same *Meridian*, where the *Watches* were set with the *Sun*. But, if the Sum be more than 12, then was she at Noon under a more *Westerly Meridian*; and, if less, then under a more *Easterly*; and that by as many Times 15 *Deg.* as that Sum Exceeds or comes short Hours of 12; as the Calculation thereof hath been already deliver'd.

Suppose, e. g. that the *Watches* A and B, as before, were set with the *Sun* at the Place whence you parted, the 20th of *February*; and the *Indexes* set to the Hour, Min. and Sec. shewed by the *Sun*, abating the *Æquation* of that day, viz. 2' 20"; the *Watch* A being Reduced to the Right Measure, and B going too slow by 7" a day. Afterwards on the 22d of *May*, desiring to know the *Longitude* of the Place to which you are come, you Observe in the Morning the *Sun* half above the *Horizon*, when the *Watch* points at—2^h 30' 10". And in the Evening, the *Sun* being half under the *Horizon*,

when the same *Watch* points at—3 8 40
To find the Time Elapsed between them, Subducing the Time
of the *Rising*—2 30 10
From—12 0 0

There Remains—9 29 50
Adding thereunto the Time of the *Setting*—3 8 40
You have for the Time Elapsed between the Observations—12 38 30
Whereof the Half—6 19 15
Being Added to the Time of *Rising*—2 30 10
You have the Time by the *Watch* A, when ☉ was in the *South* 8 49 25
And after the same manner you are to seek the Time by the

Watch B, when the *Sun* was in the *South*; which Let be—8 38 48
But this *Watch* going 7" a day too slow, it is Retarded in 91 days
(from the 20th of *Febr.* to the 22d of *May*)—0 10 37

Which therefore Added to the said Time gives—8 49 25

That is the same Time given by the *Watch* A. Now Adding
to this Time of the *Watches*, the *Æquation* of the 22d of *May*—18 10

You have—9 7 35
Vol. I. B b b b Which

Which is the same Time of the Day with that of the Place, where the *Watches* were set when the *Sun* was in the same *Meridian* with the Ship, or where the Ship was at Noon.

| | | | | |
|--|-------|----|----|----|
| The Difference is | _____ | h. | ' | " |
| | | 2 | 52 | 25 |
| Wherefore this last <i>Meridian</i> is by so much more <i>Easterly</i> , | | | | |
| than the first, which being reduced to <i>Degrees</i> , (as hath | | 0 | ' | " |
| been formerly directed (make | _____ | 43 | 6 | 15 |

'Tis manifest, that by this way you find precisely enough the *Longitude* of the Place, where you were at Noon, or the Time of the *Sun's* being in the South: Which although it differs from the *Longitude* of the Place, where you are when you observe the *Setting* of the *Sun*; yet you may Estimate near enough, how much you have advanc'd, or chang'd the *Longitude* in these few Hours, by the *Log-Line*, or other Ordinary Practices of *Reckoning* the Ship's Way; or (which is the surer Way) by the *Degrees* passed in 24 Hours by a former day's Observation.

You may also, instead of Observing the *Sun's Rising* and *Setting*, Observe the *Setting* first, and then the next Morning the *Rising*; marking at both Times the Time shew'd by the *Watches*; and find thence, after the same manner as before, the *Longitude* of the Place where the Ship was at Midnight.

Finally, You may also, instead of the *Rising* and *Setting* of the *Sun*, Observe before and after Noon two *Equal Altitudes* of the *Sun*, Noting the Time shown by the *Watches*, and Reckoning in the same manner, as hath been said of the *Rising* and *Setting*: Yet it is to be consider'd, that the *Altitudes* of the *Sun* are best taken, when it is about East and West, as hath been already intimated. But note that in Sailing North and South you make not the Observations at the *Sun's Rising* and *Setting*, but at its being due East and West.

8. But you may put the Rule here prescribed in Practice, by taking two *Equal Altitudes* of some known *Star*, that Riseth high above the Horizon. For you shall thence according to the mentioned Rule, know at what time by the *Watches* the *Star* hath been in the South, and so the *Right Ascension* of that *Star* being known, as also the *Right Ascension* of the *Sun*, you may thence easily Calculate, what Time it then was: Which, being compared with the Time of the *Watches*, as before, shall give the *Longitude* of the Place where you were, when you had the *Star* in the *Meridian*.

9. If the *Watches* that have gone exactly for a while, should come to differ from one another (as in length of time it may well happen, that the one or other fail a Minute, more or less;) in that case it will be best to Reckon by that which goes fastest; unless you perceive an apparent cause, why it goes too fast (as it may happen when the *Cheeks* retain not their proper Figures) seeing it is not so easy for these *Pendulum Watches* to move faster than at first, as it is to go slower. For the *Wire*, on which the *Pendulum* hangs, may perhaps by the violent Agitation of the Ship, come to stretch a little, but it cannot grow shorter; and the little weight of the *Pendulum* may perhaps slip downward, but cannot get up Higher.

If it should be said, that upon any Foulness the *Watch* will go faster by reason of the shorter Vibrations of the *Pendulum*, it is to be considered, that this is only True when the *Watches* have no *Cheeks*, but when they have them 'tis not so. n. 48. p. 976.

10. When you get Sight of any known Country, Island or Coast, be sure to Note the *Longitude* thereof as exactly as you can by the help of the Rules here prescribed. First, thereby to Correct the *Sea-Maps*, after the *Longitude* of a Place shall have been found at divers times to be the same; so that you doubt no more of it. For all *Maps* are very Defective as to the Situation of Places in respect of East and West, chiefly where Seas are interposed. Secondly, to be able always to know in the Prosecution of your Journey, how far you have sailed from any Place to the East or West. And, if by any Notable Mischance or Carelessness all the *Watches* should come to stand still, yet you may at any Place, whereof the *Longitude* is certainly known, set them a Going again, and Adjust them there by the *Sun*, and so reckon the *Longitudes* from that same Meridian. For you are to know, that you are not at all obliged to put one certain Meridian of any known Place as a Beginning of the *Longitude* Reckoning; this happening only in *Maps*, or *Tables of Longitude*: As when you take for that purpose the *Meridian* of the *Pico in Teneriffe*, or that of the Islands of *Corvo* and *Flores* (the most Westerly of the *Azores*) or any others. Yet it were very fit, that all *Geographers* agreed and pitched upon one and the same *First Meridian*, that so all Places might be known by the same Degrees as well of *Longitude* as *Latitude*; though in Voyaging, it is sufficient to observe only the Difference of *Longitudes*, beginning to Reckon from the *Meridian* of any Place, you please, as if it were the *First*. n. 47. p. 951.

11. If it happen that being at *Sea* all the *Watches* stop, you must as speedily as is possible, set them a Moving again, that you may know how much you Advance from that Place towards the *East* or *West*.

12. The *Watches* being distinguished by Marks, as A, B, or the like, every Day about Noon, or when most conveniently you can, Observe the Time of the Day by the *Sun*, or by the *Stars* at Night, and subduct thence the Minutes and Seconds, that are adjoined to that Day in the Table, and write the Remainder down in the Paper, wherein 9 Columns or more are marked, placing them in the Second Column, having placed the Day of the Month in the First; and at the same time write down the Hours, Minutes, and Seconds, of each *Watch* in a distinct Column, all Opposite one to another. Then in another Column write down the Difference between the Time taken by Observation, and that given by the *Watches*, or one of them. Then one Column for the *Latitude*: One for the *Longitude* by the Ordinary Way of Reckoning: Another for the *Longitude* taken from the Difference between the Time found by Observation, and that given by the *Watches*: And at last a large Column to Note the Accidents, that befall the *Watches*, &c. The Journal for the Watches.

2. Major *Holmes* having left the Coast of *Guinea*, and being come to the Isle of *St. Thomas* under the *Line*, he adjusted his *Watches* there, and put to Sea, and Sailed Westward, 7 or 800 Leagues, without Changing his Course; The Success of Pendulum Watches; by Maj. Holmes. n. 1. p. 13.

B b b b 2

after

after which finding the Wind favourable, he steered towards the Coast of *Africk* N. N. E. But, having Sailed upon that Line, a matter of two or three hundred Leagues, the Masters of the other Ships under his Conduct, apprehending that they should want Water, before they could reach that Coast, did propose to him to steer their Course to *Barbadoes*, to supply themselves with Water there. Whereupon the said *Major*, having called the Master and Pilots together, and caus'd them to produce their Journals and Calculations, it was found that those Pilots did differ in their Reckoning from that of the *Major*, one of them about 80 Leagues, another about an hundred, and the third more; but the *Major* judging by his *Pendulum Watches* that they were only some thirty Leagues distant from the Isle of *Euego*, which is one of the Isles of *Cape Verde*, and that they might reach it next Day, and having a great Confidence in the said *Watches*, resolved to steer their Course thither, and having given Order so to do they got the very next Day about Noon a Sight of the said Isle of *Euego*, finding themselves to Sail directly upon it, and so arrived that Afternoon as he had said.

ib. p. 14.

M. Hugen being informed of this Success, wrote to *Paris* to this effect; I did not imagine that the *Watches* of this First Structure would succeed so well, and I had reserveyed my main Hopes for the New Ones. But, seeing that those have already served so successfully, and that the other are yet more just and exact, I have the more Reason to believe, that the Invention of *Longitudes* will come to its Perfection. In the mean time I shall tell you, that the States did receive my Proposition, when I desired of them a Patent for these New *Watches*, and the Recompence set apart for the Invention in Case of Success; and that without any difficulty they have granted my Request, commanding me to bring one of these *Watches* into their Assembly, to explicate unto them the Invention, and the Application thereof to the *Longitudes*, which I have done to their Contentment.

Longitudes from
the Moon's
Places; by
Math. Profes.
for at Seville.
n. 118. p. 427.

III. 1. Inveni tandem Modum *Luna Locum* sciendi, exili quodam Instrumentiolo tantum adjutus ad unum vel duo Scrupula; & quod mirum est, nec *Refractiones*, nec *Parallaxes* meis Observationibus obsunt, quia Ingeniosa Methodus his Tricis me liberat. Possum hac Methodo *Terra Marique* uti; ideo Tabulis *Luna* Correctis non amplius Modus desideratas *Locorum Longitudes* captandi, in omnibus *Terra Marisque* Locis ignorabitur.

Consider'd by
Mr. Flamsteed.
ib. p. 431.

2. Quod de Instrumentiolo suo scribit *Professor Hispanensis*, Fidem meam (quod bonâ ipsius Veniâ dictum velim) superat. Privari enim *Luna* nec *Refractione*, nec *Parallaxi*, in Horizonte nostro potest, nisi ad *Zenith* aliquando posset pertingere; eoque enim extenduntur, ejusque Locum implicant *Refractiones*; Desinit in Nonagesimo Gradu *Ecliptica* semel tantum de Die *Longitudinis Parallaxis*, sed *Latitudinis* non perinde. Nec satis capio, quomodo fabricari Instrumentum possit, quod unâ cum *Parallaxi Refractionem*, cujus Incrementum longè diversam habet rationem, consideret.

IV. The

IV. The Observation of *Lunar Occultations* is of Singular Use to determine the *Longitude* of Places, especially those that are far Remote.

Longitudes from Lunar Occultations; by M. Halley. n. 181. p. 87. Longitudes by the Revolution of Jupiter upon his Axis; by M. Cassini. n. 82. p. 4042.

V. The *Revolution* of *Jupiter* upon his *Axis* being the swiftest, and the most Regular Motion that is hitherto known in the Heavens, a Traveller alone, even without having any Correspondence with other Observers, may make use of it to find the *Longitudes* of the most Remote Places of the Earth.

VI. 1. *Utrum æque bene Jovialium Eclipses ad investigandam Differentiam Meridianorum conducant, ut quidem Occultationes Fixarum à Luna, habeo fere cur dubitem; præprimis ob minus Tardum Jovialium Motum, utrum etiam accuratiori Tubo peragantur.*

Longitudes by the Satellite Eclipses; by M. Hevelius. n. 78. p. 3030.

2. The *Eclipses* of the *Satellites* of *Jupiter*, which happen almost every Day, afford a fair Way for establishing the *Longitudes* over all the Earth. For, besides that these *Eclipses* are very frequent, the *Emersion* and *Immersion* of these *Satellites*, especially in the Shadow of *Jupiter*, is so Momentary and so Sensible, that they may be observed with the greatest Exactness, being altogether Exempt from those Essential Inconveniences that accompany the *Eclipses* of the *Sun* and *Moon*, which also are Rare, and whose *Beginning* and *End* are always doubtful by reason of a certain *Ambiguous Light*.

By M. Borelli. n. 128. p. 691.

The *Longitude* of Places at *Sea*, *Capes*, *Promontories*, and divers *Islands*, being once exactly known by this Means, would doubtless be of Great Help, and considerable Use to *Navigation*.

3. The *Eclipses* of *Jupiter's Satellites* have been esteemed, and certainly are a much better Expedient for the discovery of the *Longitude* than any yet known, by reason that they happen frequently, and are easily observable with a *Telescope* of 12 Foot, or for need with one of eight.

By Mr. Flamsteed. n. 151. p. 322.

The *Longitude* might be also attained by Observations of the *Moon*, if we had *Tables* that would answer her Motions exactly; but after 2000 Years experience (for we have some Observations of *Eclipses* much Ancienter) we find the best *Tables* extant erring sometimes 12 Minutes or more in her apparent Place, which would cause a fault of half an Hour, or $7\frac{1}{2}$ deg. in the *Longitude* deduced by comparing her Place in the Heavens with that given by the *Tables*. I undervalue not this Method, for I have made it my Business, and have succeeded in it, to get a large Stock of good *Lunar Observations* in order to the Correction of Her Theory, and as a ground-Work for better *Tables*; but, if we should happily attain what we seek, yet the Calculation will be so Perplexed and Tedious, that it will be found much more Inconvenient and difficult than that I propose by observing the *Eclipses* of *Jupiter's Satellites*, which however at present I must prefer. For I am persuaded, that the *Eclipses* of the *First* will scarcely be found above 4 Minutes of Time different from my Calculations, and I hope it will scarce ever be found to Err so much. But, if the same *Eclipse* may be observed in two Distant Places at the same time, or compared with an Observation of the same

n. 154. p. 404. n. 165. p. 762.

same

same *Satellite*, made within a Week elsewhere, the *Difference* of *Meridians* will be had something better, than by comparing two Observations of the same *Phasis* of a *Lunar Eclipse*, made in Distant Places. For, whereas it is somewhat difficult by reason of the *Penumbra* to determine the True Time of the Application of either of the *Moon's* Limbs to the Shadow, the *Satellite Eclipses*, especially those of the *First*, are almost Momentany.

And, whereas there can rarely happen 4 *Eclipses* of the *Moon* Visible the same Year, those of the *Satellites* happen so frequently that there are more of them Visible in one Year than we Count Days in it, though the Planet *Jupiter* lie hid under the *Sun's* Rays every Year a whole Month together.

I know our *Navigators* will Object against this Method, that it is difficult to Practice at *Sea*, because Long *Telescopes* are required, which the Motion of the *Ship* will not permit them to manage abroad. But, if it be not Practicable at *Sea*, they cannot deny but that it is at *Land*; and that the True *Longitude* of Remote Coasts from us is the First thing desired for the Correction of their Charts: Let them attempt these First, and I doubt not but the Success will encourage them so much, that they will readily find means to put it in Practice at *Sea*; That the *French* have used this Method successfully both in *Denmark* and in their Own Country; That a Telescope of 14 Foot Long at most, or for need one of 8 Foot, with broad *Eye Glasses*, will be sufficient for this purpose; That the Difficulty cannot be known till it be tryed, and that Use renders many things easy, which our first Thoughts conceived Impracticable.

If it be Required to know whether any of those *Eclipses* which are Invisible with us, be Visible in any other Given Place, Convert the *Difference* of *Meridians* betwixt it and *London* into Time; and, if the Place Lie to the East of *London*, Add it to, if to the West, Subtract it from, the Time of the Appearance at *London*, the Sum or Difference accordingly shall be the True Time of the *Eclipse* under that *Meridian*, at which, if *Jupiter* be above the Horizon, and the *Sun* beneath it, the *Eclipse* is there Visible, otherways not.

Or, By the help of the *Ephemerides* of the Planet's Places and a *Terrestrial Globe*, the space on it in which any of these *Eclipses* will be Visible may be found thus.

First seek the true Places of the *Sun* and *Jupiter* with his *Latitude* in the *Ephemerides*, whereby you may find their Declinations and Right Ascensions, either by the Vulgar Tables, or the Globe itself, exactly enough for this Method.

Bring *London* on the *Globe* to the *Meridian*, and, detaining it there, note what *Deg.* of the *Aequator* is cut by it. From this Subtract the Time of the *Eclipse* after Noon converted into *deg.* and *min.* the Remainder shews you the *Longitude* of that *Meridian* on the Earth, where it is then Noon when the *Satellite* is *Eclipsed*, which I therefore call the *Meridional Longitude* of the *Eclipse*. Bring this *Meridional Longitude* under the *Meridian*, and Elevate the nearer Pole to the *Sun* as much as is his *Declination*, keep the *Globe* in this Position, and, if *Jupiter* be in Consequence of the *Sun*, Draw a

Line

Line of the *Globe* along the *Eastern Horizon*, it passes over all those Places where the *Sun* is Setting at that Time, but, if *Jupiter* be in *Antecedence* of the *Sun*, Draw the said Line on the *Globe* by the *Western Edge* of the *Horizon*, it passes over all those Places where the *Sun* is then Rising. *Jupiter* being in *Consequence* of the *Sun*, Add the Difference of *His* and the *Sun's* Right Ascensions to the *Meridional Longitude* afore-mentioned, bring the *Deg.* of the *Æquator* answering their Sum under the *Meridian*, Raise the *Pole* next *Jupiter* equal to his Declination, and, detaining the *Globe* in this Position, Draw a Line again to the *Eastern Horizon*, the Space intercepted betwixt This and the Line of the *Sun's* Setting, before Described on the *Globe*, comprehends all those places on the Earth where this *Eclipse* is seen from *Sun* Setting till *Jupiter* is Set. But, if *Jupiter* were in *Antecedence* of the *Sun*, Subtract the difference of *His* and the *Sun's* Right Ascensions from the *Meridional Longitude*, Set the *Degree* of the *Æquator* answering the Remainder under the *Meridian*, and Elevate the *Pole* next *Jupiter* equal to his Declination. Keeping the *Globe* in this position, Draw a Line by the *Western Edge* of the *Horizon*, the Space Included betwixt This and the Line of the *Sun's* Rising contains all those places on the Earth, where this *Eclipse* is Visible betwixt *Jupiter's* Rising and *Sun-rise*.

When any *Eclipse* of these is observed, the Difference betwixt the Noted Time and that given by the *Tables* shall be the *Difference* of *Meridians* betwixt the Place of the Observation and *London*.

As the *Sun* Removes from the Conjunction of *Jupiter*, the *Ingresses* of the *Satellites* into his Shadow become Observable. When he is about 30° from it the *Emerfions* of the *Fourth*, and at 60° of the *Third*, begin to be seen betwixt the Shadow and Body continuing so till the *Sun* be arrived within 60° of the Opposition of *Jupiter*, when the *Emerfions* of the *Third* fall behind his Body, but the *Emerfions* of the *Fourth* continue Visible till he be less than 30° Distant from δ , at which Time they also are hid behind him, all the Appearances being made really to the Right Hand or in Antecedence of *Jupiter*, tho' with Inverting Telescopes, they appear on the contrary, to the Left.

After the Opposition of the *Sun* and *Jupiter* we begin to see the *Immersions* of the *Satellites* from the Shadow now on the Left Hand, or in Consequence of *Jupiter*, but through Inverting Glasses on the Right, when the *Sun* is near 30° deg. from the Opposition, the *Ingresses* of the *Fourth*, when 60° deg. from it, of the *Third*, begin to be observable betwixt the Body and Shadow, continuing so till the *Sun* arrive at the same or rather within something a wider Distance for the Conjunction of *Jupiter*.

4. Restaurata *Geographiæ* Fundamenta, Hac Methodo facillimâ ac nullo fere By Mr. Halley.
Instrumentorum apparatu præstandâ, sed quæ minime Fallat Jaceantur. Quæ n. 191. P. 435.
huc pertinent præcepta. *Astronomice* Doctos latere non possunt; unicum monere non abs re erit, nempe, Tubo octo vel etiam septem pedum, hoc est, facile portatili, momenta harum *Eclipsium* satis distinctè observari posse, præsertim in exterioribus *Satellitibus*, si modo *Lentis* Objectivæ Apertura $2\frac{1}{2}$ vel 3 pollices pateat. Sic enim *Radiorum* maxima copia ad oculum *Refracta* perveniet, unde *Minimæ* hæ *Stellulæ* in vicinia *Jovis* conspici possint, quæ alias

alias Luce ejus nimia obfuscarentur; ac, quamvis Coloribus tingantur, ac Jovis Limbus parum nitidus videatur, tamen, cum de momento amissa vel recuperata Lucis unice agatur, sufficit eas Lumine quantum fieri possit autas in oculos certius incurrere.

n. 214. p. 237.

The Eclipses of the First Satellite of Jupiter are found by the Royal Academy at Paris, in Ascertaining the Geographical Site of the Principal Ports of France, almost Instantaneous, and with good Telescopes discernible almost to the very Opposition of Jupiter to the Sun. So that, could the Satellites be Observed with Telescopes Manageable on Shipboard, a Ship at Sea might be enabled to find the Meridian she was in to a very great Exactness, beyond what we can yet Hope to do by the Moon, tho' She seem to afford us the only Means Practicable for the Seamen. However, before Saylor's can make use of the Art of Finding the Longitude, it will be requisite that the Coast of the whole Ocean be first Laid down Truly, for which Work this Method by the Satellites is most Apposite.

Long. and Lat. of Derby; by Mr. Flamsteed. n. 55. p. 1107, 1106. n. 111. p. 237.

VII. The Longitude of Derby from London W. is 5 or 6 min. the Latitude $52^{\circ} 57'$ or $59'$.

Lat. of Ecton. n. 76. p. 2272. Long. and Lat. of Townley. n. 127. p. 664.

VIII. Ectonia, in Com. Northamptoniano, Lat. $52^{\circ} 15'$.

IX. Townleii in Com. Lanc. Latitudo Observata (ut scribit D. Townleius) $53^{\circ} 44'$; Longitudo à Meridiano Londinensi 9. circiter scr. hor. ad Occasum.

Lat. of Tredagh. n. 164. p. 749.

X. The Latitude of Tredagh in Ireland is $53^{\circ} 40'$.

Long. of Oxford and Dantzick; by Mr. Halley. n. 129. p. 724. vid. sup. Cap. IV. § LXII.

XI. Having carefully considered the Parallaxes of the Moon in the Observations of the Occultation of Mars, Aug. 21. 1676. at Dantzick and Greenwich, I find from the Immersion the Difference of Meridians between Greenwich and Oxford $4' 57''$, between Greenwich and Dantzick $1^h 14' 50''$: By the Emerision the First of these Differences is found $4' 59''$, the latter $1^h 14' 41''$; which near Agreement shews the Exactness of the Observations.

Long. of Paris; by M. Cassini. n. 117. p. 390. vid. sup. Cap. IV. § XLV.

XII. 1. Observationes D. Flamsteedii circa Luna Eclipsin Jul. 7. ft. n. 1675. cum nostris in Regio Observatorio habitis magna cum voluptate contuli. Ex iis quippe Differentiam Meridianorum, quam olim, nostrarum Observationum Collatione, Minutorum 11 definieram, nunc decem novis Comparisonibus eandem intra pauca Secunda video confirmari.

Obser-

| Observationes D. Flamstedii. | | | Observationes Nostra | | | Differentia Merid. | | |
|------------------------------|----|----|----------------------|-----------------------|----|--------------------|----|-----------------|
| | h. | " | | h. | " | | | |
| Pentadactyl. Tectus | 1 | 55 | 15 | Idem seu Seleucus. | 2 | 06 | 15 | 11 00 |
| Porphyrites Tectus. | 2 | 02 | 20 | Idem seu Aristarchus. | 2 | 12 | 40 | 10 20 |
| Sina Limbus Primus. | 2 | 05 | 30 | Ejusdem seu Tychoonis | 2 | 16 | 30 | 11 00 |
| | | | | vel* | 16 | 25 | | 10 55 |
| Etna Limb. Primus. | 2 | 06 | 00 | Ejus. seu Corpernici. | 2 | 16 | 30 | 10 30 |
| | | | | vel* | 16 | 40 | | 10 40 |
| Besbici Limbus Prior | 2 | 23 | 05 | Ejus. seu Manil Med. | 2 | 34 | 15 | Min. quam 11 05 |
| Horminius Tectus. | 2 | 26 | 03 | Ad eund. seu Dionys. | 2 | 36 | 15 | Maj. quam 10 12 |
| Tetigit Limbum Pri- | | | | | | | | |
| mum Corocondometis. | 2 | 39 | 30 | Ejus. seu Palud. Som. | 2 | 50 | 20 | 10 50 |
| Tetig. Palud. Maotid. | 2 | 45 | 00 | Eand. seu Mare Casp. | 2 | 55 | 20 | 10 20 |
| | | | | vel* | 55 | 40 | | 10 40 |
| Maotis tota Tecta. | 2 | 50 | 40 | Eadem. | 3 | 01 | 10 | 10 30 |
| Immersio. | 2 | 56 | 55 | Immersio. | 3 | 07 | 45 | 10 50 |
| | | | | vel* | 3 | 07 | 40 | 10 45 |

Tota hæc * denotat D. Cassini peculiarem Existimationem, in reliquis cum DD. Picardo & Romero consentit.

2. Medium Eclipses Lunar, Jan. 1. st. n. 1675. deductum h. ' " n. 123. p. 562.
est ex Comparatione Initii & Finis ————— 3 20 00

Duarum aequalium Phasium ————— 3 20 15. vid. sup. Cap. IV. §. XLVI.

Ex D. Flamstedii Observationibus Medium Eclipsis pari modo eruatur. Is quippe 2^h 29' 30". Distantiam Cuspidum observavit 17' 16" & 3^h 52' 45", Eclipsi decrescente, Distantiam observavit 18' 57", uno scil. Minuto 41" Majorem. Itaque Medium Eclipsis propius est posteriori Observationi quam priori. Medium Tempus inter utramque Observationem fuit 3^h 11' 7". Tardius igitur aliquanto deducitur hinc Eclipsis Medium; unde Differentia Meridianorum proveniret Minor 9'; quod minime convenit Observationibus Certioribus Eclipsis præcedentis Æstivæ, ex quibus illam deduxi Min. 10 $\frac{1}{2}$. Prior Observatio nostra cum Priori D. Flamstedii, aliquanto tardiore, comparata, Differentiam Meridianorum exhibet Majorem 8' 35". Posterior nostra, tardior Observatione posteriori D. Flamstedii, Differentiam Meridianorum exhiberet Minorem 9' 40".

Finis à D. Flamstedio Existimatus ————— h. ' " 4 07 15
Et à Nobis. ————— 4 15 25
Differentiam Meridianorum inferret ————— 0 08 10
Initium à D. Halleio Londini Observatum ————— 2 16 00
Cum Observato à Nobis ————— 2 24 35
Differentia Meridianorum faceret ————— 0 08 35

Ex hac igitur Eclipsi Differentia Meridianorum erueretur duobus circiter Minutis minor quam ex Eclipsi Æstatis præcedentis, quam tamen huic longè præfero; non solum spectatâ majori Facilitate Determinandi Tempora Appulsuum & Emerisionum in ea Eclipsi Totali, quam in hæc Partiali;

tiali; verum etiam ob Aeris serenitatem, qua utique aequaliter usi fuimus in ea Eclipsi, cum in hac Parisiis Cælum serenissimum, Londini fuit subnubilum. Priori itaque standum censeo, donec per Observationes Immersionum & Emerfionum Satellitum Jovis, quos ad hanc rem existimo maxime Idoneos, rem scrupulosius determinemus.

ib. p. 564.

By Mr. Flam-
steed. 1b.
p. 565.

3. *Differentia Meridianorum*, ab Eclipsi Luna Junii 27. 1675. Londini & Parisiis Observata deductæ, vix fidere possum, quippe licet Tempora Phasium à verbis Observatarum accuratissimè determinata credam, Ego, cum amplior non suppeteret, Quadrante usus fui 20 tantum Digitorum Radio, ad Horologium Corrigendum, quippe Nuda duntaxat habuit Pinnacidia; & propterea de Memento Phasis alicujus certior esse vix potui quam ad unum Minutum Horarium. Novissimam Eclipsin Dec. 22. 1675. Instructior observavi; cum tamen mihi Aer subnubilus extiterit, & propter Obliquam Luna in Umbram Terræ Incidentiam tardissimus fuerit ejus ad Maculas Apulsus, minus apta fuit hæc Eclipsis huic Negotio. Incerta igitur inter duo minuta Horaria manet etiamnum Meridianorum nostrorum Differentia, quam tamen nullus dubito Nos pro Votis aliquando determinaturos esse.

Long. of Stras-
burg and Pa-
ris; by M. Bu-
lialdus. n. 125.
p. 610.
vid. sup. Cap.

IV. § XLVI.

Long. of Avig-
non; by Mr. Hal-
ley. Pb. Col.
n. 5. p. 126.
Long. of Lat.
of several Places
in France.

n. 163. p. 718.

719. 710.
vid. sup. Cap.

IV. § XXXIV

XIII. Meridianus Parisiensis ab Argentoracensi distat 22' 48'. ex Fine Eclipsos Lunaris Jan. 1. st. n. 1676. Distat autem ex hac Eclipsi Meridiannus Parisiensis à Londinensi ad Ortum 6' 38'', qui ex Observatione Eclips. Jul. 7. 1675. apparuit 10', ut etiam in Eclips. Jan. 11. ejusdem Anni.

XIV. Avignon is 19' 40'', or 4' 50' to the Eastward of London.

XV. M. Cassini having Compared together the Observations of the Solar Eclipses of July 12. st. n. 1684. and made such Reductions as the Parallax requires, lays down the Longitudes from Paris to

Aix in Province 14' E. The Lat. by M. Gautier is 43° 30'.

Avignon 8½' E.

Lyons 8', or 13' E.

Roses 4' E. The Lat. by M. Chasseles 42° 10'.

Honfleur 7' W.

Pau. 11' W. The Lat. by P. Richaud 43° 30'.

Long. of Lis-
bon. n. 146.

p. 151.

vid. sup. Cap.

IV. § XLIX.

XVI. Mr. Jacobs an English Merchant residing at Lisbon, informed Mr. Flamsteed that he Observed the Beginning of the Lunar Eclipse, Feb. 11. 168½ there at 8h 31' p. m. which gives the Difference of the Meridians betwixt the Observatory at Greenwich, and Lisbon, 41½ Minutes of Time, or 10° 22'. considerably Different from our Mapps and Sea-Charts.

Lat. of Ma-
drid; by E. of
Sandwich.
n. 22. p. 390.

XVII. The Earl of Sandwich Esteem'd, by the Sun's Altitude in the Solstice, and by other Meridian Altitudes, the Latitude of Madrid to be 40° 10'; which differs considerably from that assigned by others, The General Chart of Europe giving to it 41° 30', the General Map of Spain 40° 27', and a large Provincial Map of Castile 40° 38'.

XVII

XVIII. 1. Per plures Congruentes Observationes *Lunares* & aliorum Planetarum, inveni *Distantiam* Civitatis *Hispalensis* Longitudinariam esse ab *Uraniburg* 90', five 1 $\frac{1}{2}$ h, vel intra 2' Differentem.

2. Videat D. Professor quomodo *Merediani* *Hispalensis* ab *Uraniburgico* Interstitium *Scrupulorum* 90' constituerit : Deliqui enim *Lunaris* Observationes *Jan.* 1675. *Londini* Medium ponunt 7h 11 $\frac{1}{2}$ p. m. cui Annotationes *Parifinae* consentiunt ; dicti Professoris Observationes Medium *Hispali* statuunt 6h 47' : Nostorum ergo *Meridianorum* Differentia 24 $\frac{1}{2}$ ' : At Nos inter & *Uraniburgum* non intercedunt nisi Minuta 52'. Est igitur *Meridianorum* Differentia nonnisi 1h 16 $\frac{1}{2}$ ' inter *Hispalim* & *Uraniburgum*. Vereor tamen, annon *Oculis* nudis D. Professoris facta fuerint Observationes : Incidentia quippe & *Emerfionis* Tempora faciunt 1h 5' ; cum *Nostre*, *Parifinae*, *Hevelianaque* Observationes non faciant ea Tempora plus quam 1h 1 $\frac{1}{2}$ ', forsan aliquanto minus.

XIX. Differentia *Meridianorum* *Hafniae* & *Pariforum*, Observationibus *Jovialium*, reperta est à D. Picard, oh 41' 45".

XX. An. 1680. O. 7. 23. St. v. S. Jos. Ponthia, and Marco Antonio Cellio, with a Telescope of 25 Palms, Observed the Total Immersion of the First Satellite in Jupiter's Shadow at Rome, at 10h 7' 53" ; p. m. which in our Observatory here I noted at 9h 15' 41", whose Difference is the Difference of our Meridians = 52' 12", or 13° 03'. Again, Jan. 28. 1685. S. Francis Blanchini Observed the Total Immersion of the First at Rome, at 11h 19 $\frac{1}{4}$ ' which I saw not here, but my Numbers give at 10h 27 $\frac{1}{4}$ ' : Therefore the Difference of Meridians is 52 $\frac{1}{2}$ ', and Rome lies so much more Easterly than the Observatory at Greenwich ; agreeing with the former Observation.

The Noble Tycho judged therefore not much amiss, when he placed *Uraniburg* and *Rome* under the same Meridian ; for by several Observations of Satellite Eclipses it is Evident, that the Difference of Meridians betwixt *Uraniburg* and our Observatory is 51' 10" of Time, so that Rome lies only one Minute of Time, or $\frac{1}{4}$ of a Deg. to the East of *Uraniburg*.

XXI. 1. Dantzick is by many and undoubted Observations proved to be 15' 30". more Easterly than London.

2. An. 1683. Die ipso Solstitii Aestivi 21 Junii st. n. Gedani, Altitudo Solis in Meridie fuit 59° 7', Quadrante quidem parvulo Orich. sed tamen satis accurato. Die vero Aequinoctii Autumnalis Altitudo Solis in Meridie reperta est 35° 27'.

XXII. The Longitude of Nuremburg has been formerly stated 11° from London, and since found to be so by Observations of the Eclipse of the Sun July 2^d 1684. which made it 44 $\frac{1}{2}$ ' of Time.

Long. of Seville and Uraniburg ; by . . . Math. Professor at Seville. n. 118. p. 427. By Mr. Flamsteed. ib. p. 431. vid. sup. Cap. IV. § XLIV.

Long. of Copenhagen by M. Picard. n. 146. p. 145. Long. of Rome and Uraniburg ; by Mr. Flamsteed. n. 177. p. 1215.

Long. of Dantzick ; by Mr. Halley. Ph. Col. n. 3. p. 124. Lat. of Dantzick ; by M. Hevelius. n. 151. p. 330. n. 154. p. 424.

Long. of Nuremburg ; by . . . n. 182. p. 147.

Long. of Mos-
cow, Leipfick
and Aleppo;
by
n. 192. p. 45.
vid. sup. Cap.
V. §. LIII.

v. 181. p. 86.
it is 52 Min.

Lat. of several
Places in Russia.
ib. p. 454.

XXIII. The *Duration* of the *Lunar Eclipse* *Apr. 5. 1688.* is made by *M. Timmerman* from $7^h 38'$, about $10^h 45'$, which agrees within 8 or 10 *Minutes* with our *Tables*, that never err sensibly in the continuance of *Eclipses*; and so much ought to be allowed to an *Observer* not sufficiently instructed to distinguish the *Penumbra* from the true *Shadow*, though a small *Telescope* were used in this *Observation*. Let us conclude then, that the *End* was at $10^h 40'$ at *Moscow*. We do not find that this *Eclipse* was observed at *London*: However this defect is in good part supplied by an *Observation* thereof made at *Leipfick*, by *M. Gotfrid Kirck*, and published in his *Ephemerides* for the Year 1689; Where the *End* is determined at $8^h 54'$ *p. m.* Hence *Moscow* will be $1^h 46'$ to the *Eastward* of *Leipfick*; and the *Difference* of *Meridians* between *London* and *Leipfick* being already determin'd $49'$, it will follow that *Moscow* is $2^h 35'$ to the *East* of *London*, or $38^\circ 45'$ of *Longitude*, which from other *Accounts* we find to be very near that of the *City* of *Aleppo* in *Syria*.

By the same hand we have procured the *Latitudes* of the following *Places*, observ'd, as 'tis said, with a large *Quadrant*.

| | | | |
|-------------------|-------|------------|-------|
| <i>Moscow</i> | — — — | 55° | $34'$ |
| <i>Tereslaw</i> | — — — | 57 | 44 |
| <i>Wologda</i> | — — — | 59 | 19 |
| <i>Wostak</i> | — — — | 61 | 15 |
| <i>Arch-Angel</i> | — — — | 64 | 30 |

Latitudes of
some Remarkable
Places; by Mr.
Francis Ver-
don. n. 124.
p. 582.
Jan. An. 1675.

XXIV. I have been as curious as I could in taking the *Latitudes* of some *Remarkable Places*: As I find them I shall give them you.

| | | | | | | | | |
|----------------|-------|------------|-------|-----|-----------------------------|-------|------------|-------|
| <i>Athens</i> | — — — | 38° | $05'$ | } { | <i>Patras</i> | — — — | 38° | $40'$ |
| <i>Corinth</i> | — — — | 38 | 14 | | <i>Delphos</i> | — — — | 38 | 50 |
| <i>Sparta</i> | — — — | 37 | 10 | | <i>Thebes</i> | — — — | 38 | 22 |
| <i>Corone</i> | — — — | 37 | 02 | | <i>Negropont or Chalcis</i> | — — — | 38 | 31 |

Latitudes of
Constantino-
ple and Rhodes;
directed to A. B.
Ulser; by Mr.
Greaves.
n. 178. p. 1295.
Dec. An. 1685.

XXV. Upon Intimation of your *Grace's* Desires, and upon Importunity of some *Learned Men*, having finished a *Table*, as a *Key* to your *Grace's* exquisite Disquisition, touching *Asia* properly so called; I thought my self obliged to give both you and them a reason, why in the situation of *Byzantium* and the *Island Rhodes*, (which two *Eminent Places* I have made the *naparizumala* and *Bounds* of the *Chart*.) I dissent from the *Traditions* of the *Ancients*, and from the *Tables* of our *Late* and best *Geographers*; and consequently, dissenting in these, have been necessitated to alter the *Latitudes* (if not *Longitudes*) of most of the *remarkable Cities* of this discourse. And first for *Byzantium*, the received *Latitude* of it by *Appianus*, *Mercator*, *Ortelius*, *Maginus*, and some others, is $43^\circ 5'$. And this also we find in the *Basil Edition* of *Ptolemy's Geography*, procured by *Erasmus* out of a *Greek MS.* of *Pettichius*. The same likewise is confirmed by another choice *MS.* in *Greek* of the most *Learned* and *Judicious* *Mr. Selden*, to whom for this Favour and

and several others I stand obliged. And as much is expressed in the late Edition of *Ptolemy* by *Bertius*, compared and corrected by *Sylburgius*, with a Manuscript out of the *Palatine Library*. Wherefore it cannot be doubted, having such a croud of Witnesses, but that *Ptolemy* assigned to *Byzantium*, as our best *Modern Geographers* have done, the Latitude of $43^{\circ} 5'$. And this will farther appear, not only out of his *Geography*, where it is often expressed, but also out of his *Μαγὰν Σύνταξις*, or *Almagest* as the *Arabians* term it, where describing the Parallel passing διὰ Βυζαντίου, he assigns to it $43^{\circ} 5'$. What was the Opinion concerning *Byzantium* of *Strabo* preceding *Ptolemy*, or of *Hipparchus* preceding *Strabo*, or of *Eratosthenes* Ancienter, and it may be Accurater than all of them, (for *Strabo* Lib. 2. calls him τελευσάντων παραματευσάμενον περί τῆς γεωγραφίας) though *Tully* (Lib. Ep. Ad Att.) makes *Hipparchus* often reprehend *Eratosthenes* as *Ptolemy* after him doth *Marinus*, their Writings not being now extant, (unless those of *Strabo*) cannot be determined by us. But as for *Strabo*, in our Inquiry we can expect little Satisfaction; for his Description of Places having more of the *Historian* and *Philosopher*, (both which he hath performed with singular Gravity and Judgment) than the Exactness of a *Mathematician*, who strictly respects the Position of Places, without Inquisition after their Nature, Qualities and Inhabitants, (though the best *Geography* would be a Mixture of them all, as *Abulfeda*, an *Arabian Prince* in his Rectification of Countries above 300 Years since hath done;) I say for these Reasons we can expect little Satisfaction from *Strabo*, and less may we hope for from *Dionysius Afer*, *Arrianus*, *Stephanus Byzantius*, and others. Wherefore, next having recourse to the *Arabians*, who in *Geography* deserved the second Place after the *Gracians*, I find in *Nassir Eddin* the Latitude of *Byzantium*, which he terms *Buzantiya*, *Constantiniya*, to be 45° , and in *Uleg Beg's Astronomical Tables* the same to be expressed. *Abulfeda* chiefly follows four Principal Authors as his Guides, in the compiling of his *Geographical Tables*, those are, *Alfaras*, *Albiruny*, *Hon Saiid Almagraby*, lastly *Ptolemy*, whose *Geography* he terms a Description of the Quadrant, (or the fourth Part of the Earth) Inhabited; and all these, according to his Assertion, place *Byzantium* in 45° of Latitude. And here it may justly be wondered, how this Difference should arise between the Greek Copies of *Ptolemy*, and those Translated into *Arabick* by the command of *Almamun*, the Learned Caliph of *Babylon*; for *Abulfeda* expressly relates, that *Ptolemy* was first interpreted in his Time, that is, in the Computation of *Almecinus* in *Erpenius's Edition*, and of *Emir Cond* a *Persian Historiographer*, more than 800 Years since: Concerning which *Abulfeda* writes thus, This Book (discourfing of *Ptolemy's Geography*) was translated out of the Grecian Language into the Arabick for *Almamun*: And in this I find, (by three fair MSS of *Abulfeda*) *Byzantium* to be constantly Placed in 45° . and as constantly in the Greek Copies in $43^{\circ} 5'$. But in the *πρόχειρος Κάνων* of *Chrysococca*, out of the *Persian Tables*, made about the Year 1346. in *Scaliger's Calculation*) it is Placed in 45° . To reconcile the Difference between the *Greeks* and *Arabians* may seem impossible, for the common Refuge of flying to the Corruption of Numbers by Transcribers, and laying the Fault on them which sometimes is the Author's, will not

not help us in this particular ; seeing the *Greek Copies* agree among themselves, and the *Arabick Copies* amongst themselves. The best way to end the Dispute will be, to give Credit concerning the *Latitude of Byzantium*, neither to the *Greeks* nor *Arabians*. And that I have reason for this Assertion, appears by several Observations of mine at *Constantinople*, with a Brass Sextant of above 4 Foot *Radius*. Where taking, in the *Summer Solstice*, the Meridian Altitude of the *Sun* without using any *περαφαιεσις* for the *Parallax* and *Refraction*, (which at that time was not necessary,) I found the *Latitude* to be $41^{\circ} 6'$. And in this *Latitude* in the *Chart* I have placed *Byzantium*, and not in that either of the *Greeks* or *Arabians*. From which Observation, being of singular Use in the *Rectification of Geography*, it will follow by way of Corollary, that all *Maps* for the *North East of Europe*, and of *Asia*, adjoining upon the *Bosphorus Thracius*, the *Pontus Euxinus*, and much farther, are to be Corrected; and consequently the Situation of most Cities in *Asia*, properly so called, are to be brought more Southerly than those of *Ptolemy* by almost two intire Degrees, and than those of the *Arabians* by almost four.

Concerning *Rhodes*, it may be presumed, that, having been the Mother and Nurse of so many Eminent Mathematicians, and having long flourished in Navigation, by the Direction of these, and by the Vicinity of the *Phœnicians*, they could not be Ignorant of the precise *Latitude* of their Country, and that from them *Ptolemy* might receive a true Information. Though it cannot be denied, but that *Ptolemy* in Places remoter from *Alexandria* hath much erred. I shall only instance in our Own Country, where he situates *Λονδίνιον*, that is *London*, in 54° of *Latitude*; and the *τὸ μέσον* or the middle of the *Isle of Wight*, (which in the printed Copies is falsely termed *Ὀυίνσις*, but in the MSS. rightly *Ὀυίνκλις*,) in 52° and $20'$ of *Latitude*. Whereas *London* is certainly known to have for the Altitude of the Pole, or *Latitude* of the Place, only 51° and $32'$, and the middle of the *Isle of Wight* not to exceed 50° and some Minutes.

But in my Judgment *Ptolemy* is very excusable in these and the like Errors, of several other Places far distant from *Alexandria*; seeing he must for their Position necessarily have depended either upon Relations of Travellers, or Observations of Mariners, or upon the *Longitude* of the *Day* measured in those times by *Clepsydræ*: All which how uncertain they are, and subject unto Error, if some Celestial Observations be not joyned with them, and those exactly taken with large Instruments, (in which kind the *Ancients* had not many, and *Our Times*, (excepting *Tycho Brahe*, and some of the *Arabians*) but a few,) I say no Man, that hath conversed with *Modern Travellers* and *Navigators*, can be ignorant. Wherefore to excuse those Errors of his (or rather of others fathered by him) with a greater Absurdity, by asserting the *Poles* of the *World* since his time to have Changed their site, and consequently all Countries their *Latitudes*, as *Mariana* the Master of *Copernicus*, and others after him have Imagined: Or else to Charge *Ptolemy*, being so excellent an Artist, with Ignorance, and that even of his own Country, as *Cluverius* hath done, (from which my Observations at *Alexandria*, and

Mem-

Memphis, may Vindicate him,) the former were too great a Stupidity, and the latter too great a Presumption. But to return to *Rhodes*: An Island (in *Eustathius's* Comment upon *Dionysius's* Παριήγνσις) of 920 Furlongs Circuit, where according to *Ptolemy* the Parallel passing Δία Ρόδου, hath 36° of Latitude, and so hath *Lindos*, and Ἰαλυωσις the chief Cities of the Island; the same is confirmed by the MS. but where the Printed Copy and *Eustathius* read Ἰαλυωσις, which *Mercator* renders *Talyssin*, the M S. renders Ιαλύσις. *Abulfeda* in some Copies situates the Island *Rhodes*, (for he mentions no Cities there) in the Latitude of 37 Deg. and 40 Min. and the Geography of *Said Ibn Aly Algiurgany*, commended by *Gilbertus Gaulmyn*, in 37°, if it be not by a Transposition in the MS. of the Numerical Letters in *Arabick* 37 for 36, which by reason of their Similitude are often confounded in *Arabick* MSS. By my Observations under the Walls of the City *Rhodes*, with a fair Brass Astrolabe of *Gemma Frisius*, containing 14 Inches in the Diameter, I found the Latitude to be 37° and 50'. A larger Instrument I durst not adventure to carry on Shore in a Place of so much Jealousie. And this Latitude in the Chart I have assigned to the City *Rhodes*, (from the Island so denominated, upon which on the North East side it stands situated) better agreeing with the *Arabians* than with *Ptolemy*, whom I know not how to excuse.

XXVI. In the second Book of the *Voyage de Siam des Peres Jesuites*, are ^{Long. and Lat. Cape of Good Hope. n. 185. p. 253.} related two Observations of the Satellites of *Jupiter*, capable, if well made, to ascertain the Longitude of the Cape of Good Hope. The first was there made June 2d st. n. 1685, when at 11h 29' 20". the First or Innermost Satellite touched the Western Edge of *Jupiter*, and at 11h 30' 50" it appeared no more: This Observation is said to be made with an excellent Telescope of 12 Foot. The other was on June the 4th following st. n. when the Emerfion of the same Satellite was observed at 9h 37' 40". from which Latter is concluded, that the Longitude of the Cape is 18' to the East of *Paris*, for that the said Emerfion, according to the Calculus of *Cassini*, in the Meridian of *Paris*, ought to have happen'd at 8h 26'. This same Emerfion is computed by *Mr. Flamsteed*, at 8h 19', at *London*, that is 3 min. later then by *S. Cassini*; and considering that neither is verified by Observation in *Europe*, the Longitude hence deduced is doubtful at least 3 min. if this had been the only Observation. But the former being considered will yet shew that there is a much greater Doubt still remaining: For from certain Astronomical Principles the Parallax of the Orb, or Difference between the Place of *Jupiter* seen from the Sun and Earth was, at the time of the first Observation, 9° 9'; Which Arch that Satellite moves in 1h 6'. and the utmost Duration of an Eclipse hereof in this Position of *Jupiter* being scarce 2h 20'. (as appears by the accurate Observations of *M. Cassini* and *Mr. Flamsteed*) it will follow, that from the Immerfion behind *Jupiter's* Western Edge to the Emerfion out of the Shadow, there could not be full 3h 26'. Wherefore the Emerfion out of the Shadow, on June 2d, ought according to the time of Immerfion, to be at 14h 56', at the latest at the Cape; which by *Mr.*

Mr. Flamsteed's *Calculus* was at *London* $13^h 51'$. or according to *S. Cassini* at $13^h 58'$. at *Paris*. Hence the Longitude of the Cape will be found but 14^{deg} . and a half at most to the *East* of *Paris*; so that these 2 Observations will differ in the result about a quarter of an Hour which is a little too much. However there are some Reasons that seem to argue for this Latter Longitude rather than the former; for it is much easier to observe what becomes of a Luminous Object that appears, than to wait upon the first Appearance of a Star Eclipsed: And it is probable that the *Satellite* might in the Latter time be several Minutes *Emerged* out of the Shadow, when they might first perceive it; but they could not but see the Application to the Body of *Jupiter* in the Former, if we may suppose their Telescopes so good as they are said to be. And that the *Cape of Good Hope* is not more than an Hour to the *East* of *Paris*, is proved by the constant Consent of our *Navigators*, who find by their Reckonings that the Island of *St. Helena* is about 22 or 23^{deg} . of Longitude to the *Westward* of the *Cape*: (and that Sailing both backwards and forwards, 'tis the same, which takes away the Objection of Currents) Now by Accurate Observations made at *St. Helena*, and compared with others made in *Europe* at the same time, the Longitude of that Isle is certainly about $8\frac{1}{2}^{\text{deg}}$. to the *West* of *Paris*; it follows therefore that the *Cape* cannot be much more than 14 or 15^{deg} . to the *East* of *Paris*; and undoubtedly it must be less than 18° , for 3^{deg} . is much too great an Error to be committed in so short a Distance of Sailing.

Long. of St.
Helena. ib.

The Long. of
Madagascar; by
Mr. Flamsteed,
n. 143. p. 15.
vid. sup. Cap.
IV. § XLVIII

XXVII. Mr. Thomas Heathcot was Chirurgeon to a Ship, which, Aug. 19. 1681. lay at the bottom of a deep Bay on the Western Shore of *Madagascar*, and that part which the *Portuguese* and our *Maps* call the *Terra del Gada*; He had then with him on Shore, a Quadrant of two Foot Radius, and a Telescope of 9 Foot, but no Clock; to supply which Defect, he made a *Pendulum* of a String and a Bullet 39 Inches long, that each single Vibration might answer a Second of Time. Waiting the Beginning of the Eclipse with his Glass, as soon as He saw the True Shadow enter on the *Moon's* Limb, he caused his Friends who assisted him, to make the *Pendulum* Vibrate, and count its Vibrations; of which they had numbred $140 = 2' 20''$ of Time, when he took the height of *Procyon* (then *East* of the Meridian) $25^{\circ} 39'$. The next day he observed the *Sun's* Meridional Height with the same Quadrant, whence he found the Latitude of the Place $19^{\circ} 29'$. South: hence the time when he took the height of *Procyon* is found $4^h 51'$ *more*, and subtracting the $2' 20''$, past since the observed Beginning of the Eclipse, its True Beginning was at ————— $4^h 48' 40''$
Which at the Observatory here I noted at ————— $1 \ 50 \ 40$
therefore this part of *Madagascar* is more Easterly ————— $2 \ 58 \ 00$
or $44^{\circ} 30'$, which our *Maps* make 52° ; that is $7\frac{1}{2}^{\text{deg}}$. more Remote from it than it really is.

The Long. and
Lat. of Balla-

XXVIII. Taking the Observations of the Occultation of the *Bul's Eye*, Oct. 28. 1680. under the Examination of a *Calculus*, I find that at $8^h 6'$

or

for the *Immersion* at *London*, the true Place of the *Moon* correct by *Parallax* fore in India; was $\pi 4^{\circ} 32' 24''$ but at $16^h 00'$ at *Ballafore Road* (in the *Lat.* of 21° by Mr. Edm. Halley, *rb. Col.* n. 5. p. 124. *vid. sup. Cap.* IV. § LXVI. Feb. An. 1682. N. and about 20 Miles E. S. E. from the Town) the True Place of the *Moon* was $\pi 5^{\circ} 54'$ that is $10^{\circ} 21' 36''$ more than at $8^h 6'$ at *London*. Now according to the *Moon's* Velocity at that Time, she passed an Arch of $1^{\circ} 21' 36''$ in $2^h 8' 40''$ of time, so then at $10^h 14' 40''$ at *London*, the *Moon* was in the same Place as at $16^h 00''$ at *Ballafore Road*, whence the Difference of Longitude will be $5^h 45' 20''$ or $86^{\circ} 20'$ *Ballafore* being so much to the *Eastward* of *London*.

2. By the Calculation of the *Immersion* of the *Bul's Eye*, Dec. 1680. I find that at the $14^h 49'$ at *Ballafore* the *Moon's* true Place was $\pi 6^{\circ} 30' 30''$ and at $7^h 46' 12''$ the Correct Time of the *Immersion* at *Dantzick*, the true Place was $\pi 4^{\circ} 55' 11''$ that is $1^{\circ} 35' 20''$ short of the Place deduced from the Observation at *Ballafore Road*, which make in Time $2^h 32' 40''$ whence it follows, that $10^h 18' 52''$ at *Dantzick* make $14^h 49'$ at *Ballafore Road*, and the Difference of Longitude $4^h 30' 8''$ and *Dantzick* being $1^{\circ} 15' 30''$ more *Easterly* than *London*, *Ballafore Road* will be from *London* $5^h 45' 38''$ or $86^{\circ} 24'$ and the same Difference of *Meridians* will be found $86^{\circ} 14'$. If you make use of the *Immersion* at *Dantzick*. *vid. sup. cap.* IV. § LXVII.

3. For farther Confirmation hereof, Mr. *Benj. Harry*, being ashore at *Ballafore* Town, observed with very great Care and Exactness, Nov. 18, 1680. that at $9^h 13'$ the Star which *Tycho* calls, in *Cotyla dextra Aquarii duarum precedens*, (and which was then in *Aquarius* $28^{\circ} 52'$ and *Lat* $2^{\circ} 46'$ N.) was in a Right Line with the *Cusps* of the *Moon*, then near the first Quarter. The Star's Place is confirmed by the Agreement of *Hevelius's* Observations with those of *Tycho*, and the Theory of the *Moon* cannot be considerably faulty in that Part of the *Orb*, it falling precisely on her greatest Equation: wherefore by the Theory and Numbers of *Horrox*, the true Place of the *Moon* at $2^h 53'$ at *London* is found $\approx 29^{\circ} 22' 10''$ but at $9^h 13'$ at *Ballafore*, her Place was in $\approx 29^{\circ} 41' 17''$ that is, $19' 7''$ more than at *London*, which in Time gives $36'$, so that $3^h 29'$ at *London*, was $9^h 13'$ at *Ballafore*, and the Difference of Long, $5^h 44'$ or $86^{\circ} 00'$ precisely, which the *Dutch Maps* make full out 99° . And the *French Maps* of *Sanfon*, pretending to correct them, have made them 5° worse, and the Error 18° compleatly. What then is to be thought of the Descriptions of those Places which have been but seldom Visited?

XXIX. Differentiam Longitudinum Cantonem inter & *Parisijs* deduxi *Long. of Canton; by M. Cassini, n. p. 371.* *IV. §. XCVIII*
 $7^h 23'$ ex Exitu *Mercurii* & *Solis* Disco *Cantoni* & *Norimbergæ* Observato, &
 ex *Eclipsibus* *Lunæ* Observatis *Noximbergæ* & *Parisijs*.

XXX. Ex Altitudine Meridiana Maxima *Stellæ* *Polaris* à P. P. S. 7. Observata Die 31 Dec. 1694. Correcto Instrumento $42^{\circ} 16' 50''$. Supposita Refractione $1' 17''$. & Distantia *Stellæ* *Polaris* à *Polo* tunc temporis $29^{\circ} 19' 57''$. *Lat. by Long. of Pekin; by M. Ja. Cassini, n. 237. p. 53.*
 eruitur Altitudo *Poli* $39^{\circ} 54' 56''$.

Ex ejusdem *Stella Polaris* Altitudine Meridiana Minima, observata Diebus 7, 8, 13. *Maii*, 1695. Correcto Instrumento $37^{\circ} 36' 40''$. Supposita Refractione $1' 18''$. & Distantia *Stella Polaris* à Polo $2^{\circ} 19' 50''$. Eruitur Altitudo Poli $39^{\circ} 55' 2''$.

| | | | |
|---|----|----|-----|
| Neglecta Refractione Altitudo maxima <i>Stella Polaris</i> deducta | 0 | 1 | 11 |
| ex Observatione 31 Dec. præcedentis fuisset sub Initium <i>Maii</i> — | 42 | 16 | 43 |
| Et Altitudo ejusdem minima tunc fuit ————— | 37 | 36 | 40 |
| Quare Differentia Altitudinum ————— | 04 | 40 | 03 |
| Et Distantia <i>Stella Polaris</i> à Polo ————— | 02 | 20 | 01½ |
| Et Altitudo Poli apparens ————— | 39 | 56 | 41½ |
| Ad hanc Altitudinem apparentem Refractio ex mea Tabula est | 00 | 01 | 10 |
| Quare Altitudo Poli in <i>Regia Pekinensi</i> ————— | 39 | 55 | 31½ |

Pro Longitudine *Pekinensis* Urbis Observata est Immersio Primi *Jovis* Satellitis in *Jovis* Umbram Die 18 Jan. 1695. $12^h 51' 14''$.

Tabulæ nostræ eo Die hanc Immersionem repræsentant $5^h 18' 49''$. Observationes autem eodem Mense habitæ in Observatorio *Regio Parisiensi* ostendunt Tabulas retardasse tunc Temporis $2' 30''$.

Quare fuit illa Immersio *Parisiis* $5^h 16' 19''$. Itaque Differentia Meridianorum inter *Pekinum* & Urbem *Parisiensem* erit $7^h 34' 55''$.

Cum autem ex aliis Observationibus olim deducta fuerit eadem Meridianorum Differentia $7^h 36'$. sumi poterit $7^h 35'½$.

Lat. of St. Salvadore. n. 105.

p. 91.

Lat. of Bridge-Town. n. 189.

p. 370.

A Description of Nova Zembla; by M. Nich.

Witsen. n. 101.

p. 3.

Fig. 205.

Mar. An. 1674.

XXXI. St. Salvadore in *Brasil* is in the Southern Latitude of $12^{\circ} 47'$.

XXXII. Bridge-Town in *Barbadoes* is in the Northern Latitude of $12^{\circ} 58'$.

XXXIII. I. I herewith send you what I have received out of *Muscovy*, which is a New Map of *Nova Zembla* and *Weigats*, as it hath been discover'd by the express Order of the *Czar*; and drawn by a Painter, call'd *Panelapetski*, who sent it me from *Mosco* for a Present; by which it appears, that *Nova Zembla* is not an Island, as hitherto it hath been believed to be; and that the *Mare Glaciale* is not a Sea, but a Sinus, or Bay, the Waters whereof are Fresh. Which is the same with what the *Tartars* do also assure us, who have tasted these very Waters in the midst of the Sinus. The *Samojeds*, as well as the *Tartars*, do unanimously affirm, that passing on the back of *Nova Zembla*, at a considerable Distance from the Shore, Navigators may well pass as far as *Japan*. And 'tis a great Fault in the *English* and *Dutch*, that, seeking to get to *Japan* on the South-side of *Nova Zembla*, they have almost always pass'd the *Weigats*.

The Letter O in the great River *Oby* marks the Place of a Cataract or Fall of Waters. The Letter K denotes the Conjunction of *Zembla* with the Continent. The River marked L runs from *China*, called *Kitaie*: which is not every where Navigable, by Reason of the Rocks, and other Inconveniences that Obstruct the passing of Vessels. *Weigats* it self is very difficult to pass, because of the great Quantity of Ice continually falling into it out of the River *Oby*, whereby that streight Passage is stopped up. The *Samojeds*

mojeds go every Year a Fishing upon the said Fresh Sea, and that on *Nova Zembla's* side.

2. I formerly thought *Nova Zembla* had been a *Continent* : But I have since been better informed, and retracted that Error. And, whereas the late *M. Vossius* would needs persuade himself, as well as he did others, to their Ruin, that there was a Passage to *Japan* by the *North*, and that the *Tartarian* Countries behind *Nova Zembla* did decline immediately towards the *South* ; I did always oppose it, and think I can even demonstrate the Impossibility thereof. So that what we wrote, to encourage Mariners to that Attempt, was even directing them to the Point of Death, as it afterwards ensued.

XXXIV. What is noted with a single Line, is exactly copied from the Map, which *M. Sanfon*, one of the most Illustrious Geographers of this Age, presented to the *Dauphin*, An. 1679. The Names of Cities, whose Situation is also taken from this Map, are written in *Italian* Characters ; the Correction of the Position of Coasts (which is deduced from the Observations which were made to that End) is marked with a Stroke a little shadow'd towards the *Sea*, as is commonly done ; and the Names of Cities, whose Situation is corrected, are set down in *Roman* Characters.

A Map of
France ; by *M.*
Picard, and *M.*
De La Hire.
n. 226. p. 443.
Fig. 205.
Mar. An. 1657.

The Degrees of *Latitude* are marked on both sides of the Border, and the Degrees of *Longitude* in the same Border, above and below ; but the Division of them begins at the *Meridian* that passes through the *Observatory* at *Paris*, by going to *East* and *West*, and not at the *Meridian* of the *Ile of Fer*, as hath been established, because we do not exactly know the Situation of this Island in respect of the *Observatory*.

XXXV. 1. What *Arithmetick* in whole Numbers and Fractions, as also in Decimals and Logarithms, is necessary for the same ? And what Books are best for Teaching so much thereof ? 2. What *Vulgar*, *Practical*, *Mechanical* *Geometry*, performable by the Scale and Compass, is sufficient ? 3. What *Trigonometry*, *Right Lined* and *Spherical*, will suffice ? 4. How many Stars are to be known ? 5. What *Instruments* are best for Use at *Sea*, with the Construction of them, and the Manner of using them ? 6. The whole Skill of the *Magnet*, as to the Directive Virtues thereof, and all the Accidents that may befall it ? 7. The *Hydrography* of the *Globe* of the *Earth*, the *Perspective* of the Coasts, and the Description of the under-water-bottom of the *Sea*, 8. The Knowledge of Winds and *Meteors*, so far as the same is attainable. 9. The History and Skill of all sorts of *Fishings*. 10. The Art of *Medicine* and *Chirurgery*, peculiar for the *Sea*. 11. The *Common Laws* of the *Admiralty*, and *Jurisdiction* of the *Sea*. 12. The several *Driftallings* and *Cloombings* fit for Seamen. 13. The whole Science of *Ebbing* and *Flowing*, as also of *Currents* and *Eddies* at *Sea*. 14. *Dromometry*, and the Measures of a Ship's Motions at *Sea*. 15. The Building of Ships of all sorts, with the several Riggings and Sails for each Species, and the Use of all the Parts and Motions of a Ship. 16. *Naval Oeconomy*, according to the several Voyages and Countries. 17. The Art of *Coning*, *Rowing*, and *Sailing*, of all the several sorts of Vessels.

What a Compleat Treatise of
Navigation
should contain ;
by *Sir W. Petty*,
n. 198. p. 657.
Mar. An. 1693.

sels. 18. The *Gunnery*, *Fireworks* and other *Armatures* peculiar to Sea and to Sea-Fights. 19. The Art of *Loading* and *Unloading* the chief *Commodities* to the best Advantage. 20. The Art of *weighing sunk Ships* and Goods, as also of *Diving* for sunk Goods in deep Water. 21. The General *Philosophy* of the *Motion* and *Figures* of the *Air*, the *Sea*, and of *Seasons*; of *Timber*, *Iron*, *Hemp*, *Brimstone*, *Tallow*, &c. And of their several uses in *Naval Affairs*. 22. An Account of 5 or 6 of the best *Navies* of *Europe*, with that of the *Arsenals*, *Magazines*, *Yards*, *Docks*, &c. 23. An Account of all the *Shipping* able to cross the *Seas* belonging to each Kingdom and State of *Europe*. 24. An Account of all the chief *Commercial Parts* of the World; with mention of what *Commodities* are originally carried from, and ultimately to, any of them. 25. An Account of the chief *Sea-Fights*, and all other *Naval Expeditions* and *Exploits*, relating to *War*, *Trade*, or *Discovery*, which have happened in this last Century. 26. Of the most advantageous Use of *Telescopes* for several Purposes at *Sea*. 27. Of the several *Depths* of the *Sea*, and *Heights* of the *Atmosphere*. 28. The Art of making *Sea-water Fresh* and *Potable*, and fit for all Uses in Food and Physick at *Sea*.

The Collection of
Secants, and the
true Division of
the Meridian in
the Sea-Chart;
by Dr. Wallis. n.
176. p. 1193.
Fig. 206.
Nov. An. 1685.

XXXVI. 1. Though it be well known, that, in the *Terrestrial Globe*, all the *Meridians* meet at the *Pole*, (as E P, E P,) whereby the *Parallels* to the *Equator*, as they be nearer to the *Pole*, do continually decrease.

2. And hereby a Degree of *Longitude* in such *Parallels* is less than a Degree of *Longitude* in the *Equator*, or a Degree of *Latitude* :

3. And that in such Proportion, as is the Co-Sine of *Latitude* (which is the *Semidiameter* of such *Parallel*) to the *Radius* of the *Globe*, or of the *Equator*.

4. Yet hath it been thought fit (for some Reasons) to represent these *Meridians*, in the *Sea-Chart*, by parallel straight Lines; as E p, Ep.

5. Whereby, each *Parallel* to the *Equator* (as L A) was represented in the *Sea-Chart*, (as la) as equal to the *Equator* E E; and a Deg. of *Longitude* therein, as large as in the *Equator*.

6. By this means, each Degree of *Longitude* in such *Parallels* was increased, beyond its just proportion, at such rate as the *Equator* (or its *Radius*) is greater than such *Parallel*, (or the *Radius* thereof.)

7. But, in the Old *Sea-Charts*, the Degrees of *Latitude* were yet represented (as they are in themselves) equal to each other; and to those of the *Equator*.

8. Hereby, amongst many other Inconveniencies, (as Mr. Ed. Wright observes, in his *Correction of Errors of Navigation*, first published in the Year 1599, the Representation of the Places remote from the *Equator* was so Distorted in those Charts, as that (for Instance) an *Island* in the *Latitude* of 60 Degrees, (where the *Radius* of the *Parallel* is but half so great as that of the *Equator*) would have its Length (from *East* to *West*) in Comparison of Breadth (from *North* to *South*) represented in a double Proportion of what indeed it is.

9. For



Fig. 207.

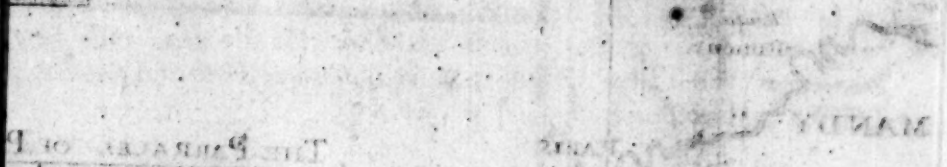


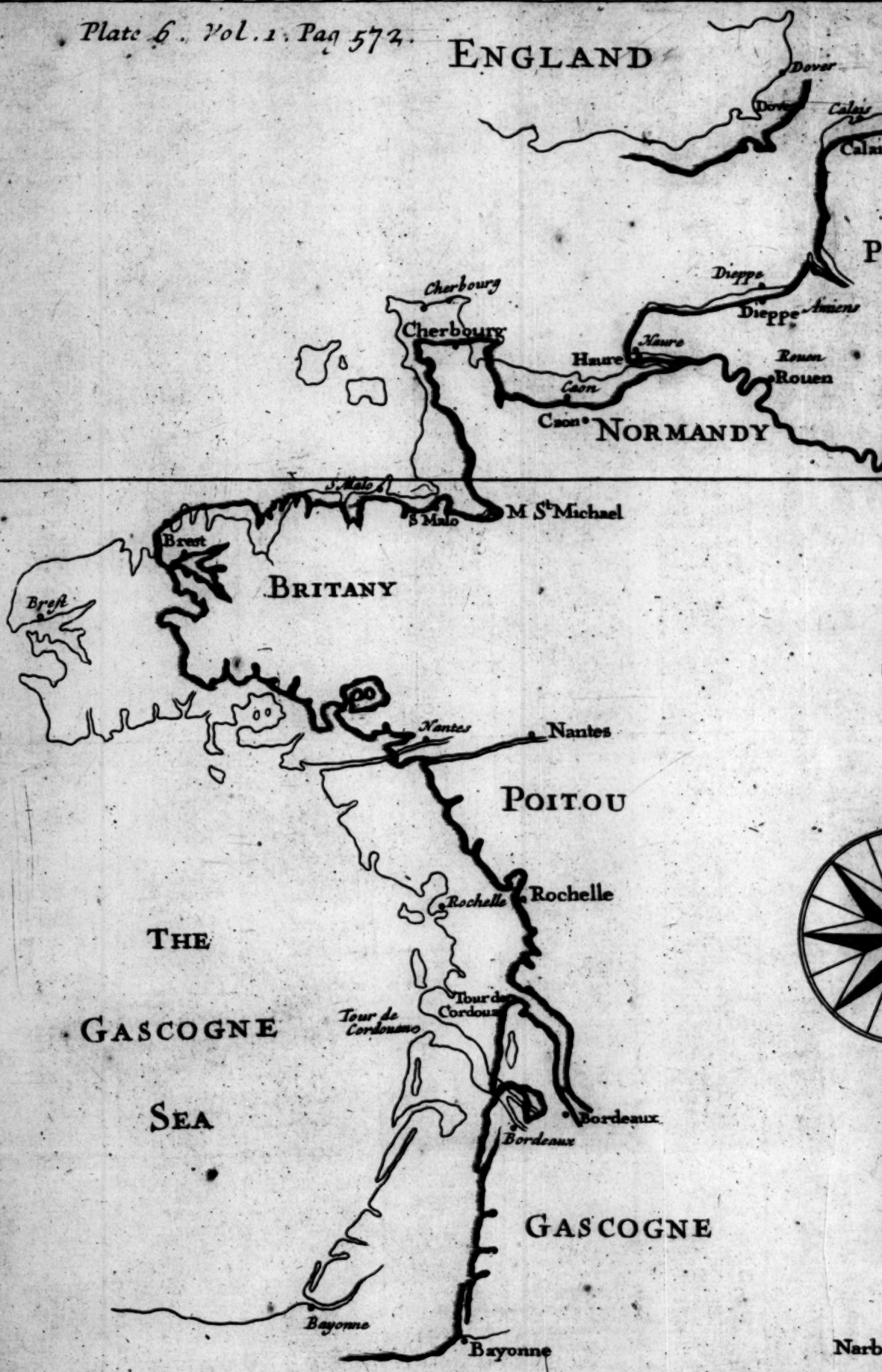
Fig. 208.

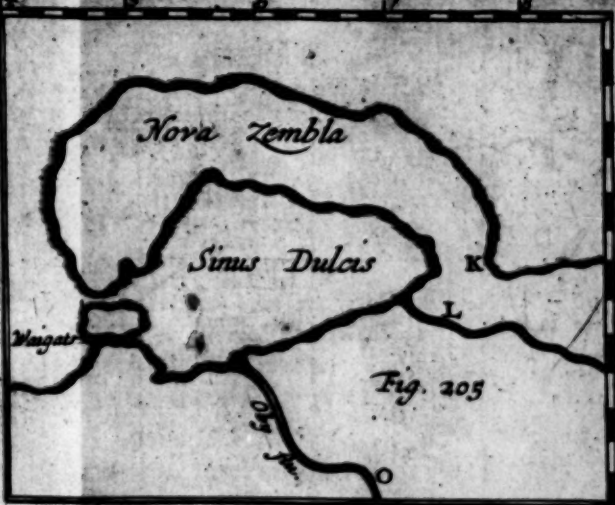
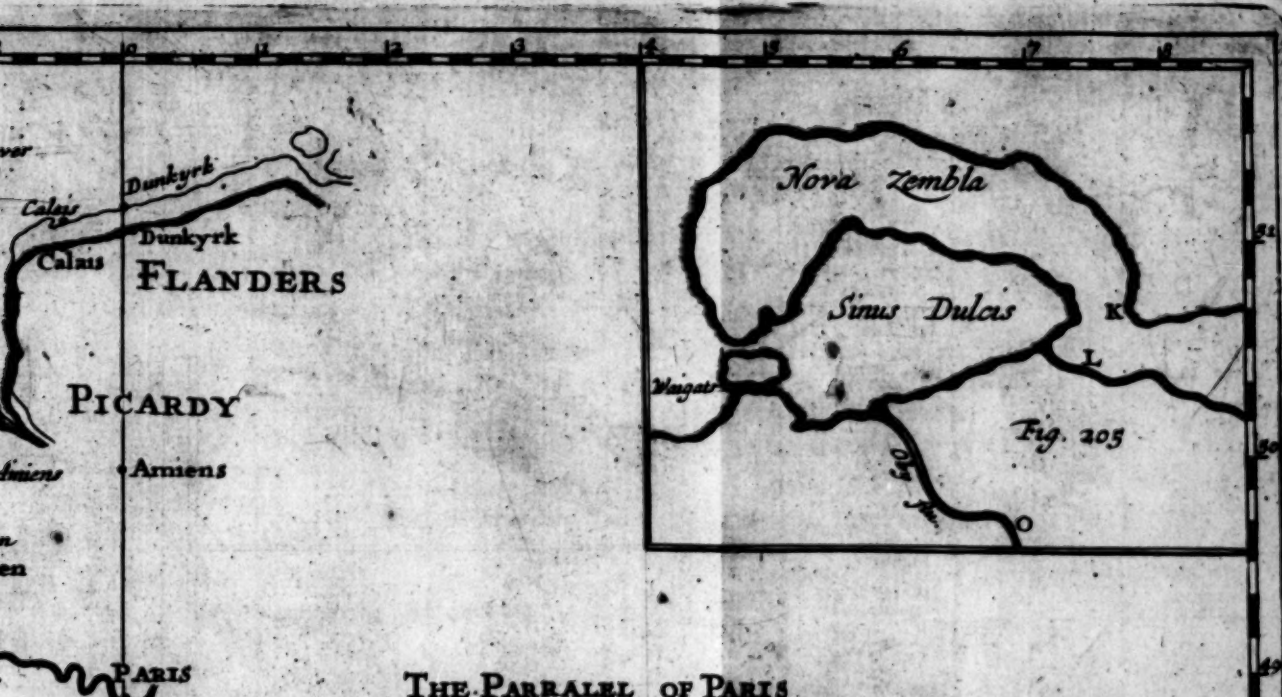


Fig. 209.

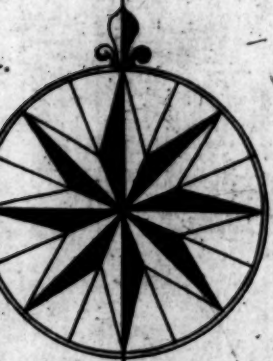
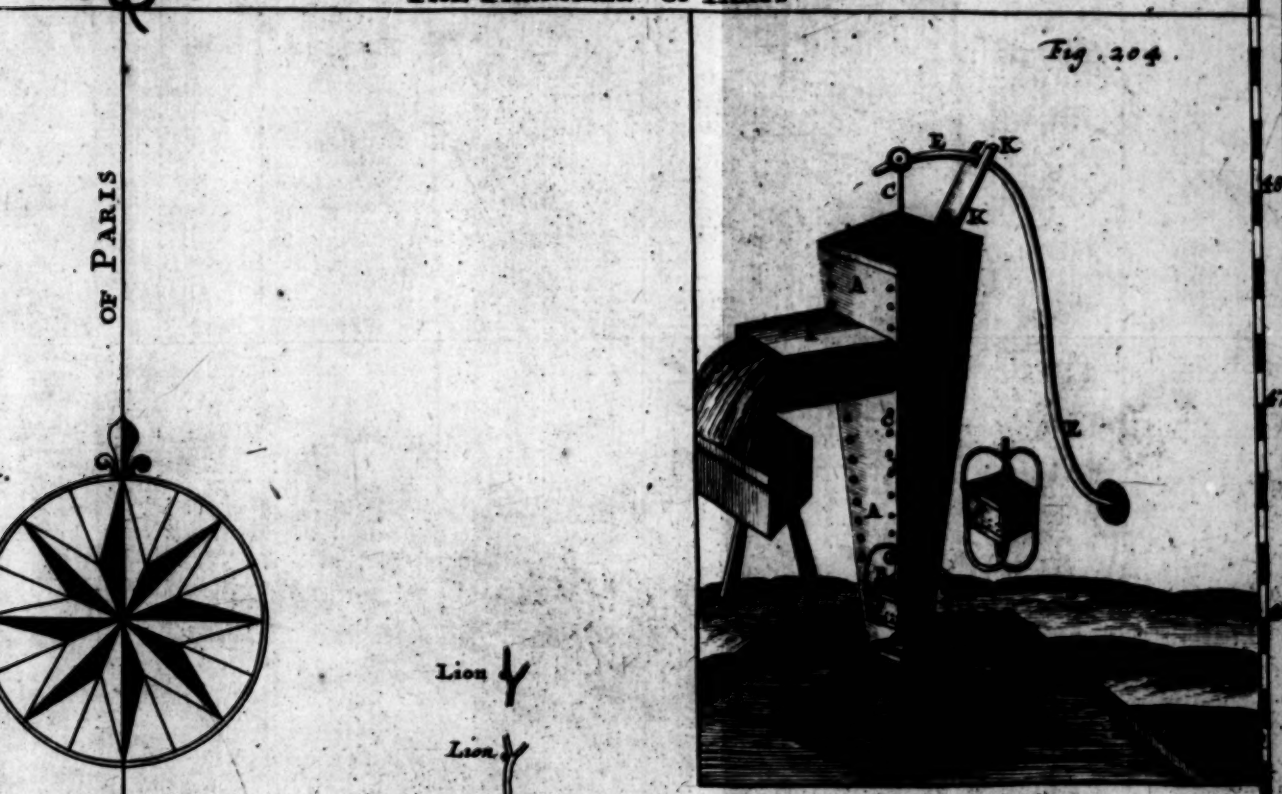
which yet (because on the Convex-side of the Curve) would be
 what too Little. 19. But

ENGLAND





THE PARRALEL OF PARIS

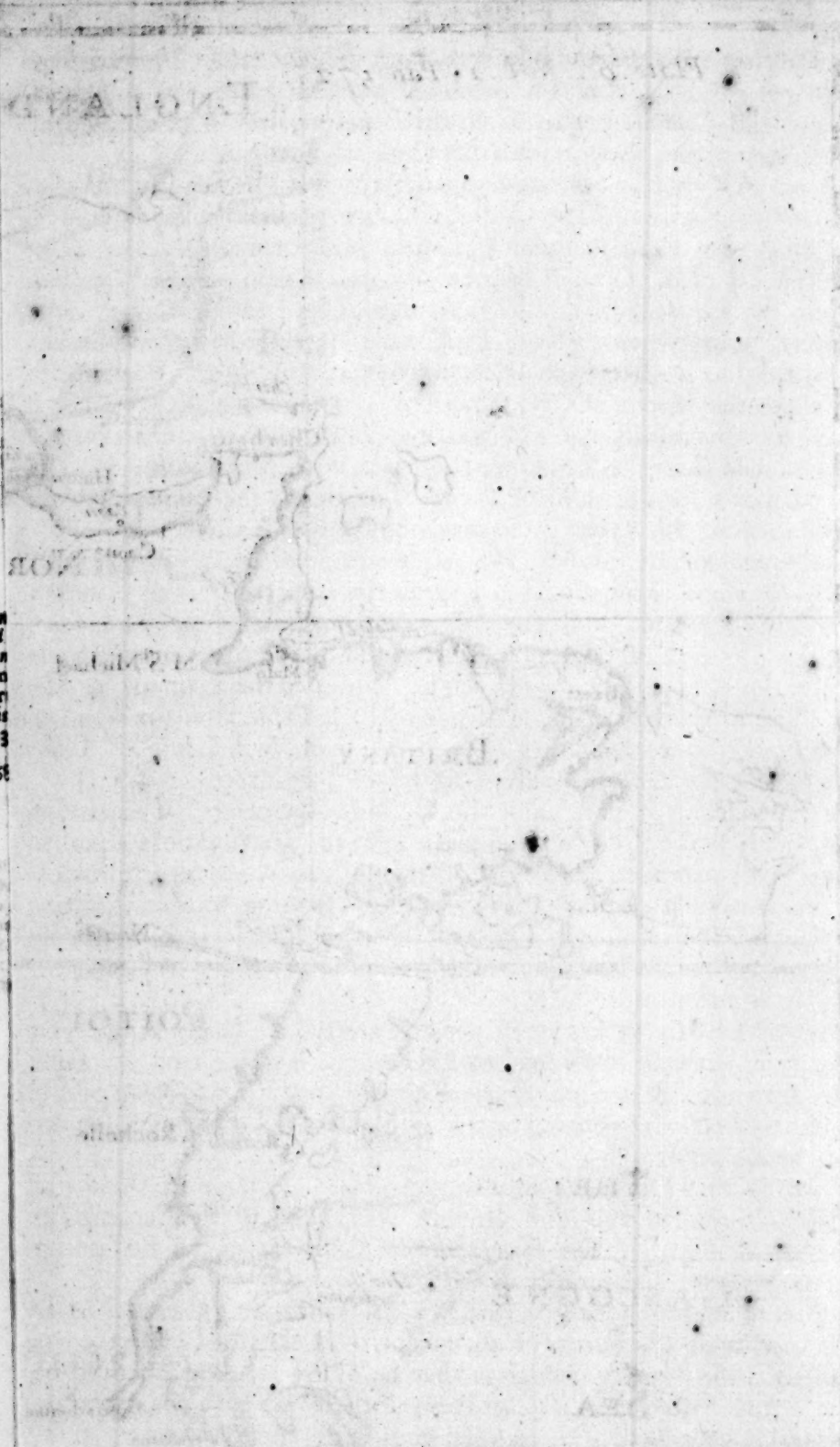


OF PARIS

THE MERIDIEN

ITALY

**The Collection
Secants, and
true Division
the Meridian
the Sea-Chart
by Dr. Wallis
176. p. 1193
Fig. 206.
Nov. An. 168**



9. For rectifying this in some Measure, (and of some other Inconveniences) Mr. *Wright* adviseth, that (the *Meridians* remaining Parallel, as before) the Degrees of the *Latitude* remote from the Equator, should at each Parallel, be protracted in like Proportion with those of *Longitude*.

10. That is ; As the Co-Sine of *Latitude*, (which is the Semidiameter of the Parallel) to the Radius of the *Globe*, (which is that of the Equator :) So should be a Degree of *Latitude* (which is every where equal to a Degree of *Longitude* in the Equator,) to such Degree of *Latitude* so protracted (at such Distance from the Equator ;) and so to be represented in the Chart.

11. That is, every where, in such Proportion, as is the respective Secant (for such *Latitude*) to the Radius. For, as the Co-Sine, to the Radius ; so is the Radius, to the Secant of the same Arch or Angle ; or $\Sigma : R :: R : f$.

Fig. 207.

12. So that (by this means) the Position of each Parallel in the Chart should be at such Distance from the Equator, compared with so many Equinoctial Degrees or Minutes, (as are those of *Latitude*) as are all the Secants (taken at equal Distances in the Arch) to so many times the Radius.

13. Which is equivalent (as Mr. *Wright* there notes) to a Projection of the Spherical Surface (supposing the Eye at the Centre) on the Concave Surface of a *Cylinder* erected at Right Angles to the Plain of the Equator.

14. And the Division of *Meridians*, represented by the Surface of a *Cylinder* erected (on the Arch of *Latitude*) at Right Angles to the Plain of the *Meridian* (or a Portion thereof.) The Altitude of such Projection (or Portion of such *Cylindrick* Surface) being (at each Point of such Circular Base) equal to the Secant (of *Latitude*) answering to such Point.

Fig. 208.

15. This Projection (or Portion of the *Cylindrick* Surface) if expanded into a Plain, will be the same with a plain Figure, whose Base is equal to a Quadrantal Arch extended (or a Portion thereof) on which (as Ordinates) are erected Perpendiculars equal to the Secants, answering to the respective Points of the Arch so extended : The least of which (answering to the Equinoctial) is equal to the Radius ; and the rest continually increasing, 'till (at the *Pole*) it be infinite.

Fig. 209.

16. So that, as ERSL, (a Figure of Secants erected at Right Angles on EL, the Arch of *Latitude* extended) to ERRL, (a Rectangle on the same Base, whose Altitude ER is equal to the Radius ; so is EL (an Arch of the Equator equal to that of *Latitude*,) to the Distance of such Parallel, (in the Chart) from the Equator.

17. For finding this Distance, answering to each Degree and Minute of *Latitude*, Mr. *Wright* (as the most obvious way) adds all the Secants (as they are found calculated in the *Trigonometrical Canon* from the Beginning to the Deg. or Min. of *Latitude* proposed.

18. The Sum of all which, except the Greatest, (answering to the Figure Inscribed) is too little : The Sum of all, except the Least, (answering to the Circumscribed) is too Great ; (which is that he follows :) And it would be nearer to the Truth than either, if (Omitting all these) we take the Intermediates ; for *Min.* $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$. &c. or (the double of these) *Min.* 1, 3, 5, 7, &c. Which yet (because on the Convex-side of the Curve) would be somewhat too Little.

19. But

19. But any of these ways are exact enough for the Use intended, as creating no sensible Difference in the Chart.

20. If we would be more exact; Mr. Oughtred directs, (and so had Mr. Wright done before him) to divide the Arch into Parts yet smaller than Minutes, and calculate Secants suiting thereunto.

21. Since the *Arithmetick* of *Infinities* introduced, and (in pursuance thereof) the *Doctrine* of *Infinite Series*, (for such Cases as would not, without them, come to a determinate Proportion;) Methods have been found for squaring some such Figures.

Fig. 207.

22. In order to a *Quadrature* for this Figure of Secants (by an Infinite Series fitted thereunto) put we, for the *Radius* of a Circle, R ; the *Right Sine* of an Arch or Angle, S ; the *Versed Sine*, V ; the *Co-Sine*, (or *Sine* of the Complement) $\Sigma = R - V = \sqrt{R^2 - S^2}$; the *Secant*, f ; the *Tangent*, T .

23. Then is $\Sigma : R :: R : f$. That is, $\Sigma) R^2 (f = \frac{R^2}{\Sigma}$; the *Secant*.

24. And $\Sigma : S :: R : T$. That is, $\Sigma) SR (T = \frac{SR}{\Sigma}$; the *Tangent*.

Fig. 210.

25. Now, if we suppose the *Radius* CP , divided into equal Parts, and each of them $= \frac{1}{n} R$; and on these, to be erected the *Co-Sines* of Latitude LA .

26. Then are the *Sines* of Latitude in *Arithmetical Progression*.

27. And the *Secants* answering thereunto, $L f = \frac{R^2}{\Sigma}$.

28. But these *Secants*, (answering to the *Right Sines* in *Arithmetical Progression*;) are not those that stand at equal Distances on the *Quadrantal Arch* extended. Fig. 209.

Fig. 211.

29. But, standing at Unequal Distances (on the same extended Arch;) Namely, on those Points thereof, whose *Right Sines* (whilst it was a Curve) are in *Arithmetical Progression*. As Fig. 211.

30. To find therefore the Magnitude of $RELf$, Fig. 209. which is the same with that of Fig. 211, (supposing EL of the same Length in both; however the Number of *Secants* therein may be unequal) we are to consider the *Secants*, though at Unequal Distances, Fig. 211. to be the same with those of Equal Distances in Fig. 210. answering to *Sines* in *Arithmetical Progression*.

31. Now, these *Intervals*, (or *Portions* of the *Base*) in Fig. 211. are the same with the *Intercepted Arches*, (or *Portions* of the *Arch*) in Fig. 210. For this *Base* is but that *Arch* extended.

Fig. 212.

32. And these *Arches*, (in Parts infinitely small) are to be reputed equivalent to the *Portions* of their respective *Tangents* intercepted between the same *Ordinates*. As in Fig. 210, 212.

33. That is, Equivalent to the *Portions* of the *Tangents* of Latitude.

34. And these *Portions* of *Tangents* are to the *Equal Intervals* in the *Base*, as the *Tangent* (of Latitude) to its *Sine*.

35. To

35. To find therefore the true Magnitude of the Parallelograms (or Segments of the Figure ; we must either protract the Equal Segments of the Base, *Fig. 210.* (in such Proportion as are the respective Tangents to the Sine) to make them Equal to those of *Fig. 211.*

36. Or else, (which is equivalent) retaining the equal Intervals of *Fig. 210.* protract the Secants in the same Proportion. (For either way the Intercepted Rectangle or Parallelograms will be equally increased) as LM. *Fig. 212.*

37. Namely, as the Sine (of Latitude) to its Tangents ; so is the Secant, to a Fourth ; which is to stand (on the Radius equally divided) instead of that Secant.

Fig. 212.

$$\frac{SR}{S} (:: \Sigma : R) :: \frac{R^2}{\Sigma} : \frac{R^3}{\Sigma^2} = R^2 - S^2 = LM.$$

38. Which therefore are as the Ordinates in (what I call *Arith. Infin. Prop. 104.*) *Reciproca Secundanorum* : Supposing Σ^2 to be Squares in the Order of Secundans.

39. This (because of $\Sigma^2 = R^2 - S^2$; and the Sines S, in Arithmetical Progression) is Reduced (by Division) into this Infinite Series

$$R : + \frac{S^2}{R} + \frac{S^4}{R^3} + \frac{S^6}{R^5} \text{, } \&c.$$

40. That is, (putting $R = 1$.) $1 + S^2 + S^4 + S^6 \text{, } \&c.$

41. Then, (according to the *Arithmetick of Infinites*) we are to interpret S, successively by $1S, 2S, 3S, \&c.$ till we come to S, the greatest. Which therefore represents the Number of all.

And, because the first Member doth represent a Series of Equals, the second of Secundans ; the third of Quartans, $\&c.$ Therefore the first Member is to be multiplied by S ; the second by $\frac{1}{2}S$; the third by $\frac{1}{3}S$; the fourth by $\frac{1}{4}S$; $\&c.$

43. Which makes the Aggregate, $S + \frac{1}{2}S^2 + \frac{1}{3}S^3 + \frac{1}{4}S^4 + \frac{1}{5}S^5 \text{, } \&c.$ = ECLM.

44. This, (because S is always less than $R = 1$) may be so far continued, till some Power of S become so small, as that it (and all which follow it) may be safely neglected.

45. Now, (to fit this to the *Sea-Chart*, according to Mr. *Wright's* Design) having the proposed Parallel (of Latitude) given ; we are to find (by the *Trigonometrical Canon*) the Sine of such Latitude ; and take Equal to it, $CL = S$. And by this find the Magnitude of ECLM, *Fig. 212.* that is, of REL f, *Fig. 211.* that is, of REL f, *Fig. 209.* And then, as RRLE (or so many times the Radius) to REL f, (the Aggregate of all the Secants ;) so must be a like Arch of the *Aequator* (equal to the Latitude proposed,) to the Distance of such Parallel, (representing the Latitude in the *Chart*) from the *Aequator* : Which is the Thing required.

46. The same may be obtained in like manner, by taking the Versed Sines in Arithmetical Progression. For, if the Right Sines (as here) beginning at the *Aequator*, be in Arithmetical Progression, as, 1, 2, 3, $\&c.$ Then will the

the Verfed Sines, beginning at the *Pole*, (as being their Complements to the Radius) be so alfo.

47. The same may be applied in like Manner, (though that be not the present Business) to the Aggregate of *Tangents*, (answering to the Arch divided into equal Parts.

48. For those answering to the Radius so divided are $\frac{SR}{\Sigma}$; (taking S in Arithmetical Progression.)

49. And then Inlarging the Base (as in *Fig. 211.*) or the Tangent (as in *Fig. 212.*) in Proportion of the Tangent to the Sine.

$$S : \frac{SR}{\Sigma} (:: \Sigma : R) :: \frac{SR}{\Sigma} : \frac{SR^2}{\Sigma^2} = \frac{SR^2}{R^2 - S^2}$$

50. We have by Division this Series, $S + \frac{S^3}{R^2} + \frac{S^5}{R^4} + \frac{S^7}{R^6} + \frac{S^9}{R^8}$, &c.

51. That is, (putting $R=1$) $S + S^3 + S^5 + S^7 + S^9$, &c.

52. Which (multiplying the respective Members by $\frac{1}{2}S, \frac{1}{4}S, \frac{1}{8}S, \frac{1}{16}S$, &c.) becomes $\frac{1}{2}S^2 + \frac{1}{4}S^4 + \frac{1}{8}S^6 + \frac{1}{16}S^8 + \frac{1}{32}S^{10}$, &c.

Which is the Aggregate of Tangents to the Arch whose Right Sine is S.

53. And this Method may be a Pattern for the like *Process* in other Cases of like Nature.

Two Problems in
Navigation, pro-
posed; by Mr.
Nich. Mercator.
n. 13. p. 215.
Jun. An. 1666.

XXXVII. The Line of *Artificial Tangents*, or the *Logarithmical Tangent Line*, beginning at 45° . and taking every half Deg. for a whole one, is found to agree pretty near with the *Meridian-Line* of the *Sea-Chart*, they both growing, as it were, after the same proportion. But the Table of *Meridional Degrees* being calculated only to every sexagesimal Minute of a Degree, shews some small Difference from the said *Logarithmical Tangent-line*. Hence it may be doubted, whether the Difference do not arise from that little Error which is committed by calculating the Table of *Meridional Degrees* only to every Minute.

But, if a certain Rule could be produced, by which the Agreement or Disagreement of the said two Lines might be shewn, the *Helix* or *Spiral Line* of the *Ship's Course* would be reduced to a more precise Exactness than ever was pretended by any.

The same Rule would also discover a far easier way of making *Logarithms*, than ever was practised or known; and therefore might serve, whenever thete should be Occasion, to extend the *Logarithms* beyond the Number of Places that are yet extant.

Moreover, such a Rule would enable Men to draw the *Meridian Line* Geometrically, that is, without Tables or Scales; which indeed might also be done by setting of the *Secants* of every whole or half Degree, if there were not this Inconveniency in it, (which is not in my Rule :) That a Line composed of so many small Parts, would be subject to many Errors, especially in a small Compass.

The

The same Rule will also serve to find the Course and Distance between two Places assigned, as far as Practice shall require it; and that without any Table of *Meridional Parts*, and yet with as much Ease and Exactness.

And, seeing all these things do depend on the Solution of this Question, *Whether the Artificial Tangent Line be the True Meridian Line*; it is therefore that I undertake, by God's Assistance, to resolve the said Question. And, to let the World know the Readiness and Confidence I have to make good this Undertaking, I am willing to lay a Wager against any one or more Persons that have a Mind to engage, for so much as another Invention of mine (which is of less Subtily, but of a far greater Benefit to the Publick) may be worth to the *Inventor*.

As for the great Advantage, that all *Merchants, Mariners*, and consequently the *Common-Wealth*, may receive from this other *Invention*, it is, in my Judgment, highly valuable, seeing it will oftentimes make a *Ship* sail, though, according to the common way of *Sailing*, the Wind be quite contrary, and yet as near to the Place intended, as if the Wind had been favourable: Or, if you will, it will enable one to gain something in the Intended Way, whether the Wind be good or no, (except only when you go directly *South* or *North*) but the Advantage will be most where there is most need of it) that is, when the Wind is contrary: So that one may very often gain a fifth, fourth, third Part, or more of the Intended Voyage; according as it is longer or shorter, viz. always more in a longer Voyage, where the Gain is more considerable, and more welcome, not only by saving Time, but also *Victuals, Water, Fuel, Mens Health*, and so much Room in the *Ship*.

XXXVIII. It was first discovered by chance, and, as far as I can learn, first published by Mr. Henry Bond, as an Addition to Norwood's *Epitome of Navigation*, about 50 Years since, that the *Meridian Line* was analogous to a Scale of *Logarithmick Tangents* of half the Complements of the Latitudes.

The Analogy of Logarithmick Tangents to the Meridian Line Demonstrated; by Mr. Edm. Halley. n. 219. p. 202. Feb. An. 1696.

For the Demonstration of that Proposition, it is requisite to premise these four Lemmata.

Lemma 1. *In the Stereographick Projection of the Sphere upon the Plain of the Equinoctial, the Distances from the Centre, which in this Case is the Pole, are laid down by the Tangents of half those Distances; that is, of half the Complements of the Latitudes. This is evident from Eucl. 3. 20.*

Lemma 2. *In the Stereographick Projection, the Angles, under which the Circles intersect each other, are in all Cases equal to the Spherical Angles they represent; which is a very valuable Property of this Projection.*

Demonst. Let EPBL be any great Circle of the Sphere, E the Eye placed in its Circumference; C its Centre, P any Point thereof; and let FCO be supposed a Plain erected at Right Angles to the Circle EPBL, on which FCO we design the Sphere to be Projected. Draw EP crossing the Plain FCO in p, and p shall be the Point P projected. To the Point P draw the Tangent APG, and on any Point thereof, as A, Erect a Perpendicular AD, at right Angles to the Plain EPBL, and draw the Lines PD, AC, DC; and the Angle APD shall be equal to the Spherical Angle contained between the

Fig' 213.

Plains APC, DPC. Draw also AE, DE, intersecting the Plain FCO in the Points a and d ; and join $a d$, $p d$: I say, the Triangle $a d p$, is similar to the Triangle ADP, and the Angle $a p d$ equal to the Angle APD. Draw PL, AK, parallel to FO, and by reason of the Parallels $a p$ will be to $a d$, as AK, to AD: But, (by Eucl. 3. 32.) in the Triangle AKP, the Angle AKP = LPE, is also equal to APK = FPG: wherefore the Sides AK, AP, are equal, and 'twill be as $a p$, to $a d$; so AP to AD. Whence the Angles DAP, $d a p$, being Right, the Angle APD, will be equal to the Angle $a p d$, that is, the Spherical Angle is equal to that on the Projection, and that in all Cases. Q. E. D.

This Lemma I lately received from Mr. Ab. de Moivre, though I since understand from Dr. Hook, that he long ago produced the same thing before the Society. However, the Demonstration, and the rest of the Discourse is my own.

Lemma 3. On the Globe, the Rhumb Lines make equal Angles with every Meridian, and, by the foregoing Lemma, they must likewise make equal Angles with the Meridians in the Stereographick Projection on the Plain of the Equator: They are therefore in that Projection, Proportionall Spirals about the Pole-point.

Fig. 214.

Lemma 4. In the Proportional Spiral it is a known Property, that the Angles BPC, or the Arches BD, are Exponents of the Rationes of BP to PC, : For, if the Arch BD be divided into innumerable equal parts, Right Lines drawn from them to the Centre P, shall divide the Curve B c c C into an Infinity of Proportionals between PD and PC, whose Number is equal to all the Points d, d , in the Arch BD: Whence, and by what I have delivered concerning the Construction of Logarithms, it follows, that, as BD to B d, or, as the Angle BFC. to the Angle BP c, so is the Logarithm of the Ratio of PB to PC, to the Logarithm of the Ratio of PB to P c.

vid. sup. Cap. 1.
§ XXVIII.

From these Lemmata our Proposition is very clearly Demonstrated: For, by the First PB, P c, PC, are the Tangents of half the Complements of the Latitudes in the Stereographick Projection: And, by the Last of them, the Differences of Longitude, or Angles at the Pole between them, are Logarithms of the Rationes of those Tangents one to the other. But the Nautical Meridian Line is no more than a Table of the Longitudes, answering to each Minute of Latitude on the Rhumb Line, making an Angle of 45 Degrees with the Meridian. Wherefore the Meridian Line is no other than a Scale of Logarithmick Tangents of the Half Complements of the Latitudes. Q. E. D.

Corol. 1. Because that in every Point of any Rhumb-Line the Difference of Latitudes is to the Departure, as the Radius to the Tangent of the Angle that Rhumb makes with the Meridian; and those equal Departures are every where to the Differences of Longitude, as the Radius to the Secant of the Latitude; it follows that the Differences of Longitude are on any Rhumb, Logarithms of the same Tangents, but of differing Species; being proportioned to one another, as are the Tangents of the Angles made with the Meridian.

Coroll. 2.

Coroll. 2. Hence any Scale of Logarithm Tangents (as those of the Vulgar Tables made after *Brigg's* Form ; or those made to *Napier's*, or any other Form whatsoever) is a Table of the Differences of Longitude, to the several Latitudes, upon some determinate *Rhumb* or other ; and therefore, as the Tangent of the Angle of such *Rhumb*, to the Tangent of any other *Rhumb* : So the Difference of the *Logarithms* of any two Tangents, to the Difference of Longitude on the proposed *Rhumb*, intercepted between the two Latitudes, of whose half Complements you took the Logarithm Tangents.

Now the *Momentary Augment*, or *Fluxion* of the *Tangent-line* at 45 Degrees, is exactly double to the *Fluxion* of the Arch of the Circle, (as may easily be proved) and the Tangent of 45 being equal to the Radius, the *Fluxion* also of the *Logarithm Tangent* will be double to that of the Arch, if the *Logarithm* be of *Napier's* Form : But, for *Brigg's* Form, it will be as the same doubled Arch multiplied into 0,43429, &c. or divided by 2,30258, &c. yet this must be understood only of the Addition of an Indivisible Arch, for it ceases to be true, if the Arch have any determinate Magnitude.

Hence it appears, that, if one Minute be supposed Unity, the Length of the Arch of one Minute being 0,000290888208665721596154, &c. in parts of the Radius, the Proportion will be as Unity to 2,908882, &c. So Radius to the Tangent of $71^{\circ} 1' 42''$, whose *Logarithm* is 10,46372611720718-325204, &c. and under that Angle is the *Meridian Intersected* by that *Rhumb-line*, on which the Differences of *Napier's* Logarithm Tangents of the Half Complements of the Latitudes are the true Differences of Longitude, estimated in Minutes and Parts, taking the first 4 Figures for Integers. But for *Vlacq's* Table we must say,

As 2302585, &c. to 2908882, &c. so Radius to 1,2633114387244-569212, &c. which is the Tangent of $51^{\circ} 38' 9''$, and its *Logarithm* 10,10-1510428507720941162, &c. wherefore in the *Rhumb-line*, which makes an Angle of $51^{\circ} 38' 9''$ with the Meridian, *Vlacq's* Logarithm Tangents are the true Differences of Longitude. And this, compared with our *Second Corollary*, may suffice for the Use of the Tables already computed.

But, if a Table of *Logarithm Tangents* be made by Extraction of the Root of the *Infinith Power*, whose Index is the Length of the Arch you put for Unity, (as for the Minutes the 0,0002908882th, &c. Power) which we will call *a*; such a Scale of Tangents shall be the true *Meridian Line*, or Summ of all the Secants taken infinitely many. Here the Reader is desired to have Recourse ^{vid. sup. Cap.} to my little Treatise of *Logarithms*, that I may not need to repeat it. By I. § XXVIII. what is there delivered it will follow, that putting *t* for the Excess or Defect of any Tangent above or under the Radius or Tangent of 45° ; the *Logarithm* of the Ratio of Radius to such Tangent will be

$$\frac{1}{m} \text{ into } t - \frac{1}{2} t^2 + \frac{1}{3} t^3 - \frac{1}{4} t^4 + \frac{1}{5} t^5, \&c.$$

When the Arch is greater than 45, or

$$\frac{1}{m} \text{ into } t + \frac{1}{2} t^2 + \frac{1}{3} t^3 + \frac{1}{4} t^4 + \frac{1}{5} t^5, \&c.$$

when it is less than 45° . And by the same Doctrine putting T for the Tangent of any Arch, and t for the Difference thereof from the Tangent of another Arch, the Logarithm of their Ratio will be $\frac{1}{m}$ into $\frac{t}{T} + \frac{t^2}{2T^2} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}$, &c. When T is the greater Term; or, $\frac{1}{m}$ into $\frac{t}{T} - \frac{t^2}{2T^2} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}$, &c. when T is the lesser Term.

And, if m be supposed, 0,0002908882, &c. = a , its reciprocal $\frac{r}{a}$ will be 3437,7467707849392526, &c. which multiplied into the aforesaid Series shall give precisely the Difference of the Meridional Parts, between the two Latitudes to whose half Complements the assumed Tangents belong. Nor is it material from whether Pole you estimate the Complements, whether the Elevated or Depressed; the Tangents being to one another in the same Ratio as their Complements, but inverted.

In the same Discourse I also shewed, that the Series may be made to converge twice as swift, all the even Powers being omitted; and that, putting τ

for the Summ of the two Tangents, the same Logarithm would be $\frac{2}{m}$ or $\frac{2r}{a}$

into $\frac{\tau}{\tau} + \frac{\tau^3}{3\tau^3} + \frac{\tau^5}{5\tau^5} + \frac{\tau^7}{7\tau^7} + \frac{\tau^9}{9\tau^9}$, &c. but the Ratio of τ to c , or of

the Summ of two Tangents to their Difference, is the same as that of the Sine of the Summ of the Arches, to the Sine of their Difference. Wherefore, if S be put for the Sine Complement, of the Middle Latitude, and s for the

Sine of half the Difference of Latitudes, the same Series will be $\frac{2r}{a}$ into $\frac{s}{S}$

$+ \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}$, &c. wherein as the Differences of

Latitude are smaller, fewer Steps will suffice. And, if the Equator be put for the Middle Latitude, and consequently $S=R$, and s the Sine of the

Latitude, the Meridional Parts reckoned from the Equator will be $\frac{s}{a} +$

$\frac{s^3}{3rra} + \frac{s^5}{5r^4a} + \frac{s^7}{7r^6a}$, &c. which is Co-incident with Dr. Wallis's

wid. §.XXXVI Solution. And this same Series, being half the Logarithm of the Ratio of $R+s$ to $R-s$, that is of the Versed Sines of the Distances from both Poles, does agree with what Dr. Barrow had shewn in his XI. Lecture.

The

The same *Ratio* of r to t may be expressed also by that of the Summ of the Co-sines of the two *Latitudes*, to the *Sine* of their Difference: As likewise by that of the *Sine* of the Summ of the two *Latitudes*, to the Difference of their Co-sines: Or by that of the *Versed Sine* of the Summ of the *Co-Latitudes*, to the Difference of the *Sines* of the *Latitudes*: Or as the same Difference of the *Sines* of the *Latitudes*, to the *Versed Sine* of the Difference of the *Latitudes*; all which are in the same *Ratio* of the Co-sine of the *Middle Latitude*, to the *Sine* of half the Difference of the *Latitudes*. As it were easy to demonstrate, if the Reader were not supposed capable to do it himself, upon the bare Inspection of a Scheme duly representing these Lines.

This Variety of *Expression* of the same *Ratio* I thought not fit to be omitted, because by help of the *Rationality* of the *Sines* of 30° , in all Cases where the Summ or Difference of the *Latitudes* is 30° , 60° , 90° , 120° , or 150° ; some one of them will exhibit a simple *Series*, wherein great Part of the Labour will be saved. But the former seems for all Uses the most convenient, whether we design to make the whole *Meridian-Lines*, or any Part

thereof, viz. $\frac{2r}{a}$ into $\frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7} + \frac{s^9}{9S^9}$, &c. wherein a

is the Length of any Arch, which you design shall be the Integer or *Unity* in your *Meridional Parts* (whether it be a Minute, League or a Degree, or any other,) S the Co-sine of the *Middle-Latitude*, and s the *Sine* of half the Difference of *Latitudes*; but, the *Secants* being the Reciprocals of the Co-sine

$\frac{s}{S}$ will be equal to $\frac{f s}{r r}$ putting f for the *Secant* of the *Middle-Latitude*; and

$\frac{2r}{a}$ into $\frac{s}{S}$ will be $= \frac{2 f s}{r a}$. This multiplied by $\frac{f s}{3 S S}$ that is by $\frac{f f s s}{3 r r r r}$

will give the second Step; and that again by $\frac{3 f f s s}{5 r r r r}$, the third Step; and

so forward till you have compleated as many Places as you desire. But, the *Squares* of the *Sines* being in the same *Ratio* with the *Versed Sines* of the double Arches, we may instead of $\frac{s s}{3 S S}$ assume for our Multiplier $\frac{v}{3 V}$, or the

Versed-sine of the Difference of the *Latitudes* divided by thrice the *Versed sine* of the Summ of the *Co-Latitudes*, &c. which is the utmost *Compendium*

I can think of for this Purpose, and the same *Series* will become $\frac{2 s r}{a S}$ into 1

$+ \frac{v}{3 V} + \frac{v^3}{5 V^3} + \frac{v^5}{7 V^5} + \frac{v^7}{9 V^7}$. Hereby we are enabled to estimate

the Default of the Method of making the *Meridional Line*, by the continual Addition of the *Secants* of *Equi-different Arches*, which, as the Differences of those Arches are smaller, does still nearer and nearer approach

approach the Truth. If we assume, as Mr. *Wright* did, the Arch of one Minute to be Unity, and one Minute to be the common Difference of a Rank of *Arches*: It will be in all Cases, as the Arch of one Minute, to its Chord; so the Secant of the middle Latitude, to the First Step of our Series. This by Reason of the near Equality between a and $2s$, which are to one another in the Ratio of Unity to $1 - 0,00000000352566457713$, &c. will not differ from the Secant s , but in the 9th Figure; being less than it in that Proportion. The next Step being $+\frac{2s^3}{3arr}$ will be Equal to the

Cube of the Secant of the *Middle Latitude* multiplied into $\frac{2s^3}{3arr} = 0,0000$

0000705132908715; which therefore, unless the Secant exceed Ten Times Radius, can never amount to 1 in the Fifth Place. These two Steps suffice to make the *Meridian Line*, or *Logarithm Tangent*, to far more Places than any of Natural Secants yet extant, are computed to; but, if the

Third Step be required it will be found to be $+\frac{2s^5}{5arr^4} = 0,00000$

000000000089498; by all which it appears that Mr. *Wright's* Table does no where exceed the true *Meridian Parts* by fully half a Minute; which small Difference arises by his having added continually the Secants $1', 2', 3', \&c.$ instead of $0', 1', 2', 3', \&c.$ But, as it is, it is abundantly sufficient for *Nautical Uses*. That in Sir *Jonas Moor's New System of the Mathematicks* is much nearer the Truth, but the Difference from *Wright* is scarce sensible, till you exceed those Latitudes where *Navigation* ceases to be practicable, the one exceeding the Truth about half a Minute, the other being a very small Matter deficient therefrom.

For an Example easy to be imitated by whoso pleases, I have added the true *Meridional Parts* to the first and last Minutes of the *Quadrant*.

The First Minute. 1,00000001410265862178.

The Second, ——— 2,00000005641063806707.

The Last, or $89^\circ 59'$. 30374,9634311414228643, and not 32348,5279 as Mr. *Wright* has it, by the Addition of the Secants of every whole Minute: Nor 30249,8 as Mr. *Oughtred's* Rule makes it, by adding the Secants of every half Minute. Nor 30364,3 as Sir *Jonas Moor* had concluded it by I know not what Method, though in the rest of his Table he follows *Oughtred*.

The same may be deduced independently from the Arch it self. For if the Latitude from the Equator be estimated by the Length of its Arch A , Radius being Unity, and the Arch put for an Integer be a , as before; the *Meridional*

Parts answering to that Latitude will be $\frac{1}{a}$ into A , $+\frac{1}{6} A^3 + \frac{1}{24} A^5 +$

$\frac{1}{60} A^2$ or $\frac{61}{5640} A^2 + \frac{11}{2880} A^3$ or $\frac{1335}{362880} A^3$ &c. which Converges much swifter than any of the former Series, and besides has the Advantage of A increasing in Arithmetical Progression, which would be of great Ease, if any should undertake *de novo* to make the Logarithm Tangents, or the Meridian-line to any more Places than now we have them. The Logarithm Tangent to the Arch of $45^\circ + \frac{1}{2} A$ being no other than the aforesaid Series $A + \frac{1}{2} A^3 + \frac{1}{24} A^5$, &c. in Napier's Form, or the same multiplied into 0,43-429, &c. for Brigg's.

But, because all these Series towards the latter End of the Quadrant do converge exceedingly slowly, so as to render this Method almost useless, or at least very tedious: It will be convenient to apply some other Arts, by assuming the Secants of some intermediate Latitudes; and you may for s , or the Sine of n , the Arch of half the Difference of Latitudes, substitute $a - \frac{1}{2} a^3 + \frac{1}{24} a^5 - \frac{1}{720} a^7 + \frac{1}{40320} a^9$, &c. according to Mr. Newton's Rule for giving the Sine from the Arch; and, if a be no more than a Degree, a very few Steps will suffice for all the Accuracy that can be desired.

And, if a be commensurable to a , that is, if it be a certain Number of those Arches which you make your Integer, then will $\frac{a}{a}$ be that Number which if we call n , the Parts of the Meridional Line will be found to be

$$\begin{aligned}
 & 1 + \frac{\int a a}{3 r^4} + \frac{\int^2 a^4}{5 r^8} + \frac{\int^3 a^6}{7 r^{12}}, \text{ \&c.} \\
 & \frac{\int n}{- \text{into } r} \left[\frac{a a}{6 r^4} - \frac{\int a^4}{6 r^6} - \frac{\int^2 a^6}{6 r^{10}}, \text{ \&c.} \right. \\
 & \quad \left. + \frac{1}{120} \frac{a^4}{r^4} + \frac{13}{360} \frac{\int^2 a^6}{r^8}, \text{ \&c.} \right. \\
 & \quad \left. - \frac{1}{5040} \frac{a^6}{r^6}, \text{ \&c.} \right]
 \end{aligned}$$

In this the First two Steps are generally sufficient for Nautical Uses, especially when neither of the Latitudes exceed 60 Degrees, and the Differences of Latitudes do not pass 30 Degrees.

To conclude, I shall only add, that Unity being Radius, the Co-sine of the Arch A , according to the same Rules of Mr. Newton, will be $1 - \frac{1}{2} A^2 + \frac{1}{24} A^4 - \frac{1}{720} A^6 + \frac{1}{40320} A^8 - \frac{1}{362880} A^{10}$, &c. from which and

< the

the former *Series* exhibiting the *Sine* by the Arch, by Division it is easie to conclude, that the *Natural Tangent* to the Arch A is $A + \frac{1}{3} A^3 + \frac{2}{15} A^5 + \frac{17}{315} A^7 + \frac{62}{2835} A^9$, &c. and the *Natural Secant* to the same Arch $1 + \frac{1}{2} A^2 + \frac{5}{24} A^4 + \frac{61}{720} A^6 + \frac{227}{8064} A^8$, &c. And from the *Arithmetick of Infinites*, the Number of these *Secants* being the Arch A , it follows that the Sum Total of all the *Infinite Secants* on that Arch is $A + \frac{1}{6} A^3 + \frac{1}{24} A^5 + \frac{61}{5040} A^7 + \frac{277}{72576} A^9$, &c. the which, by what foregoes, is the *Logarithm Tangent* of *Napier's Form*, for the Arch of $45^\circ + \frac{1}{4} A$, as before.

And collecting the *Infinite Summ* of all the *Natural Tangents* on the said Arch A , there will arise $\frac{1}{2} A A + \frac{1}{12} A^4 + \frac{1}{45} A^6 + \frac{17}{2520} A^8 + \frac{31}{14175} A^{10}$, &c. which will be found to be the *Logarithm* of the *Secant* of the same Arch A .

To find the Variation of the Compass at Sea. by . . . n. 24. p. 435. Apr. An. 1667.

XXXIX. The Height of the Pole, and the *Sun's Declination* being known, a large Ring-Dial, truly wrought, having a Box with a Compass or Needle fixt to its Meridian below, may go as near as any other Instrument, to shew the Variation of the Needle at Sea. For, when it is set to the just Hour and Minute of the Day, the Meridian of it stands just in its due Place; and so shews how far the Needle varies from it, as exactly as the largeness of the Card will permit.

But, because these Dials are so rarely just, &c. though they may be used and taken Notice of, yet they are not to be relied on. The Thing therefore is to be performed, as followeth:

Find out the *Sun's Azimuthal Distance* from the Meridian some Hours before or after Noon, and then its *Magnetical Azimuth*, or Distance from the Meridian pointed at by the Needle, and the Difference of those two Distances is the Variation of the Needle.

To find the *Sun's true Azimuth*, or by how many Degrees, &c. of the Horizon, it is distant from the Meridian: Its Declination, its Altitude, and the Elevation of the Pole, must all three be known; and thence the true *Azimuth* may be easily calculated. The true *Azimuth* of the *Sun* being thus found, and the *Magnetical Azimuth* of it, according to your Needle, being observed, subtract the lesser Number from the greater, and the Remainder is the Variation of the Needle. If the *Magnetical Azimuth* be less than the other, then the Variation is towards the same side of the Meridian, where the *Sun* is; if greater, on the other.

To

To observe the *Sun's Azimuth* by the *Needle*, and the *Needle's Variation*, to Degrees, any *Needle* long enough to afford upon a Card under it a Circle divided into Degrees, put into a square Box, after the ordinary manner of *Clinatories*, will serve turn; by placing the Box so, as the Sun may shine upon any two opposite sides of it, at the same time when the Sun's Height, &c. are taken. For then the *Needle's Distance* from the Diameter of the Circle on the Card, that is parallel to those Sides, is the *Magnetick Azimuth* required.

The same may be done with an ordinary *Sea-Compass*, so it have a Circle towards the Limb of the Card divided into Degrees, by fastening a small Thread, Lute-string or Wire (not of Iron) so upon it, as to pass just over the Centre of that Circle: and placing a strait Piece of Wood or Brass-wire perpendicular on the Edge of the Box at the end of the Thread, and turning it to the Sun till the shadow of it fall just upon the Thread; then observe what Degrees of the Circle on the Card the Thread cuts, by looking plum upon it; and that is the Sun's *Magnetical Azimuth*.

But to have the *Variation* to Degrees and Minutes, (which is most desirable) then the Observation last mentioned must be made with a *Quadrant*, *Sextant*, or some such other Instrument, so large as to admit of the Division of a Degree into Minutes: which will require the Radius to be about 3 Foot; the larger the better. If a *Quadrant*, then, it being laid flat, and the square Box with the *Needle* placed upon it, move the *Quadrant* to and again, till that side of it, on which the Box is placed, lie parallel to the *Needle* when at quiet: Then the Sight of the *Quadrant* being slid along the Limb of it, till the Sun shine on both its sides at the same time, the Mid-line, that divides equally the Sight, when the Sun shines upon it through the Slit, will mark the Degree and Minute of the Sun's *Magnetical Azimuth*. All which is easie to be put in Practice.

To find this Variation by the Stars, is so easie, that every Master can do it.

XL. It is a received Error, in the Practice of observing the Variation at Sea, to take it by the Amplitude of the *Rising* and *Setting Sun*, when his Centre appears in the Visible Horizon, whereas he ought to be observed when his Under-Limb is still above the Horizon about $\frac{1}{3}$ of his Diameter, or 20 Minutes, upon the Score of the *Refraction*, and the Height of the Eye of the Observer above the Surface of the Sea: Or else they are to work the *Amplitude* as they do the *Azimuth*, reckoning the Sun's Distance from the Zenith $90^{\circ} 36'$.

A Caution for Observing the Variation at Sea by Mr. Edm. Halley, n. 195. P. 571. Oct. An. 1692.

This, though it be of little Consequence near the Equinoctial, will make a great Error in *High Latitudes*, where the Sun rises and sets obliquely.

XLI. The *Latitudes* of the *Lizard* and *Scilly*, are laid down too far Northerly by near 5 Leagues: For, from undoubted Observation, the *Lizard* lies in $49^{\circ} 55'$. the middle of *Scilly* due West therefrom, and the South Part thereof nearest $49^{\circ} 50'$. Whereas in most Charts and Books of

A Caution to Seamen bound up the English Channel; by Mr. Halley, n. 247. P. 725. Nov. An. 1702.

Navigation they are laid down to the Northward of 50° . and in some full $50^{\circ} 10'$. Nor was this without a good Effect, as long as the *Variation* continued *Easterly*, as it was when the *Charts* were made. But since it is become considerably *Westerly*, (as it has been ever since the Year 1657.) and is at present about $7\frac{1}{2}$ Deg. all Ships standing in, out of the Ocean, *East* by the *Compass*, go two thirds of a Point to the Northward of their true Course, and in every 80 Miles they sail, alter their *Latitude* about $10'$. So that if they miss an Observation for two or three Days, and do not allow for this *Variation*, they fail not to fall to the Northward of their Expectation, especially if they reckon *Scilly* in above 50° , and to run up the *Bristol Channel*, not without great Danger of all, and the Loss of many of them. This has been by some attributed to the *Indraught* of *St. George's Channel*: But, the *Variation* being allowed, it hath been found, that the said *Indraught* is not sensible. It is therefore recommended to all Masters of Ships, that they steer two Watches E. by S. for one E. which will exactly keep their Parallel; as also, that they come in, out of the Sea, on a Parallel not more *Northerly*, than $49^{\circ} 40'$. which will bring them fair by the *Lizard*.

XLII. Papers of less General Use, omitted.

Pendulum Watches, n. 118. p. 440. *vid. sup.*
Cap. V. Sec. V.

n. 128. p. 710.
n. 129. p. 749.

Mr. Oldenburg having published from *Journal de Scavans*, an Account of M. *Hugen's Portable Watches*, Dr. *Hook*, in the Postscript to his *Description of Helioscopes*, complains of it, for not having taken Notice, That this Invention was first found out by an Englishman, and long since published to the World. To this Mr. Oldenburg answers, by relating the Plain Truth of the Matter: Whereupon, Dr. *Hook*, in a Postscript to his *Lampas*, further complains, and reflects on Mr. Oldenburg's Integrity and Faithfulness in his Management of the Intelligence of the Royal Society. This gave Occasion to the Council of that Society to declare, That Mr. Oldenburg had carried himself Faithfully and Honestly and given no just Cause of such Reflections; To which Mr. Oldenburg adds Part of a Letter from Mr. *Hugens* to him, Offering, (if Mr. Oldenburg believes a Patent in England might be worth something) all he might there pretend to. So that if Mr. Oldenburg had a Desire to take out a Patent, it was for no other Contrivance than Mr. *Hugen's*.

XLIII. Account of Books and Emendations omitted.

- n. 231. p. 670. 1. *Volumen Primum Geographorum Gr. Minorum. Oxon. in 8vo.*
- n. 231. p. 671. 2. *Dionysii Periegesis, Græce & Latine, cum Scholis Gr. tam Editis quam Ineditis. Cura Edw. Thwaites, M. A. Oxon. in 8vo.*
- n. 91. p. 5172. 3. *Bernhardi Vareni, M. D. Geographia Generalis; aucta & illustrata ab Isaaco Newtono, R. S. S. Cantab. 1672. in 8vo.*
- n. 231. p. 666. 4. *Philippi Cluverii Introductio in Universam Geographiam, tam Veterem quam Novam: Tabulis Geographicis 46. ac Notis olim ornata a Joanne Burnone; jam vero locupletata Additamentis & Annotationibus Jo. Frid. Hekelii & Jo. Reiskii. Amst. 1697. in 4to.*

5. *Geography Anatomiz'd*; or a Compleat Geographical Grammar. Being a short and exact Analysis of the whole Body of Modern Geography, after a New and Curious Method; by Pat. Gordon, M. A. F. R. S. The Second Edition. n. 256. p. 335.

6. An Account of the Measure of a Degree of a Great Circle of the Earth; by M. Picart. Paris. 1671. Fol. Translated into English; by Mr. Waller, R. S. Sec. Lond. 1687. This Book is here abridg'd; and the Sum of the whole amounts in short to this, M. Picart measured on a plain and straight Ground a space of 5663 Toises, to serve for the First Basis to divers Triangles; by which he hath concluded the Length of a Meridian Line equivalent to a Degree of Latitude, to be 57060 Toises or Fathoms, that is, $28\frac{1}{2}$ Leagues and 60 Toises. n. 112. p. 261. n. 124. p. 595. n. 169. p. 376.

7. The Seaman's Practice; by Mr. Richard Norwood, Lond. 1636. in 4to. n. 126. p. 636. The Measure of a Degree is here extracted from that Book. Mr. Norwood, An. 1635. having actually measured, for the most Part, the Way from York to London, and having observed the Meridian Altitudes of the Sun in both Places, he found the Difference of Latitude to be $20^{\circ} 28'$. and the Distance of their Parallels 905751. English Feet; and therefore one Degree of a Great Circle is 367196 Feet, or Numero Rotundo 367200 Feet, which is equal to $69\frac{1}{2}$ English Miles, and 14 Poles; Whereas the French make it no more than 365000 such Feet.

8. Longitude found; by Hen. Bond, Sen. Lond. 1676. in 4to. A Mistake in that Book is here corrected. n. 95. p. 6065. n. 130. p. 774.

9. A Book published by Mr. Jo. Moxon, describing a new sort of Terrestrial Globes, invented by the E. of Castlemain. n. 139. p. 9883. Ph. Col. n. 1. p. 43. Ph. Col. n. 28. p. 39.

10. The English Atlas. Oxford, for Moses Pitt. 1680. Fol.

11. A new Map of England, full 6 Foot Square, wherein Computed and Measured Miles are entered in Figures; by Mr. Jo. Adams. n. 135. p. 886.

12. A large and curious Map of Great Tartary; by M. Nich. Witson. n. 193. p. 492.

C H A P. VIII.

Architecture. Ship-Building.

*Stones fit for
Building; by
.....
n. 53. p. 6010.
Apr. An. 1673.*

I. **T**H E R E is a sort of grey *Freestone* at *Paris*, every where on the South-side of the River *Seine*, which is of a reasonable course Greet, and so soft when first taken out of the *Quarry*, that 'tis drest and hewn with broad sharp Axes, almost as easily as dried Clay, but grows harder and harder in the Air; 'tis very durable, and exceeding fit for *Building*. The *Portland Stone* is of a fine Chalky Greet, fit for all curious hewn and carved Work, though not fit for Water or Fire. On the contrary, the *Freestone* in *Kent*, of a whitish grey Colour, lasts well in Air and Water; the Greet thereof less fine and chalky than that of *Portland*. The *Derbyshire Freestone*, though it endure the fiercest Fire, is yet brittle, and so unfit for fine and curious Workmanship.

*The Choice and
Charges of Slate
for covering
Houses; by Mr.
Sam. Colepres.
n. 50. p. 1009.
Aug. An. 1669.*

II. 1. Take the thin cleft Stone, Slat or Shindle, and so knock it against any hard Matter, as to make it yield a Sound; if the Sound be good, and clear, that sort of Stone is not crazy, but firm and good. Or, If in hewing it does not break before the Edge of the *Setts*, (the Hewing Instrument of the *Slatters*), you may not much doubt of the Firmness of the Slate. But,

3. If after it hath been exactly weighed (and the Accompt thereof laid by) it be put, and for 2, 4, or 8 Hours left to remain all under Water in a Vessel; and afterward taken up and wiped very clean with Cloths, if then it weigh more than before, 'tis of that kind which imbibes Water, and therefore not so fit to endure any considerable time without rotting the Lathes and Timber.

4. These Stones may be pretty well guessed at, whether they be of a close or loose Texture, by their Colour: For the over blackish Blue is aptest to take in Water; but the lighter Blue is always the firmest and closest. To which may be added the Touch; for a good Stone feels somewhat hard and rough; whereas an open Stone feels very smooth, and as it were Oily.

5. Place your Stone long-ways perpendicular in the midst of a Vessel of Water, (no matter how shallow the Water be, so it exceed half a Foot depth;) and be sure, the upper un-immersed Part of the Stone be not accidentally wetted by the Hand, or otherwise; and so let it remain a Day, or half a Day, or less. If it be a good firm Stone, it will not draw (as they speak)

speak) Water more than half an Inch above the Level of the Water, and that perhaps but at the Edges only, the Parts of which might be somewhat loosened in the Hewing. But a bad Stone will draw Water up to the very top, be the Stone as long as it will, all over.

As for the Charge of covering Houses with Slate, they may be thus computed.

| | s. | d. |
|--|----|----|
| 1000 of <i>Efford Small Blue</i> at the Ship's side in <i>Plymouth Harbour</i> — | 5 | 6 |
| 1000 of <i>Efford Large Blue</i> — | 9 | 9 |
| 1000 of <i>Can Pelmel</i> — | 7 | 0 |
| 1000 of <i>Small Blue</i> of other Quarries — | 4 | 0 |
| 1000 of <i>Large Blue</i> — | 8 | 0 |

3000 of *Small Blue*, accounted two Tuns in Carriage by Water.

1000 of *Large Blue*, 1 Tun.

3000 of *Small* will cover 1 *Pole* of Work at the 5 Pin Plain.

Every *Pole* of Work is either 6 Foot broad and 14 up, on both sides, or 168 Foot in Length, and one in Breadth.

3000 of *Large*, will cover two *Poles* of plain Work.

Hewing of all sorts of plain *Pelmel* per 1000 — 1 6

Pinning per 1000, 8 d. Pins per 1000. 8 d. — 1 4

Three Bushels (*Winchester Measure*) of good *Lime*, will take 6 Bushels of fresh Water Sand, and serve to lay on one *Pole* of Work ; though much less may serve the Turn.

300 of Lathes to every *Pole* of Work.

1000 of Lathe Nails to every 3000 of Lathes.

An able Workman may { Lath one *Pole* of Work
Lay on 2000 or more of *Slate*
Hew 1500 plain
Pin 4000 } by the Day.

Chequer-work consists in Angles, Circles, and Semicircles, &c. which require no common Skill and Time in hewing and laying.

It is worthy Observation, That, if a side Wall happen to take Wet by the beating of the Weather, or the like, when nothing else will cure it, our *Kersyeing* with *Slate* (which is much used in the curious Fronts of Houses, especially in Towns) will quickly remedy it.

We have some sorts, which by the Conjectures of the more Experienced *Helliers*, (or Coverers with *Slate*) have continued on Houses several Hundreds of Years, and yet as firm as when first put up.

III. The Custom of *Felling Timber* here in the South of England, differs from that of *Staffordshire*, in the Time of *Felling*, and manner of *Barking*. It is *Felled* here in the *Spring*, as soon as the *Sap* is found to be fully up, by the Trees putting out, and then *Barked* after the Trees are prostrate, the *Sap* yet remaining in the Bodies of them : Whereas there it is first *Bark'd*, (in the *Spring* as here) but before it is *Felled*, the Trees yet living and standing all the *Summer*, and not *Felled* till the following *Winter*, when the *Sap* is fully in *Repose*.

In

The best Time of Felling of Timber; by Dr. Rob. Plot, n. 192. p. 455. Jan. Ann 1691.

In the *Spring* Season, and some time after, all *Trees* are pregnant and spend themselves (as *Animals* do in their respective *Offsprings*) in the *Production* of *Leaves* and *Fruits*, and so become weaker than at other times in the *Year*; their *Cavities* and *Pores* being then turgid with *Juices*, or *Sap*, which (the *Trees* being felled at that time) still remain in the *Pores*, having no manner of *Means* of being otherwise spent, and there putrifie; not only leaving the *Tree* full of those *Cavities* which render the *Timber* weak; but, 2dly, breeding a *Worm*, as both *Pliny* and *Mr. Evelyn* testify, that will so exceedingly prejudice it, that it becomes altogether unfit for strong *Incumbencies*, or other robust *Uses*. 3ly, All *Timber* fell'd at this *Time* of the *Year*, whether the *Juices* putrifie, or otherwise sweat forth, or dry away, is not only subject to rift and gape, but will shrink so considerably, that a *Piece* of such *Timber* of a *Foot* square will usually shrink in the *Breadth* of $\frac{1}{4}$ of an *Inch*; than which, says *Vegetius*, nothing is more pernicious, if us'd for the *Building* of *Ships*. To which, 4thly, the *First* and *Greatest* *Roman* *Emperor*, *Julius Caesar*, adds, that tho' *Ships* may be made of such moist *Timber*, felled in the *Spring*, yet they will certainly be *Sluggs*, not near so good *Sailers* as *Ships* made of *Timber* felled later in the *Year*.

In all which *Circumstances*, I find, most of the *Ancients* so very well agree, that none of them advise the felling of *Timber*, for any sort of *Use*, before *Autumn*, at soonest; others, not till the *Trees* have born their *Fruit*; which, says *Theophrastus*, must always be proportionably later, as their *Fruits* are ripe later in the *Year*. A third sort, not till *Mid-winter*; not till *November*, says *Palladius*, nay, not till the *Winter Solstice*, says the wise *Cato*; and then too in the *Decrease* or *Wane* of the *Moon*, between the 15th and 23d *Day* of her *Age*, says *Vegetius*, or rather, according to *Columella*, between the 20th and the *New-Moon*. In general, says *Theophrastus*, the *Oak* must be fell'd very late in the *Winter*, not till *December*, as the *Emperor Constantine Pogonatus* positively asserts, the *Moon* too being then under the *Earth*, as 'tis for the most part in the *Day* time in the first *Part* of its *Decrease*. And the felling of *Oak* within those *Limits* they call *Tempestriva Casura*, *Felling Timber* in *Season*, which they all unanimously pronounce (if thus felled) will neither shrink, warp, nor cleave, nor admit of *Decay*, in many *Years*; it being tough as *Horn*, and the whole *Tree* in a manner (as *Theophrastus* asserts) as hard and firm as the *Heart*; with whom also agrees our *Countryman* *Mr. Evelyn*, if you fell not *Oak* (says he) till the *Sap* is in *Repose*, as 'tis commonly about *November* and *December*, after the *Frost* has well nipped them, the very *Saplings* thus cut will continue without *Decay*, as long as the *Heart* of the *Tree*.

And the *Reason* of this is given in short by *Vitruvius*, quia *Aeris Hybernus comprimit & consolidat Arbores*, because the *Winter* *Air* doth close the *Pores*, and so consequently consolidates all *Trees*, by which means, the *Oak*, (as he and *Pliny* both express it) will acquire a sort of *Eternity* in its *Duration*; and much more will it so, if it be *Bark'd* in the *Spring*, and left standing all the *Summer*, exposed to the *Sun* and *Wind*, as is usual in *Staffordshire*, and the adjacent *Countries*, whereby they find, by long *Experience*, the *Trunks* of their

their Trees so dried and harden'd, that the sappy Part, in a manner, becomes as firm and durable as the Heart itself.

Which way of *Barking* and *Felling* of *Timber*, tho' it were unknown to the Ancients (as perhaps it is to all the World besides these few Counties) yet they seem not unacquainted with the Rationality of the Practice. The great *Vitruvius* prefers the *Timber* on the *South-side* the *Apennine* (where it winds about, and incloses *Tuscany* and *Campania*, and strongly reflects the constant Heat of the *Sun* upon it, as it were from a Concave) incomparably before that which grows upon the *North-side* of the same Hill, in the shady moist Grounds: Of which his Opinion he renders us this Reason, for that the *Sun* does not only lick up the superfluous Moisture of the *Earth*, whence the *Trees* are supplied in such shady Places with too great a Quantity, but in great measure exhales the remaining Juices (after the Production of Leaves and Fruits) out of the *Trees* themselves, rendring the *Timber* of them the more close, substantial and durable; which certainly it would do also much more effectually, if the *Bark* were taken off in the Spring of the Year, as is accustomed in *Staffordshire*, where the People are content to use this Method in their Provision of *Timber*, though but for private Uses:

Much rather then should it be done in so publick a Concern as the *Building* of *Ships*, where tough and solid *Timber* is much more necessary than in ordinary *Buildings*. There is indeed an *Act* of *Parliament*, 1 *Jac.* 1. *Chap.* 22. which forbids the *Felling* of *Timber* for ordinary Uses (in consideration of the *Tan*) at any other time but between the 1st of *April*, and the last of *June*, when the Sap is up, and the *Bark* will run; made on Supposition, (I guess) that should they have admitted *Felling Timber* in any other Season, the *Tanners* would have wanted a Supply of *Bark*. To which I readily answer, That I fear the *Legislators* that pressed the making that *Act*, were ignorant that the *Bark* might be taken off in the Spring, and that the *Tree* would notwithstanding live and flourish till the *Winter* following, as I have seen many in *Staffordshire*: So that though the *Tree* be not *Fell'd* till the *Winter Solstice*, or *January* following, yet the *Tanner* is not at all defeated of his *Tan*, but has it here in as-due Season, as in any of the Southern Countries. The *Legislators*, I say, were ignorant of this, otherwise they would never have made an *Act* so pernicious to the whole Kindom, as *Felling Timber* at this Season is, for the sake of a few *Tanners*.

But notwithstanding this Ignorance, yet then they were so wise as to except in that *Act* the *Timber* to be used in building of *Ships*, which may be *Fell'd* in *Winter*, or any other time; as I am told all the ancient *Timber* remaining in the *Royal Sovereign* was, it being still so hard, that 'tis no easy matter to drive a *Nail* into it.

'Tis true indeed, that the *barking* and *peeling* the *Tree* standing is somewhat more troublesome, and therefore somewhat more chargeable, than when they are prostrate; and that 'tis likely, People therefore have usually *fell'd* their *Timber*, as well for *Shipping* as other Uses, in the Spring of the Year, for the sake of the more easy and cheap *Barking* it only, rather than any thing else. 'Tis too true, that *Timber* is harder to *Fell* in the *Winter*, it being now so compact

pack and firm, that the Ax will not make so great Impression as it doth in the *Spring*, which will also increase the Price of the *Felling* some small Matter, and its *Sawing* afterwards; but how inconsiderable these things are in comparison of the great Good of this manner of *Felling*, I think is self-evident.

The greatest Objection, that I can foresee will be urged here in the *South* against this Practice, is, that if the *Timber* be not fell'd till *Mid-winter*, or *January*, where it grows in *Copses* and *Woods*, they cannot perhaps inclose their young *Sprigs* so soon as some may imagine needful, and therefore will be backward to fell their *Timber* (so growing) at that Season. To which I answer, That the *Timber* so fell'd in *Woods* or *Copses* may be easily carried off before the second *Spring*, and so the Prejudice small, and the first it must be there, wherever it is fell'd. But Secondly, That which will quite remove this inconsiderable Difficulty is, that perhaps it may be expedient that no *Timber* whatsoever growing in *Woods* or *Copses*, be at all bought in the *King's Yards*, for that *Timber* growing in such shady Places, and so fenced from the *Sun* and *Wind*, as *Timber* in *Woods* for the most part is, cannot be so good as that which comes from an exposed Situation, such as it usually has in *Forests*, *Parks*, *Hedge-rows*, and open *Fields*; where too it is indifferent at least, if not better, for the Proprietor, that it be fell'd in *Winter*, (when the *Grass* and *Corn* is gone) than in the *Spring* it self: and the Officers designed for that purpose may buy all their *Timber* under such Conditions as to be fell'd in *Winter*, enjoining the Proprietor to take off the *Bark* in the *Spring* in due Time, making him some small Allowance for the Trouble he will have in peeling it standing.

*The Difference of
Timber in different
Countries,
and Fell'd at
different Seasons;
by M. Ant. Van
Leuwenhoek. n.
213. p. 224. Sep.
An. 1694.*

IV. It is the common Opinion, That *Timber* which is fell'd in *Winter*, is stronger and more lasting, as being more close and firm than that which is fell'd in *Summer*: But, M. *Leuwenhoek's* Sentiment is, That there is no Difference, except in the *Bark*, and outermost Ring of the Wood, which in the *Summer* are softer, and so more easily pierced by the *Worm*; *Wood* consisting of hollow Pipes, which in the *Summer* and *Winter* both, are full of Moisture, they do not shrink in the *Winter*, and therefore the *Wood* cannot be closer at one time than another, for otherwise it would be full of Cracks and Clefs. The sudden and unexpected rotting of some *Timber*, he conceives to proceed from some inward Decay in the Tree before it was fell'd; having observed all Trees to begin to decay at first in the Midst, or Heart of the Tree, tho' possibly the Tree may stand and grow for near an Hundred Years afterwards, and increase in Bigness all along.

2. He says, he was once of Opinion, that Trees growing in good Ground, but increasing slowly, were the best and strongest *Timber*; and that those Trees, which in few Years grew large, were the softest and brittlest; the contrary to which, upon Enquiry of experienced Workmen, he found to be true, and instances of an *Elm* of 80 Years Growth, which was 11 Foot in Circumference, and proved excellent tough *Timber*.

3. The *Age of Trees* is to be known by the Number of Rings to be seen when the *Tree* is cut athwart, in each of which Rings is one Circle of large open *Pipes*; now the fewer of these large *Pipes* the stronger the *Timber* is: wherefore by Consequence these *Trees* that make the largest Growth in a Year, must be the closer and stronger, and therefore those *Trees* that grow in warm Countries grow fastest, and are the best and toughest *Timber*; which he confirms by *Riga* and *Danzick* Oak, which is of slow Growth, and proves spongy and brittle *Timber*, whereas the contrary is observable in *English* and *French* Oak, which grows faster, and is excellent *Timber*.

V. 1. This Famous *Roman Bridge* at *Pont St. Esprit*, is very crooked, bowing in many Places, and making several unequal Angles, especially in those Places where the Torrent runs strongest, as where the *Turret* stands. 4. In which Place the Angle is most unequal, and the greatest; the *Arches* are very wide, and have their Feet secured by two *Pedestals* that encompass them. Both these *Pedestals* have their several Degrees or Ranks of *Jettings* out, like so many Rows of Stairs or Steps, the lowermost Order pushing out most, the others being less, and going gradually more in; the second or uppermost *Pedestal* is much less than the first or lowermost, being built a little within its Lines of Circumference; 1, 2. Between the great *Arches* there are *Windows*, or, (as it were) small *Arches*; 3. that come down to the very Plane of the second, or uppermost *Pedestal* dividing the Feet of the great *Arches*. From this my rude Description it appears to me, that the *Romans* have here contrived all possible ways to break gradually the mighty Force of the *Rosne*, and to render its Passage easy, and inoffensive to the Feet of the great *Arches*; for here we see so many several *Palisadoes* and *Sluices*, as may be sufficient to defend this wonderful *Fabrick* against all Storms of the *Torrent*; the several Ranks of *Stairs* jetting from the *Pedestals* (for the most part Triangularly built, and faced well with *Free-stone*) opposing and breaking the Stream severally, I mean, not altogether, or at the same time, by reason of their various Inequalities in standing out: In case the Flood should swell so high (as it frequently does) as to cover both the *Pedestals*, then the small *Arches*, dividing the Feet of the great ones, help to convey the Water through, which otherwise might endanger the great *Arches*.

2. That which seems the Foot of the *Arch* is an *Horizontal Arch* gradually contracted, every Stone being of vast Length and Wedge-like, laid level with the Water. This I speak by Memory.

3. The stately *Modern Bridge* at *Avignon* hath yielded in many Places to the extream Rapidity and Violence of the *Rhosne*. Its Fall in my Opinion may be ascribed to three Defects. First, It was not so multangular, as that at *St. Esprit*: Secondly, It wanted in three or four Places, the little *Arches* dividing the Feet of the great ones, and in those Parts it hath suffered most; for where those useful *Sluices* are, there I observed the Bridge to stand still the most intire. Thirdly, The *Pedestals* (or as you very properly call them *Horizontal Arches*) were not so Geometrically and exactly laid, as those of *Pont St. Esprit*, there *Jettings* out were few, and they not gradually contracted; so that the Force of the Stream must be greater upon the *Fabrick*.

Vol. I. G g g g

Though

The Bridge at
St. Esprit in
France; by Dr.
Tankred Ro-
binson, n. 160.
p. 584.
Fig. 215.

By Dr. Lister.
ib. p. 585.

Compared with
some
other Bridges;
by Dr. Tank.
Robinson.
n. 163. p. 712.

Though the *Tiber* be not so swift as the *Rhofne*, yet it is subject to greater Inundations, as many Inscriptions assure us. No River ever had so many Bridges built with that Magnificence and Art, as this; and tho' they were more pompous and rich in rare Stones, in Sculpture, &c. than that I formerly sent you a Draught of from *Montpelier*; yet they had the like Provision for their Security and Preservation, and their Design was much the same; which may be seen at *Rome* this very Day at the old *Pons Milvius* (now *Ponte Nolle*) near the *Via Flaminia*, in the Marble Remains of the *Pons Æmilius*, repaired with rich Materials by *Antoninus Pius* on the Side of the *Ripa*, or *Trastevere*, near the Root of the *Aventine Hill*, where first the *Pons Sublicius* stood; as also in the *Pons Fabritius* and the *Cestius*, that leads over to the *Insula Tiberina*; in all which there are still very fair Marks of the old Roman Structure and Design; and if that prodigious City had not been knocked so oft to pieces by *Barbarous Sackers*, we might have had still as clear Proofs from the other Bridges. *viz.* the *Pons Triumphalis*, the *Senacotius*, &c. But *Goths* and *Northern Torrents* broke all before them.

A Bridge without any Pillar under it; from the Journal of the Phil. Society of Oxford. ib. p. 714. Fig. 216.

VI. A *Timber Bridge* may be built 70 Foot long, or somewhat more, without any *Pillar* under it, which may be useful in some Places where *Pillars* cannot be conveniently built, after this manner; AC, and BO are Beams 28 Foot long, and AB is 32 Foot long. Under the Angles are set two large Braces EL, and SR. At each End is a Wall, on which are laid two Beams BH, and AD, each 20 Foot long; under these two are two Braces DE, and RH. There may also be Braces at the End of the Arches, that may lie obliquely cross the Bridge. It may be laid with Planks and Railed. Behind the Walls are *Caisses* FD, and AN. The Length of the Bridge CMO, is 70 Foot; the Height KM is 19 Foot.

An Aqueduct near Versailles. n. 171. p. 1016. May. AD. 1615.

VII. 1. The *Aqueduct* which is to be made near *Maintenon*, for the carrying the River *Bure* to *Versailles*, will have in Length 7000 Fathom; 462 whereof will be 35 Fathom and 4 Foot high, the rest will be lower, according to the Difference of the Ground, but no less than 5 Foot and 6 Inches high. There will be to the said *Aqueduct* 861 *Arches*, which, where they are highest, will have 12 Fathom in Breadth, and 8 Fathom in Thickness, diminishing to 14 Foot at the Top. The other *Arches* will be lesser in Breadth, as well as Thickness, according to the Nature of the Ground. The said *Aqueduct* will have 15 Inches Fall to every Thousand Fathom in Length; so that for the 7000 Fathom, there will be 8 Foot 8 Inches Fall. The River is to pass by *Maintenon*, *le Parc Espernon*, *Gajeran*, *Rambouillet*, *les Essarts*, *le Perrey*, *Cognieres*, and from thence to *Versailles*. There are 14000 Soldiers that work there, under the Command of the *Marquess d'Uxelles*, with three *Commissaries of War* for their Conduct.

n. 176. p. 1206.

2. A *Magazine* for the Waters upon the Mountain *Monthoron*, is already cut, which will have 2200 *Perches* of Surface (each *Perch* being 8 French Foot) and 12 Foot in Depth. In another Place, much lower, will be another *Magazine*, to receive the Waters of many *Pools*, the most part of which,

as yet have no Water in them. In the Valler of *Buc* will be an *Aqueduct*, the Middle whereof will be raised 22 Fathom high, for conveying the Pools of *Sarle*, which it's said contain much Water, though there be nothing but Rain to fill them; this *Aqueduct* is 300 Fathom long, and passes thro' two Mountains which have been cut through upon that Account. The Valley also on both sides of the *Aqueduct* is raised 11 Fathom high, to make Passages.

An *Aqueduct* is also making near the Tower of *Stone*, (where the Mills raise the Water) which will now pass without Force to the Top of the Mountain; and there be part of it distributed into several great *Cisterns*, which are making about *Marli*, for that Place.

The Elevation of the *Aqueduct* of *Maintenon* is now set forth at but 2560 Fathom; whereas it was designed to be carried on more than 8000 Fathom, and the Remainder will be made of Earth, which must be brought thither: This Opinion prevails, in regard it gives a quicker Dispatch, tho' it may be doubted, it will not be for the Better.

These 2560 Fathoms contain 242 *Arcades*, whose *Aperture* is 6 Fathom and $\frac{1}{2}$, and the *Face* of each *Pillar* sustaining the *Arches*, 4 Fathom; there will be then on the side of *Maintenon* 33 *Single Arches*, afterwards 71 *Double* ones; (as having one *Arch* upon another) then 46 *Treble* ones; which will generally be 216 Foot 6 Inches high, (*viz.* up to the Floor of the *Channel*) afterwards 72 *Double* ones; then 20 *Single*, which will reach to the Mount of *Earth*, that is to be 50 Foot high.

From the Ground up to the *Second Arcade* are 16 Fathom, from the *Second* to the *Third*, or upper *Arcade*, are 14 Fathom, (which *Arcades* are *Double* in Number to those they stand upon) and 6 Fathom 6 Inches more, to the Floor of the *Channel*, which will at least be 7 Foot high, besides the *Parapet*.

The *Pillars* by the Ground are 8 Fathom thick, but, what with the *Slopes* and *Shortnings*, which are made in every Story, the Top, where the *Channel* goes, will be but 20 Foot broad. There will likewise be at each *Pillar* a *Buttress* jetting out one Fathom, and two Fathom wide.

The intelligent Observer, tho' well skill'd in Things of this Nature, as being no Stranger to the *Writings* of the Ancients, or the famous Ruins and Remains of their *Fabricks* in *Italy*, and other Places, professes himself surpriz'd with the Greatness of this Undertaking at *Versailles*, and *Maintenon*; for the Magnificence of the Design, the Number of Labourers, the Excessiveness of the Expence, and the admirable Industry of the Work.

VIII. Having been lately at *Edgecot* in *Northamptonshire*, at the House of *Tobias Clancey Esq.* he shewed me in an Ancient Kitchen (now disused) two *Chimneys*, vastly large, of *Stone-Work*: Which I took the more Notice of, because of a peculiar Way of *Arch-Work* in the Front of them; whereby, without the Advantage of a Discharge of *Timber* (which is usual, in such Cases, to defend the *Arch-Work* from being overburden'd,) an *Arch* of massy *Stone*, (in each of them) sustains itself at a great Length, tho' almost upon

A very large
Stone Chimney,
with a peculiar
sort of Arch-
work; by Dr.
J. Wallis, p.
166. p. 800.
Fig. 217.

a Flat, being very little rais'd in the Middle. Over this Arch (after some walling interposed) there is another Arch (to defend the former) more raised from the Flat. The Dimensions of all, I have thought fit here to subjoin.

A B. The Breadth between the *Jambs*, from Inside to Inside, 18 Foot.

C D. The Depth of the *Stones* in the Lower Arch, 22 Inches ; locked one into another, with a crooked Joint.

D E. The Distance in Walling, between the Arches, 2 Foot and 7 Inches.

E F. The Depth of the *Stones* for the Upper Arch, 15 Inches : With a streight Joynt.

G H. The Place of two vast *Tunnels* of Stone.

K. A *Window* between them.

*A new Kind of
Stairs ; by M.
Weighelius.*
74. P. 2212.

IX. M. *Weighelius* hath lately invented an odd *Bridge*, or kind of *Stairs*, by which a Man shall descend, and yet really be raised upward ; and going as 'tere upon a Plain, shall from a lower, by gently subsiding, arrive to an upper Story.

*Preserving of
Ships from be-
ing Wormeaten ;
by*
p. 190. Apr.
An. 1666.

X. In the *Indian Seas*, there is a kind of small *Worms* that fasten themselves to the *Timber* of the Ships, and so pierce them, that they take Water every where ; or if they do not altogether pierce them through, they so weaken the Wood, that it is almost impossible to repair them. Some have imployed *Deal*, *Hair* and *Lime*, &c. and therewith Lined their Ships ; but besides that, this does not altogether affright the *Worms*, it retards much the Ship's Course. The *Portugals* scorch their Ships, in so much that in the *Quick Works* there is made a Coaly Crust of about an Inch thick. But as this is dangerous, it happening not seldom that the whole Vessel is burnt ; so the Reason why the *Worms* eat not through *Portugal* Ships, is conceived to be the exceeding Hardness of the *Timber* employed by them. There is *Holland* a in Man that pretends to have found an admirable Secret to remedy this Evil. And a very worthy Person in *London*, suggests the *Pitch*, drawn out of *Sea-Coms*, for a good Remedy to scare away those noysome *Insects*.

*An Account of
Lead Sheathing ;
by Mr. J. Bulteel.*
p. 190. p. 6192.
Jan. An. 1674.

XI. Some few Years since, Sir *Phil. Howard* and Major *Watson*, with great Charge and Industry found out a new Way, by a Manufacture of our own, to preserve the Hulls of Ships from the *Worms*, &c. which is much smother and consequently better for Sailing, and more cheap and durable than the Way of *Boards*, *Pitch*, *Tar*, *Rosin*, *Brimstone*, or any *Sheathing* or *Graving* hitherto used. The King and *Parliament* being satisfied, upon Examination, of the great Benefit that might redound thereby to his Majesty and Subjects in general, for the Inventors Incouragement to make the same publick, were pleased, almost 4 Years since, to grant them an *Act* of *Parliament* for the sole Use of this their Invention, with Penalty and Prohibition to all others. In Prosecution whereof, Experiments have been made upon several of his Majesty's

Majesty's Ships, viz. the Phoenix, done three Years ago, has made two Voyages into the *Streights*, &c. and when she was lately taken into the *Dock at Woolwich* to be repaired, upon View of the *Master Shipwright* and others, her *Sheathing* was found to be in as good Condition, as at the first doing; and the Ship so tight during the whole time, that they were forced to in Water to keep her sweet. The *Dreadnought*, a Third Rate, done ~~done~~ *done*, 1671; the *Henrietta*, *Lyon*, and *Mary*, all three of the Third Rate, and done a Year and half since, being lately laid on Ground at *Sheerness* and *Portsmouth*, are found to be all in as good Condition, and the *Sheathing* to continue as firm and as well as at the first doing; as the *Master-Builder* and *Assisant* at *Portsmouth*, and others, have certified.

The *Bread Rooms* also of some of these, and many others of his *Majesty's* Ships, have been Lined within, almost in the same manner the *Sheathing* is without; which has prov'd a great Preservation of the Bread, as several of the *Pursers* and *Officers* of the said Ships have Certified; and by Reason of its Duration must be much cheaper and better than *Tin*, which is so liable to rust, or any Way yet used.

Also the Lead itself (which is the Principal thing used herein) they make so Close pressed, Smooth, and Equal, or of what Thickness or Thinness desired, that great Use may be made thereof about several other things relating to Shipping.

XII. A Paper of less General Use Omitted, viz.

Directions for Inquiries concerning Stones and other Materials for the Use n. 93. p. 6010. of Building.

XIII. Accounts of Books omitted.

1. *Vitruvius* done into English; by Mr. Chr. Wase. n. 72. p. 2190.
n. 112. p. 279.
Les dix Livres d'*Architecture* de *Vitruve*, corrigez, & traduits nouvellement en Francois, avec des Notes & des Figures; par Claude Perrault. Paris 1673. in Folio.
2. *Cours d'Architecture*, enseigne dans l'*Academy Royale d'Architecture*, premiere Partie; par M. Francois Blondel. a Paris, 1675, in Fol. n. 112. p. 549.
3. *Raphaelis Fabretti Urbinatis* de Aquis & Aquæductibus Veteris Romæ, Dissertationes tres. Roma 1680. in 4to. n. 155. p. 456.
4. *Modern Fortification*, &c. by Sir Jonas Moor. 1673. in 8vo. n. 95. p. 6071.
5. *Novelle Maniere de Fortifier les Places*; par M. Blondel. Hague, 1684. n. 158. p. 586.
6. *Marci Meibomii* de Fabrica *Trivemium* Liber. Amsterlodami, 1671. in 4to. n. 79. p. 3072.
7. *Scheeps-Boew en Bestier*, that is, *Naval Architecture and Conduct*; by N. Witsen. Amsterdam. 1671. in Folio. n. 77. p. 3006.
8. *L'Architecture Navale*, avec le Routier des Indes Orientales & Occidentales; par le Sieur Daffiè. à Paris. 1677. in 4to. n. 135. p. 879.

C H A P. IX.

Perspective. Sculpture. Painting.

*A Perspective
Instrument; by
Sir Chr. Wren.
n. 45. p. 893.
Fig. 218.
Mar. An. 1669.*

I. **A** Is a small Sight with a short Arm B, which may be turned round about, and moved up and down the small Cylinder C D, which is screwed into the Piece ED, at D : This Piece ED moving round about the Center E ; by which means the Sight may be removed either towards R, or F.

EF is a Ruler fastened on the two Rulers GG, which Rulers serve both to keep the square Frame SSSS perpendicular, and, by their sliding through the square Holes T T, they serve to stay the Sight, either farther from, or nearer to the said Frame; on which Frame is stuck on with a little Wax the Paper OOOO, whereon the Picture is to be drawn by the Pen I. This Pen I is by a small Brass Handle V so fix'd to the Ruler HH, that the Point I may be kept very firm, so as always to touch the Paper. H H is a Ruler, that is always, by means of the small Strings a a a, b b b, moved Horizontally, or Parallel to it self; at the End of which is stuck a small Pin, whose Head P is the Sight, which is to be moved up and down on the out Lines of any Object.

The Contrivance of the Strings is this. The two Strings a a a, b b b, are exactly of an equal Length. Two Ends of them are fasten'd into a small Leaden Weight Q Q, which is moved in a Socket on the back side of the Frame, and serves exactly to counterpoise the Ruler H H, being of equal Weight with it. The other two Ends of them are fasten'd to two small Pins H H, after they have rolled about the small Pulleys N, M, M, L, L, K K; by means of which Pulleys, if the Pen I be taken hold of, and moved up and down the Paper, the Strings moving very easily, the Ruler will always remain in an Horizontal Position.

The Manner of using it is this : Set the Instrument upon a Table, and fix the Sight A, at what Height above the Table, and at what Distance from the Frame SSSS, you please. Then, looking through the Sight A, and holding the Pen I in your Hand, move the Head of the Pin P up and down the out Lines of the Object, and the Point I will describe on the Paper O O O O, the Shape of the Object so traced.

*A new way of
Delineating by
Parallel Visual
Rays, exactly*

II. *Parallelogrammum Presopographicum est ABCD, Stylus Centralis HF, Calamus Designator LC, Index KA, five Regula oblonga, Plano Parallelogrammi, ope Clavi Striati ex ære, secundum Specimen E, ad Angulos rectos aptata. Huic Regulæ insiguntur duæ Dioptræ P. R, S V.; in medio P R, pertusum*

perforatum est foramen O, in medio SV, erigitur Filum perpendiculare Regula R A, in cujus medio est Globus quidam parvus, per quem, & Foramen O, Radius ad oculum (quem inter delineandum non oportet esse fixum, sed Libertum & Solutum) ab Objecto protenditur.

Observing the
Symmetry; by
Mr. St. Clare,
n. 56. p. 6080.
Fig. 219.
July. An. 1673.

1. Animadvertendum, Radium per Foramen O, & Globulum, protensum, semper fore perpendicularem Plano *Parallelogrammi*, sive ejus Diametro, quæ ex Recta Linea extensa per Stylum Designatorium L C, & Centrum fixum HF, & dictum Globulum parvum, in qua Linea semper versatur iste Globulus, qualiscunque sit *Parallelogrammi* motus.

2. Not. Planum Delineatorium sensibile super quod volutatur apex L, Styli Pictorii LC, ad amissim describentis Imaginem ad motum Indicis K A, & in quod infixus est Stylus Centralis HF, esse QYX'T; Planum vero merè Rationale, sive *Mathematicum*, priori continuum, esse $\alpha \beta \gamma$.

3. Not. Omnes Radios ab Objecto per Globulum & Foramen O, protensos ad Oculum, (in tot medii Diaphani punctis, duce Indice K A, collocatum, quot sunt puncta in superficie visibili Objecti describendi quæ sunt infinita) semper fore sibi invicem *Parallelos*.

Objicient forsan quidam, in Objectis longe diffitis Dioptrarum nullum fore usum. Quid vero illud nostrâ interest, cum ad tollendas tantum in *Protopographia* difficultates, quibus hætenus Scheineri *Parallelogrammum* laboravit, hæc nostra Methodus comparata est? Sæpius enim expertus sum (licet ab hoc non est quod sequius de illius *Instrumenti* præstantia ab Artifice statuatur) nequaquam inter Partes Eætypi in Plano eam esse *Symmetriam*, quæ inter Partes Prototypi diffiti.

III. I here send you a Method of Casting Statues in Metal, in Obedience to the Commands of the Royal Society; It is as follows. First, I form out of good Clay, that will endure the Fire, and not crack either in Drying or Burning, such a Figure or Statue as I desire to cast; when this is well dry, I make, all over the Figure, little Holes of no great Depth (but both Size and Depth proportionate to the Bigness of the Statue) into which I let small Pieces of Metal, and with some of the same Clay fix them firmly in the Holes; the Use of these Bits of Metal, $a a a a$, is to keep the Core and Mould from touching one another, or falling together when the Wax runs out; and that they may remain constantly in the same fixt Posture. This done, I scrape away with some proper Instrument, as much of the Clay in Thickness as I design for the Thickness of my Statue; and then laying it in a Furnace, I burn the Core till it be red hot: (by the Core is meant always the Statue first made in Clay.) When it is cold, I rub the Core all over with that sort of Earth or Colour, which our German Potters use to colour the Joints of the Tiles, when they set Stoves of Tiles or (*Kachel-Ofens*;) this Colour much resembles Black Lead which is used to Design on Paper, and easily wipes out with Bread, but it is not the same. This Colour I mix with Water, and daub all over the Core, because the Metal is found to run freely upon it. There are other Substances proper for this Use, but I have always made use of this, especially for thin Statues. This done, I lay upon the Core as much yellow Wax mixed

A Method of casting Statues of an extraordinary Thinness; by M. John Weirichard Valvasor, n. 186. p. 259. Fig. 220. Jan. An. 1687.

ed with Pitch or Rosin, as will make the Thickness of the intended Statue, which I form in the Wax, with all the Exactness possible.

Here note, that the Particles of *Metal* mentioned to be set into the *Core*, to keep it a Distance from the *Mould*, must be so set as to fall in with the Surface of the Wax exactly ; and that the Reason of mixing Pitch or Rosin with the Wax is, because that when it is burnt out, it makes a great Smoke, and that Smoke adhering to the *Mould*, occasions the *Metal* to run more freely ; as I have experienced it. Next I put all over upon the Surface of this *Statue* of Wax, little Pieces of Wax which I call the little *Channels c c c c c*, (all which must be contrived so as to enter the great *Channels d d d*.) This done, I cover the *Core* and Wax all over with the same sort of Clay, that will endure the Fire without cracking ; and so I have my *Concave Statue* or *Mould* made. Upon this I lay the great *Channels* marked *d d d d*, both upright and transverse, formed likewise in Wax, and placed according to Judgment, so as best to receive the Ends of the little *Channels c c c c c*, for the more easy distribution of the *Metal*. The great *Channels* must all meet at the top of the *Statue*, so as come out by one Hole, as at *E*, where the *Metal* is to be poured in : It is also necessary to have a *Channel* or two to let out the Air as the *Metal* enters, as those marked *ff*, and there must be a Hole or two left at the Foot, as *g g*, where the great *Channels* and *waxen Statue* joyn ; and whereat when the *Mould* is burnt, the Wax as well of the *Statue* as of the *Channels* may run out. The great *Channels* being thus placed, the *Mould* must be again laid over with the same sort of Clay ; (I use constantly to bind about the *Mould* with Iron Wire, and then lay on more Clay) and when this *Mould* is well dry, then I heat it red-hot ; as I did before the *Core*, so now both together.

I burn the *Core* first, that there may not need so strong a Fire to burn the *Mould* as will melt the small Bits of *Metal* : But for small manageable *Statues*, of not above a Foot or two high, they may be both burnt together, and there is no need of the Holes *g g*, but the *Mould* may be inverted, and the Wax run out by the *Channels ff*, and *E*.

The *Mould* being thus Burnt, I stop with the same Clay the two Holes *g g*, and then I bury it in a Pit, and proceed as is usual in *Casting of Bells*, and the like ; but Care must be taken, that the *Metal* be very well in Fusion.

If it be a small *Statue*, not above a Foot or two high, whose *Mould* may be managed in ones Hands ; then I make me a *Concave Statue* of Wax, of the Thickness I desire, and then place upon it all those great and lesser *Channels*, as afore : Which done, I put it all together, into a liquid Substance made of Plaister and Tile, or Brick-Dust tempered with Water.

If the *Statue* be intended very thin, then I take Copper, and when it is well in Fusion I mix with it a good quantity of Zinc, without observing any certain Proportion of Weight ; the more Zinc the better the *Metal* Runs. I have sometimes for small and thin *Statues* put in above a third part of Zinc. I have found by Experience, that this Mineral makes the *Metal* run most freely, and gives it a fair Golden Colour.

The

The Statue being cast, I take off the *Mould*, and cut off all the little *Channels*; all which, both great and small, are filled with *Metal*, which may be kept for farther Use: In these there is much more *Metal* than in the whole Statue; for, if the Statue be very thin, there must be more and bigger *Channels*, and so, the cheaper the Statue, the more weighty the *Channels*, and the more *Metal* remaining.

To know the Quantity of *Metal* requisite for any intended Work, I take a Lump of the same Mixture of Wax and Pitch, with which I make the *Mould* of my Statue; and, having weighed it, I make a *Mould* upon it, and cast in the same a Lump of *Metal* of the same Size; which I weigh, and thereby compute the Proportion of the Weight of the *Metal* and *Wax*; and then, observing how many Pounds of Wax I use about the *Figure* and *Channels*, I can calculate to a small matter how much *Metal* I need to melt.

Hitherto I have cast no Statue above 9 Foot high, but I doubt not but I could, by the same Methods, cast one of any Bigness desired.

IV. 1. *Spanish White* is made of Chalk and Allum burnt together.

2. I take the *Lapis Armenius* to be the *Blue Bice* sold in the Shops, for it is light and friable; formerly brought out of *Armenia*, now from the Silver Mines of *Germany*, called *Melochites*, in *High Dutch*, *Berghblaw*.

3. *Ultramarine* is made of the bluest *Lapis Lazuli*, which is free from Gold Veins, by Calcination.

4. *Smalt* is made of *Zaffer* and *Pot-Asbes* calcined together in a Glass Furnace.

5. *Litmase* or *Litmose*, I suppose the Juice of a Plant.

6. *Indigo*, said by *Pliny* to be brought from *India*; a kind of *Mud* adhering to the *Froth* about *Reeds*; and that when tried with a Coal, the true burns with a purple *Flame*, and smells of the *Sea*. *Linshoten* says, it is called *Anil*, that it grows in *Cambai*, and is a Plant like *Rosemary*, which is gathered and dried, then wetted with fair Water, and beaten to a Mud.

7. *Indian Ink*, its use is known to *Pliny*, tho' not its Composition; which is yet undiscovered, except it should be burnt *Rice*, as hath been thought.

1. *Ceruse* is the *Rust* of *Lead*, made by a vaporous Calcination; *Pliny* writes thus of it in *Cap. 34. Lib. 18. Ceruse Psmithium* is made in the Plumbers Shops, of small Plates of *Lead* laid upon a Vessel of strong *Vinegar*, what falls into the *Vinegar* is taken out and dried in the *Sun*: and in *Cap. 6. Lib. 35*. He says it was made at *Rome* of burnt *Marble Flint* quenched in *Vinegar*.

2. *Masticot* is a kind of improper *Calx* or *Tin*.

3. *Gutta Camba*, or *Cambodia*, the inspissated Juice of a Plant, not well known; it comes from both the *Indies*. Some think it the Juice of *Euphorbium*; others *Scammony* or *Tithymal*; others *Ricinus*, others refer it to the greater *Cataputia*, *Esula*, or the Flowers of the *Indian Ricinus*, and will have it coloured with *Turmerick*; as *Schroder*.

4. *Oker*, a kind of natural Earth. There are two sorts thereof, the one *Native*, formerly brought out of *Africa*, now from *Dacia* and *Hungary*, and from many Places in *England*, especially in the Forest of *Dean*: The other a factitious Substance of *Lead* burnt and quenched in *Vinegar*. In *Pliny's* Time it was made of *Rubrica*, or *Reddle* burnt.

A Description of
some Simple Colours;
by Mr. Rich. Waller.
n. 179. p. 26, 30.
Jan. An. 1686.
Simple Blues.

Simple Yellows
and Reds.

5. *Orpiment*, a flat inflammable Mineral, justly ranked amongst Poysons for its extream corrosive Quality. *Pliny* says, it was dug up in *Syria* on the Surface of the *Earth*: and that the Emperor *Caligula* had hopes of getting *Gold* out of it; wherefore he caused 14 Pounds of it to be tried, which afforded him very good *Gold*, but in so small a Proportion, that he lost by the Tryal.

6. *Umber* is a Native Earth.

7. *Red-Lead*, a Colour unknown to the Ancients, made of *Litharge*, or burnt *Lead*, by a *Reverberatory Calcination*, or of *Ceruse* put in a Platter over the Fire, which must be continually stirred till it has acquired a *Red-Lead* Colour. Dr. *Charleton de Fof*.

8. *Burnt Oker* is the common *Yellow Oker* burnt in the open Fire.

9. *Cinnabar*, or *Vermilion*. There are two sorts, *Native*, or the *Minium* of the Ancients, which is the Mineral that yields *Quicksilver*; whereof and of *Sulphur* it chiefly consists; it is found in the Mines of *Istria*. This Colour was amongst the *Ancient Romans* used to sacred Purposes, and on Festivals *Jupiter's Face* was painted therewith, as likewise the Bodies of those that entred in *Triumph*. The *Facitious Cinnabar* is that which we now use, and is made by a Sublimation of *Mercury* and *Sulphur*.

10. *Carmin*, made of *Cochineel*.

11. *Lake*, thought to be an *Arabick Word*: It is made of *Flocks Dyed*, or Shavings of *Scarlet Cloth*, or of the *Cochineel Insect*, or else of *Kermes-Berries*, their Tincture being extracted with a Lye of *Pot-Ashes*, and then precipitated with a Solution of *Roch-Allom*. After the same Manner a *Lake* may be made of any Plant or Flower. There is another sort of *Lake* made of *Gum-lac*, by extracting its Tincture with *Urine*.

12. *Sanguis Draconis* is the Gum of a Tree, which looks like dry'd Blood; 'tis brought out of several Places in the *East-Indies*.

13. *English Reddle*, or *Ruddle*, is found in many Places of *England*; amongst the rest, near *Witney* in *Oxfordshire*.

14. *Lamp-black*, by *Pliny* thus described: 'Tis made of the Soot of *Rosin*, or *Pitch* burnt, Houses being built on purpose for it, that keep in the Smoak.

To make China Varnishes; by Dr. Will. Sherrard. n. 262. p. 525. Mar. An. 1700.

V. This Way of making several *China Varnishes* was first sent from the *Jesuits* in *China* to the Great Duke of *Tuscany*.

Take of *Crude Varnish* 60 Ounces, ordinary *Water* 60 Ounces, mix them well together till the *Water* disappears, afterwards put this Matter into a wooden Vessel 5 or 6 Palms long, and 2 or 3 broad, mix them with a wooden *Spatula*, for a whole Day in the *Summer's Sun*, and for two in the *Winter*. It is afterwards kept in Earthen Vessels with a Bladder over it, and cool. This is the *Varnish prepared in the Sun*.

Boyling the Oyl of Wood. To give the first Grounds called Camiscia.

Take 20 Ounces of the Oyl, called *Oyl of wood*, of that of the *Fruit* 10 Drams, give them 5 or 6 Boils, till it comes to be a little *Yellow*. Let it cool, and put to it 5 Drams of *Quick-lime* powdered.

Take *Swines Blood* and *Quick-lime* powder'd, mix them well, lay this Mixture on the *Wood*, and, when it is dry, smooth it with *Pumice-Stones*.

Take

Take of the *Varnish* prepared in the Sun, 60 Ounces, *Stone-black Allum*, To make Black Varnish. (supposed to be a sort of *Copperas*) dissolved in a little Water, 3 Drams; 70 Drams of *Lamp-Oyl*, call'd by the *Portuguese Azeite de Candea*. It is prepared in a wooden Vessel, as the prepared *Varnish*, observing to put in the *Lamp-Oyl* at twice.

Take of the *Oyl* of *Wood* crude, (called by the *Portuguese Azeite de Pao*) Pitch-colour'd Varnish. 40 Drams, of the *Lamp-Oyl*, called *de Candea*, Crude 40 Drams: It is prepared in the Sun in a wooden Vessel, as the prepared *Varnish*.

Take 10 Drams of *Cinnabar*, 20 Drams of *Varnish* prepared, a little *Oyl* Red Varnish. *de Candea*, or *Lamp-Oyl*; mix them well.

Take of the *Yellow Colour* 10 Drams, 30 Drams of the prepared *Varnish*, Yellow Varnish. with some *Lamp-Oyl*.

Take of the *Red Varnish* 10 Drams, of the *Black Varnish* 4 Drams; mix Musk-colour'd Varnish. them well.

VI. M. Colbert being pleas'd some time since to visit the *Academy Royal* for the Improvement of *Painting* and *Sculpture*, express'd himself to this Effect, That he thought it proper from time to time that the Works of the most Excellent *Painters* should be Examined, and such Observations made thereon as would inform others wherein the *Perfection* of a *Picture* consists. Which hath been ever since practis'd among them, as the best Means to carry the Art of *Painting* to its highest *Perfection*; such an *Examen* of the best *Pictures* disclosing many *Secrets* of that Art, for which there are no *Rules*, and opening a Door to debate many important *Questions* not hitherto treated of. An Examen of Pictures propos'd; by M. Colbert. n. 47. p. 953. May. An. 1669.

VII. Here is a Man who makes more lively *Counterfeits* of Nature in *Wax*, than ever I yet saw in *Painting*, having an extraordinary Address in *Modelling* the *Figures*, and mixing the *Colours* and *Shadows*; making the *Eyes* so lively, that they seem to kill, excelling all things of this Art I ever beheld. Wax Work, and a new kind of Maps in Low Relievo, in France; by . . . n. 6. p. 99. Nov. An. 1665.

I have also seen a new kind of *Maps* in *Low Relievo*, or *Sculpture*; for Example, the Isle of *Antibe*, upon a Square about 8 Foot, made of Boards, with a Frame like a *Picture*: There is represented the Sea, with Ships, and other Vessels artificially made, with their Cannons, and other Tackle of Wood fix'd upon the Surface, after a new and most admirable manner. The Rocks about the Island exactly form'd, as they are upon the natural Place; and the Island itself with all its Inequalities, and Hills and Dales; the Town, the Forts, the little Houses, Plat-form, and Cannons mounted; and even the Gardens, and Plat-forms of Trees, with their green Leaves standing upright, as if they were growing, in their natural Colours; in fine, Men, Beasts, and whatever you may imagine to have any Protuberancy above the Level of the Sea. This new delightful, and most instructive Form of a Map, or wooden Country, you are to look upon either Horizontally, or side-long, and it affords equally a very pleasant Object.

VIII. Whether the Way mentioned by Kircher in his *Mundus Subterraneus* will succeed or not, is much doubted by some experienced Men. But 'tis certain, To Colour Marble; by . . . n. 7. p. 125. tain, Dec. An. 1665.

tain that a *Stone-cutter* in *Oxford*, *Mr. Bird*, hath many Years since found out a Way of doing the same thing, in effect, that is there mentioned; and hath practised it for many Years. That is, he is able so to apply a *Colour* to the out-side of polished *Marble*, as that it shall sink a considerable *Depth* into the Body of the Stone, and there represent like Figures or Images as those are on the out-side; deeper or shallower, according as he continues the Application a longer or lesser while.

An Extraordi-
nary Tincture
given to a Stone;
by Dr. Salo-
mon Reifel, p.
179. p. 22.

IX. *Aurifaber Stutgardianus*, qui & Gemmis & Metallicis Typis Nummorum Cudendorum Insculpendis Artificiosus est, Nomine *Christophorus Muller*, An. 1685. Aurum Aqua Regis solutum, Oleo Tartari præcipitatum atque edulcoratum, quod *Aurum fulminans* dicunt, dum in Scutella, quam Maturellam vocant, ex *Lapide Chalcedonico* coloris unici Pellucidi *Onychini*, seu Cornei, vitro pro Fusione præparato Rubro mixtum & Aqua Fontana Imbutum tereret, ad facienda Encausta seu *Smalta*; de quibus *Ant. Neri*, vertente *Andrea Frisio*, egit *Lib. 6. Artis Vitruvianæ*, invenit iterato tertium eodem Labore, quod Color Pulveris istius Puniceus, qui per dies aliquot siccatus in Vasculo manserat, quousque inter terendum etiam ad *Marginem* effluxit, relictis tamen puris hinc inde Spatiis *Onychini* Coloris, durissimam hanc *Gemmam*, quæ Limam spernit, ita profunde penetraverit, non tantum in Scutella, sed & ipso Pistello, & distrinxerit Maculis atque Circulis sat ordinate ductis, ut Color hic neque simplici Aqua, neque Lixivia, vel acriori alio Liquore potuerit deleri, & quidem sine Polituræ elegantioris Detrimento. Talis itaque Tinctura per repetitas Trituras dicti Pulveris tentata denuo aliquoties, in similis Coloris alio Vasculo, neque vero apparuit postea ut ante unquam. Sed hoc imprimis circa Tinctionem hujus Vasculi observandum est, quod secundum Texturam *Gemma*, tam nudo quam armato Oculo, in Tincta Interna, & Sincera Externa parte Vasis, notentur Fibre seu ductus Circulares; juxta quos Bracteis Succu Lapidei Novi per Intervalla impositis, in ejusmodi Mole excrevisse credendum est; uti *Bezoar* aliique Lapides Laminis super accrescentibus augentur, & Ligna, in quorum ultimorum Trunco Circuli seu Annuli designant succi annui Numerum & Incrementa: adeo ut hic *Purpureus* ille Color Lineis pallidioribus & obscurioribus, prout vel densiores vel rariores Poras molliorem vel duriorem Texturam offendit, Circulares Ambitus circa Verticem aliquam, veluti circa Medullam, seu Cor ut appellant, aut Granum, aut Paleam in aliis Lapidibus & Lignis, signaverit; intermistis quoque hinc inde Maculis & Spatiis obscurioribus. Veluti Illust. *Boyle Specim. de Orig. & Virt. Gemmarum. Sect. 1. p. 22, 23.* in *Adamante* & *Granatis* acies & commissuras tenuium Bractearum aut Planorum observavit; quod Granum Artifices, seu planam Contexturam non dissimilem Fissilitati Ligni, vocant.

Jam vero Tingi posse quoque *Marmora* & *Alabastra* & *Ossa* per Lixiviatos & acres Succos, hinc inde scriptum est: quod fortassis de *Gemmis* sperandum est, quando *Rob. Boyle, Cit. Sect. 2. p. 123.* ex iis Tincturam manifestam extractam esse scribit, *alibi. p. 43. & 190.* per Vapores Minerales tinctos esse *Crystallus Petrosos*, atque *p. 45.* ipsum *Sapphirum* per Vapores subterraneos.

Cum

Cum denique ex Observatione nostra manifestum sit, revera *Tinctam* esse *Gemmam Chalcedoniam*, quamvis fortuito acciderit, neque repetito Processu simile quid evenierit, merebitur tamen Meditationem, an ex Astrorum Fluxu aliave abscondita potius vi venerit, & Tentamen, an ex Mixtura Salium & Succorum Acrium possit imitando produci ejusmodi Tinctura, & quidem sine Igne, ut Splendor & Pelluciditas Gemmæ non destruat, Durities autem maneat, adeoque ipsa Gemmæ pretiositas non tantum servetur; sed & per Tincturam novam crescat.

X. Papers omitted.

1. A Description of *Scheiner's Stereographick Parallelogram*, and its Imperfections considered; by Mr. J. St. Clare, vid. sup. Sect. II. n. 96. p. 608.
2. A Table of *Simple and Mix'd Colours*, in *Latin, Greek, French, and English*: with a Specimen of each Colour prefix'd to its proper Name; by Mr. Rich. Waller. n. 179. p. 24, 29.

XI. Accounts of Books omitted.

n. 21. p. 383.

1. *Entretiens sur les Vies & sur les Ouvrages des plus excellens Peintres, Anciens & Modernes*; par M. Felibien. n. 39. p. 784.
2. An Idea of the Perfection of Painting: Originally written in French, by Roland Freart, Sieur de Cambray; and render'd English by J. Evelyn, Esq; F.R.S. Lond. 1668. in 8vo. n. 47. p. 954.
3. A General Idea of the Art of Painting, and Relation of Seven Conferences held at Paris in the Academy Royal, for the Improvement of the Arts of Painting and Sculpture. n. 86. p. 5048.
4. *Optique de Portraiture & Peinture, contenant la Perspective, Speculative, & Pratique Accomplie*; &c. Par Gregoire Huret, de l'Academie Royale de Peinture & Sculpture. A Paris, 1677. in Fol.

CHAP. X.

Musick.

*Of the Trembling
of Consonant
Strings; by Dr.
Wallis. n. 134.
p. 839. Mar.
An. 1677.*

Fig. 221.

IT hath been long since observed, That if a *Viol-string*, or *Lute-string*, be touched with the Bow or Hand, another *String* on the same or another Instrument not far from it, (if an *Unison* to it, or an *Octave*, or the like) will at the same time tremble of its own Accord. But I can now add, That not the whole of that other String doth thus tremble, but the several Parts severally, according as they are *Unisons* to the whole, or the Parts of that String so struck. For Instance, Supposing A C to be an upper *Octave* to $\alpha \gamma$, and therefore an *Unison* to each Half of it, stopped at β . If, while $\alpha \gamma$ is open, A C be struck; the two Halves of this other, that is, $\alpha \beta$, and $\beta \gamma$, will both tremble; but not the middle Point at β . Which will easily be observed, if a little Bit of Paper be lightly wrapt about the String $\alpha \gamma$, and removed successively from one End of the String to the other.

Fig. 222.

In like manner, If A D be an Upper *Twelfth* to $\alpha \delta$, and consequently an *Unison* to its three Parts equally divided in β, γ ; if $\alpha \delta$, being open, A D, be struck, its three Parts $\alpha \beta, \beta \gamma, \gamma \delta$, will severally tremble, but not the Points $\beta \gamma$. In like manner, if A E, be a *double Octave* to $\alpha \epsilon$, the four

Fig. 223.

Fig. 224.

Quarters of this will tremble when that is struck, but not the Points β, γ, δ . So if A G, be a *Fifth* to $\alpha \eta$; and consequently each Half of that stopped in D, an *Unison* to each third Part of this stopped in $\beta \gamma$; while that is struck, each Part of this will tremble severally, but not the Points β, γ ; and while this is struck, each of that will tremble, but not the Point D. The like will hold in lesser *Concords*; but the less remarkably, as the Number of Divisions increases.

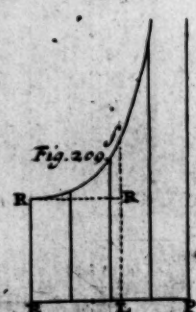
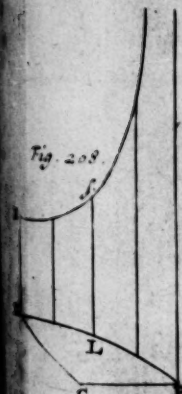
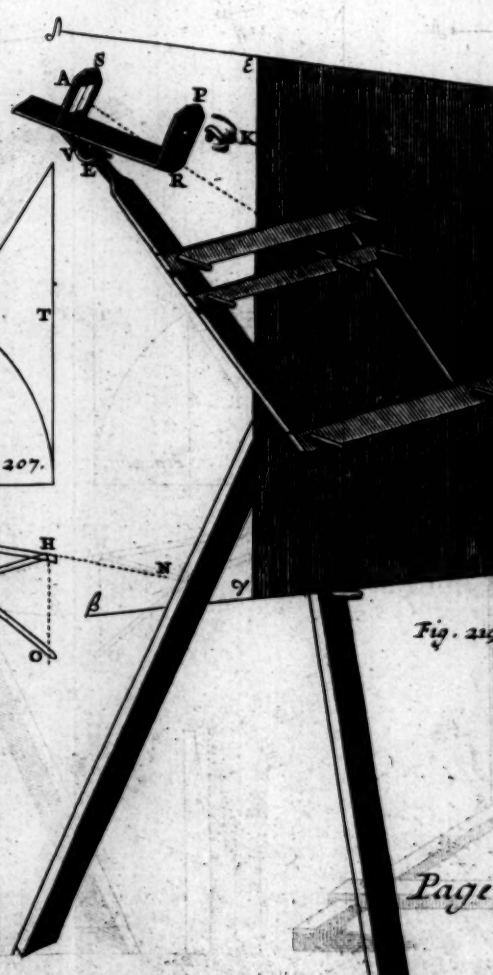
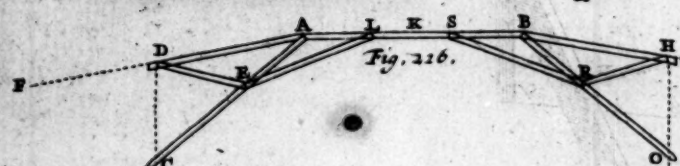
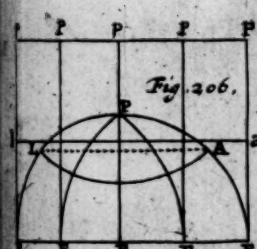
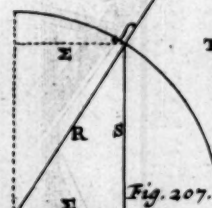
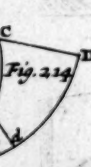
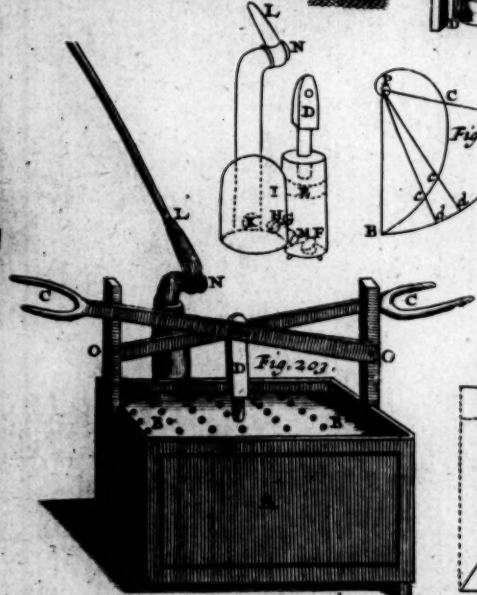
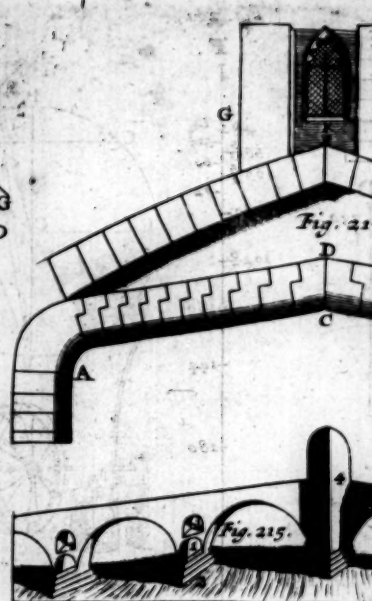
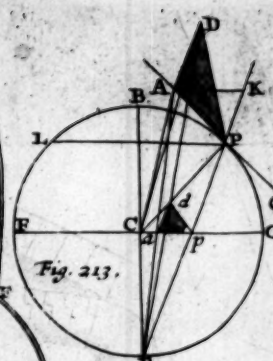
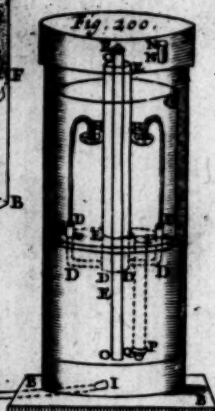
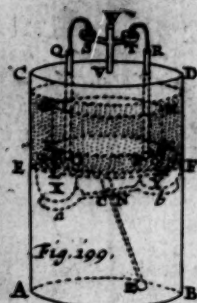
This was first of all, (as I know of) discovered by Mr. Will. Noble, M. A. of Merton College; and by him shewed to some of our Musicians, about three Years since; and after him by Mr. Tho. Pigot, A. B. of Wadham College, without knowing that Mr. Noble had discover'd it before. I add this further, (which I took Notice of upon Occasion of making Tryal of the other,) that the same String, as $\alpha \gamma$, being struck in the midst of ϵ , each Part being *Unison* to the other, will give no clear Sound at all, but very confused. And not only so (which others have observed, that a String doth not sound clear if struck in the midst;) but also, if $\alpha \delta$ be struck at β , or γ , where one Part is an *Octave* to the other; and in like manner, if $\alpha \epsilon$ be struck at β or δ ; the one Part being a *double Octave* to the other. And so if $\alpha \zeta$ be struck in γ or δ ; the one Part being a *Fifth* to the other; and so in other like *Consonant* Divisions; but still the less remarkable, as the Number of

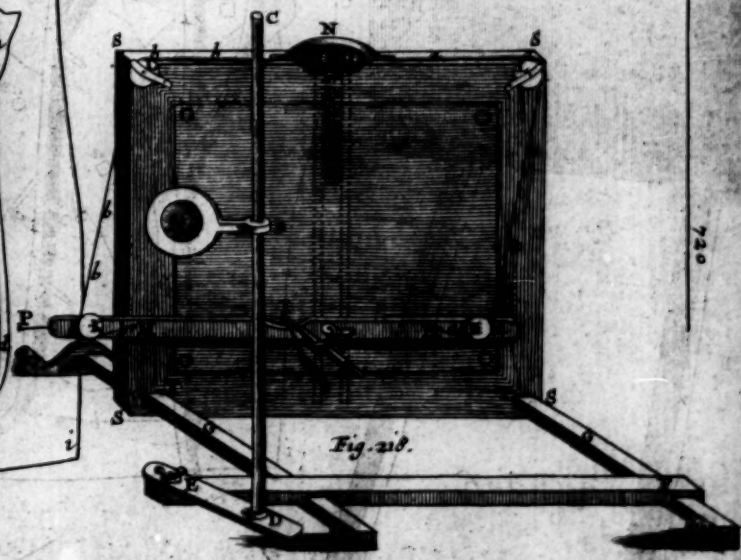
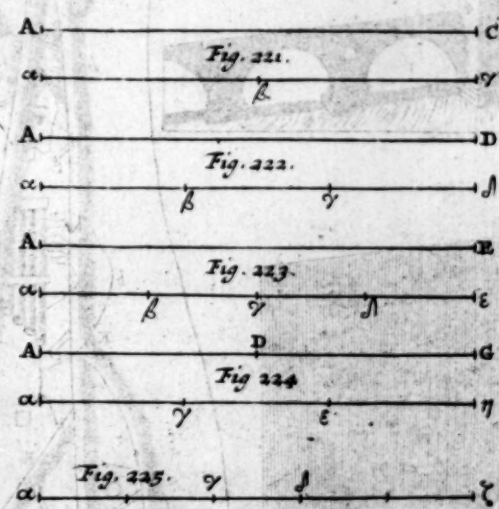
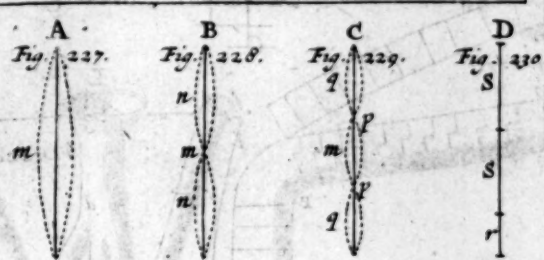
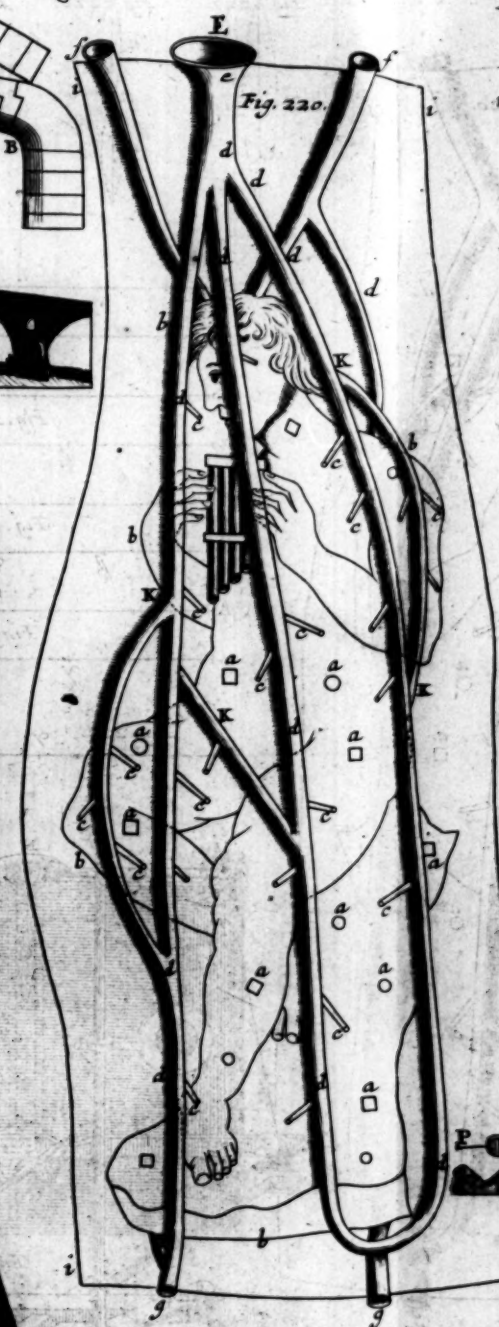
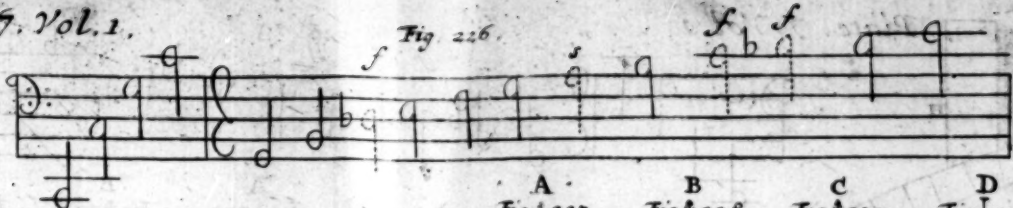
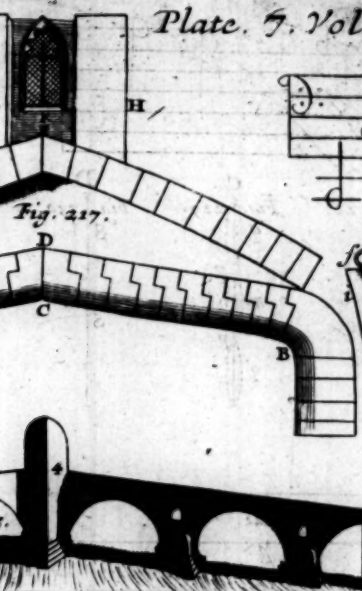
Fig. 225.

Divi-

135. p. 879.

*The Defects of
Trumpet, and
Trumpet-Ma-
chine, by Mr.
John Roberts.
1895. p. 559
Fig. 226.
Oct. An. 1692.*





Of the Trembling
of Consonant
Strings; by Dr.
Wallis. n. 134.
p. 839. Mar.
An. 1677.

Fig. 221.

Fig. 222.

Fig. 223.

Fig. 224.

Fig. 225.

t
a
f
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Divisions encreaseth. This and the former I judge to depend upon one and the same Cause, viz. the Contemporary Vibrations of the several Unison Parts, which make the one tremble at the Motion of the other: But, when struck at the respective Points of Divisions, the Sound is incongruous, by Reason the Point is disturbed, which should be at Rest.

A Lute-string or Viol-string will also thus answer to a Consonant Note in Wind Instruments: But not so remarkably to the Wire-strings of an Harpsicord. And we feel the Wainscot Seats, on which we sit or lean, to tremble constantly at certain Notes on the Organ, or other Wind Instruments; as well as at the same Notes on a Bass Viol. I have heard also (but cannot aver it) of a thin fine Venice Glass cracked with the strong and lasting Sound of a Trumpet, or Cornet, (near it) sounding an Unison or a Consonant Note to that of the Tone or Ting of the Glass.

Concerning these Phenomena, an exquisite Solution is given by Dr. Nairn, n. 135. p. 879. cissus Marsh, in Dr. Plot's Natural History of Oxfordshire.

II. The Extent of the Trumpet cannot be strictly determined; it reaches as High as the Strength of the Breath can force it: But by considering its Notes within the ordinary Compass of the Scale of Musick (from Double C-fa-ut to C-sol-fa in alt) the Nature of the higher Notes will plainly appear. These are all set down in the Table; only take Notice, that the Prick'd Notes are imperfect, not exactly in Tune, but a little Flatter or Sharper than the Places where they stand, according as f or s is set over them.

The Defects of a Trumpet, and Trumpet-Marine; by Mr. Fran. Roberts. n. 195. p. 559 Fig. 226. Oct. An. 1692.

Here we may make two Enquiries.

1. Whence it comes to pass, that the Trumpet will perform no other Notes (in that Compass) but only those in the Table, which are usually called by Musicians Trumpet-Notes.

2. What is the Reason that the 7th, 11th, 13th and 14th Notes are out of Tune, and the others exactly in Tune.

In this Matter we may receive some Light from the Trumpet Marine, an Instrument, though as unlike as possible to the Trumpet in its Frame, one being a Wind-Instrument, the other a Monochord, yet has a wonderful Agreement with it in its Effect.

The Sound is so like, as not to be easily distinguished by the nicest Ear, and, as it performs the very same Notes, so it has the same Defects as a Trumpet, for, if the Strings be stopt in any Part but such as produces a Trumpet-Note, it yields a harsh and uncouth (not a Musical) Sound.

Let us therefore proceed to our first Inquiry, and examine what is the Reason that the Trumpet Marine will perform no other but the Trumpet-Notes. It is a known Experiment of two Unison Strings, that striking one of them moves the other, which probably proceeds from hence, that the Impulses of the Air, which are made by one String, do more easily set another in Motion, which lies in a Disposition to have its Vibrations Synchronous to them, than a Third, whose Motion would be cross.

We may improve this a little farther, by observing that a String will move not only at the Striking of an Unison, but an 8th or 12th, though after a different Manner. If

Fig. 227.

If an *Unison* be struck, it makes one intire Vibration in the whole String, and the Motion is most sensibly in the midst at *m*, for there the Vibrations take the greatest Scope.

Fig. 228.

If an 8th is struck, it makes two Vibrations ; and the Point *m* is in a manner Quiescent, and the most sensible Motion at *n, n*.

Fig. 229.

If a 12th be struck, then it makes three Vibrations ; and the greatest Motion at *q, m, q* ; and hardly to be perceived at *p, p*. So that in short, this Experiment holds, when any Note is struck which is an *Unison* to half the String, and a 12th to a Third Part of it.

Fig. 230.

In this Case, (the Vibrations of the equal parts of a String being synchronous) there is no Contrariety in the Motion to hinder each other, whereas it is otherwise, if a Note is *Unison* to *S*, that does not divide the String into equal Parts, for then the Vibrations of the Remainder *r*, not suiting with those of the other Parts, immediately make a Confusion in the whole.

Now in the *Trumpet-Marine*, you do not stop close, as in other Instruments, but touch the String gently with your Thumb, whereby there is a mutual Concurrence of the upper and lower part of the String to produce the Sound. This is sufficiently evident from that, That if any thing touches the String below the Stop, the Sound will be as effectually spoiled, as if it were laid upon that Part which is immediately struck with the Bow. From hence therefore we may collect, that the *Trumpet-Marine* yields no Musical Sound, but when the Stop makes the upper Part of the String an *Aliquot* of the Remainder, and consequently of the whole : Otherwise, as we just now remarked, the Vibrations of the Parts will stop one another, and make a Sound suitable to their Motion, altogether confused.

Now, that these *Aliquot Parts* are the very Stops which produce the *Trumpet-Notes*, shall be plainly shewn in the treating of the second Inquiry, viz. What is the Reason that the 7th, 11th, 13th, 14th Notes are out of Tune ; and the rest exactly in Tune ?

All Writers of the Mathematical Part of *Musick* agree,

| | | | |
|-------------|---|---------------------|--|
| That by | $\left\{ \begin{array}{l} \text{Half} \\ \text{a Third Part} \\ \text{a Fourth} \\ \text{a Fifth} \\ \text{a Sixth} \end{array} \right\}$ | the Sound is raised | $\left\{ \begin{array}{l} \text{an Eight} \\ \text{a Fifth} \\ \text{a Fourth} \\ \text{a Sharp Third} \\ \text{a Flat Third.} \end{array} \right\}$ |
| Shortning a | | | |
| String | | | |
| | | | |
| | | | |

From this Foundation all the other Notes are derived. The Flat and Sharp Sixth are to be the Flat and Sharp Third to the Fourth, and the 7th the like to the 5th : The Second to be a Fifth to the Fourth below, &c. By this Rule let us examine what Notes a *Monochord* fretted in its *Aliquot Parts* will produce.

Fig. 231.

Suppose the *Monochord* F to consist of 720 Parts, and its Tone Double C-fa-ut, the first Note in the Table ; then Half of it will be 360, and a third Part 240, &c.

Now

Now I say, *Fretting*, (or *Stopping* with the Thumb) at 360 must produce *C-fa-ut*; because 360 being Half 720, the Sound will Rise an *Eight* from double *C-fa-ut*. Again 360 being *C-fa-ut*, 240 must make *G-sol-re-ut*, the Third Note in the Table; because 240 being just a Third-part less than 360, the Sound will Rise a *Fifth* from that Note. After the same manner proceeding Step by Step it will be evident that,

| | | | | | | | |
|-----|--------------------|-----|---------|----------|----------|------------------------|--------------------|
| 180 | which is less than | 240 | by just | a Fourth | produces | C-sol-fa-ut the fourth | Note in the Table. |
| 144 | | 180 | | a Fifth | | E-la-mi fifth | |
| 120 | | 144 | | a Sixth | | G-sol-re-ut 6th | |
| 90 | | 180 | | Half | | C-sol-fa 8th | |
| 80 | | 120 | | a Third | | D-la-sol 9th | |
| 72 | | 90 | | a Fifth | | E-la 10th | |
| 60 | | 90 | | a Third | | G-sol-re-ut 12th | |
| 48 | | 60 | | a Fifth | | B-fa-bi-mi 15th | |
| 45 | | 90 | | Half | | C-sol-fa 16th | |

By the same Reason,

| | | | | | | |
|------------------|--------------------|------------------|---------|-------|----------|------------------|
| 100 | which is less than | 120 | by just | a 6th | produces | B-fa-bi-mi Flat, |
| 67 $\frac{1}{2}$ | | 90 | | a 4th | | F-fa-ut |
| 54 | | 67 $\frac{1}{2}$ | | a 5th | | A-la-mi-re |
| 50 | | 100 | | Half | | B-fa-bi-mi Flat, |

And Consequently,

| | | | | |
|-------------------|---------------------------------------|---------|------|------------------|
| 102 $\frac{2}{7}$ | the 7th Note in the Table is a little | Flatter | than | B-fa-bi-mi Flat, |
| 65 $\frac{1}{7}$ | | | | F-fa-ut, |
| 55 $\frac{1}{7}$ | | | | A-la-mi-re, |
| 51 $\frac{3}{7}$ | | | | B-fa-bi-mi Flat, |

Which answers the Second Inquiry.

Now to apply this (in a few words) to the *Trumpet*, where the Notes are produced only by the Different Force of the *Breath*; it is reasonable to imagine that the strongest Blast *Raises* the Sound by breaking the Air within the *Tube* into the shortest Vibrations, but that no *Musical* Sound will raise, unless they are suited to some *Aliquot Part*, and so by Reduplication exactly measure out the whole Length of the *Instrument*, as in *Fig. 229.* for otherwise a Remainder will cause the same Inconvenience in this Case, as in *Fig. 230.* To which if we add that a *Pipe*, being shortened according to the Proportions we even now discoursed of in a *String*, *Raises* the Sound in the same Degrees, it renders the Case of the *Trumpet* just the same with the *Monochord.*

For a *Corollary* to this Discourse, we may observe that the Distances of the *Trumpet Notes*, Ascending continually, decreased in Proportion of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$, in *infinitum.* For,

The { Second
Third
Fourth, &c. } Note in the Ta-
ble differs from { First
Second
Third, &c. } by { $\frac{1}{2}$
 $\frac{1}{3}$
 $\frac{1}{4}$ } of the String, &c.

*The Division of
the Mono-
chord; by Dr
J. Wallis.
n. 238. p. 80.
Mar. An. 1698.*

III. Any String or Chord of a Musical Instrument Open (or at its full length) will Sound (what we call) an *Octave* (or *Diapason*) to that of the same String stopt in the Middle, or at half its Length. Hence it is that we commonly assign, to an *Octave*, the *Duple* Proportion (or that of 2 to 1) because such is the Proportion of Lengths (taken in the same String) which give those Sounds. And (upon a like Account) we assign to a *Fifth* (or *Diapente*) the *Sesqui-alter* Proportion (or that of 3 to 2.) And to a *Fourth* (or *Diatesseron*) the *Sesqui-tertian* (or that of 4 to 3.) And to a *Tone* (which is the Difference of a *Fourth* and *Fifth*) the *Sesqui-octave* (or that of 9 to 8 :) Because Lengths (taken in the same String) in these Proportions, do give such Sounds.

And (universally) whatever Proportion of Lengths (taken in the same String equally stretched) do give such and such Sounds ; such Proportions (of Gravity) we assign to the Sounds so given.

But, when an *Eight* (or *Octave*) is said (in common Speech) to consist of 12 *Hemi-tones*, or 6 *Tones* ; this is not to be understood according to the Utmost Rigour of *Mathematical Exactness*, (of such 6 *Tones*, as what they call the *Diazeutick Tone*, or that of *la*, *mi*, which is the Difference of a *Fourth* and *Fifth*;) but, as exact enough for common Use. For 6 such *Tones*, (that is, the Proportion of 9 to 8, 6 times repeated) is somewhat more than that of an *Octave*, (or the Proportion of 2 to 1) : And consequently, such an *Hemi-tone*, is somewhat more than the Twelfth-part of an *Eight* or *Octave*, or *Diapason*. But the Difference is so little, that the Ear can hardly distinguish it: And therefore (in common Speech) it is usual so to speak.

And, accordingly, when we are directed to take the Lengths (for what are called the 12 *Hemi-tones*) in *Geometrical Proportion*, it is to be understood (not to be so in the utmost Strictness, but) to be accurate enough for common Use ; for placing the *Frets* on the Neck of a *Viol*, or other *Musical Instrument* ; wherein a greater Exactness is not thought necessary. And this is very convenient, because (thus) the Change of the *Key* (upon altering the Seat of *mi*) gives no new Trouble, for this doth indifferently serve any *Key* ; and the Difference is so small, as not to offend the Ear.

But those who chuse to treat of it with more Exactness go this way to work.

Presupposing the Proportion for an *Octave* or *Diapason* to be that of 2 to 1 ; they divide this into two Proportions; not just Equal (for that would fall upon the *Surd Numbers*, as $\sqrt{2}$ to 1;) but near equal (so as to be expressed in small Numbers.) In order to which, instead of taking 2 to 1, they take (the Double of these Numbers) 4 to 2 ; (which is the same Proportion as before; and interpose the Middle Number 3. And of these three Numbers, 4, 3, 2, that of 4 to 3, is the Proportion of a *Fourth* (or *Diatesseron*.) And that of 3 to 2, the Proportion for a *Fifth* (or *Diapente*.)

pente.) And these two, put together, make up that of an *Octave* (or *Diapason*;) that of 4 to 2, (or 2 to 1.) And the Difference of those two, that of a *Tone*, or 9 to 8. As will plainly appear in the ordinary Method of

Multiplying and Dividing Fractions, That is, $\frac{4}{3} \times \frac{3}{2} = \frac{4}{2} = \frac{2}{1}$. And

$$\frac{4}{3} \div \frac{3}{2} \left(\frac{2}{8} \right).$$

Thus in the common *Scale* (or *Gam-ut*) taking an *Octave*, in these Notes, *la, fa, sol, la, mi, fa, sol, la*; suppose from E to e (placing *mi*, *B-fa-b-mi*, which is called the *Natural Scale*;) the Lengths for the Extremes *la, la*, an *Octave*, are as 2 to 1, or 12 to 6. Those for *la, la*, (in *la, fa, sol, la*;) or *mi, la*, (in *mi, fa, sol, la*;) a *Fourth*, as 4 to 3, or 12 to 9, or 8 to 6. Those for *la, mi*, (in *la, fa, sol, la, mi*;) or *la, la*, (in *la, mi, fa, sol, la*;) a *Fifth* as 3 to 2, or 12 to 8, or 9 to 6. Those for *la, mi*, the *Diazeugtick-Tone* (or Difference of a *Fourth* and *Fifth*;) as 9 to 8. So have we for these four Notes *la, la, mi, la*, their Proportionate Length in the Numbers 12, 9, 8, 6.

Then, if we proceed in like manner to Divide a *Fifth* (or *Dia-pente*;) *la, fa, sol, la, mi*, or *la, mi, fa, sol, la*, or the Proportion of 3 to 2, into near Equals, (taking double Numbers in the same Proportion, 6, 4; and interposing the middle Number 5;) of these three Numbers, 6, 5, 4; that of 6 to 5, is the Proportion of a *Lesser Third*, (called a *Tri-hemitone*, or *Tone and half*;) as *la, fa*, (in *la, mi, fa*.) And that of 5 to 4, is the Proportion of the *Greater Third*, (commonly called a *Ditone*, or two *Tones*, (as *fa, la*, (in *fa,*

sol, la;) which two put together make a *Fifth*, as 3 to 2; that is $\frac{6}{5} \times \frac{5}{4}$

$$= \frac{6}{4} = \frac{3}{2}; \text{ and their Difference is, as 25 to 24: That is } \frac{6}{5} \div \frac{5}{4} \left(\frac{25}{24} \right)$$

So have we for these 3 Notes, *la, fa, la*, their Proportionate Lengths in Numbers, as 6, 5, 4.

In like manner, if we divide a *Ditone*, (or *Greater Third*;) as *fa, la*, (in *fa, sol, la*;) whose Proportion is as 5 to 4 (or 10 to 8.) into two near Equals by help of a middle Number 9;) then have we (in these three Numbers 10, 9, 8,) that of 10 to 9, for (what they call) the *Lesser Tone*: And that of 9 to 8, for (what they call) the *Greater Tone*.

But, whether *fa, sol*, shall be made the *Lesser* (as 10 to 9,) and *sol, la*, the *Greater*, (as 9 to 8;) or, This the *Lesser*, as 10 to 9,) and That the *Greater*, (as 9 to 8,) or some time This, some time That, as there is Occasion, (to avoid what they call a *Schisim*;) is somewhat indifferent. For, either way, the Compound will be as 5 to 4; and the Difference

(which they call a *Comma*;) as 81 to 80. That is $\frac{9}{8} \times \frac{10}{9} = \frac{10}{9} \times \frac{9}{8} =$

$$\frac{10}{8} = \frac{5}{4}. \text{ And } \frac{10}{9} \div \frac{9}{8} \left(\frac{81}{80} \right).$$

Lastly, if from that of the *Tri-hemi-tone* (or *Lesser Third*) *la, mi, fa*; whose Proportion is as 6 to 5; we take that of the *Tone, la, mi*, (which is the Difference of a *Fourth* and *Fifth*) as 9 to 8; there remains for the

Hemi-tone, mi, fa, (or *la, fa*;) that of 16 to 15. That is $\frac{9}{8} \times \frac{6}{5} = \frac{48}{45}$;
 $= \frac{16}{15}$.

Or, the *Tri-hemi-tone* (or *Lesser Third*) whose Proportion is as 6 to 5; may be divided into three near Equals; (by taking Triple-Numbers, in the same Proportion 18, 15; and interposing the two Intermediates 17, 16;) which will therefore be as 18 to 17, and as 17 to 16, and as 16 to 15;

That is, $\frac{18}{17} \times \frac{17}{16} \times \frac{16}{15} = \frac{18}{15} = \frac{6}{5}$.

Where also the *Greater Tone*, whose proportion is as 9 to 8 or 18 to 16, is divided into its two near Equals (commonly called *Hemi-tones*;) that of

18 to 17, and that of 17 to 16: That is, $\frac{18}{17} \times \frac{17}{16} = \frac{18}{16} = \frac{9}{8}$.

And the *Lesser Tone*, that of 10 to 9, or 20 to 18, may be in like manner Divided into that of 20 to 19, and that of 19 to 18: That is,

$\frac{20}{19} \times \frac{19}{18} = \frac{20}{18} = \frac{10}{9}$.

Which Divisions of the *Greater* and *Lesser Tone*, answer to what is wont to be designed by *Flats* and *Sharps*.

So that (by this Composition,) of these Eight Notes, *la, fa, sol, la, mi, fa, sol, la*; their Proportions stand thus; that of *la, fa*, (or *mi, fa*) is as 16 to 15. That of *fa, sol*, as 10 to 9, and that of *sol, la*, as 9 to 8: (or else that of *fa, sol*, as 9 to 8, and that of *sol, la*, as 10 to 9.) That of *la, mi*, as 9 to 8. And, if either of the *Tones* (*Greater* or *Lesser*) chance to be divided (by *Flats* or *Sharps*) into (what they call) *Hemi-tones*; their Proportions are to be such as is already mentioned.

There may be a like Division of a *Fourth*, (or *Dia-tesseron*) into two Near Equals: And of some others of these, into three Near Equals. Which might be of use for (what they were wont to call) the *Chromatick* and *Enarmonick Musick*. But, those sorts of *Musick* having been long since laid aside, there is now no need of these Divisions, as to the *Musick* now in Use.

*The Imperfection
of an Organ;
by Dr. J. Wal-
lia, n. 242.
p. 249.
July An. 1698.*

IV. I think 'tis Evident that the *Pipe* in the *Organ* is intended to Express a Distinct Sound at such a *Pitch*; that is, in such a Determinate Degree of *Gravity* or *Acuteness*; or (as it is now called) *Flatness* or *Sharpness*; And the Relative or Comparative Consideration of Two (or more) such Sounds or Degrees of *Flatness* or *Sharpness*, is the Ground of (what we call) *Concord*, and *Discord*; that is, a Soft, or Harsh Coincidence.

Now

Now, concerning this, there were amongst the Ancient Greeks, Two (the most considerable) Sects of Musicians: the *Aristoxenians*, and *Pythagoreans*.

They both agreed thus far; that *Dia-teseron*, and *Dia-pente*, do together make up *Dia-pasem*; That is (as we now speak, a *Fourth* and *Fifth*) do together make an *Eighth* or *Octave*: And, the Difference of those two, of a *Fourth* and *Fifth*, they agreed to call a *Tone*, which we now call a *Whole Note*.

Such is that, (in our present *Musick*;) of *la, mi*, (or, as it was wont to be called, *re, mi*.) For *la, fa, sol, la*, or *mi, fa, sol, la*, is a perfect *Fourth*: And *la, fa, sol, la, mi*, or *la, mi, fa, sol, la*, is a perfect *Fifth*: The Difference of which is *la, mi*, which is what the Greeks call the *Dia-zeugma-Tone*; which doth Disjoin two *Fourths* (on each side of it;) and being added to either of them doth make a *Fifth*; Which was, in their *Musick*, that from *Mese* to *Paramese*; that is in our *Musick*, from *A* to *B*: Supposing *mi* to stand in *B-fa-bi-mi*, which is accounted its Natural Position.

Now in order to this, *Aristoxenus* and his Followers did take that of a *Fourth*, as a Known Interval, by the Judgment of the Ear; and that of a *Fifth* likewise; And consequently that of an *Octave*, as the Aggregate of both; and that of a *Tone*, as the Difference of those Two.

And this of a *Tone* (as a Known Interval) they took as a common Measure, by which they did Estimate other Intervals: And accordingly they accounted a *Fourth* to contain two *Tones* and a Half; a *Fifth* to contain three *Tones* and a Half, and consequently an *Eighth* to contain six *Tones*; or five *Tones* and two Half *Tones*.

And at this Rate our *Practical Musicians* talk of *Notes* and Half *Notes* at this Day; supposing an *Octave* to consist of Twelve *Hemi-tones*, or Half *Notes*.

But, *Pythagoras* and those who follow him, not taking the Ear alone to be a Competent Judge in a Case so nice; chose to distinguish these, not by Equal Intervals, but by Due Proportions: And this is followed by *Zarlino*, *Kepler*, *Comenius*, and others, who treat of *Speculative Musick* in this and the last Age: Accordingly they accounted that of an *Octave*, to be, when the Degree of *Gravity*, or *Acuteness* of the one Sound to that of the other, is Double, or as 2 to 1; that of a *Fifth*, when it is *Sesqui-alter*, or as 3 to 2; that of a *Fourth* (when *Sesqui-tertian*, or as 4 to 3. Accounting that the Sweetest Proportion, which is express'd in the smallest Numbers, and therefore (next to the *Unison*) that of an *Octave*, 2 to 1, then that of a *Fifth*, 3 to 2, and then that of a *Fourth*, 4 to 3.

And thus that of a *Fourth* and *Fifth* do together make an *Eighth*; For $\frac{4}{3} \times \frac{3}{2} = \frac{4}{2} = \frac{2}{1} = 2$, or the Proportion of 4 to 3, compounded with that of 3 to 2, is the same with that of 4 to 2, or 2 to 1: And consequently, the Difference of those two, which is that of a *Tone*, or Full *Note*,

is that of 9 to 8. For $\frac{4}{3} \times \frac{3}{2} \left(\frac{9}{8} \right)$; or, if out of the Proportion of 3 to 2, we take that of 4 to 3; the Result is that of 9 to 8. Now,

Now, according to this Computation, it is manifest, That an *Octave* is somewhat less than Six *Full Notes*. For (as was first demonstrated by *Euclid*, and since by others) the Proportion of 9 to 8, being six Times com-

pounded, is somewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8}$
 $\times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Case; they allowed (indisputably) to that of the *Diazeutick Tone* (*la, mi,*) the full Proportion of 9 to 8, as a thing not to be altered; being the Difference of the *Dia-pente* and *Dia-tesseron*, or the *Fifth* and *Fourth*.

All the Difficulty, was, How the remaining *Fourth* (*mi, fa, sol, la,*) should be divided into three parts, so as to answer (pretty near) the *Aristoxenians* Two *Tones* and a half: And might, altogether make up the Proportion of 4 to 3, which is that of a *Fourth* or *Dia-tesseron*.

Many Attempts were made to this purpose: And according to those they gave Names to the Different *Genera* or *Kinds* of *Musick*, (the *Diatonick*, *Chromatick*, and *Enarmonick* Kinds,) with the several *Species*, or lesser Distinctions under those Generals.

The first was that of *Euclid* (which did most generally obtain for many Ages:) Which allows to *fa, sol*, and to *sol, la*, the full Proportion of 9 to 8; And therefore to *fa, sol, la*, (which we call the *Greater Third*) that of 81 to

64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$. And, consequently, to that of *Mi, fa*, which

is the Remainder to a *Fourth*) that of 256 to 243. For $\frac{81}{64} \times \frac{4}{3} = \frac{256}{243}$; that is, if out of the Proportion of 4 to 3 we take that of 81 to 64, the Result is that of 256 to 243. To this they gave the Name of *Limma* (*λῑμμα*) that is, the Remainder (to wit, over and above two *Tones*.) But, in common Discourse (when we do not pretend to speak nicely, nor intend to be so understood) it is usual to call it an *Hemi-tone*, or *Half-Note*, (as being very near it) and the other, Two *Whole Notes*. And this is what *Ptolemy* calls *Diatonum Ditonum*, (of the *Diatonick* kind with Two *Full Notes*.)

Against this it is Objected (as not the most convenient Division) that the Numbers of 81 to 64, are too great for that of a *Ditone*, or *Greater Third*; which is not Harsh to the Ear; but is rather Sweeter than that of a *Single Tone*, whose Proportion is 9 to 8. And, in that of 256 to 243, the Numbers are yet much greater. Whereas there are many Proportions (as

$\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$, in smaller Numbers than that of 9 to 8; of which, in this Division, there is no Notice taken.

To Rectifie this, there is another Division thought more convenient ; which is *Ptolemy's Diatonum Intensum* (of the *Diatonick* Kind, more *Intense* or *Acute* than that other.) Which instead of two *Full Tones* for *fa, sol, la*, assigns (what we now call) a *Greater* and a *Lesser Tone* ; (which by the more Nice *Musicians* of this and the last Age seems to be more embraced;) Assigning to *fa, sol*, that of 9 to 8, (which they call the *Greater Tone* ;) and to *sol, la*, that of 10 to 9, (which they call the *Lesser Tone* :) And therefore to *fa, la*, (the *Ditone* or *Greater Third*) that of 5 to 4. (For

$$\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}.) \text{ And consequently to } mi, fa, \text{ (which is remaining}$$

of the *Fourth*) that of 16 to 15. For $\frac{5}{4} \times \frac{4}{3} = \frac{16}{15}$. That, if out

of that of 4 to 3 we take that of 5 to 4, there remains that of 16 to 15.

Many other ways there are (with which I shall not trouble you at present) of dividing the *Fourth* or *Dia-tesseron*, or the proportion of 4 to 3, into three parts, answering to what (in a looser way of Expression) we call an

Half Note, and two *Whole Notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that

which is now received as the most proper.

To which therefore I shall apply my Discourse ; Where $\frac{16}{15}$ is (what we

call) the *Hemi-Tone*, or *Half-Note*, in *mi, fa* ; $\frac{9}{8}$ that of the *Greater Tone*, in

fa, sol, and $\frac{10}{9}$ the *Lesser Tone*, in *sol, la*.

Only with this Addition ; That each of those *Tones*, is (upon Occasion by *Flats* and *Sharps* (as we now speak) divided into two *Hemi-tones*, or *Half Notes* : Which answers to what by the *Greeks* was called *Mutatio quoad Modos* (the Change of Mood ;) and what is now done by removing *mi* to another

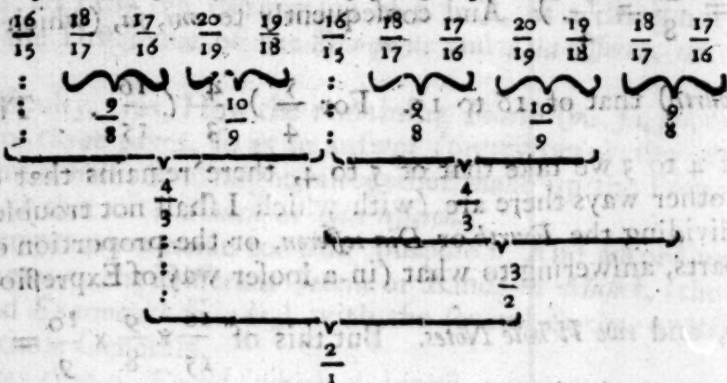
Key. Namely $\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16}$; and $\frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18}$.

Thus by the help of *Flats* and *Sharps* (dividing each *whole Note*, be it the *Greater* or *Lesser*, into two *Half Notes*, or what we call so,) the whole *Octave* is divided into Twelve parts or *Intervals* (contained between *Thirteen Pipes*) which are commonly called *Hemi-tones* or *Half Notes*, not that each is precisely *Half a Note*, but some what near it, and so called. And I say, by *Flats* and *Sharps* ; For sometime the one, sometime the other is used. As for Instance, a *Flat* in *D*, or a *Sharp* in *C*, do either of them denote a Middling Sound (though not precisely in the Midst) between *C* and *D* ; *Sharper* than *C*, and *Flatter* than *D*.

Accor-

Accordingly, supposing *mi* to stand in *B-fa-bi-mi*, which is accounted its *Natural Seat*, the Sounds of each Pipe are to bear these Proportions to each other, *viz.*

B . C . $\sharp$. D . $\sharp$. E . F . $\sharp$. G . $\sharp$. a . $\sharp$. b
mi . fa . $\sharp$. sol . $\sharp$. la . fa . $\sharp$. sol . $\sharp$. la . $\sharp$. mi



And so in each *Octave* successively following. And, if the Pipes in each *Octave* be fitted to Sounds in these Proportions of Gravity and Acuteness, it will be supposed (according to this *Hypothesis*) to be perfectly Proportioned.

But, instead of these successive Proportions of each *Hemi-tone*; it is found necessary (if I do not mistake the Practice) so to order the 13 Pipes (containing 12 Intervals which they call *Hemi-tones*) as that their Sounds (as to Gravity and Acuteness) be in Continual Proportion, (each to its next following, in one and the same Proportion;) which all together shall compleat that of an *Octave* or *Dia-pason*, as 2 to 1. Whereby it comes to pass that each Pipe doth not express its proper Sound, but very near it, yet somewhat varying from it; Which they call *Bearing*; which is somewhat of Imperfection in this *Noble Instrument* the Top of all.

It may be asked, 'Why may not the Pipes be so ordered, as to have their Sounds in just Proportion, as well as thus *Bearing*?'

I answer, It might very well be so, if all *Musick* were Composed to the same Key, or (as the *Greeks* call it) the same *Mode*; As for Instance, if, in all Compositions, *mi*, were always placed in *B-fa-bi-mi*. For then the Pipes might be ordered in such Proportions as I have now designed.

But *Musical Compositions* are made in a great Variety of *Modes*, or with great Diversity in the Pitch. *Mi* is not always placed in *B-fa-bi-mi*; but sometimes in *E-la-mi*, sometimes in *A-la-mi-re*, &c. And (in Summ) there is none of those 12 or 13 Pipes but may be made the *Seat* of *mi*. And, if they were exactly

exactly fitted to any one of these Cases, they would be quite out of Order for all the rest.

As for Instance, If *mi*, be removed from *B-fa-b-mi*, (by a *Flat* in *B*) to *E-la-mi*: Instead of the Proportions but now designed, they must be thus ordered,

| | | | | | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| B . ♯ . | C . ♯ . | D . ♯ . | E . | F . ♯ . | G . ♯ . | a . | b | | | | |
| fa . ♯ . | sol . ♯ . | la . ♯ . | mi . | fa . ♯ . | sol . ♯ . | la . | fa . | | | | |
| $\frac{18}{17}$ | $\frac{17}{16}$ | $\frac{20}{19}$ | $\frac{19}{18}$ | $\frac{18}{17}$ | $\frac{17}{16}$ | $\frac{16}{15}$ | $\frac{18}{17}$ | $\frac{17}{16}$ | $\frac{20}{19}$ | $\frac{19}{18}$ | $\frac{16}{15}$ |

where 'tis manifest, that the Removal of *mi* doth quite disorder the whole Series of Proportions. And the same would again happen, if *mi* be removed from *E* to *A* (by another *Flat* in *E*.) And again, if removed from *A* to *D*. And so perpetually. But the *Hemitones* being made all equal, they do indifferently answer all the Positions of *mi* (though not exactly to any :) Yet nearer to some than to others. Whence it is, that the same Tune sounds better at one *Key* than at another.

It is asked, Whether this may not be remedy'd ; by interposing more *Pipes* ; and thereby dividing a *Note*, not only (as now) into *Half-Notes*, but into *Quarter-Notes*, or *Half-Quarter-Notes*, &c.

I answer; It may be thus remedy'd in Part; (that is, the Imperfection might thus be somewhat less, and the Sounds somewhat nearer to the Just Proportions :) but it can never be exactly true, so long as their Sounds (be they never so many) be in continual Proportion ; that is, each to the next Subsequent in the same Proportion.

For it hath been long since demonstrated, that there is no such thing as a just *Hemitone* practicable in *Musick* (and the like for the Division of a *Tone* into any Number of equal Parts ; three, four or more.) For, supposing the Proportion of a *Tone* or *Full Note*, to be $\frac{9}{8}$ (or, as 9 to 8) that of the *Half Note* must be as $\sqrt{9}$, to $\sqrt{8}$; that is as 3 to $\sqrt{8}$, or 3 to $2\sqrt{2}$.)

which are *Incommensurable Quantities*. And that of a *Quarter-Note*, as $\sqrt[4]{9}$ to $\sqrt[4]{8}$, which is yet more *Incommensurate*. And the like for any other Number of equal Parts, which will therefore never fall in with the Proportions of Number to Number.

So that this can never be perfectly adjusted for all *Keys* (without somewhat of *Bearing*) by multiplying *Pipes* ; unless we would for every *Key*, or every different Seat of *mi* have a different Set of *Pipes*, of which this or that is to be used, according as (in the Composition) *mi* is supposed to stand in this or that Seat. Which vast Number of *Pipes* (for every *Octave*) would vastly increase the Charge : And (when all is done) make the whole Impracticable.

A new Tuning of
the Lyra-Viol;
by S. Salvetti.
n. 87. p. 3064.
Aug. An. 1672.

V. S. Salvetti, about 4 Years ago invented a New Tuning of the Ancient *Lyra-Viol* with the usual 13 Strings; by means of which Tuning it is render'd wholly perfect, so that you may express upon it all *Concords*, *Discords*, and also the *Imperfect Concords*, as *Sevenths*, *Sixths*, &c. as well as upon any *Virginal* that hath the Quarters of Notes upon it. 'Tis true, 'tis only for melancholly and passionate Matter, and not for *Division*, as is the proper Nature of the *Lyra*. I shall only add, that with the above said Tuning, it ascends in *Alt* as high as *G-sol-re-ut*; and descends as low as *Double C-fa-ut*; and can make every where the same *Concords* as above.

The strange Effects reported of
Musick in former Times, examined; by Dr.
Wallis, n. 243.
p. 297. Aug.
An. 1698.

VI. 1. I take it for granted, that much of the Reports concerning the great Effects of *Musick* in former Times, beyond what is to be found in latter Ages, is highly *Hyperbolical*, and next Door to *Fabulous*; and therefore great Abatements must be allowed to the *Elogies* of their *Musick*.

2. We must consider, That *Musick* (to any tolerable Degree) was then (if not a new, at least) a rare Thing, which the *Rusticks*, on whom it is reported to have had such Effects, had never heard before; and on such a little *Musick* will do great Feats; as we find at this Day, a *Fiddle* or a *Bag-pipe* at a Country *Morice Dance*.

3. We are to consider, that their *Musick* (even after it came to some good Degree of Perfection) was much more plain and simple than ours now-a-days, They had not *Consorts* of two, three, four or more *Parts* or *Voices*: But one single *Voice* or single *Instrument* a-part; which to a rude Ear, is much more taking than more compounded *Musick*. For that is at a Pitch not above their Capacity; whereas this other confounds it, with a great Noise, but nothing distinguishable to their Capacity.

4. We are to consider, that *Musick* with the *Ancients* was of a larger Extent than what we call *Musick* now-a-days: For *Poetry* and *Dancing* (or comely Motion) were then accounted *Parts* of *Musick*, when *Musick* arriv'd to some Perfection. Now we know that *Verse* of it self, if in good Measures and affectionate Language, and this set to a *Musical Tune*, and sung by a decent *Voice*, and accompanied but with soft *Instrumental Musick*, if any, such as not to drown or obscure the *emphatick* Expressions (like what we call *Recitative Musick*) will work strangely upon the Ear, and move all Affections suitable to the *Tune* and *Ditty*; (whether brisk and pleasant, or soft and pitiful, or fierce and angry, or moderate and sedate) especially if attended with a *Gesture* and *Action* suitable. For, 'tis well known, that suitable *Acting* on a *Stage* gives great Life to the Words. Now all this together (which were all Ingredients in what they called *Musick*) must needs operate strongly on the Fancies and Affections of ordinary People, unacquainted with such kind of Treatments. For, if the deliberate reading of a Romance (when well penn'd) will produce Mirth, Tears, Joy, Grief, Pity, Wrath, or Indignation, suitable to the respective Intents of it, much more would it so do, if accompanied with all those Attendants.

5. You

5. You will ask, perhaps, why may not all this be now done, as well as then? I answer, no doubt it may, and with like Effect, if an Address be made in proper Words, with moving Arguments, in just Measures, (*Poetical* or *Rhetorical*) with the *emphatick Words*, Words set in signal Places, pronounced with a good Voice, and a true Accent, and attended with a decent Gesture; and all these suitably adjusted to the Passion, Affection, or Temper of Mind, particularly designed to be produced, (be it Joy, Love, Grief, Pity, Courage or Indignation) will certainly now as well as then, produce great Effects upon the Mind, especially upon a Surprise, and where Persons are not otherwise pre-engaged: And if so managed, as that you be, (or seem to be) in earnest; and if not over-acted by apparent Affectation.

6. We are to consider, that the usual Design of what we now call *Musick*, is very different from that of the *Ancients*. What we now call *Musick*, is but what they called *Harmonick*; which was but one Part of their *Musick*, (consisting of Words, Verse, Voice, Tune, Instrument, and Acting) and we are not to expect the same Effect of one Piece as of the whole.

7. When *Musick* arrived to great Perfection, it was applied to particular Designs of exciting this or that particular Affection, Passion, or Temper of Mind; the *Tunes* and *Measures* being suitably adapted to such Designs. But such Designs seem almost quite neglected in our *present Musick*. The chief Design now, in our most accomplished *Musick*, being to please the Ear; when by a sweet Mixture of different *Parts* and *Voices*, with *Cadencies* and *Concord* intermixed, a grateful Sound is produced, which only the Judicious *Musician* can discern and distinguish.

8. 'Tis true, that even this *Compound Musick* admits of different *Characters*, some are more brisk and airy; others more sedate and grave; others more languid; as the different Subjects do require. But that which is most proper to excite particular *Passions* or *Dispositions*, is such as is more *Simple*, and *Uncompounded*: such as a Nurse's *languid Tune*, lulling her Babe to sleep; or a continual Reading in an *Even Tone*; or even the soft Murmur of a little Rivulet, running upon Gravel or Pebbles; inducing a quiet Repose of the Spirits. And contrariwise, the Briskness of a *Jig*, on a *Kit* or *Violin*, exciting to Dance. Which are more Operative to such particular Ends, than an *Elaborate Composition* of *Full Musick*.

9. To conclude; If we aim only at pleasing the *Ear*, by a *sweet Consort*, I doubt not but our *Modern Compositions* may be equal, if not exceed those of the *Ancients*: Amongst whom I do not find any Footsteps of what we call several *Parts* or *Voices*, (as *Base*, *Treble*, *Mean*, &c. sung in Consort) answering each other, to compleat the *Musick*. But if we would have our *Musick* so adjusted, as to excite particular *Passions*, *Affections*, or *Temper* of Mind, (as that of the *Ancients* is supposed to have done) we must apply more simple Ingredients, fitted to the Temper we would produce. And this, I doubt not, but a Judicious Composer may so effect, that (with the Help of such *Hyperboles*, as with which the *Ancient Musick* is wont to be set off) our *Musick* may be said to do as great Feats as any of theirs.

VII. Account of Books Omitted.

- n. 143. p. 20. 1. *Claudio Ptolemai Harmonicorum Libri tres, Ex Cod. MSS. undecim, nunc primum Græcè, Editi. Jo. Wallis, S. S. Th. D. Resensuit, Edidit, Versificavit & Notis Illustravit, & Auctuarium adjecit. Oxon 1682. in 4to.*
- n. 231. p. 668. 2. *Borphyri Commentarius in Librum Primum Harmonicorum Claudii Ptolemai. (Qui foliæ restant ex Græcis Musica Scriptoribus nondum Editi) Græcè ac Latine, Cura Jo. Wallis, S. Th. D. Fol.*
- n. 80. p. 3095. 3. *An Essay to the Advancement of Musick, by Tho. Salmon, M. A. Lond. 1672. in 8vo.*
- n. 90. p. 5153. 4. *Synagoga Musica; Treating of Musick Philosophically, Mathematically, and Practically; by J. Birchenha, Esq; This Book was preparing for the Press, 1674.*
- n. 100. p. 6194. 5. *Musica Speculativa del Mengoli. in Bologna 1670. in 4to.*
- n. 133. p. 835. 6. *A Philosophical Essay of Musick. Lond. 1677. in 4to.*
- n. 208. p. 67. 7. *A Treatise of the Natural Grounds and Principles of Harmony; by Will. Holder, D. D. Lond. 1694. in 8vo.*

F I N I S.